



جامعة الطفيلة التقنية
Tafila Technical University



EE 0113416 Wind Energy Systems

Chapter 1: Aerodynamics

Dr. Abdullah Awad

Email:

abdullah.awad@ttu.edu.jo



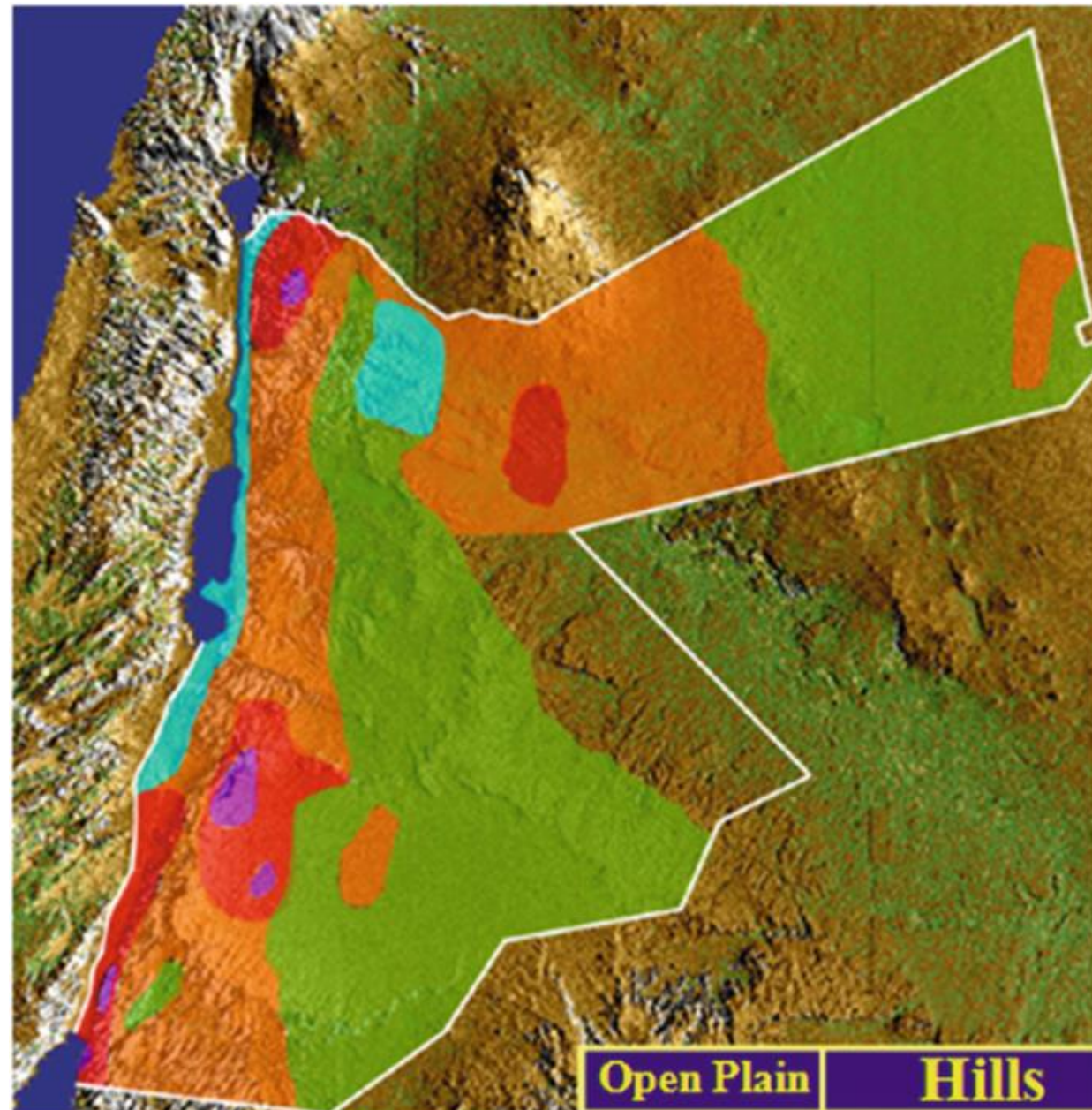
Classification of Wind Speed

Beaufort scale	10-minute sustained winds in knots (1 knot=0.5144 m/s)	General term
0	<1	Calm
1	1–3	Light air
2	4–6	Light breeze
3	7–10	Gentle breeze
4	11–16	Moderate breeze
5	17–21	Fresh breeze
6	22–27	Strong breeze
7	28–29	Moderate gale
	30–33	
8	34–40	Fresh gale
9	41–47	Strong gale
10	48–55	Whole gale
11	56–63	Storm
12	64–72	Hurricane
13	73–85	
14	86–89	
15	90–99	
16	100–106	
17	107–114	
	115–119	
	>120	

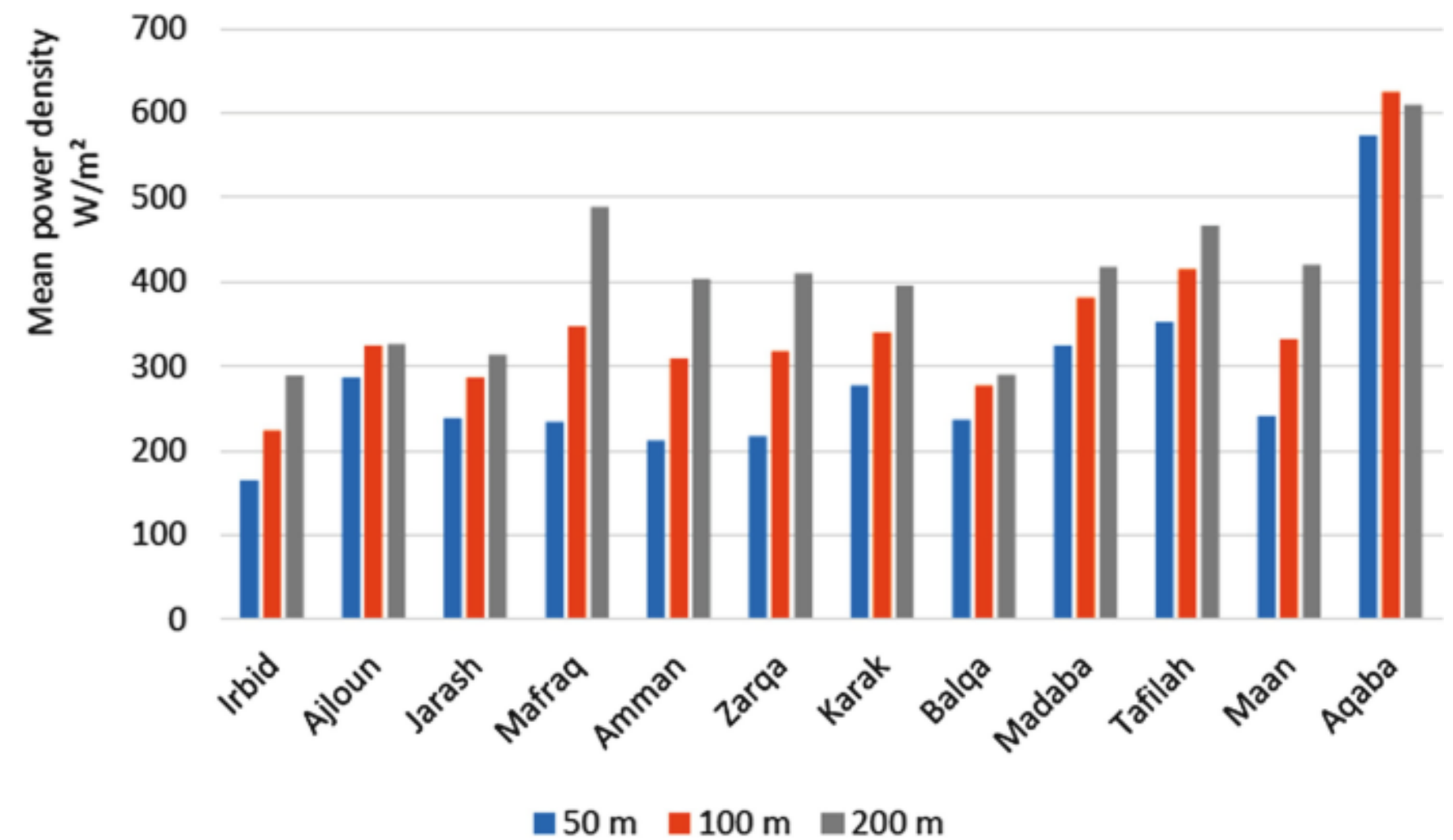
NREL Class System for Wind Energy Sites

Wind Power Class	10 m		50 m	
	Wind Power Density (W/m ²)	Speed (m/s)	Wind Power Density (W/m ²)	Speed (m/s)
1	0	0	0	
2	100	4.4	200	5.6
3	150	5.1	300	6.4
4	200	5.6	400	7.0
5	250	6.0	500	7.5
6	300	6.4	600	8.0
7	400	7.0	800	8.8
	1000	9.4	2000	11.9

Wind Power Generation in Jordan



	Open Plain	Hills
	>7.5 m/s	>11.5 m/s
	6.5-7.5m/s	10-11.5m/s
	5.5-6.5m/s	8.6-10m/s
	4.6-5.5m/s	7-8.6 m/s
	<4.5 m/s	<7.0 m/s



[Source Click here](#)

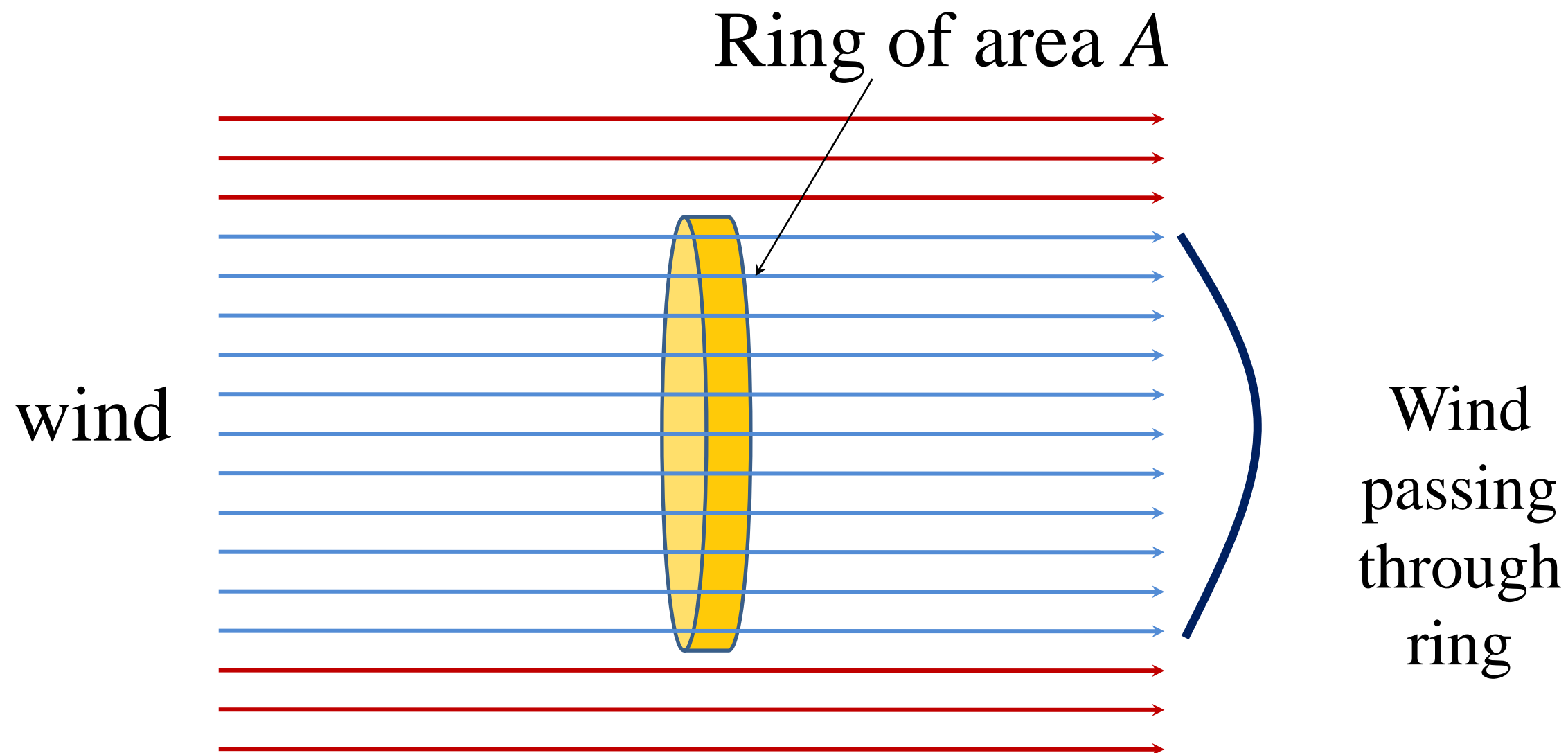


Kinetic Energy of Moving Object (KE)

$$KE = \frac{1}{2} m v^2$$

m: mass of object
v: speed of object

Kinetic Energy of Air



$$KE = \frac{1}{2} m w^2$$

$$m = volume * \delta$$
$$m = (A w t) \delta$$

m : mass of wind passing through
 A

w : speed of wind

A : sweep area

δ : air density (kg/m³)

t : time

Kinetic Energy of Air



$$KE = \frac{1}{2} m w^2$$

$$m = volume * \delta = A w t \delta$$

$$KE = \frac{1}{2} A \delta t w^3$$

KE is proportional to the cube of wind speed



Wind Speed

- Wind power increases with the cube of wind speed
 - significant benefit from even a moderate increase in wind speed
- Wind Speed increases with height
 - Air friction is higher near ground
- Smooth surfaces such as water reduces air friction
- Forests and buildings slow down the wind substantially

Air Density: Formula 1



$$\delta = \frac{353}{T + 273} e^{\frac{-h}{29.3(T+273)}} \quad \text{kg/m}^3$$

T : Air temperature in Celsius.

h : elevation of the wind above the sea level in meters.



Air Density: Formula 2

$$\delta = \frac{P_r M_w 10^{-3}}{R(T + 273)} \quad \text{kg/m}^3$$

- P_r = absolute pressure
- M_w = molecular weight of air = 28.97 g/mol
- T = absolute temperature (Celsius)
- R = ideal gas constant = $8.2056 \cdot 10^{-5} \cdot \text{m}^3 \cdot \text{atm} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$

Air Power Density (ρ)

$$KE = \frac{1}{2} A \delta t w^3$$

Wind Power $P_{wind} = \frac{KE}{t} = \frac{1}{2} A \delta w^3$

$$\rho = \frac{P_{wind}}{A} = \frac{1}{2} \delta w^3 \Rightarrow \rho \sim w^3$$

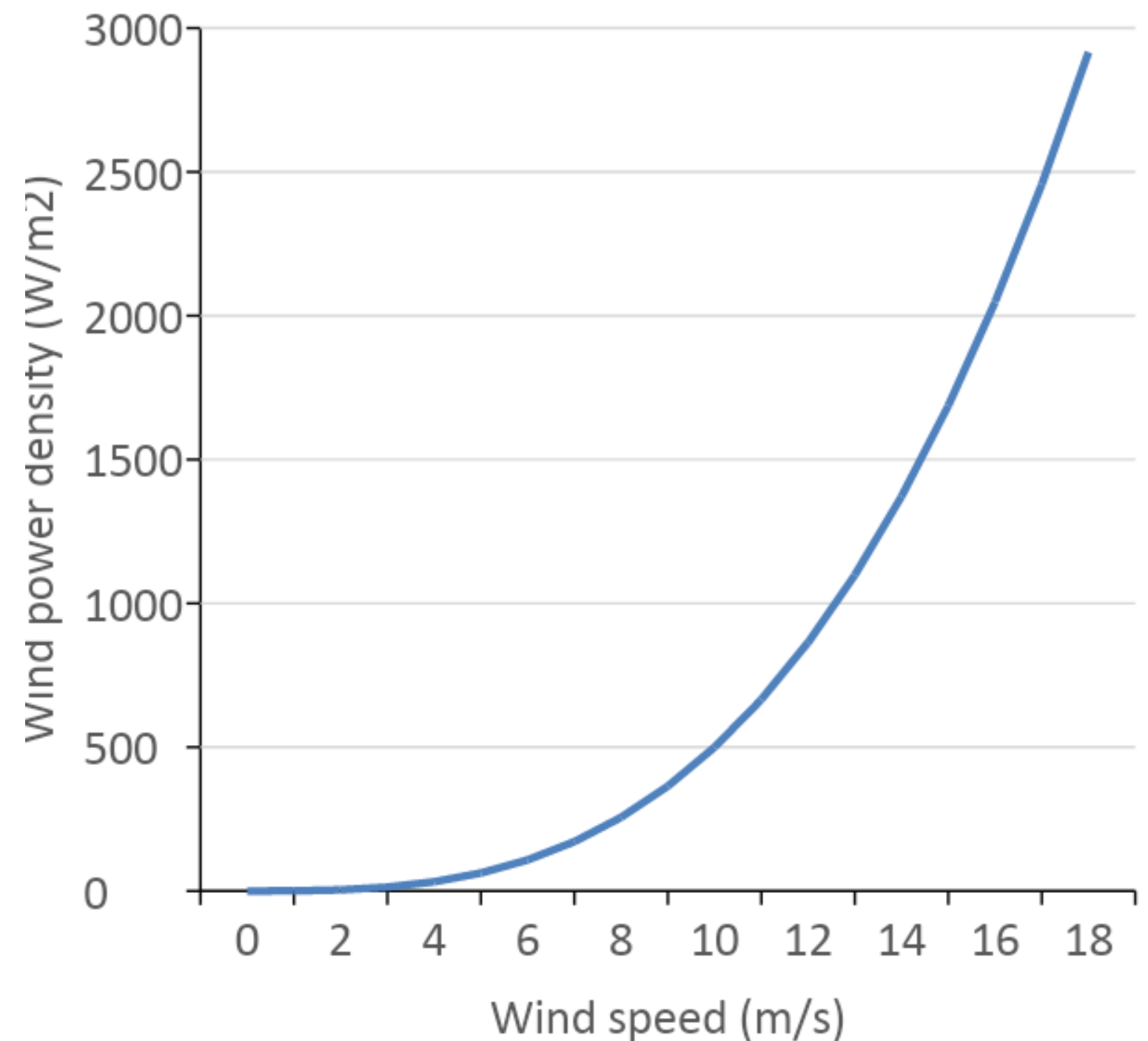
Wind Speed & The Power of Wind



$$\rho \sim w^3$$

Power increases proportional to the cube of wind speed

- Doubling wind speed increases the power by a factor of eight
- Energy in 1 hour of 20 mph winds is the same as energy in 8 hours of 10 mph winds



Blade Diameter and the Power of Wind



$$P_{wind} = \frac{1}{2} A \rho w^3$$

- Power in the wind also proportional to A .
- A is proportional to the square of the blade length.

