Electrical Circuit (1)

Discharge RC and RL (week12 class1)

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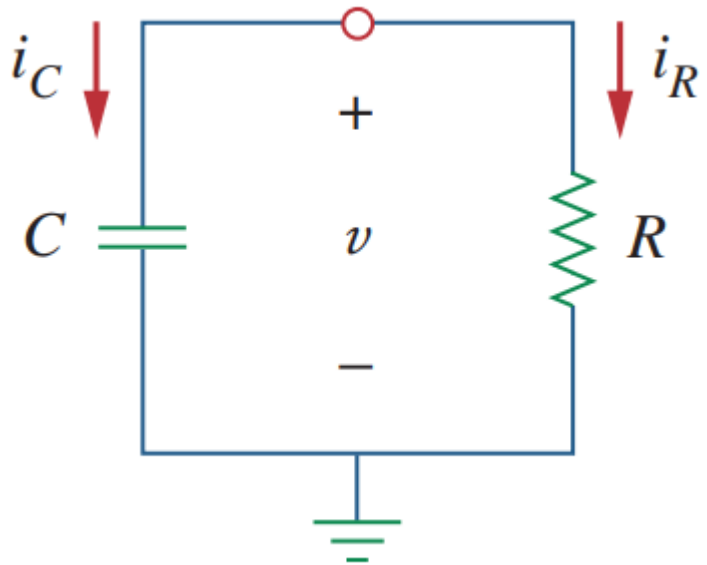
The Source-Free *RC* Circuit



