Electrical Circuit (1)

Discharge RC and RL (week13 class1)

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Chapter 7 First-Order Circuits 253

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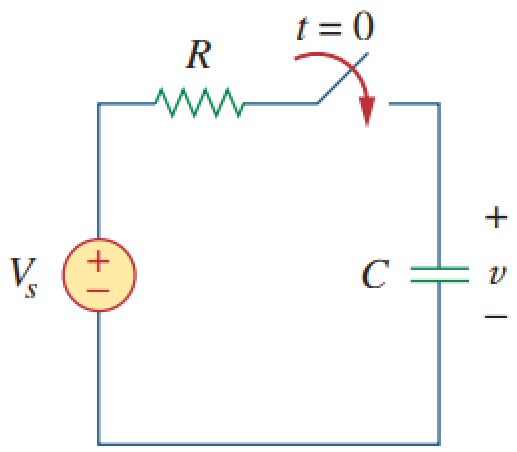
Step Response of an *RC* Circuit

The **step response** of a circuit is its behavior when the excitation is the step function, which may be a voltage or a current source.

$$\underset{\text{}}{\text{Complete response}} = \underset{\text{stored energy}}{\text{natural response}} + \underset{\text{independent source}}{\text{forced response}}$$



$$\text{Complete response} = \underset{\text{temporary part}}{\text{transient response}} + \underset{\text{permanent part}}{\text{steady-state response}}$$

The **transient response** is the circuit's temporary response that will die out with time.

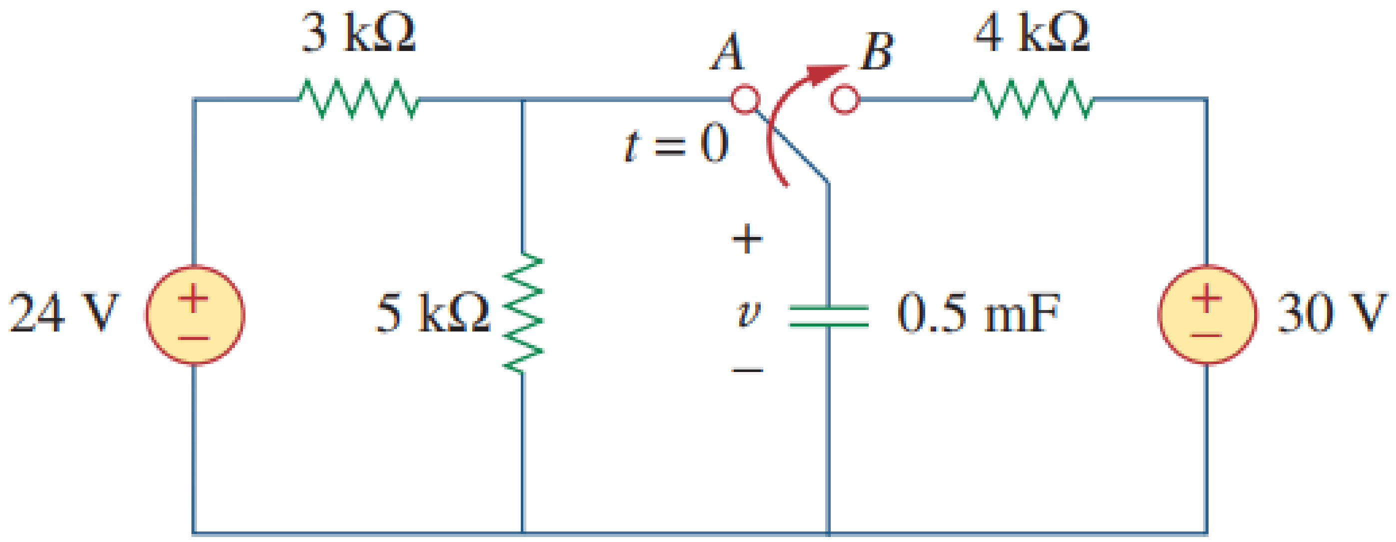
The **steady-state response** is the behavior of the circuit a long time after an external excitation is applied.