Example

Compute the wind energy in 1 m² area (density 1 kg/m³) under the following conditions:

1. After 5 hours of 4 m/s winds speed
2. After 5 hours of 8 m/s wind speed
3. After 5 hours of 9 m/s winds



Solution

$$KE = \frac{1}{2} A \delta t w^3$$

$$KE_1 = \frac{1}{2} (1)(1)(5) 4^3 = 0.16 \text{ kWh}$$

$$KE_2 = \frac{1}{2} (1)(1)(5) 8^3 = 1.28 \text{ kWh}$$

$$KE_3 = \frac{1}{2} (1)(1)(5) 9^3 = 1.822 \text{ kWh}$$



Effect of Elevation and Earth's Roughness on Wind Speed

$$\frac{w}{w_0} = \left(\frac{h}{h_0} \right)^\alpha$$

- α = friction coefficient, typical values are
 - open terrain, $\alpha = 0.143$
 - large city, $\alpha = 0.4$
 - calm water, $\alpha = 0.1$
- w = wind speed at height h
- w_0 = wind speed at height h_0 (h_0 is usually 100 m)



Impact of Elevation and Earth's Roughness on Wind Power

$$\frac{P_w}{P_{w0}} = \left(\frac{w}{w_0} \right)^3 = \left(\frac{h}{h_0} \right)^{3\alpha}$$

Example



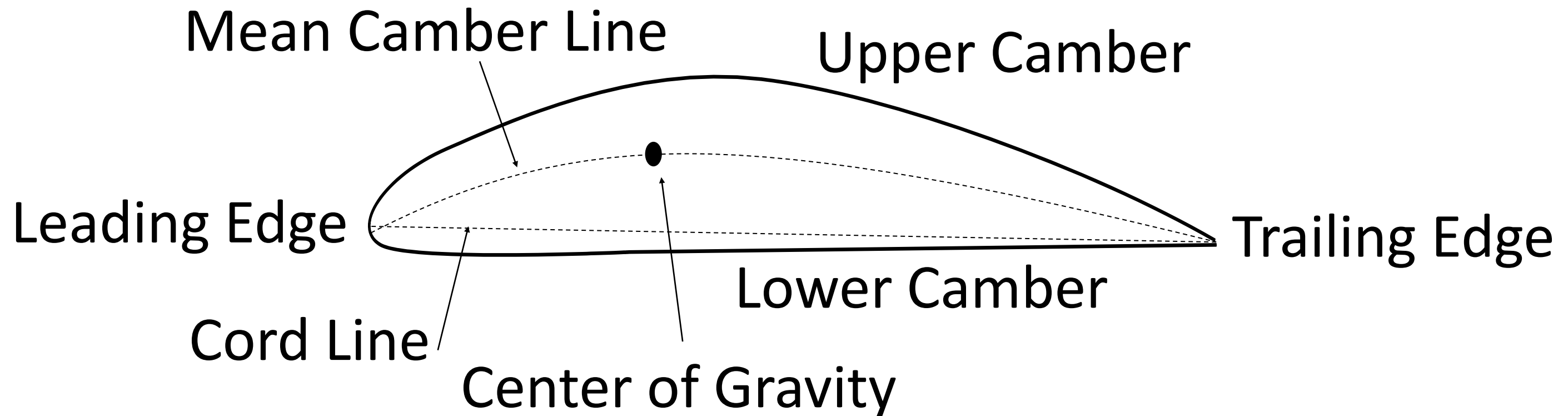
Wind power density at 100m is **2.5kW/m²** when wind speed is 10 m/s. Compute wind power density at 50m in open terrain.

Solution:

$$\frac{P_w}{P_{wo}} = \frac{\rho_w}{\rho_{wo}} = \left(\frac{h}{h_o}\right)^{3\alpha}$$

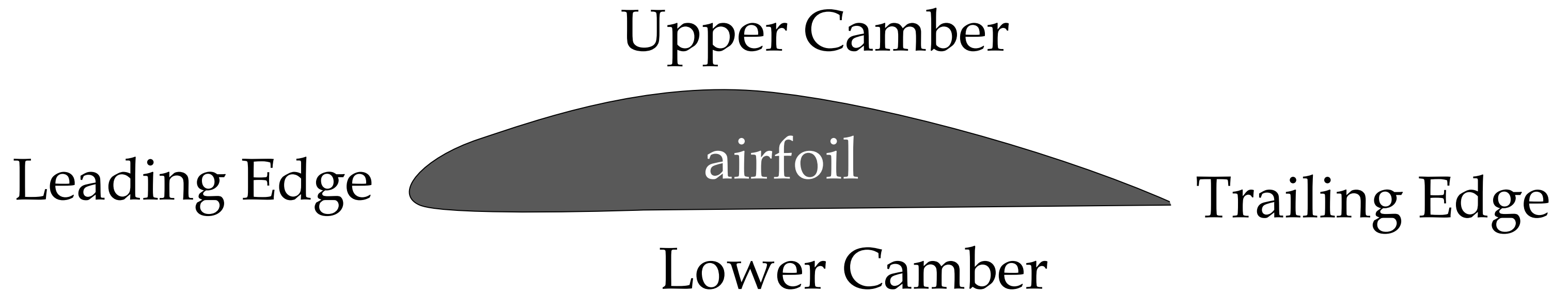
$$\rho_w = \rho_{wo} \left(\frac{h}{h_o}\right)^{3\alpha} = 2.5 \left(\frac{50}{100}\right)^{3*0.143} = \mathbf{1.86 \text{ kW/m}^2}$$

Airfoil Configuration



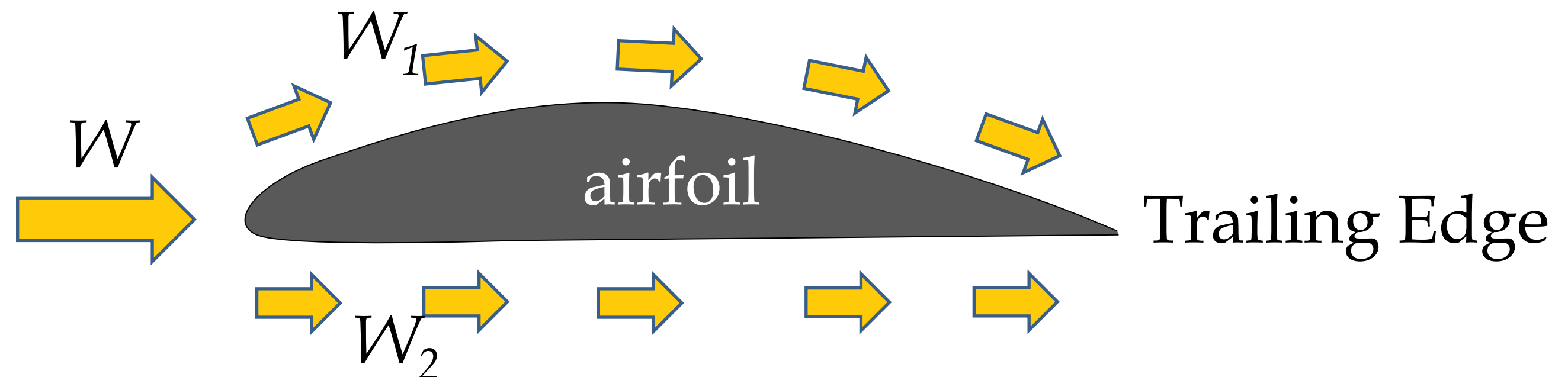
- Airfoil has a mean camber line and a center of gravity.
- Points on the mean camber line are the mid points between the upper and lower cambers.
- Center of gravity is located on the mean camber line.
- Cord line is a line connecting the leading edge to the trailing edge.

Wind Turbine Blade



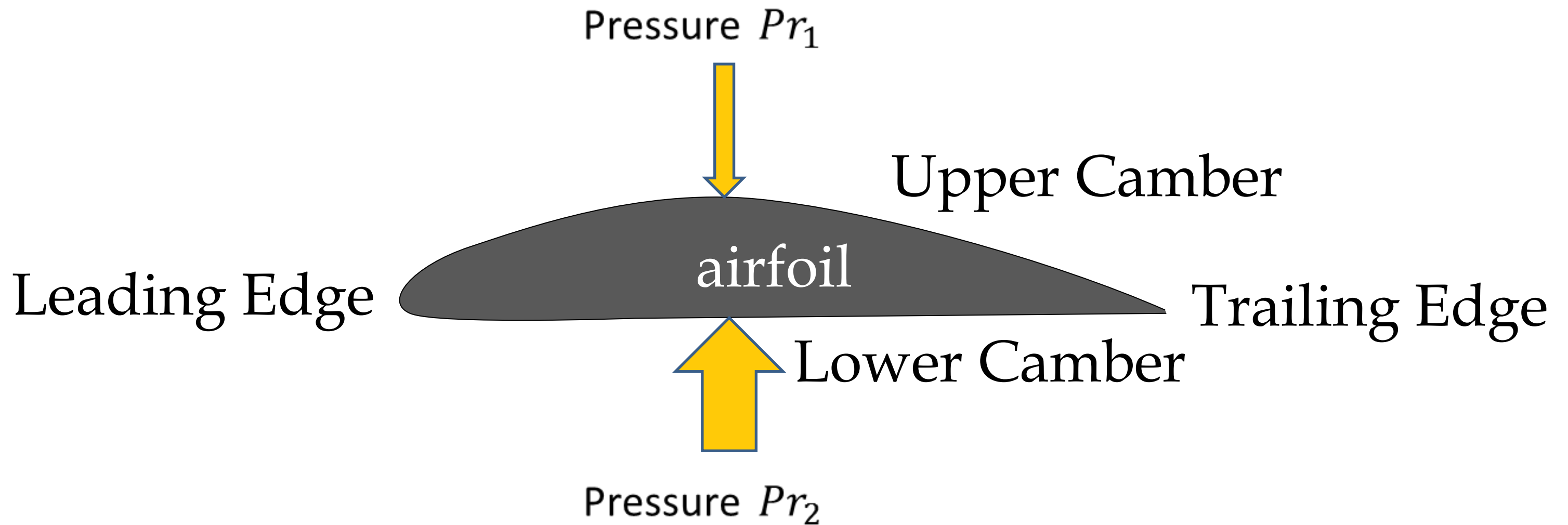
- The cross section of the blade shows longer upper camber than the lower camber
- Air coming to the leading edge will split into two components, one moves along the upper camber and the other along the lower camber

Law of Continuity



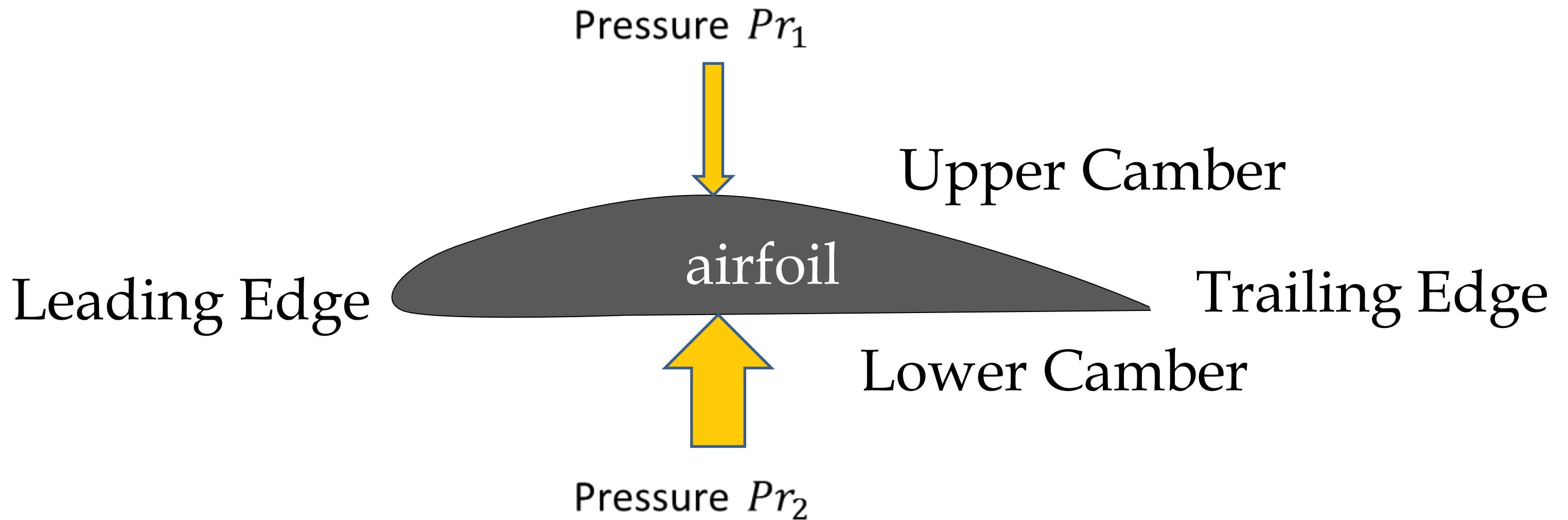
- “The air molecules separated at the leading edge to the upper and lower camber paths meet at the trailing edge at the **same time**.”
- Since the path of the upper camber is longer, the speed of air is higher

Bernoulli's Principal



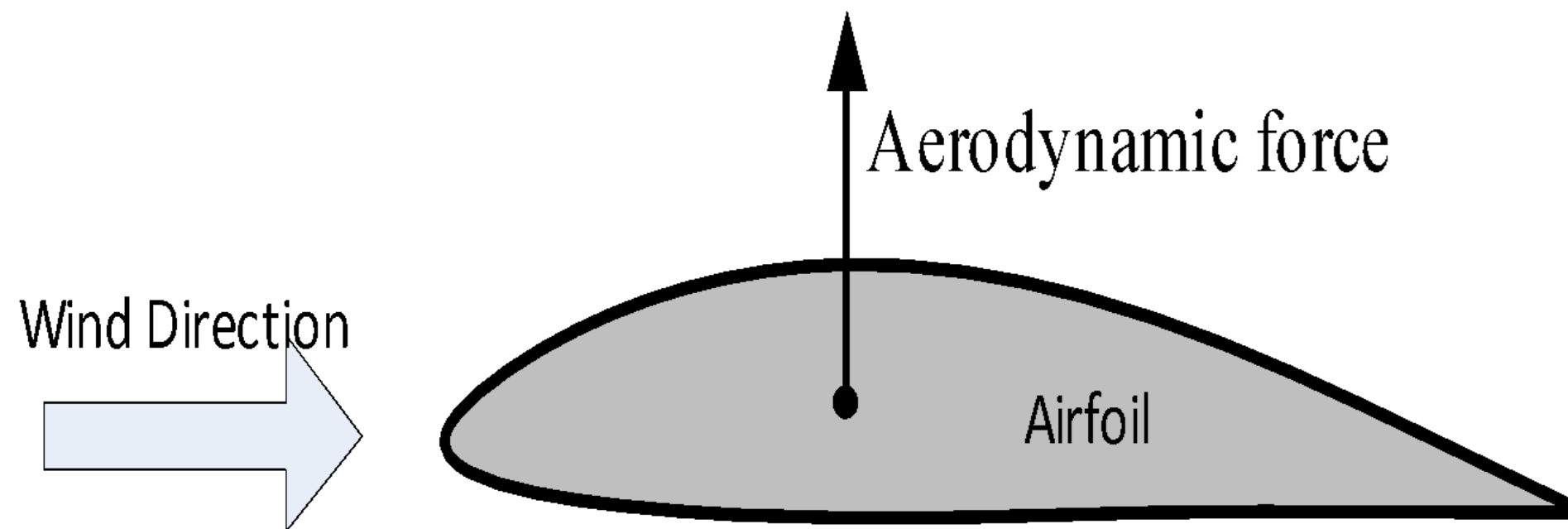
- “As the velocity of air increases, pressure decreases”.
- Hence the change in pressure causes lift

Bernoulli's Principal



- If the trailing end of the blade is fixed at the hub of a turbine, the change in pressure causes a twist that rotates the blade around the hub axis.