$$v(0) = V_0$$

$$w(0) = \frac{1}{2}CV_0^2$$

$$i_C + i_R = 0$$

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

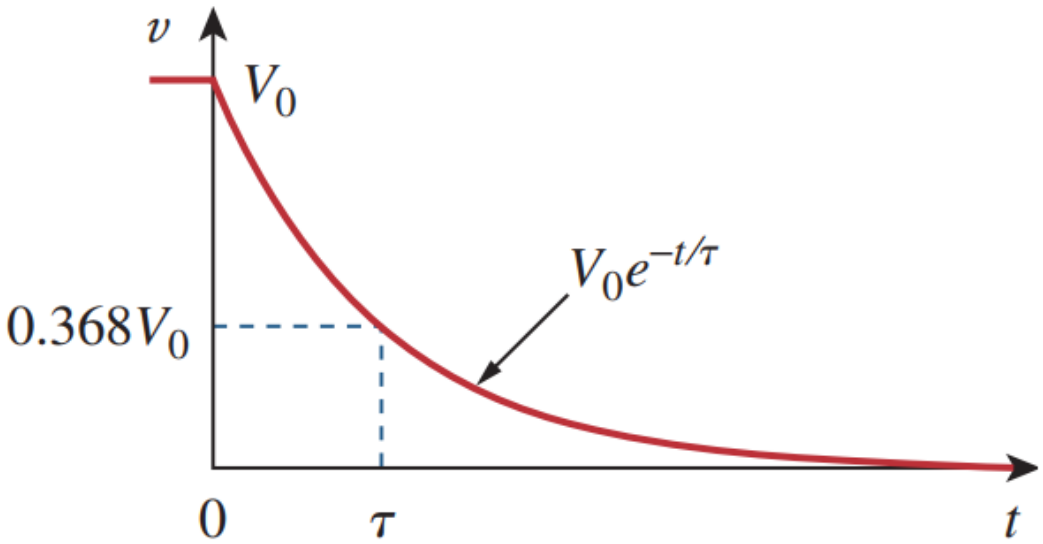
$$\frac{dv}{v} = -\frac{1}{RC}dt$$

$$\ln v = -\frac{t}{RC} + \ln A$$

$$v(t) = Ae^{-t/RC}$$

$$v(t) = V_0 e^{-t/RC}$$

The Source-Free *RC* Circuit



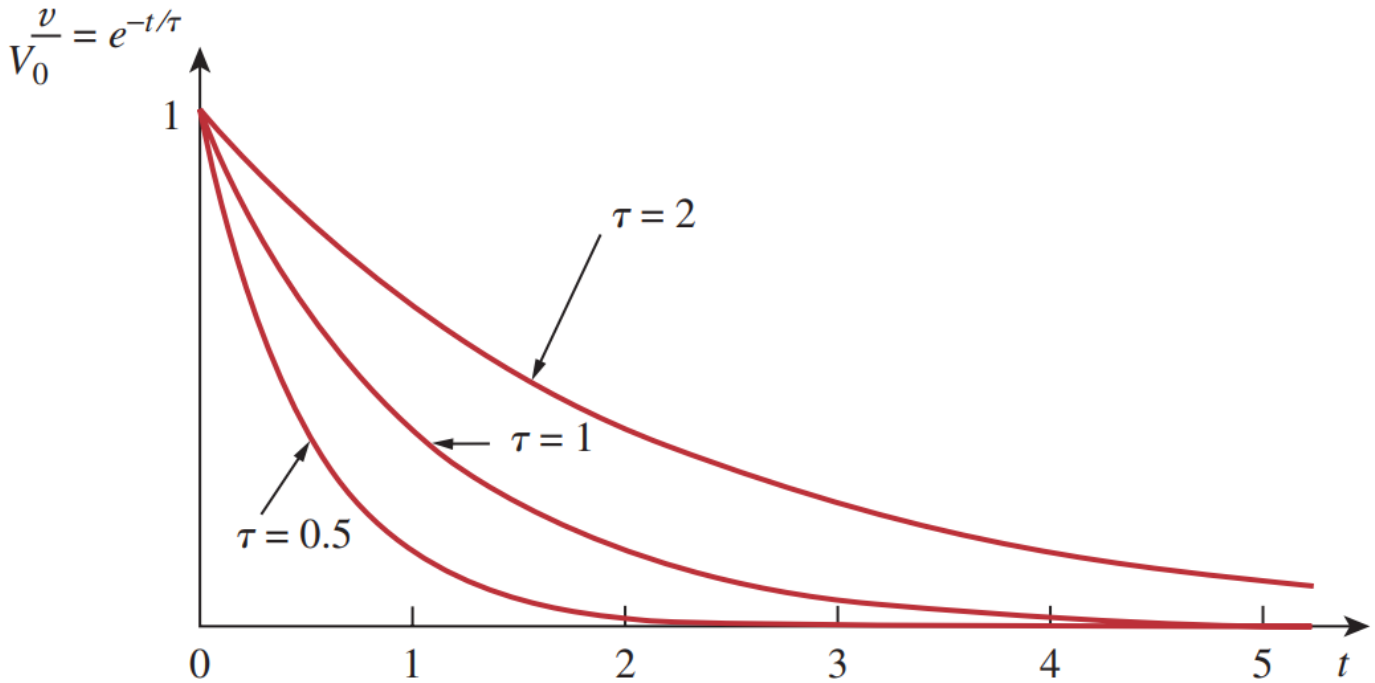
$$\tau = RC$$

TABLE 7.1

Values of $v(t)/V_0 = e^{-t/\tau}$.

t	$v(t)/V_0$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

The Source-Free *RC* Circuit



The Key to Working with a Source-Free *RC* Circuit Is Finding:

1. The initial voltage $v(0) = V_0$ across the capacitor.
2. The time constant τ.

The Source-Free *RC* Circuit

In Fig. 7.5, let $v_C(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.

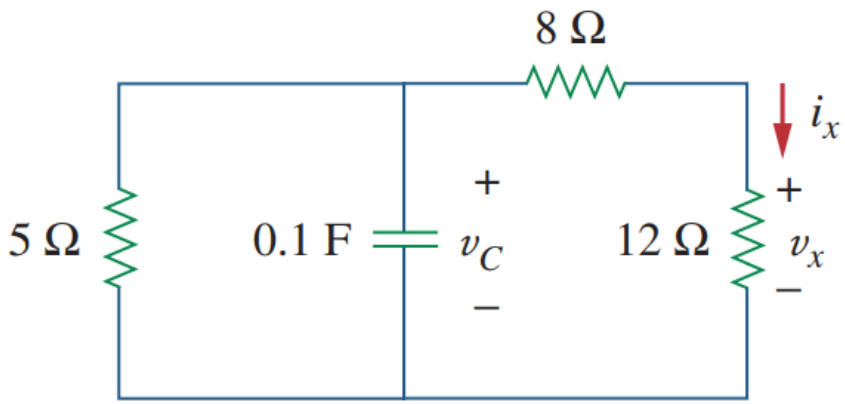


Figure 7.5
For Example 7.1.