Step Response of an *RC* Circuit

1. The initial capacitor voltage $v(0)$.
2. The final capacitor voltage $v(\infty)$.
3. The time constant τ.

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

Step Response of an *RC* Circuit



$$v(\infty) = 30\ V$$

$$v(0) = 24\ \frac{5k}{3k + 5k} = 15\ V$$

$$R_{eq} = 4\ ohm$$

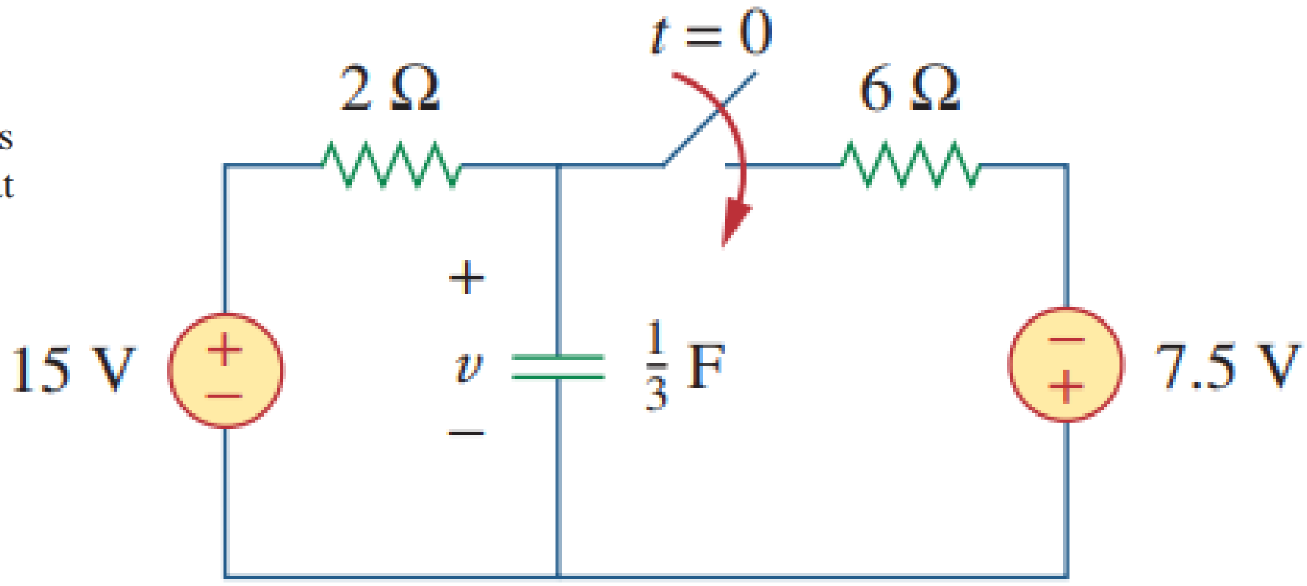
$$\tau = RC = 0.5 * 4 = 2$$

$$v(t) = v(\infty) + [v(0)\ - v(\infty)]e^{\frac{-t}{\tau}}\ V$$

$$\mathrm{v}(t) = 30 + [15 - 30]e^{\frac{-t}{2}}\ V$$

Step Response of an *RC* Circuit

Find $v(t)$ for $t > 0$ in the circuit of Fig. 7.44. Assume the switch has been open for a long time and is closed at $t = 0$. Calculate $v(t)$ at $t = 0.5$.