Aerodynamic force



$$F = P_{net} A$$

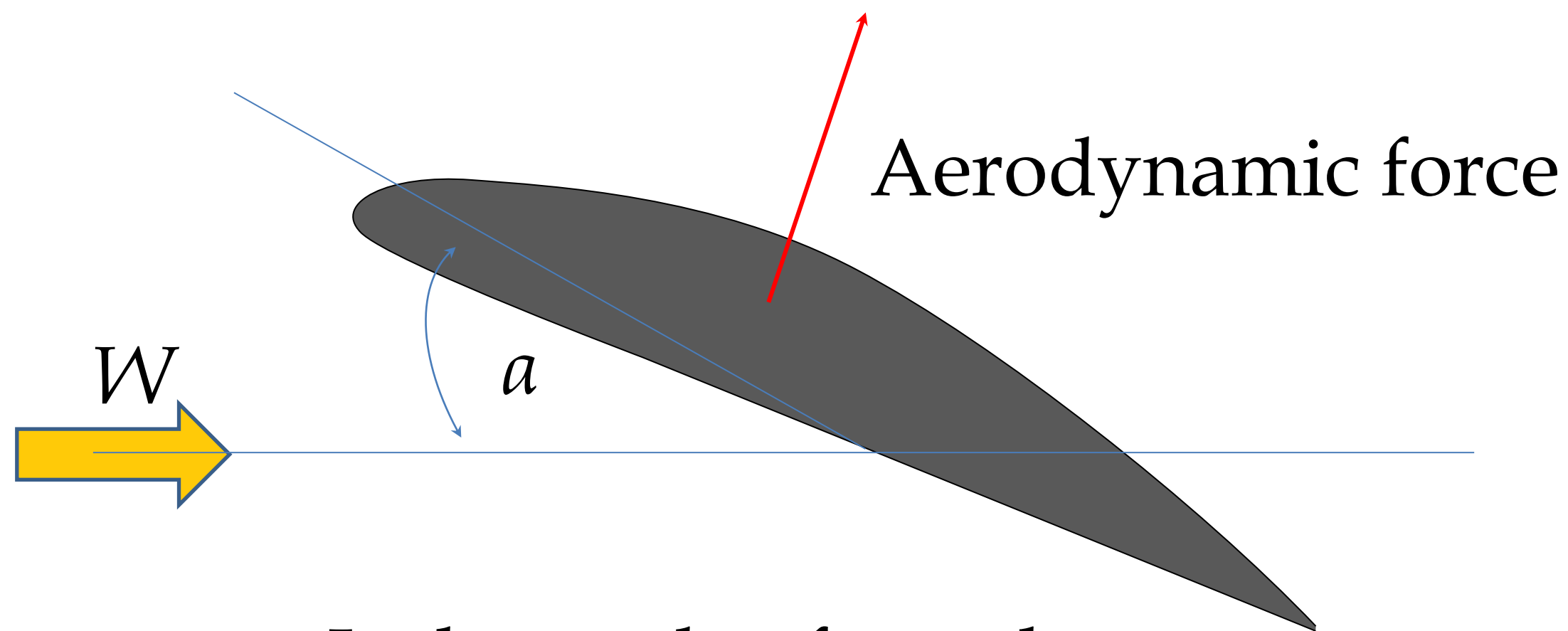
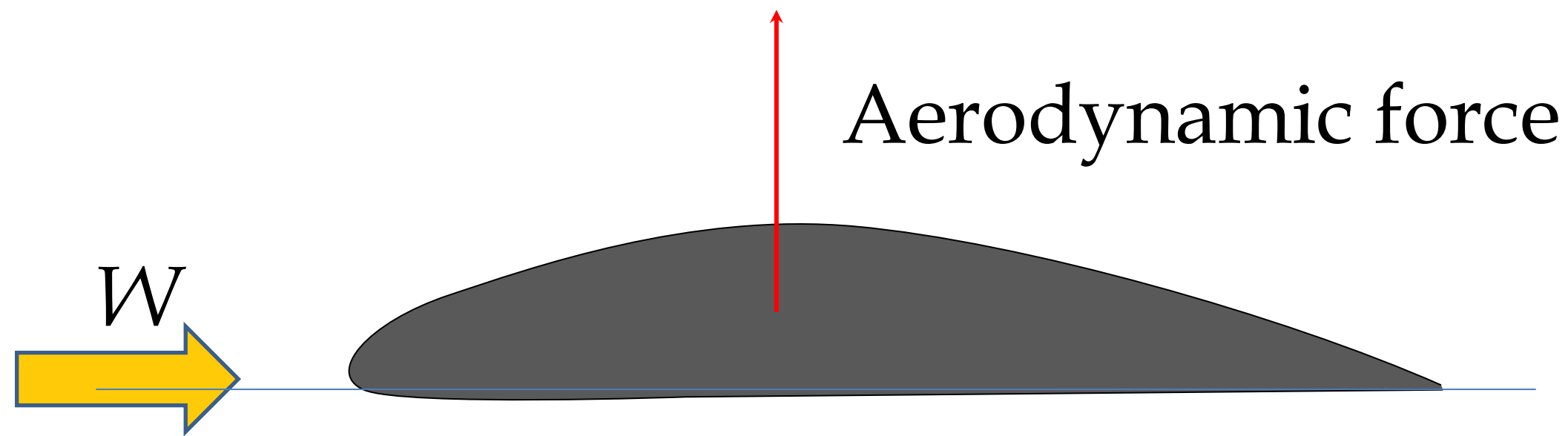
P_{net} is the net pressure exerted on an object

A is the surface area of the object

F is the aerodynamic force created by the net pressure

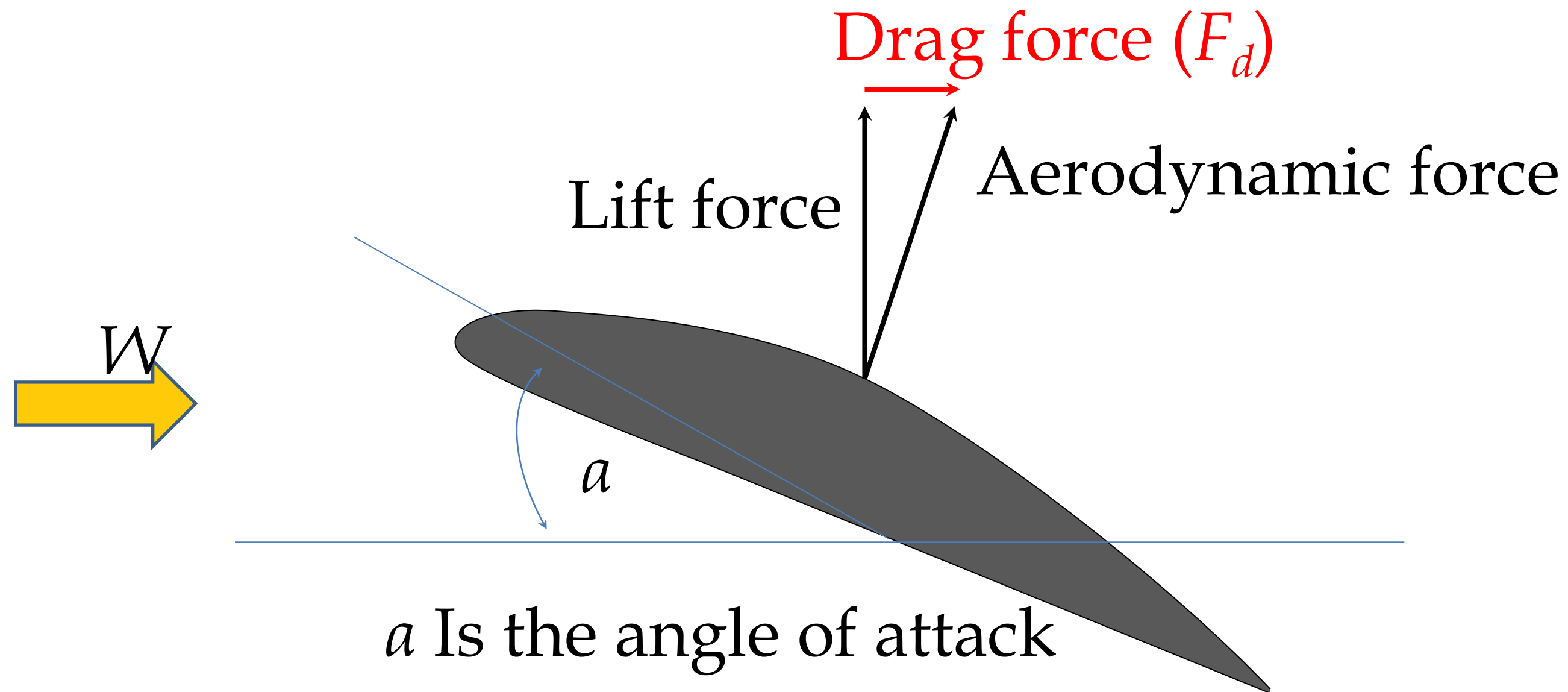
- Aerodynamic force is perpendicular to the surface of the airfoil

Angle of attack



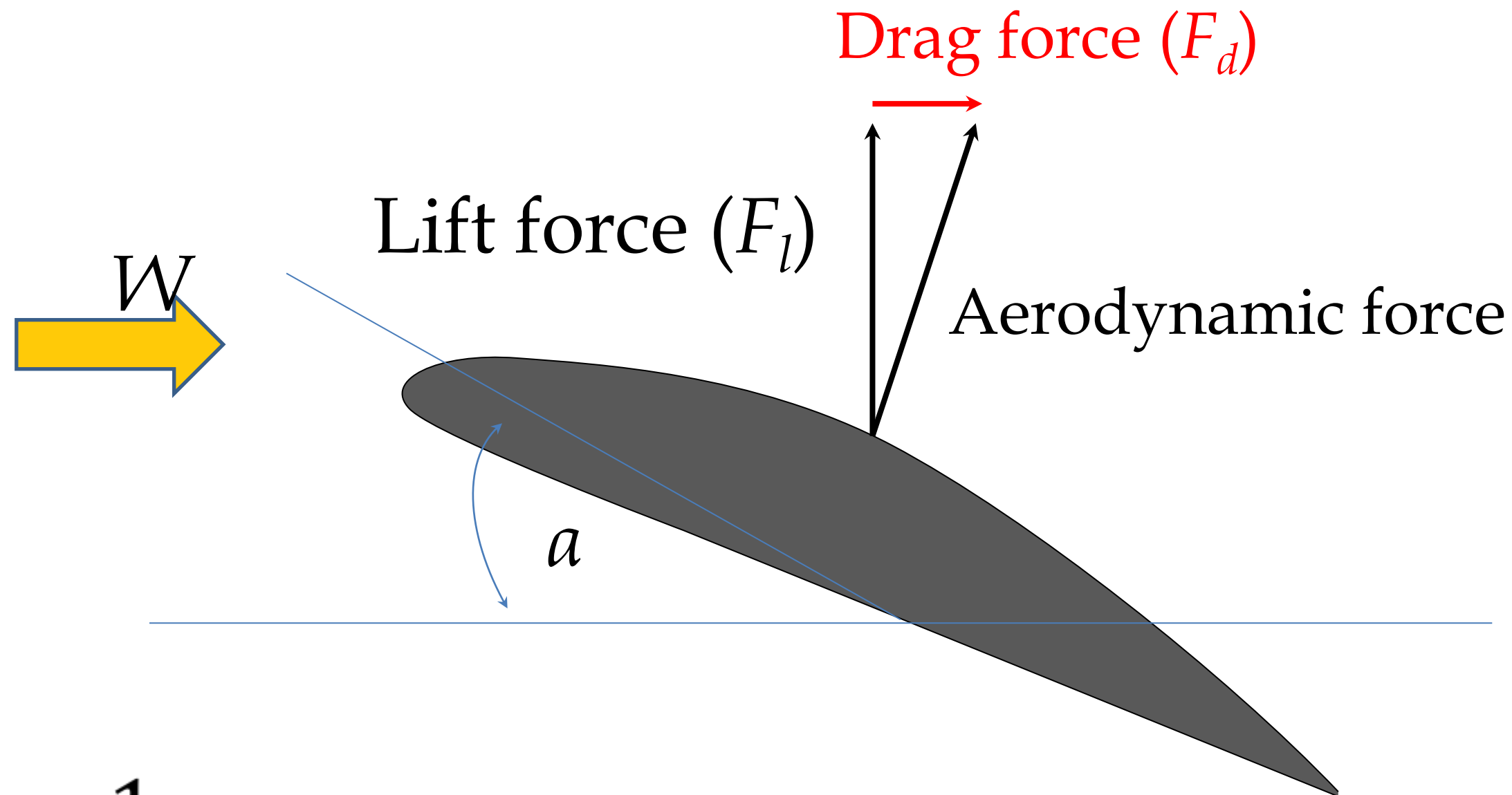
a Is the angle of attack

Drag and Lift Forces



- **Lift force** is the component of aerodynamic force perpendicular to wind direction.
- **Drag force** is the component of aerodynamic force parallel to wind direction.

Drag and Lift Forces



$$F_d = \frac{1}{2} C_D \delta A_f w^2$$

$$F_l = \frac{1}{2} C_L \delta A_f w^2$$

F_d = drag force (N)

C_D = drag coefficient

C_L = lift coefficient

δ = air density

w = air velocity

A_f = frontal area of airfoil



Lift and Drag coefficients

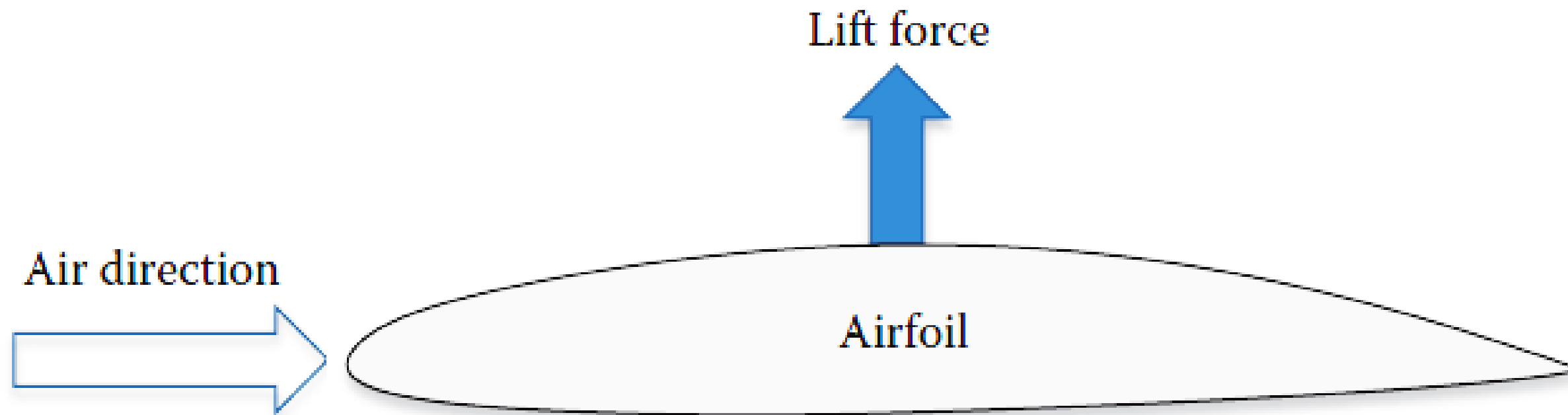
$$C_L = \frac{F_L}{F}$$

C_L : Lift coefficient

$$C_D = \frac{F_D}{F}$$

C_D : drag coefficient

Angle of attack



- Slightly increasing the angle of attack increase lift
 - air travels longer distance through the upper camber

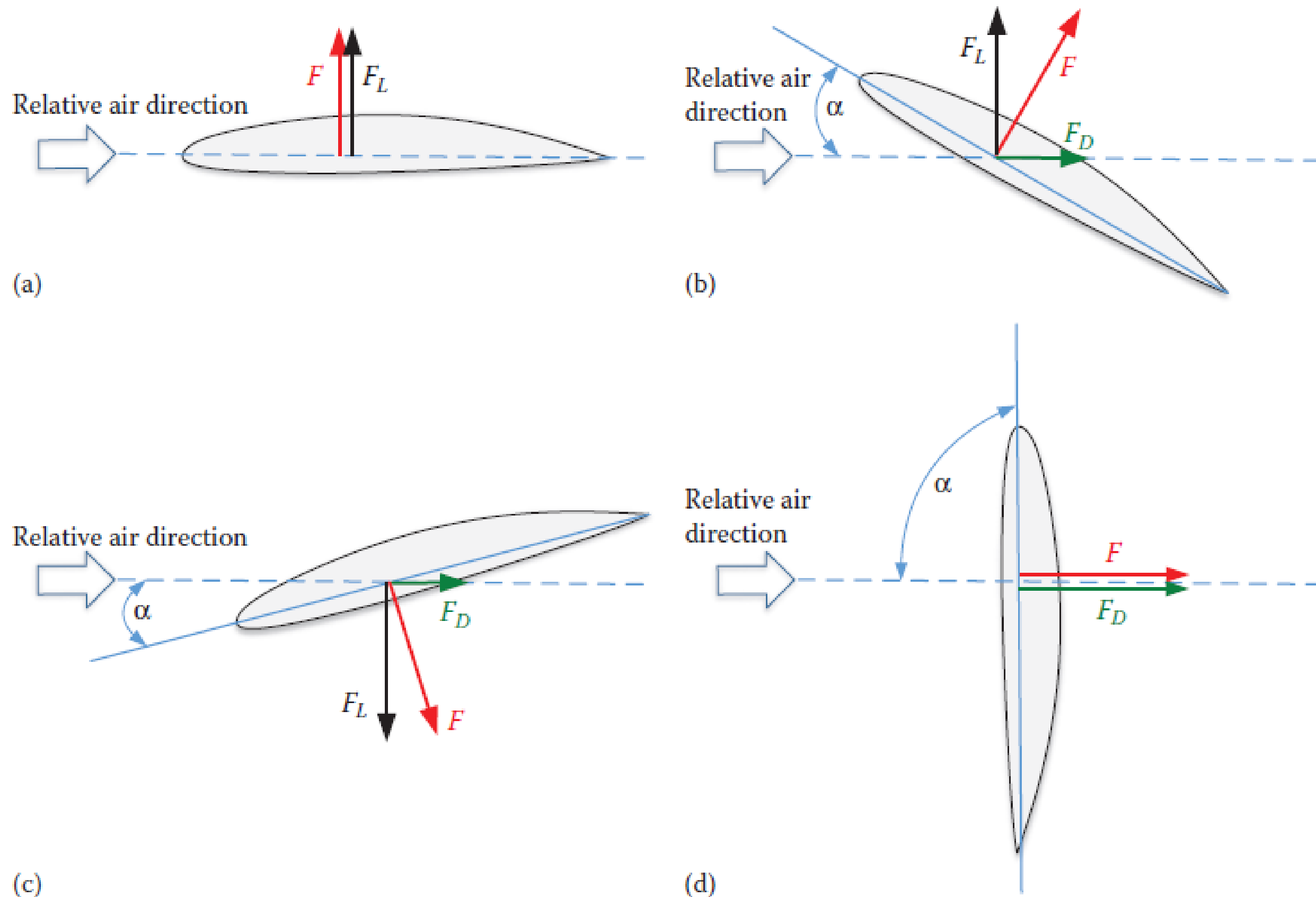
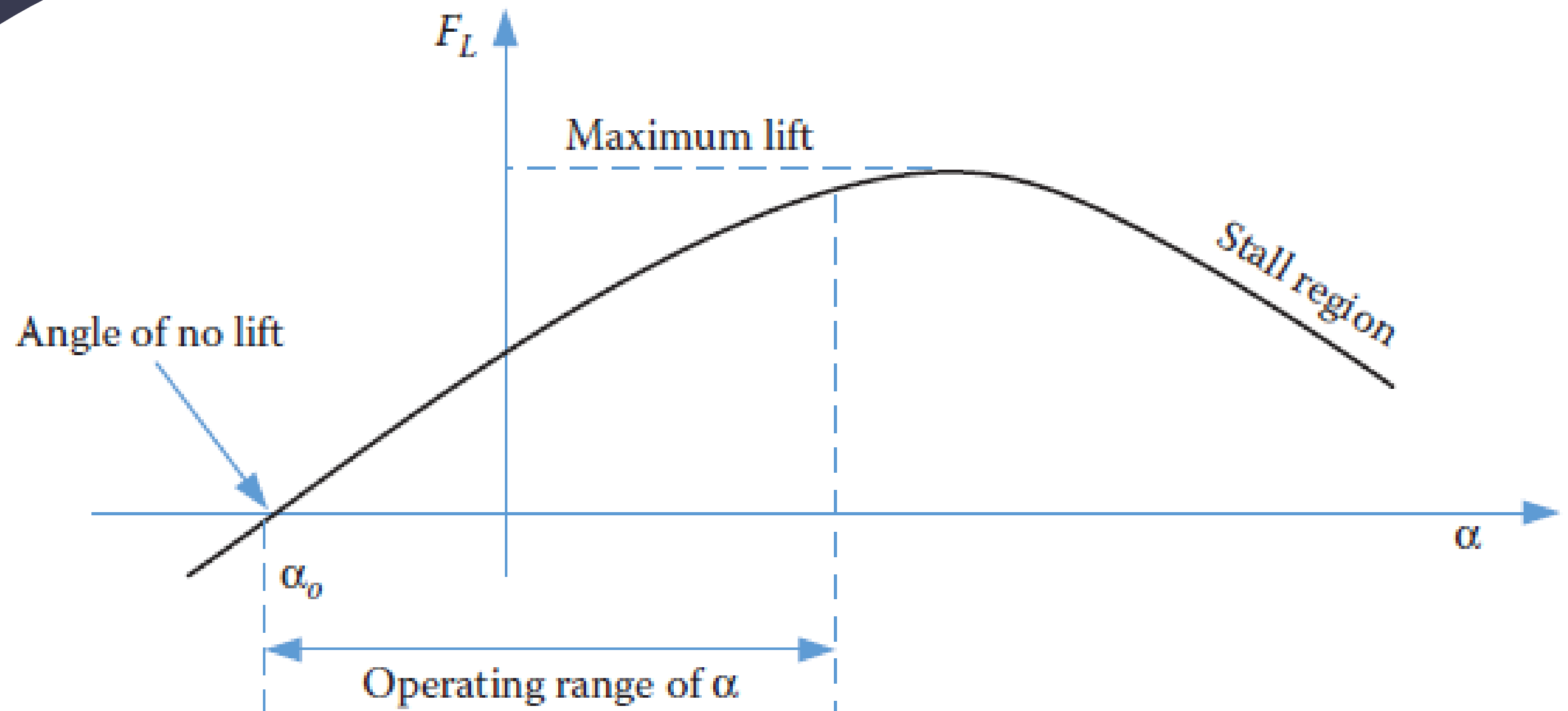