$$R_{\text{eq}} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$

$$\tau = R_{\text{eq}}C = 4(0.1) = 0.4\ \text{s}$$

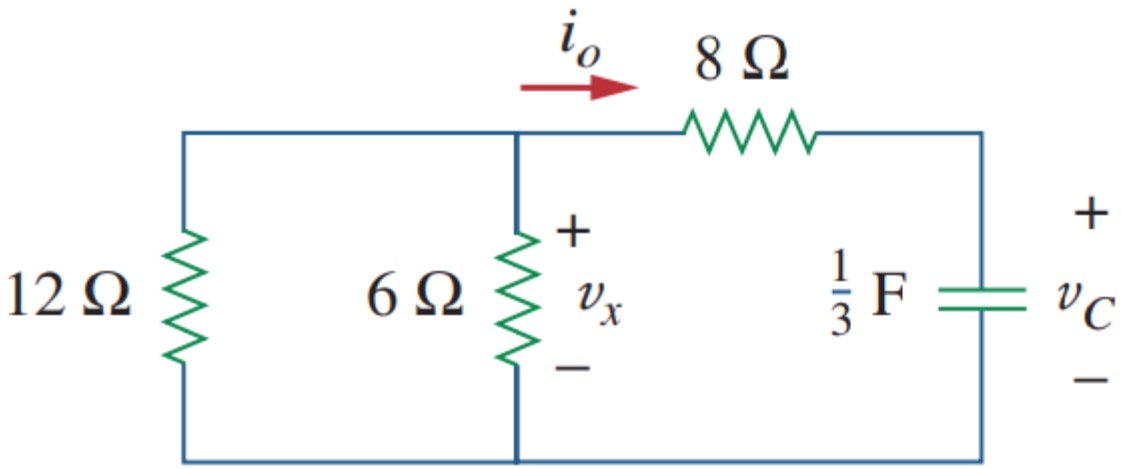
$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\ \text{A}$$

The Source-Free *RC* Circuit

Refer to the circuit in Fig. 7.7. Let $v_C(0) = 60$ V. Determine v_C, v_x, and i_o for $t \geq 0$.



$60e^{-0.25t}$ V, $20e^{-0.25t}$ V, $-5e^{-0.25t}$ A.