Answer: $(9.375 + 5.625e^{-2t})$ V for all $t > 0$, 7.63 V.

$$v(\infty) = \frac{v - 15}{2} + \frac{v + 7.5}{6} = 0 \;\rightarrow v(\infty) = 9.375\ V$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{\frac{-t}{\tau}}\ V$$

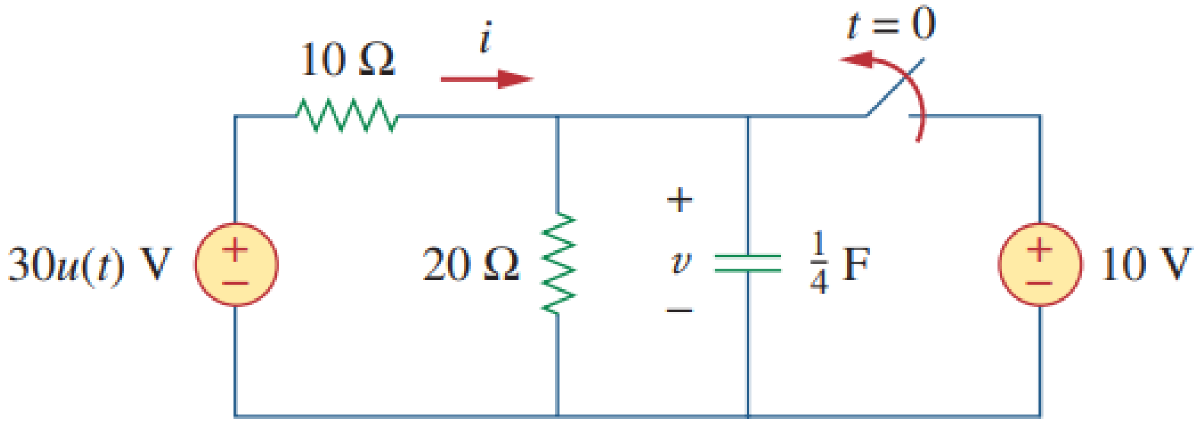
$$v(0) = 15\ V$$

$$R_{eq} = 2||6 = \frac{3}{2}\ ohm$$

$$\tau = RC = \frac{3}{2} * \frac{1}{3} = 0.5\ sec$$

$$v(t) = 9.375 + [15 - 9.375]e^{-2t}\ V$$

Step Response of an *RC* Circuit



$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{\frac{-t}{\tau}}\ V$$

$$v(\infty) = 30\frac{20}{10+20} = 20V$$

$$v(0) = 10\ V$$

$$R_{eq} = 20||10 = 6.666\ ohm$$

$$\tau = RC = 6.666 * \frac{1}{4} = 1.66\ sec$$

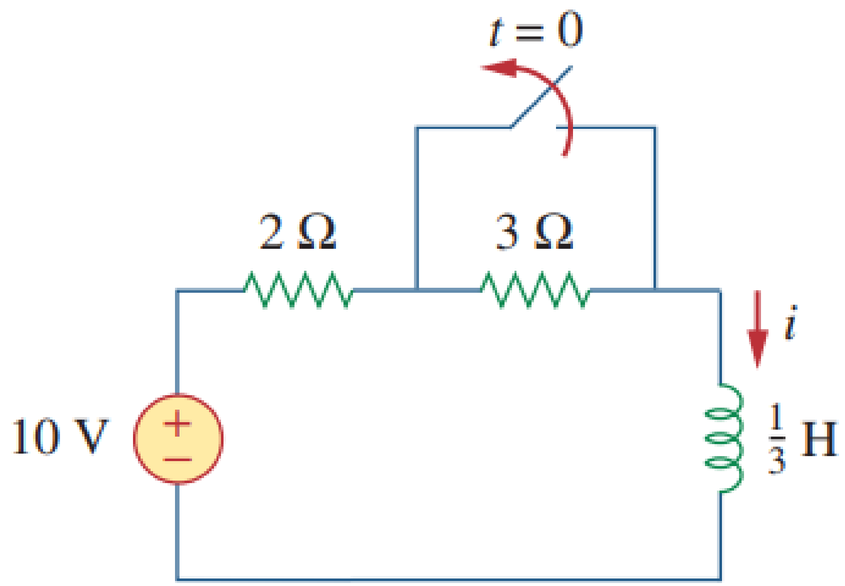
$$v(t) = 20 + [10 - 20]e^{\frac{-t}{1.66}}\ V$$