FIGURE 2.8

Aerodynamic forces and angle of attack: (a) horizontal position—all aerodynamic force is lift; (b) positive angle of attack—aerodynamic force has lift and drag; (c) negative angle of attack—lift is reversed; and (d) increasing positive angle of attack—until aerodynamic force is all drag.

Lift as a function of Angle of Attack



Example



A three-blade WT is operating at a given angle of attack and the aerodynamic force exerted by wind on each blade is 2000N. At the given angle of attack, the lift coefficient is 0.95. If the center of gravity of the blade is at 30m from the hub, compute the torque generated by the three blades. If the blades rotate at 30 r/min, compute the mechanical power generated by the blades.

Solution



torque is due to the lift force $F_L = C_L F = 0.95 * 2000 = 1,900 \text{ N}$

torque of one blade $T = F_L d$

For three-blade system $T_{total} = 3F_L d = 3 * 1900 * 30 = 171 \text{ kNm}$

power generated by the blade

$$P_{total} = T_{total} \omega_{blade} = 171 \left(2\pi \frac{n_{blade}}{60} \right) = 171 \left(2\pi \frac{30}{60} \right) = 537.2 \text{ kW}$$



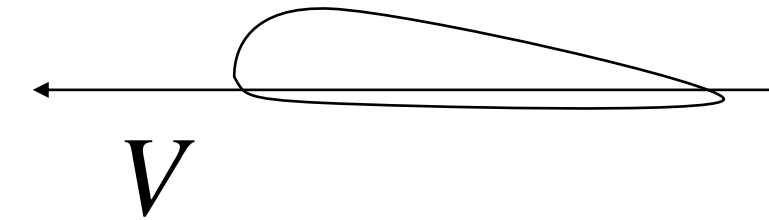
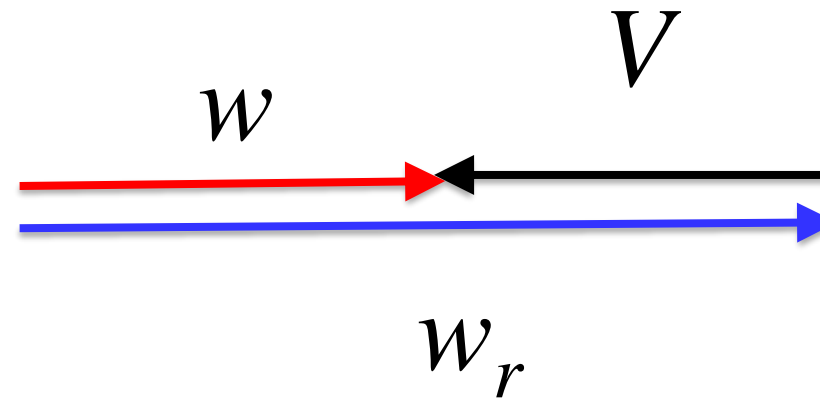
True and Relative Wind Speed

- If an instrument for measuring wind speed and wind direction is **mounted on the ground**, the readings obtained are those of the speed and direction of the **true wind**.
- If we mount the same instrument on a **moving object**, like airplane, then the readings will be quite **different from those taken on the ground**.
- Wind speed relative to a moving object is called **relative speed** or **apparent speed**.

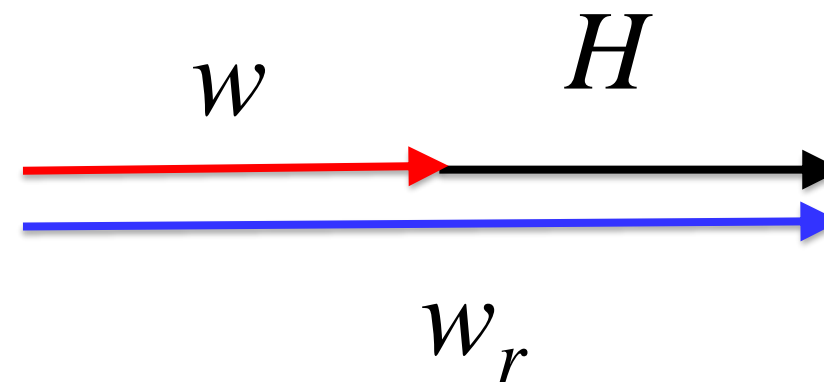
True and Relative Wind Speed



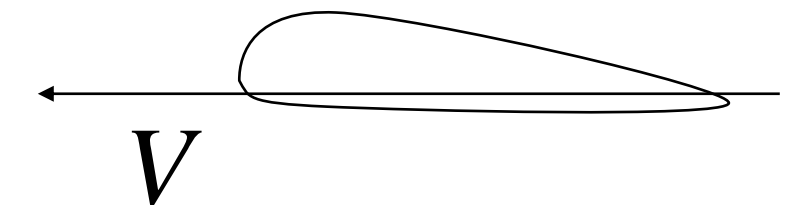
$$H \equiv -V$$



$$\overline{w_r} = \overline{H} + \overline{w}$$



Airplane wing



w : True (actual) wind direction

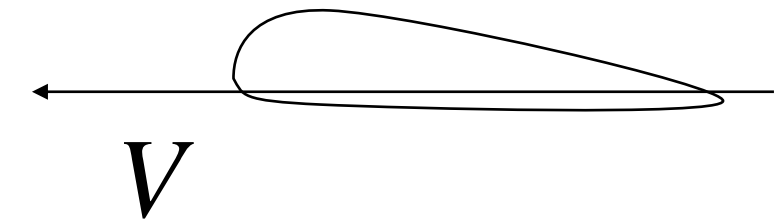
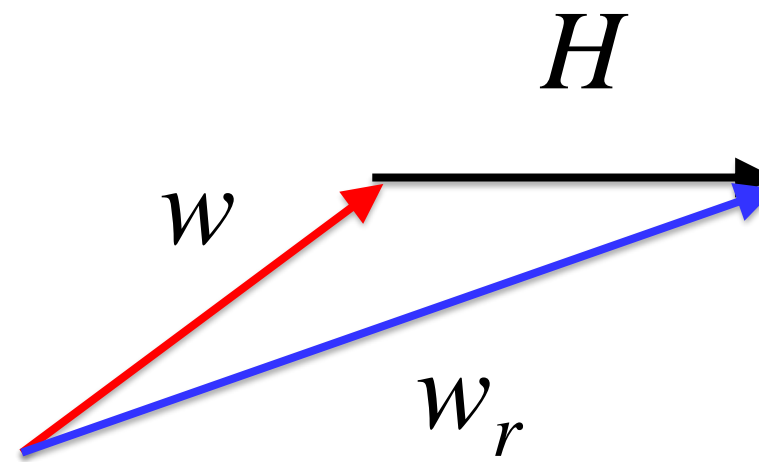
w_r : Relative wind speed

V : Velocity of wing relative to earth

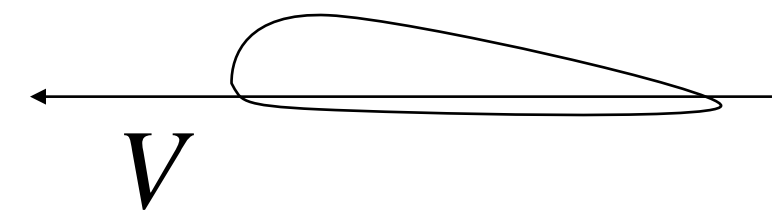
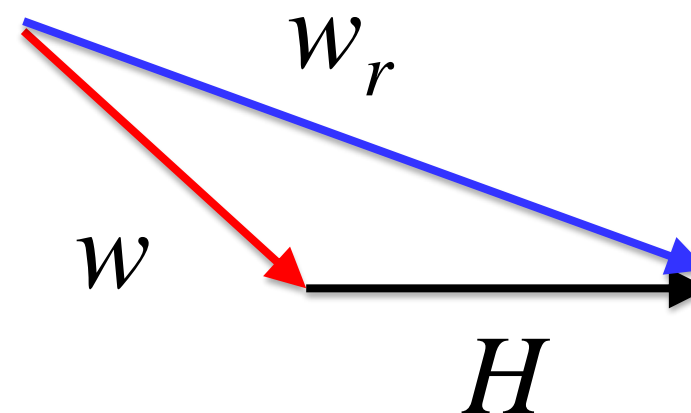
H : Head speed (speed of earth relative to wing)

True and Relative Wind Speed

$$\overline{w_r} = \overline{H} + \overline{w}$$



Airplane wing



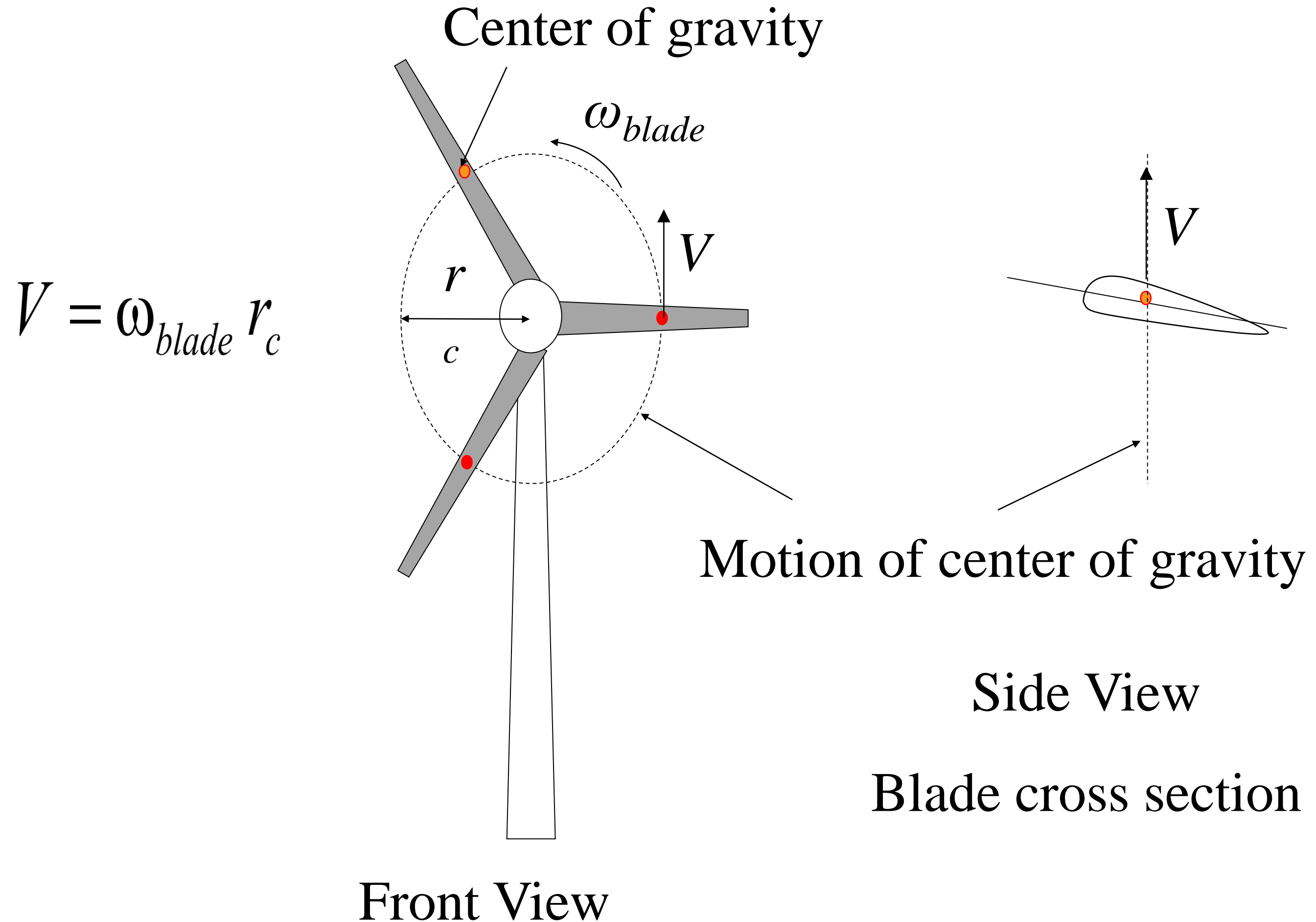
w : True (actual) wind direction

w_r : Relative wind speed

V : Velocity of wing relative to earth

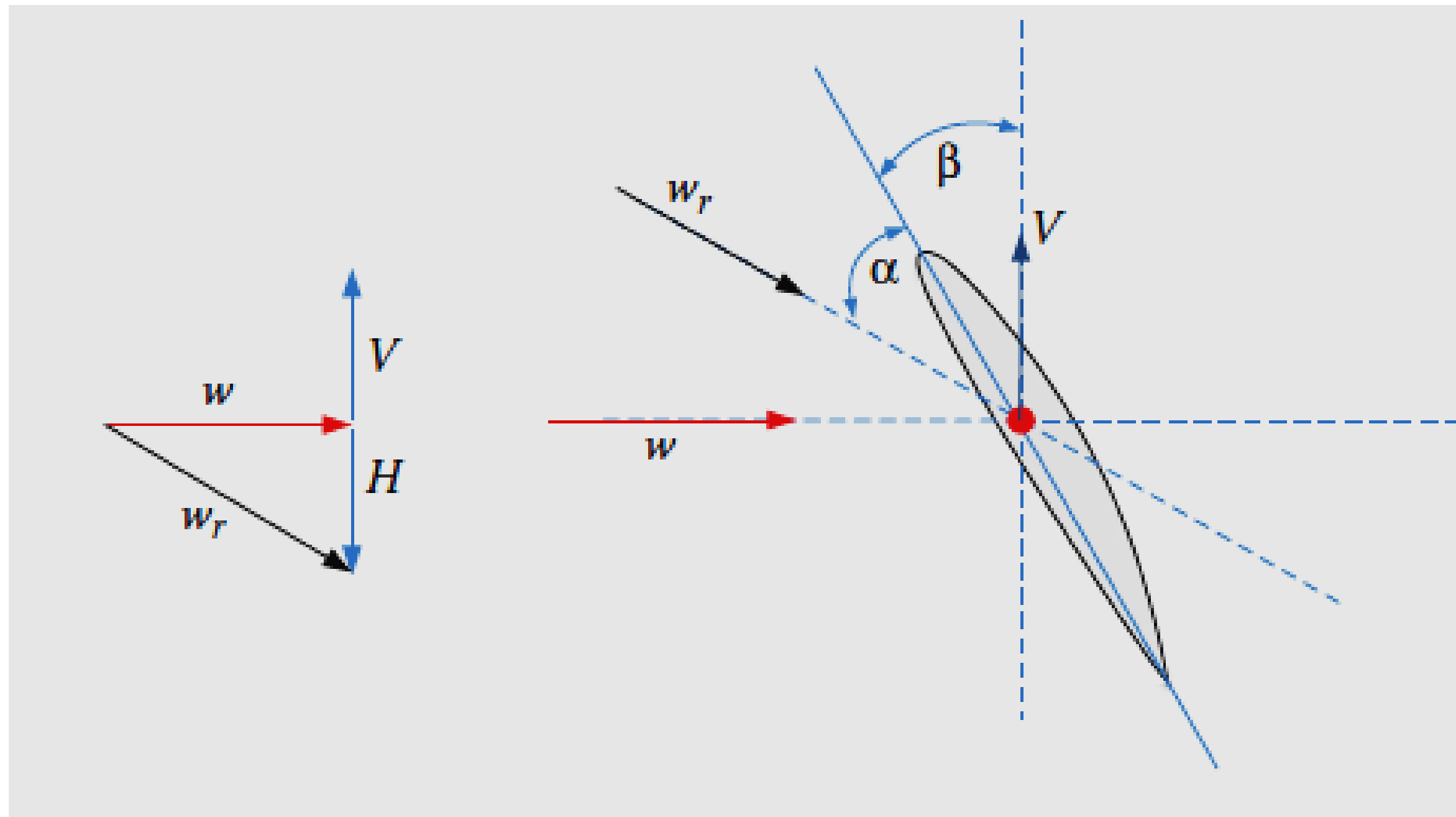
H : Head speed (speed of earth relative to wing)