$$R_{eq} = \frac{12 * 6}{12 + 6} + 8 = 12$$

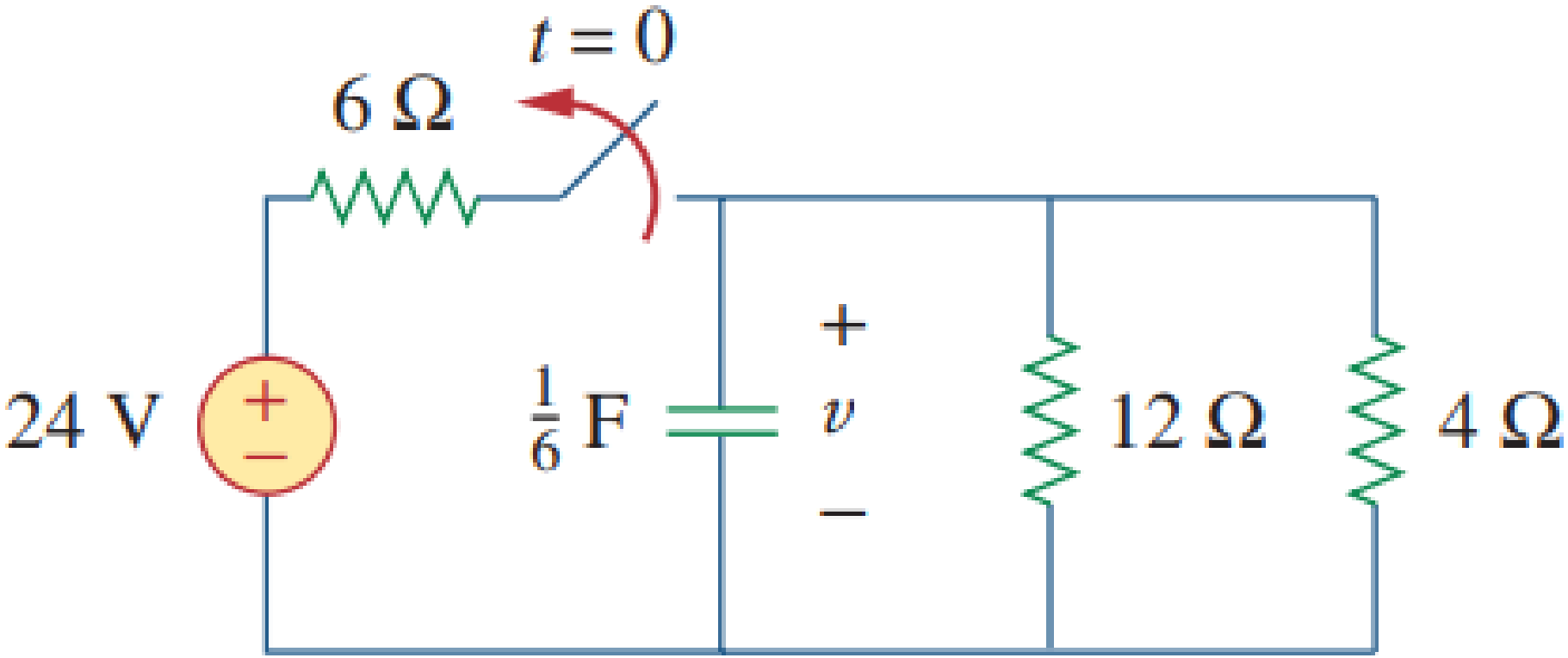
$$V_c = 60e^{\frac{-1}{12*\frac{1}{3}}} = 60e^{-0.25t}$$

$$V_x = \frac{4}{8 + 4} * 60 * e^{\frac{-1}{12*\frac{1}{3}}} = 20e^{-0.25t}$$

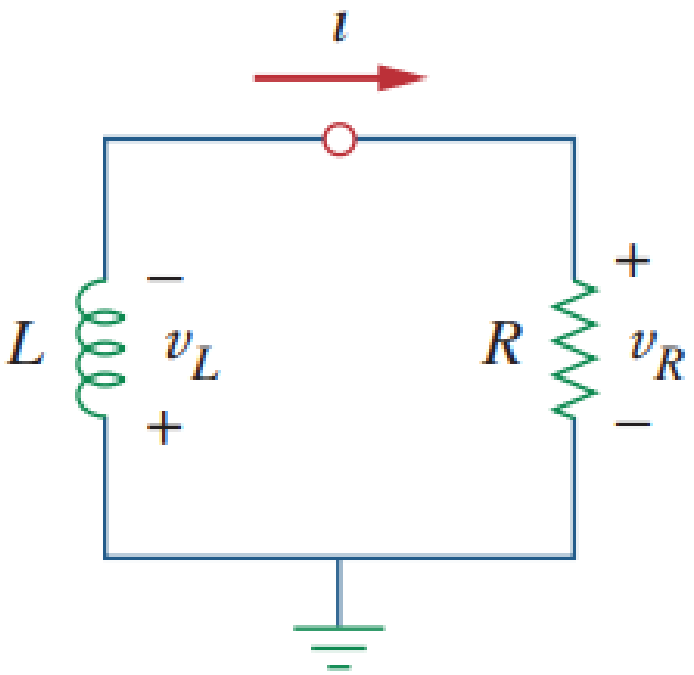
$$i_o = \frac{-60}{12} * e^{\frac{-1}{12*\frac{1}{3}}} = -5e^{-0.25t}$$