Step Response of an *RL* Circuit

1. The initial inductor current $i(0)$ at $t = 0$.
2. The final inductor current $i(\infty)$.
3. The time constant τ.

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

Step Response of an *RL* Circuit



$$i(t) = i(\infty) + [i(0) \; - i(\infty)]e^{\frac{-t}{\tau}} \; A$$

$$i(\infty) = \frac{10}{2+3} = 2A$$

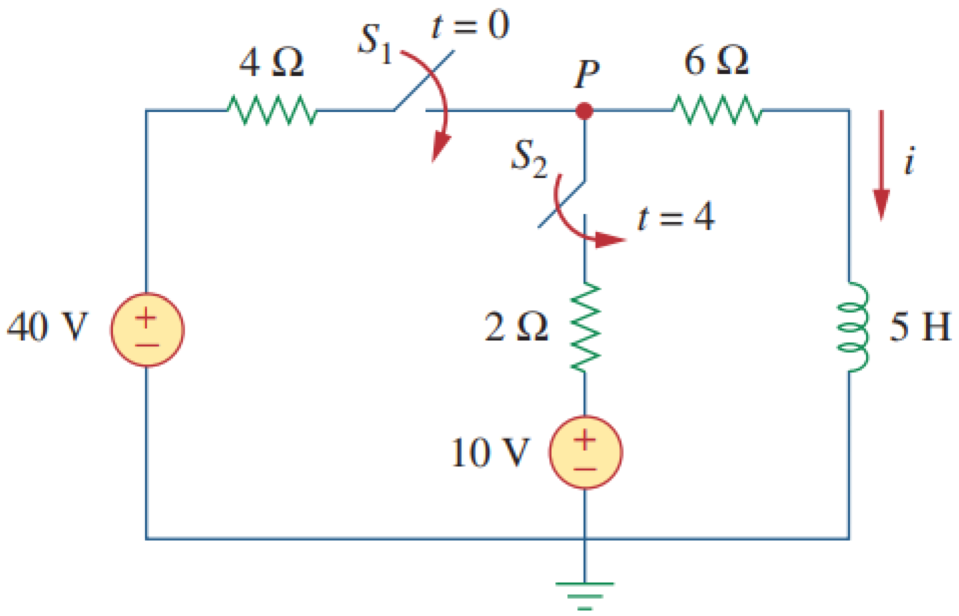
$$i(0) = \frac{10}{2} = 5A$$

$$R_{eq} = 2 + 3 = 5 \; ohm$$

$$\tau = \frac{L}{R} = \frac{\frac{1}{3}}{5} = \frac{1}{15} sec$$

$$i(t) = 2 + [5 - 2]e^{-15t} \; A$$

Step Response of an *RL* Circuit



$$i(t)_{t<0} = 0A$$

$$i(0)_{0<t<4} = 0A$$

$$i(\infty)_{0<t<4} = \frac{40}{10} = 4A$$

$$R_{eq(0<t<4)} = 4 + 6 = 10\ ohm$$

$$\tau_{(0<t<4)} = \frac{L}{R} = \frac{5}{10} = 0.5sec$$

$$i(t)_{0<t<4} = 4 + (0-4)\,e^{-2t}\ A$$

$$i(0)_{4<t} = 4 + (0-4)\,e^{-2t}\ A = 4A$$

$$i(\infty)_{4<t} \rightarrow \frac{v-40}{4} + \frac{v-10}{2} + \frac{v}{6} = 0$$

$$v = 16.36V$$

$$i(\infty)_{4<t} = \frac{16.36}{6} = 2.72A$$

$$R_{eq(4<t)} = 4||2 + 6 = \frac{22}{3}\ ohm$$

$$\tau_{(4<t)} = \frac{L}{R} = \frac{5}{\frac{22}{3}} = \frac{15}{22} = 0.681\ sec$$

$$i(t)_{4<t} = 2.72 + (4-2.72)\,e^{\frac{-t}{0.681}}\ A$$

$$i(t) = \left[\begin{array}{cc} t<0 & 0 \\ 0<t<4 & 4(0-4)\,e^{-2t} \\ 4<t & 2.72 + (4-2.72)\,e^{\frac{-t}{0.681}}\ A \end{array}\right]$$