Relative Wind Speed for Turbine



Relative Wind Speed for Turbine

w : True wind velocity at the blade

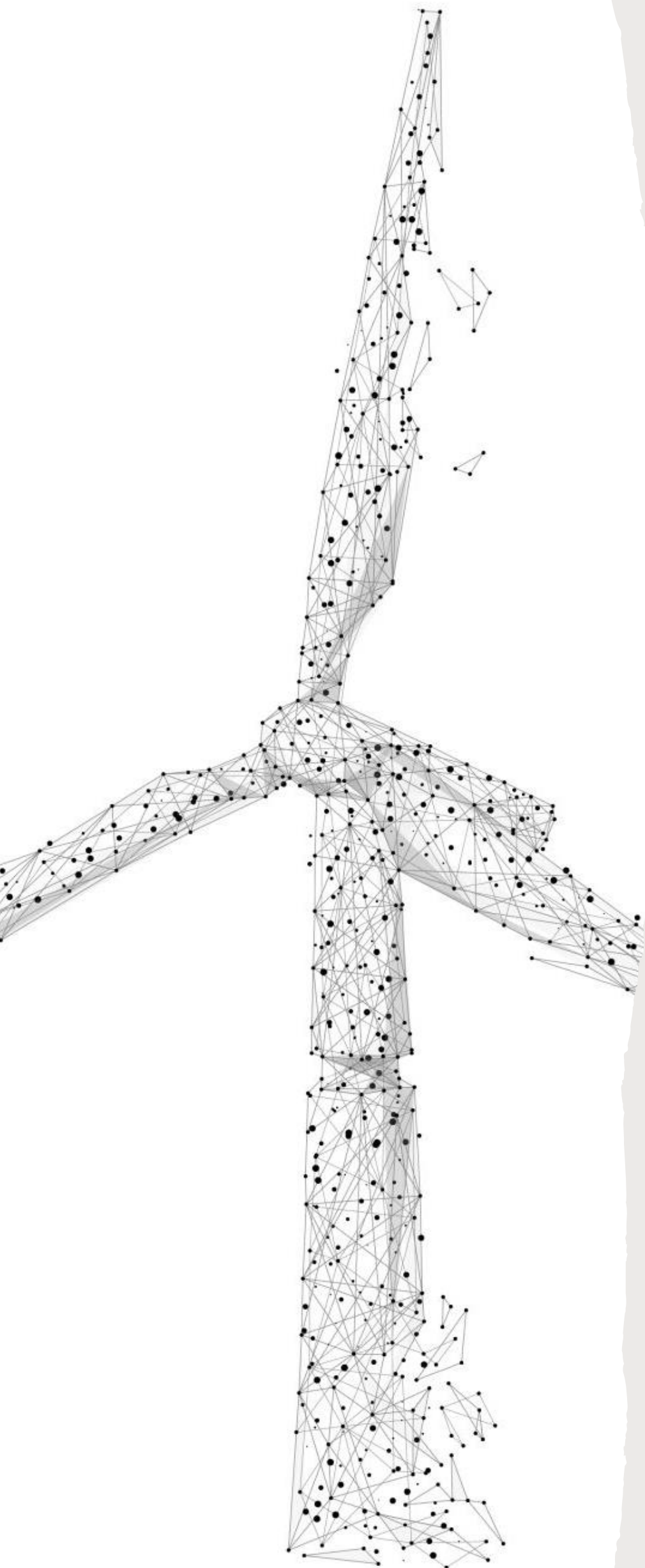


Angle of attack is measured with respect to relative wind direction, not true wind direction

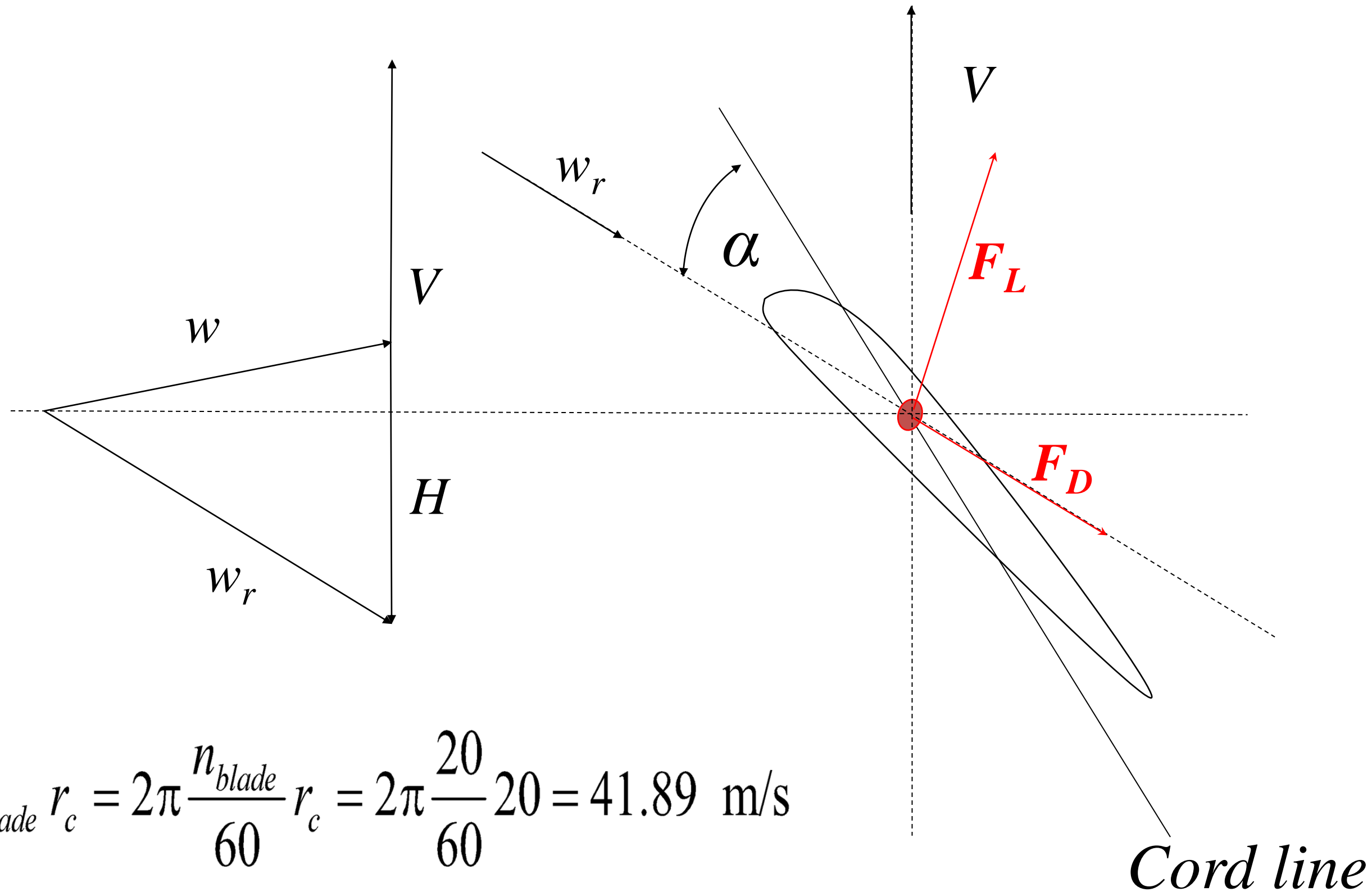


Example

- The true wind speed is 15 m/s at an angle of 20° with respect to the horizontal plane. The center of gravity of the blade is 20m from the center of the hub and is rotating at 20 r/min. Compute the relative wind speed and its direction.



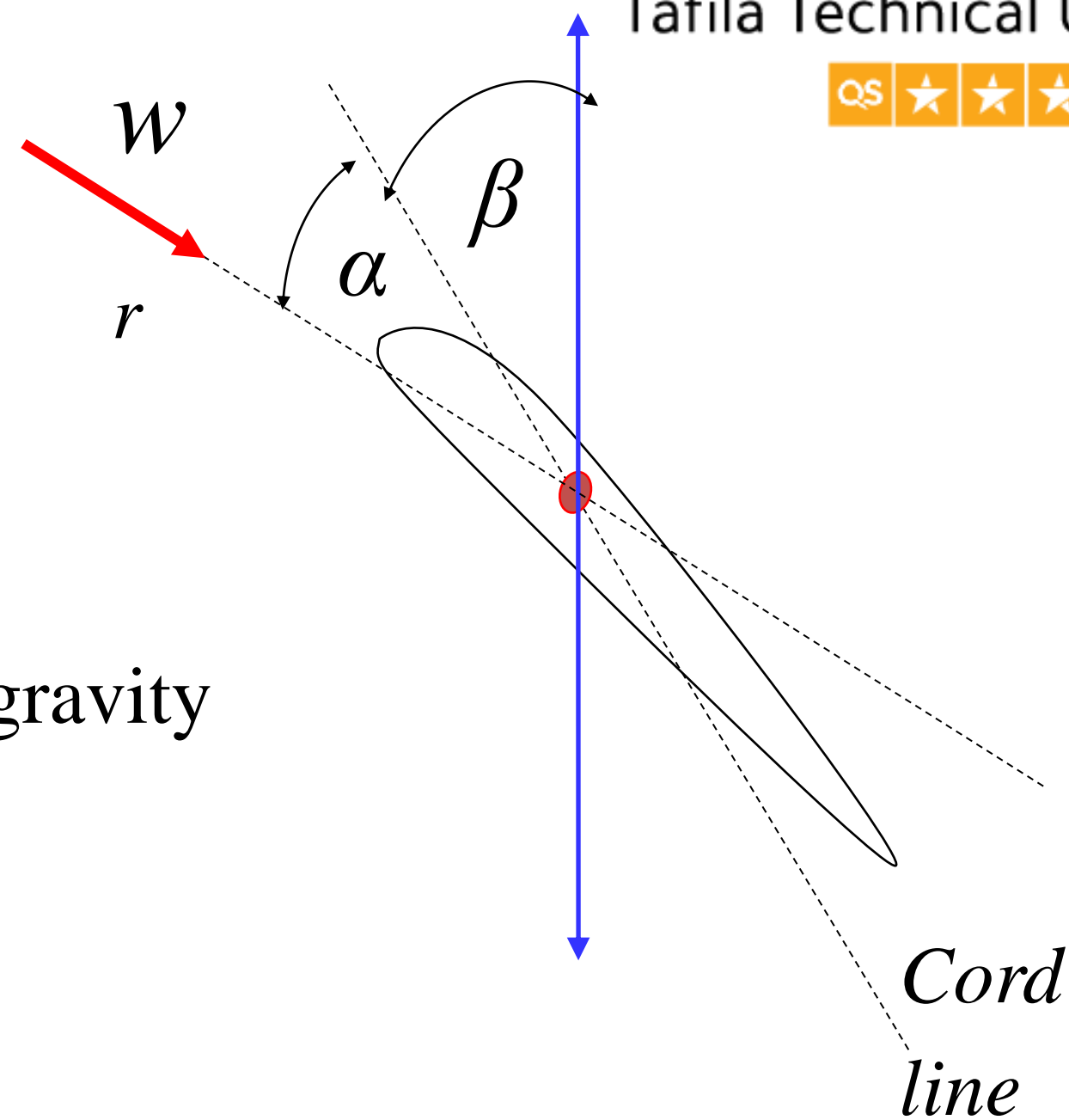
Solution



$$V = \omega_{blade} r_c = 2\pi \frac{n_{blade}}{60} r_c = 2\pi \frac{20}{60} 20 = 41.89 \text{ m/s}$$

$$\vec{w}_r = \vec{H} + \vec{w} = 41.89 \angle -90^\circ + 15 \angle 20^\circ = 39.37 \angle -69.02^\circ \text{ m/s}$$

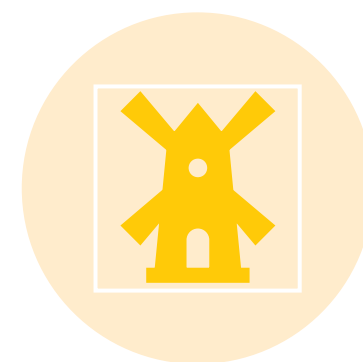
Pitch Angle



Vertical motion of the center of gravity



To compute the angle of attack you need to know the direction of relative wind speed, which may not be available with enough accuracy



For wind turbine blade, the motion of any point on the blade is circular and its locus is known



The pitch angle is the angle β between the cord line and vertical motion of the blade's center of gravity



Increasing β reduced α and vice versa.

Example

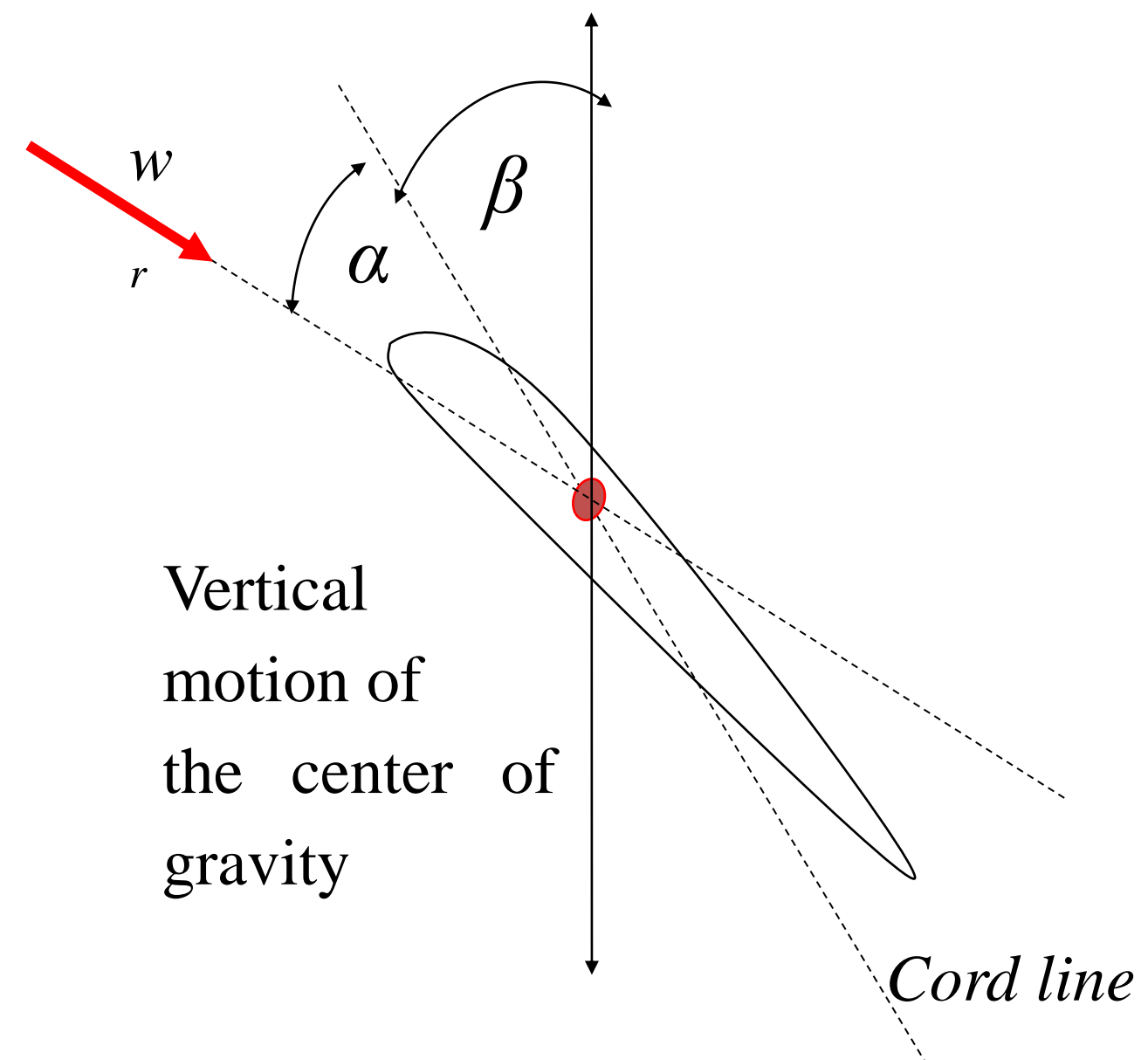


- Continuation from the previous example where the true wind speed is 15 m/s at an angle of 20° with respect to the horizontal plane.
- If the pitch angle is 5° , compute the angle of attack