The Source-Free *RC* Circuit

QUIZzzz



The Source-Free *RL* Circuit



$$i(0) = I_0$$

$$w(0) = \frac{1}{2} L I_0^2$$

$$v_L + v_R = 0$$

$$L\frac{di}{dt} + Ri = 0$$

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$\int_{I_0}^{i(t)} \frac{di}{i} = -\int_0^t \frac{R}{L} dt$$

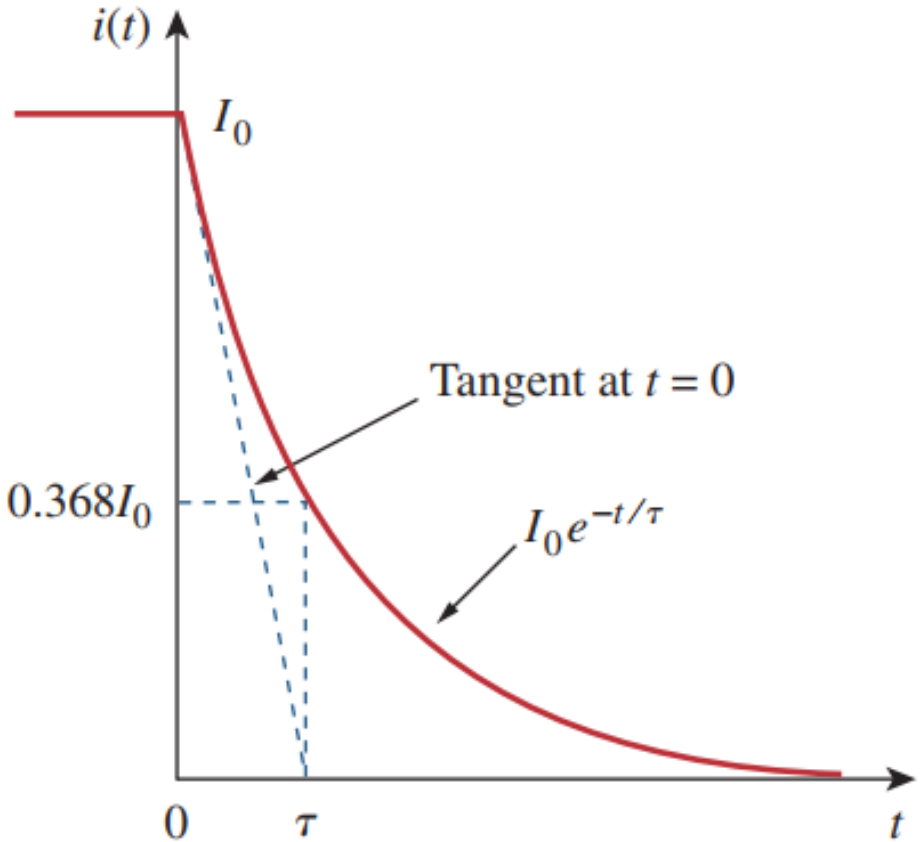
$$\ln \frac{i(t)}{I_0} = -\frac{Rt}{L}$$

$$i(t) = I_0 e^{-Rt/L}$$

$$\tau = \frac{L}{R}$$

$$i(t) = I_0 e^{-t/\tau}$$

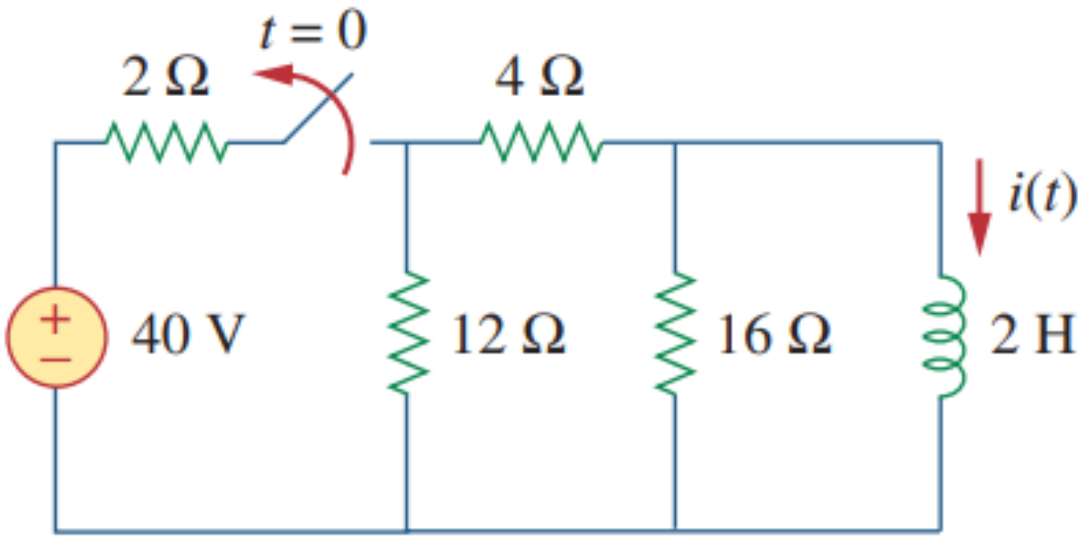
The Source-Free *RL* Circuit



The Key to Working with a Source-Free *RL* Circuit Is to Find:

1. The initial current $i(0) = I_0$ through the inductor.
2. The time constant τ of the circuit.

The Source-Free *RL* Circuit



$$\frac{4 \times 12}{4 + 12} = 3\ \Omega$$

$$i_1 = \frac{40}{2 + 3} = 8\ \text{A}$$

$$i(t) = \frac{12}{12 + 4} i_1 = 6\ \text{A}, \qquad t < 0$$

$$R_{\text{eq}} = (12 + 4) \parallel 16 = 8\ \Omega$$

$$\tau = \frac{L}{R_{\text{eq}}} = \frac{2}{8} = \frac{1}{4}\ \text{s}$$

$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t}\ \text{A}$$

The Source-Free *RL* Circuit

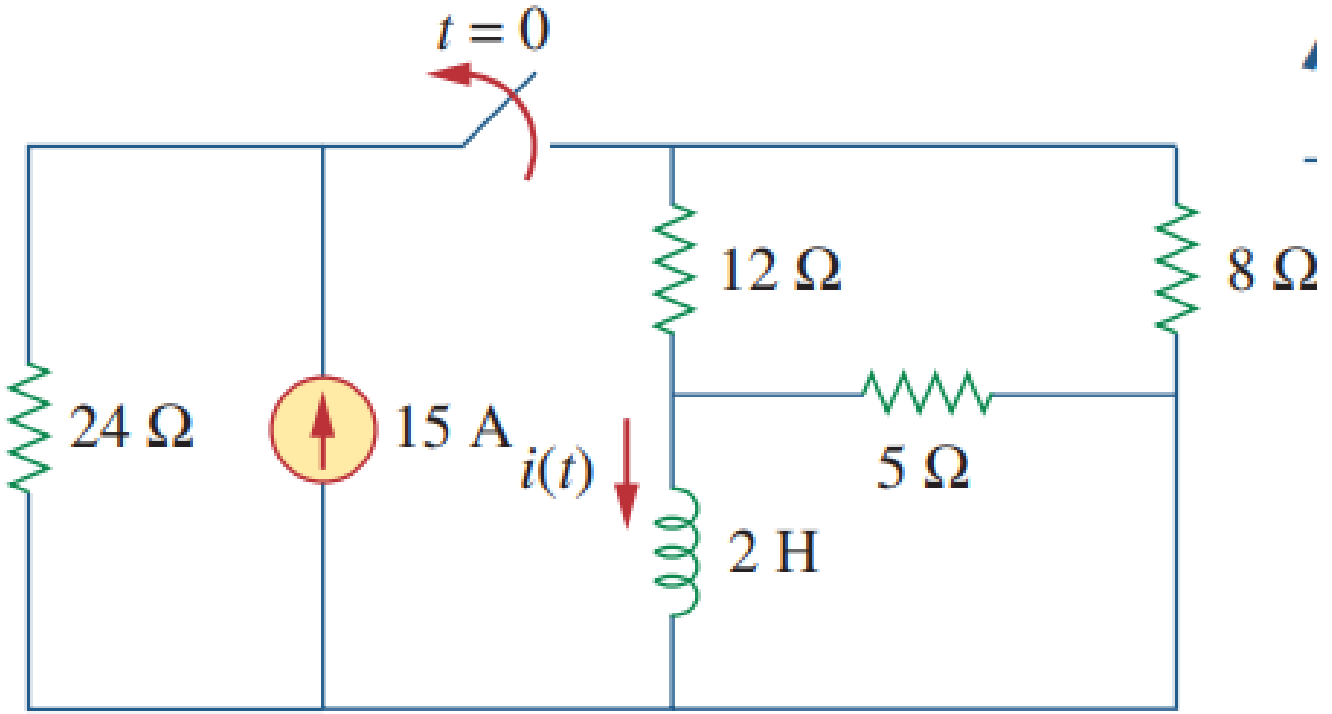
$$R_{eq} = 24||12||8 = 8||8 = 4\ ohm$$

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5$$

$$R_{24||8} = \frac{24 * 8}{24 + 8} = 6\ ohm$$

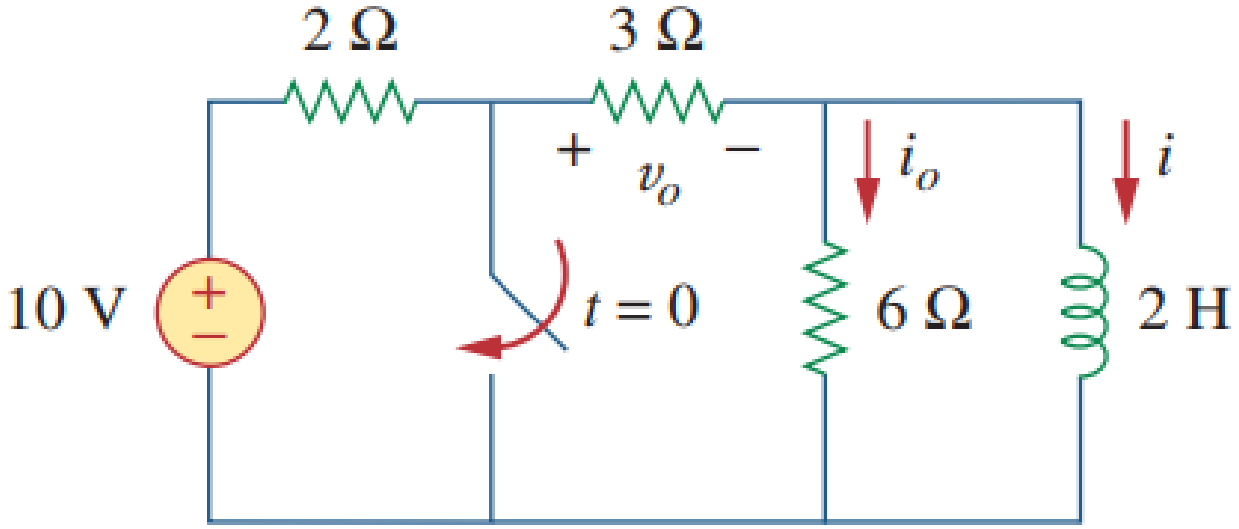
$$i(t)_{t=0} = 15\frac{6 * 12}{6 + 12} = 5\ A$$

$$i(t) = 5e^{-2t}\ A$$



Answer: $5e^{-2t}$ A, $t > 0$.

The Source-Free *RL* Circuit



$$R_{eq} = 6||3 = 2\ ohm$$

$$\tau = \frac{L}{R} = \frac{2}{2} = 1\ sec$$

$$i(t)_{t=0} = \frac{10}{2+3} = 2\ A$$

$$i(t) = 2e^{-t}\ A$$