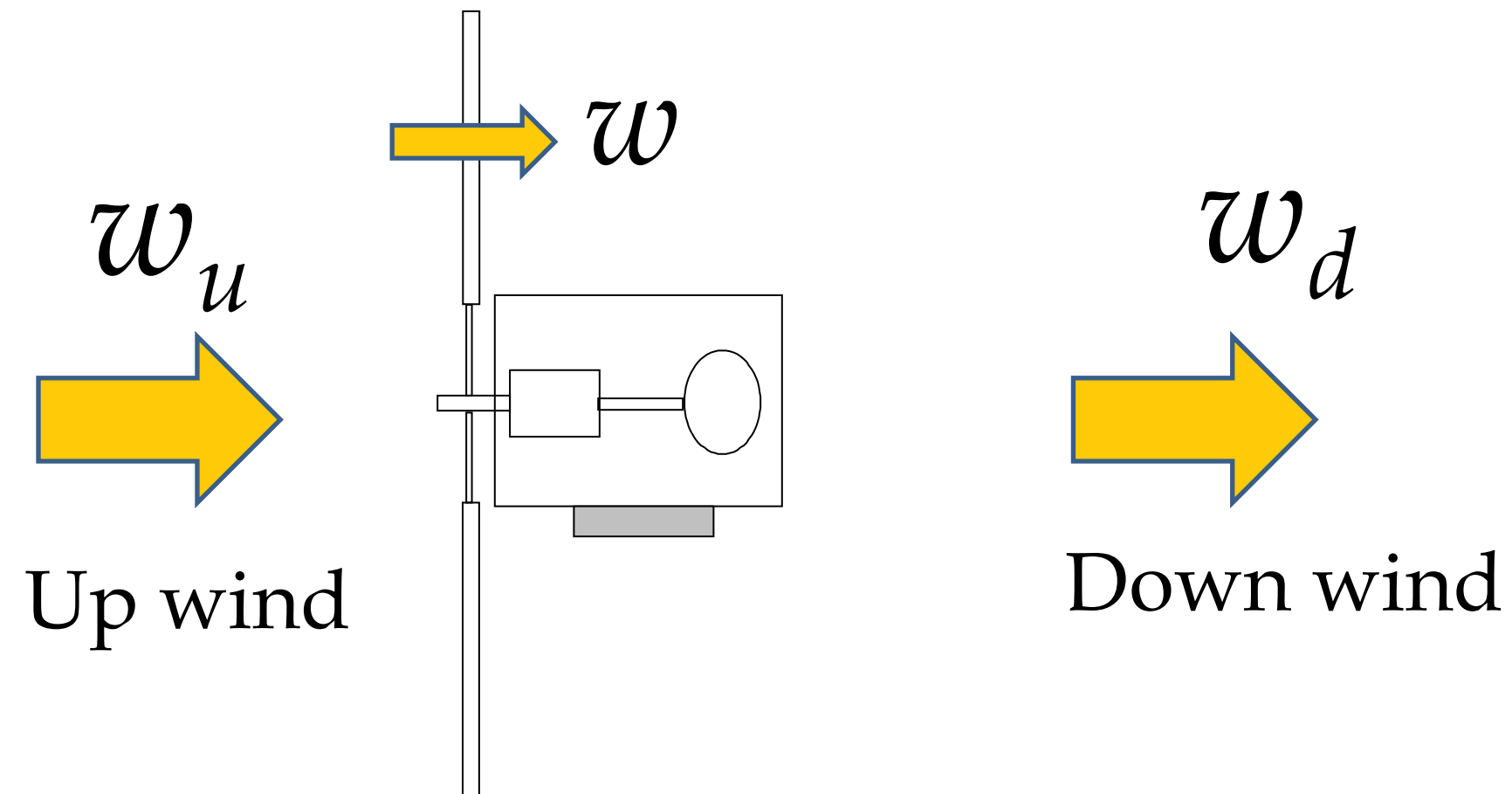
Solution:

$$90^\circ - 69.02^\circ = \alpha + \beta$$

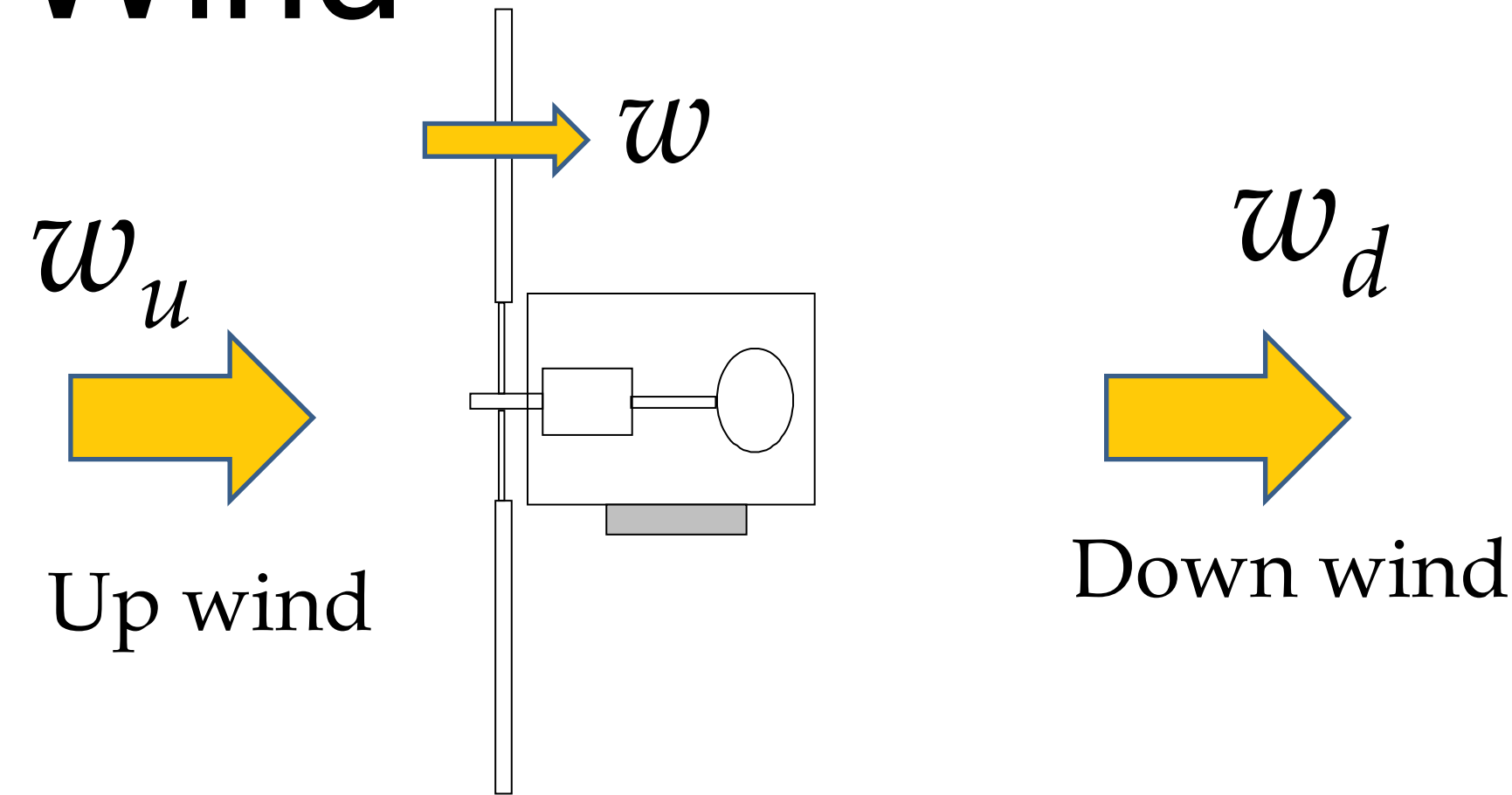
$$\alpha = 20.98^\circ - 5^\circ = 15.98^\circ$$



Change in Wind Speed



Speed and Pressure of Wind



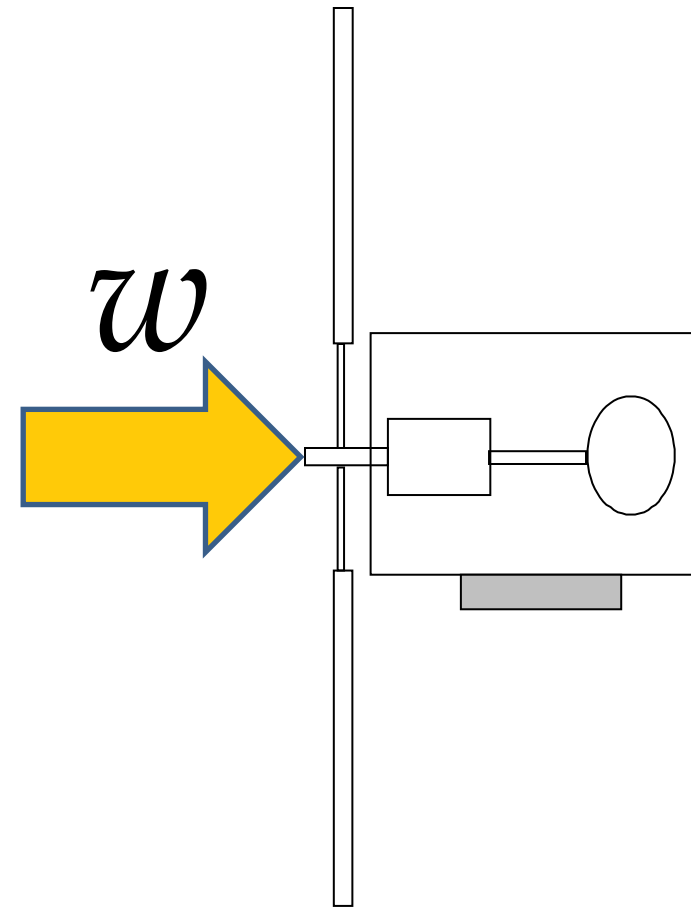
$$KE_{blade} = KE_{up\ wind} - KE_{down\ wind}$$

$$KE_{up\ wind} > KE_{down\ wind}$$

$$\frac{1}{2} m w_u^2 > \frac{1}{2} m w_d^2$$

$$w_u > w_d$$

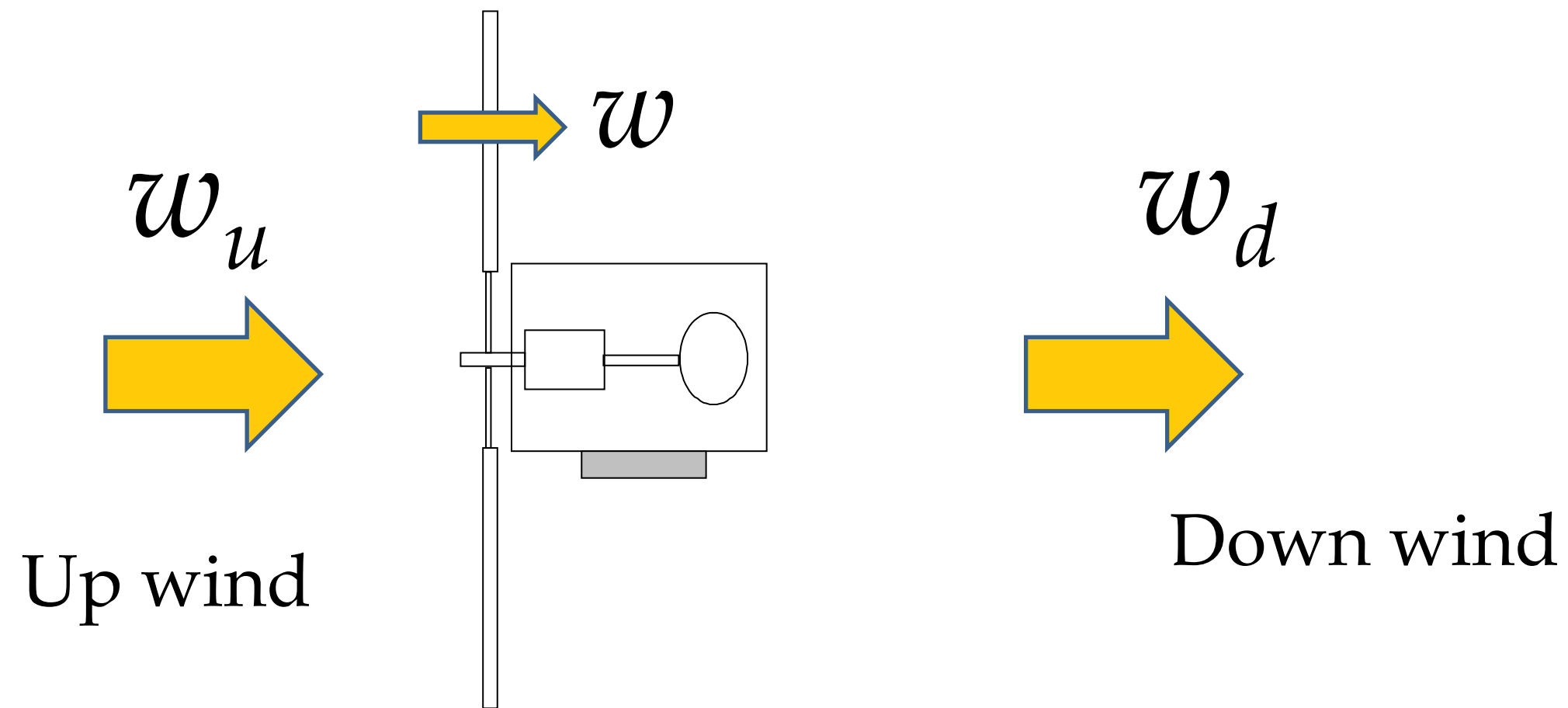
Mass Flow Rate



Mass flow rate: mass of air passing through the blade in a fixed time

$$f_{mass} = \frac{m}{t} = A w \delta \approx A w = \text{constant}$$

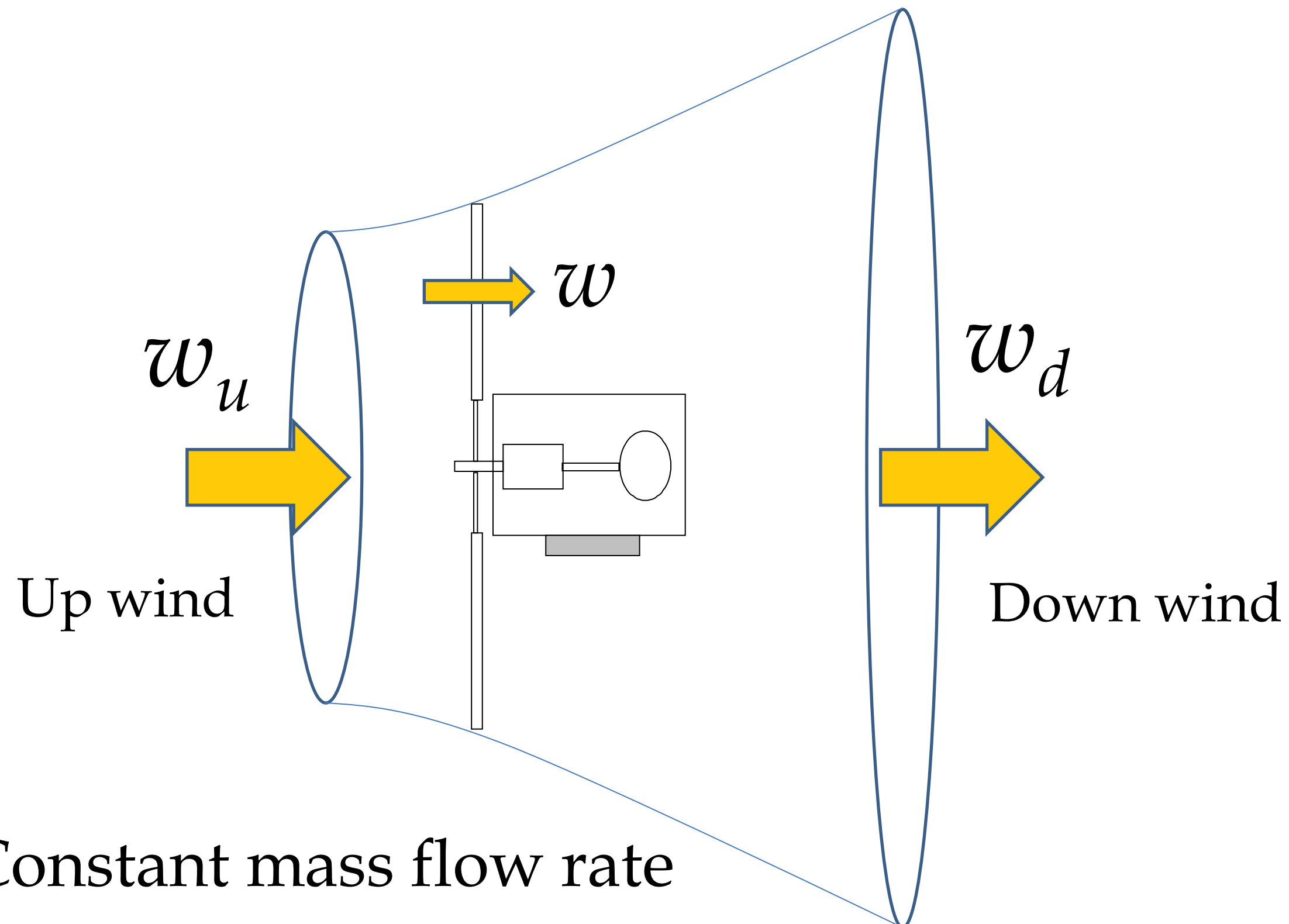
Ares of Mass of Air



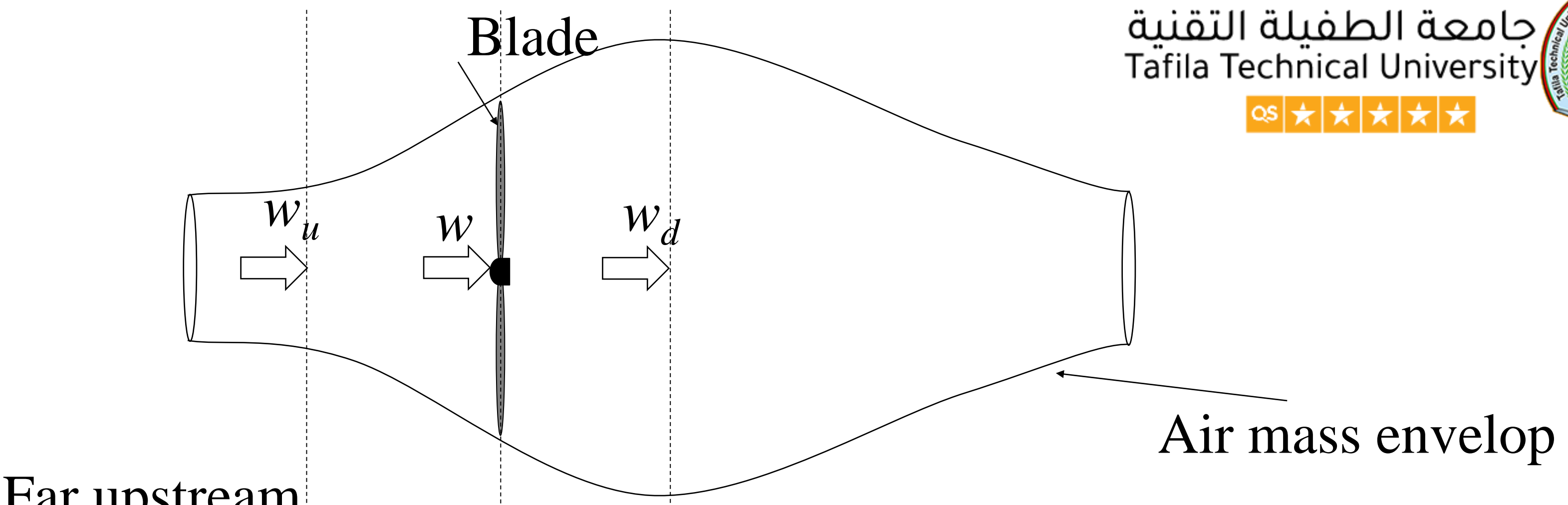
Constant mass flow rate

$$A_u w_u = A_b w = A_d w_d$$

Air Mass Boundary

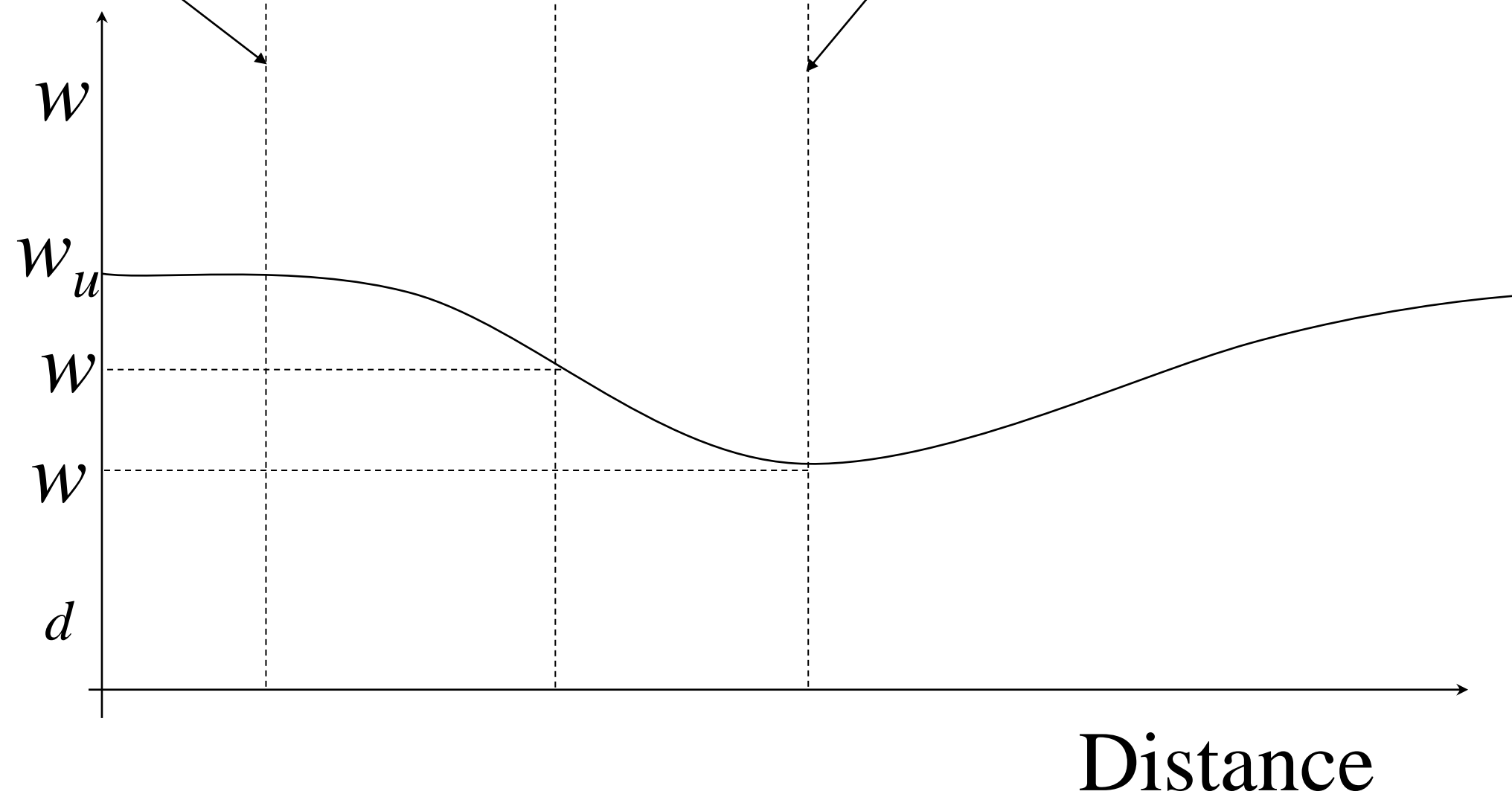


$$A_u w_u = A_b w = A_d w_d$$



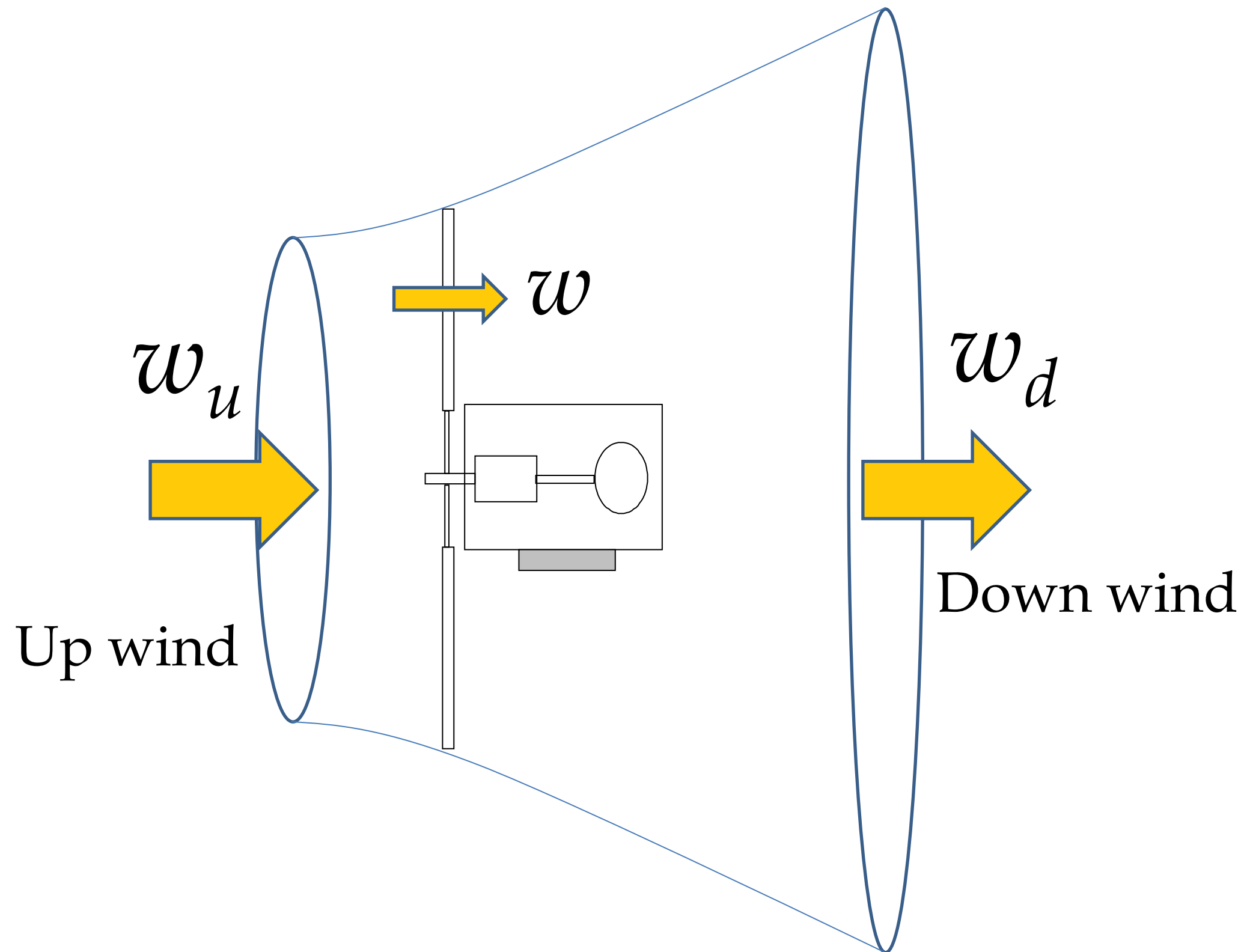
Far upstream

Far downstream



Wind Speed at Blade

$$w = \frac{w_u + w_d}{2}$$



Power Extracted by The Blades

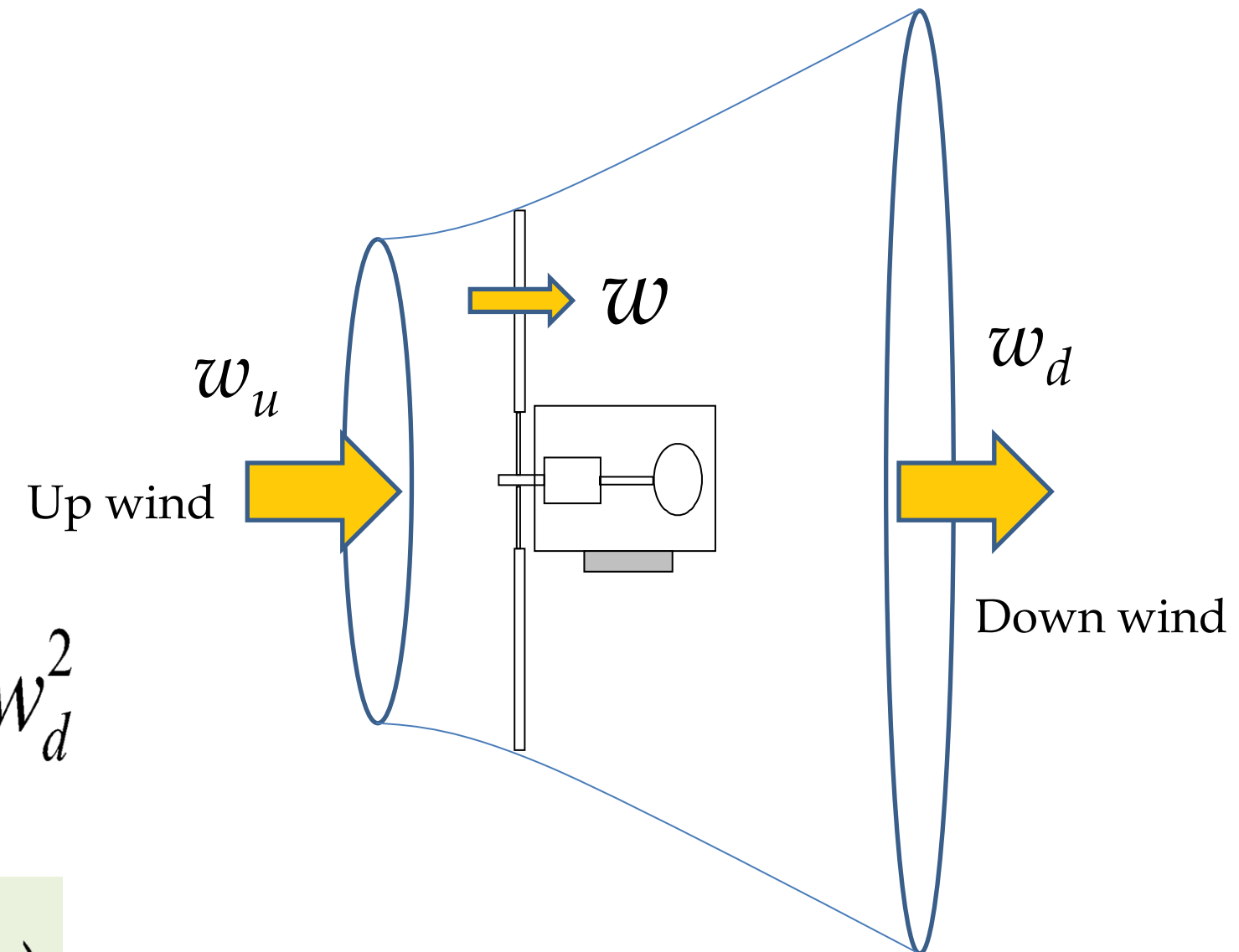


$$P_{blade} \approx P_{upwind} - P_{downwind}$$

$$P_{blade} = \frac{\Delta KE}{t} = \frac{1}{2} \frac{m}{t} w_u^2 - \frac{1}{2} \frac{m}{t} w_d^2$$

$$P_{blade} = \frac{1}{2} f (w_u^2 - w_d^2)$$

flow rate at blade $f = \frac{m}{t} = \delta A_b w = \delta A_b \left(\frac{w_u + w_d}{2} \right)$



Power Extracted by the Blades (Coefficient of Performance)



$$P_{blade} = \frac{1}{2} f (w_u^2 - w_d^2)$$

$$f = \delta A_b \left(\frac{w_u + w_d}{2} \right)$$

$$P_{blade} = \frac{1}{2} f (w_u^2 - w_d^2) = \frac{1}{2} \delta A_b \frac{w_u + w_d}{2} (w_u^2 - w_d^2)$$

Let $\gamma = \frac{w_d}{w_u}$

$$P_{blade} = \frac{1}{2} \delta A_b \left(\frac{w_u + \gamma w_u}{2} \right) (w_u^2 - \gamma^2 w_u^2)$$

$$P_{blade} = \underbrace{\frac{1}{2} \delta A_b w_u^3}_{P_w} \left[\underbrace{\frac{1}{2} (1 + \gamma)(1 - \gamma^2)}_{C_p} \right]$$

Power Extracted by the Blades



$$P_{blade} = \frac{1}{2} \delta A_b w_u^3 \left[\frac{1}{2} (1 + \gamma)(1 - \gamma^2) \right]$$

$$P_u = \frac{1}{2} \delta A_u w_u^3 \quad \text{Up wind power}$$

$$P_d = \frac{1}{2} \delta A_d w_d^3 \quad \text{Down wind power}$$

Define: $P_w = \frac{1}{2} \delta A_b w_u^3$ Wind Power

$$C_p = \frac{P_{blade}}{P_w} = \frac{P_{blade}}{P_u} \frac{A_u}{A_{blade}}$$

Example



- A WT has a mass flow rate of 20,000 kg/s. The upwind speed is 20m/s and the downwind speed is 18.7m/s.
- Compute the following:
 1. Diameter of the air mass boundary in the upwind and downwind regions
 2. Wind Power in the upwind and downwind areas
 3. Power captured by the blades
 4. Coefficient of performance computed using the upwind and downwind powers
 5. Coefficient of performance using wind speeds

Solution



Part 1

$$A_u = \frac{f}{w_u \delta} = \frac{20000}{20} = 1000 \text{ m}^2$$

$$d_u = 2\sqrt{\frac{1000}{\pi}} = 35.7 \text{ m}$$

$$A_d = \frac{f}{w_d \delta} = \frac{20000}{18.7} = 1070 \text{ m}^2$$

$$d_d = 2\sqrt{\frac{1070}{\pi}} = 37 \text{ m}$$

Solution



Part 2

$$P_u = \frac{1}{2} \rho A_u w_u^3 = \frac{1}{2} 1000 * 20^3 = 4 \text{ MW}$$

$$P_d = \frac{1}{2} \rho A_d w_d^3 = \frac{1}{2} 1070 * 18.7^3 = 3.5 \text{ MW}$$

Part 3

$$P_{blade} = P_u - P_d = 4 - 3.5 = 500 \text{ kW}$$

Solution



Part 4

$$A_b = \frac{f}{w\delta} = \frac{f}{\left(\frac{w_u + w_d}{2}\right)\delta} = \frac{20000}{19.35} = 1033.6 \text{ m}^2$$

$$C_p = \frac{P_{blade}}{P_u} \frac{A_u}{A_b} = \frac{0.5}{4} \frac{1000}{1033.6} = 0.121$$

Solution



Part 5

$$C_p = \frac{1}{2}(1 + \gamma)(1 - \gamma^2)$$

$$\gamma = \frac{w_d}{w_u} = \frac{18.7}{20} = 0.935$$

$$C_p = \frac{1}{2}(1 + 0.935)(1 - 0.935^2) = 0.121$$

Tip Speed Ratio (TSR)

$$\lambda = \frac{v_{tip}}{w_u}$$

$$v_{tip} = \omega r = (2\pi n)r$$

$$\lambda = \frac{\omega r}{w_u}$$

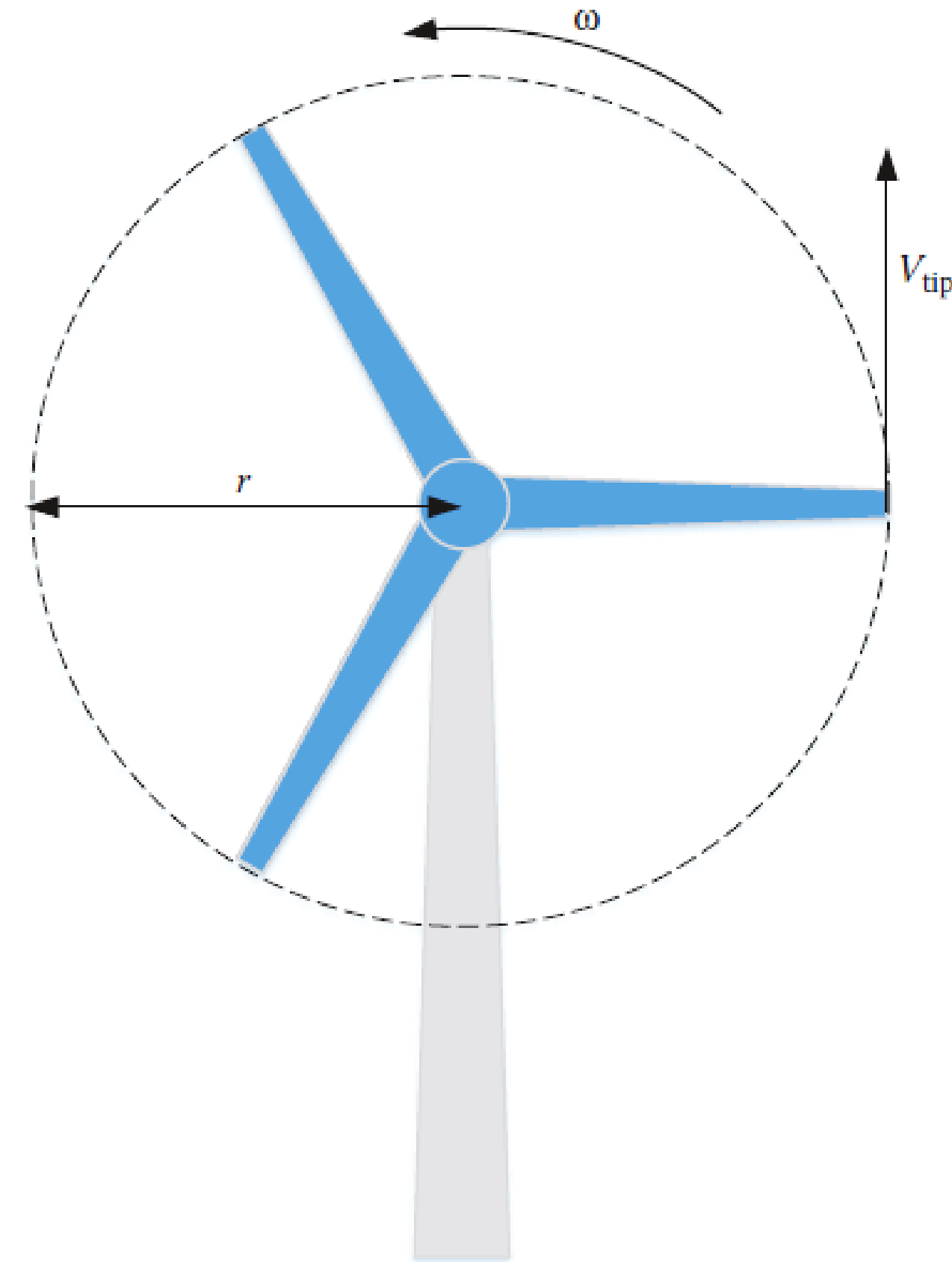
λ : TSR

w_u : up wind speed (undisturbed)

v_{tip} : tip velocity of blade

ω : angular speed of blade's shaft in rad/s

n : blade speed is rps



Coefficient of Performance

$$C_p = \frac{1}{2}(1 + \gamma)(1 - \gamma^2)$$

$$\gamma = \frac{W_d}{W_u}$$

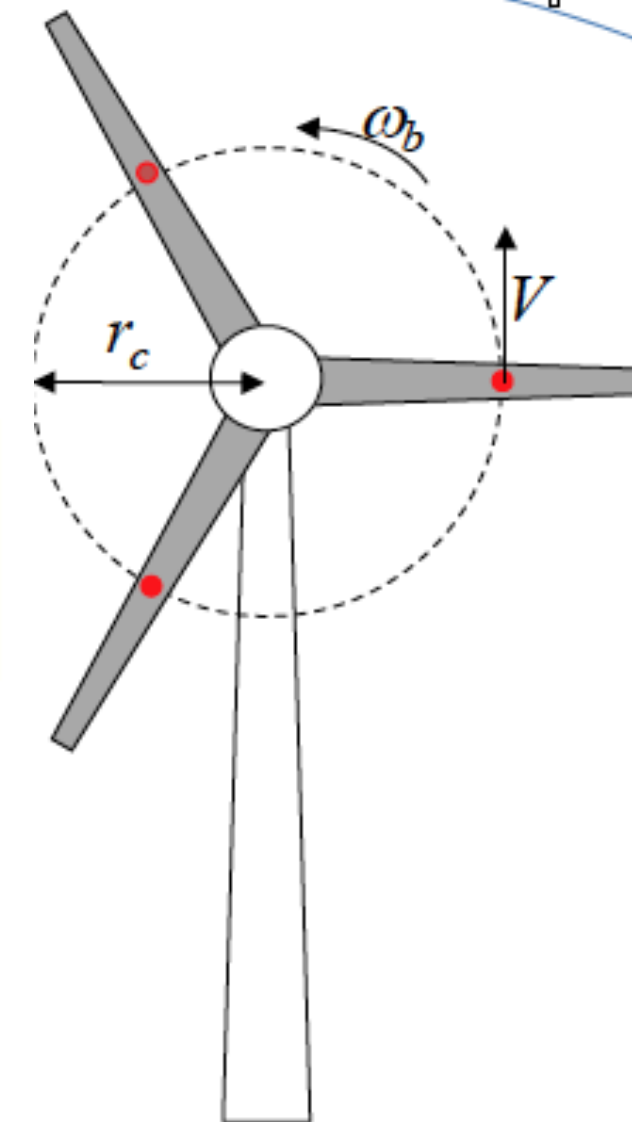
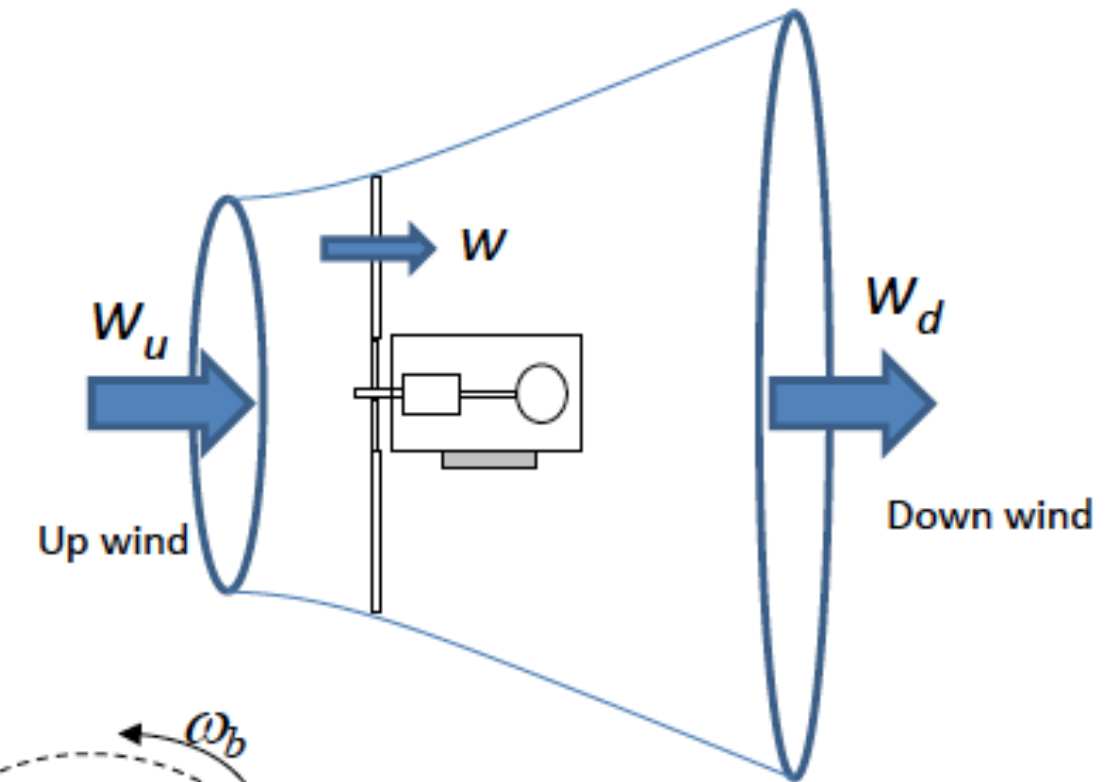
$$w \approx \frac{W_u + W_d}{2}$$

$$\gamma = 2 \frac{w}{W_u} - 1$$

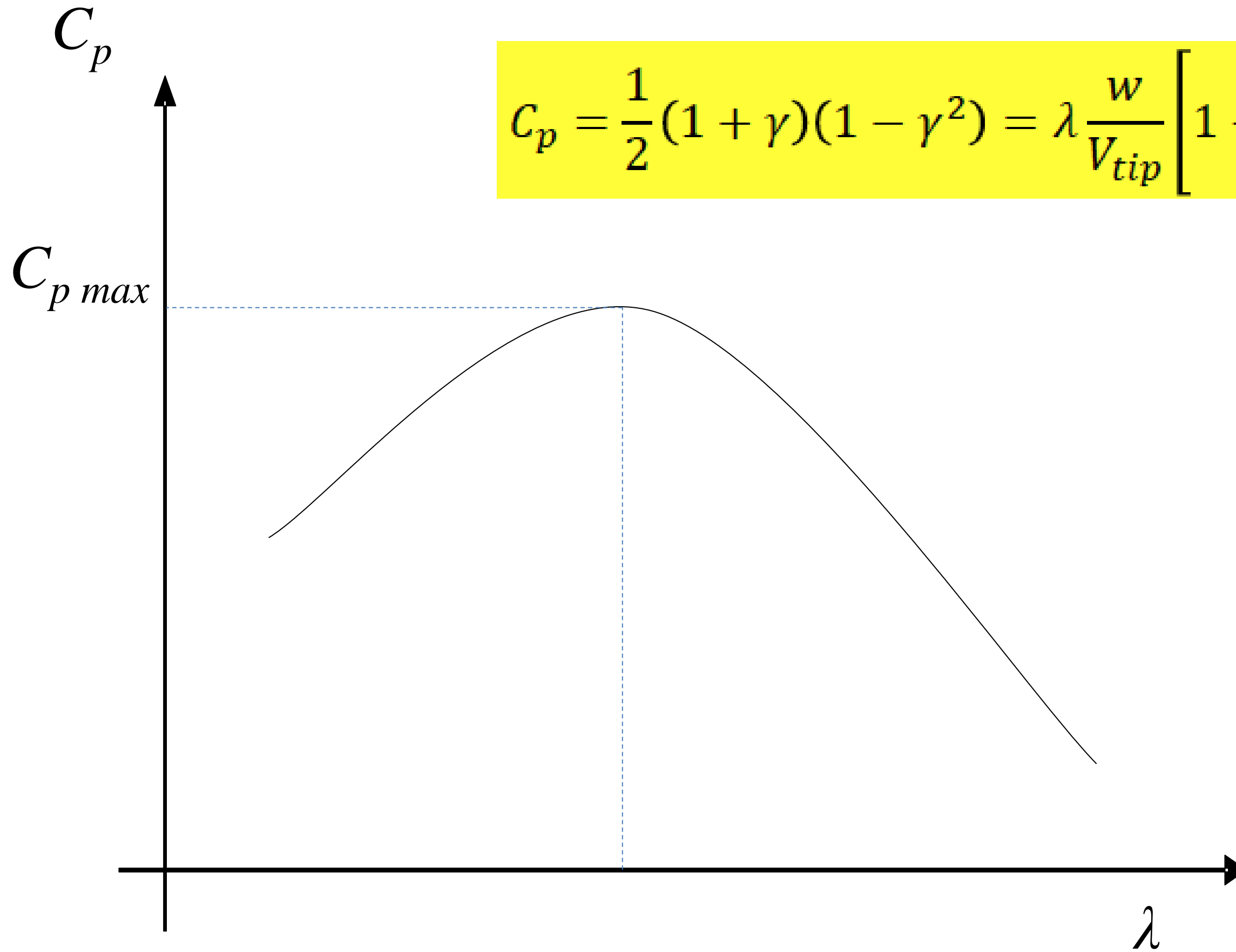
$$\lambda = \frac{v_{tip}}{W_u}$$

$$\gamma = 2\lambda \frac{w}{V_{tip}} - 1$$

$$C_p = \frac{1}{2}(1 + \gamma)(1 - \gamma^2) = \lambda \frac{w}{V_{tip}} \left[1 - \left(2\lambda \frac{w}{V_{tip}} - 1 \right)^2 \right]$$



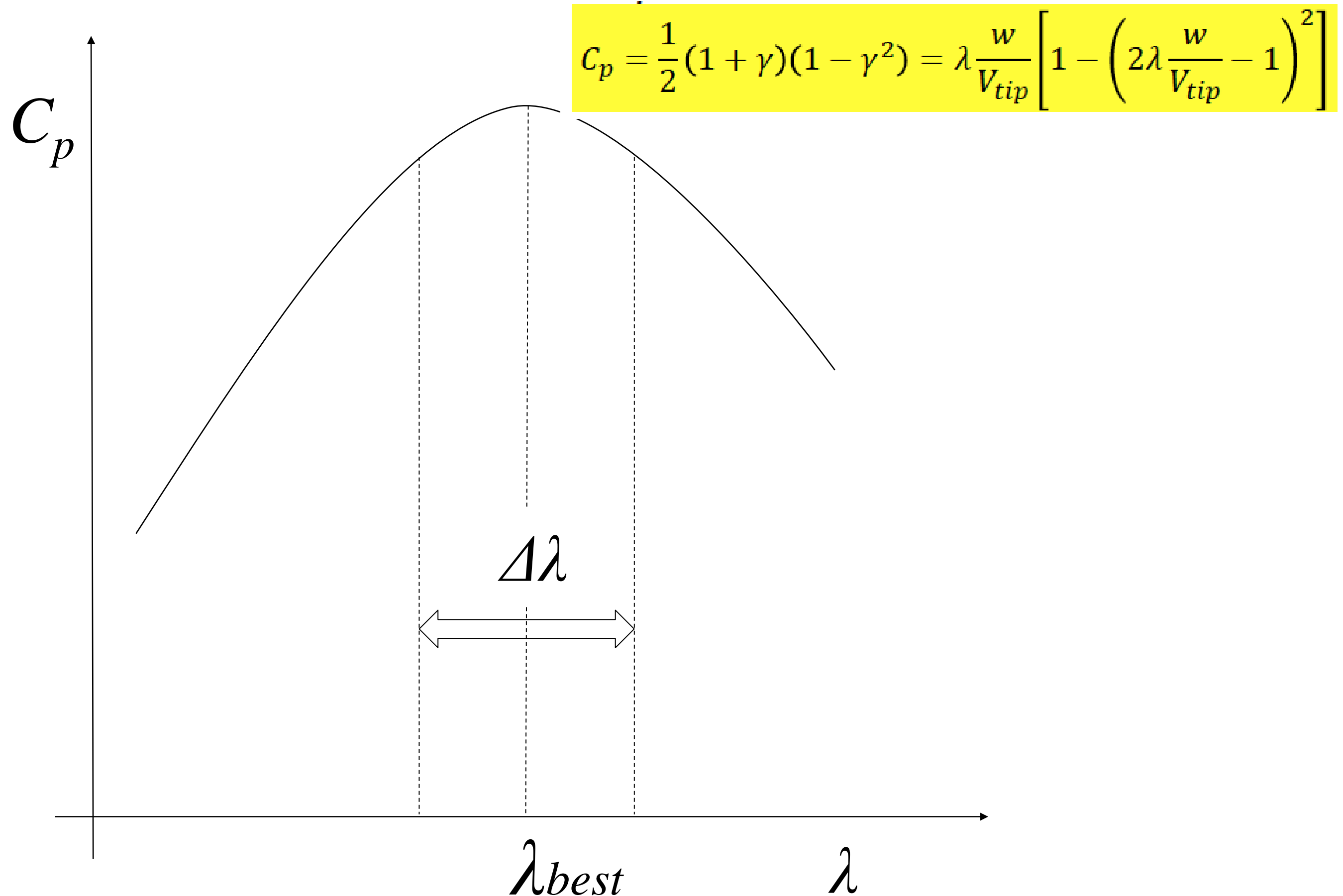
Coefficient of Performance



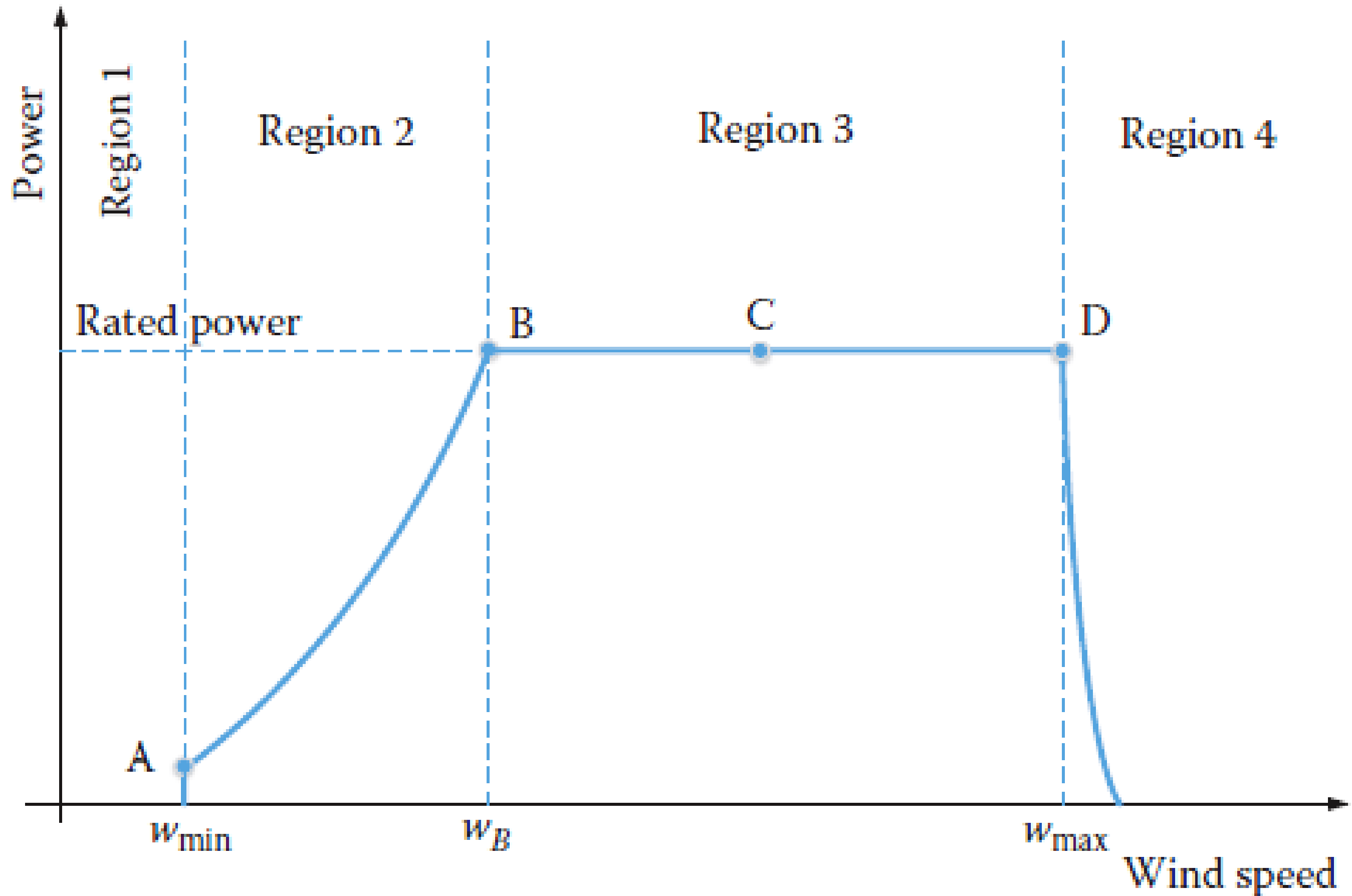
$$C_p = \frac{1}{2} (1 + \gamma)(1 - \gamma^2) = \lambda \frac{w}{V_{tip}} \left[1 - \left(2\lambda \frac{w}{V_{tip}} - 1 \right)^2 \right]$$

λ_{best}

Maximum C_p Range



Regions of Operation

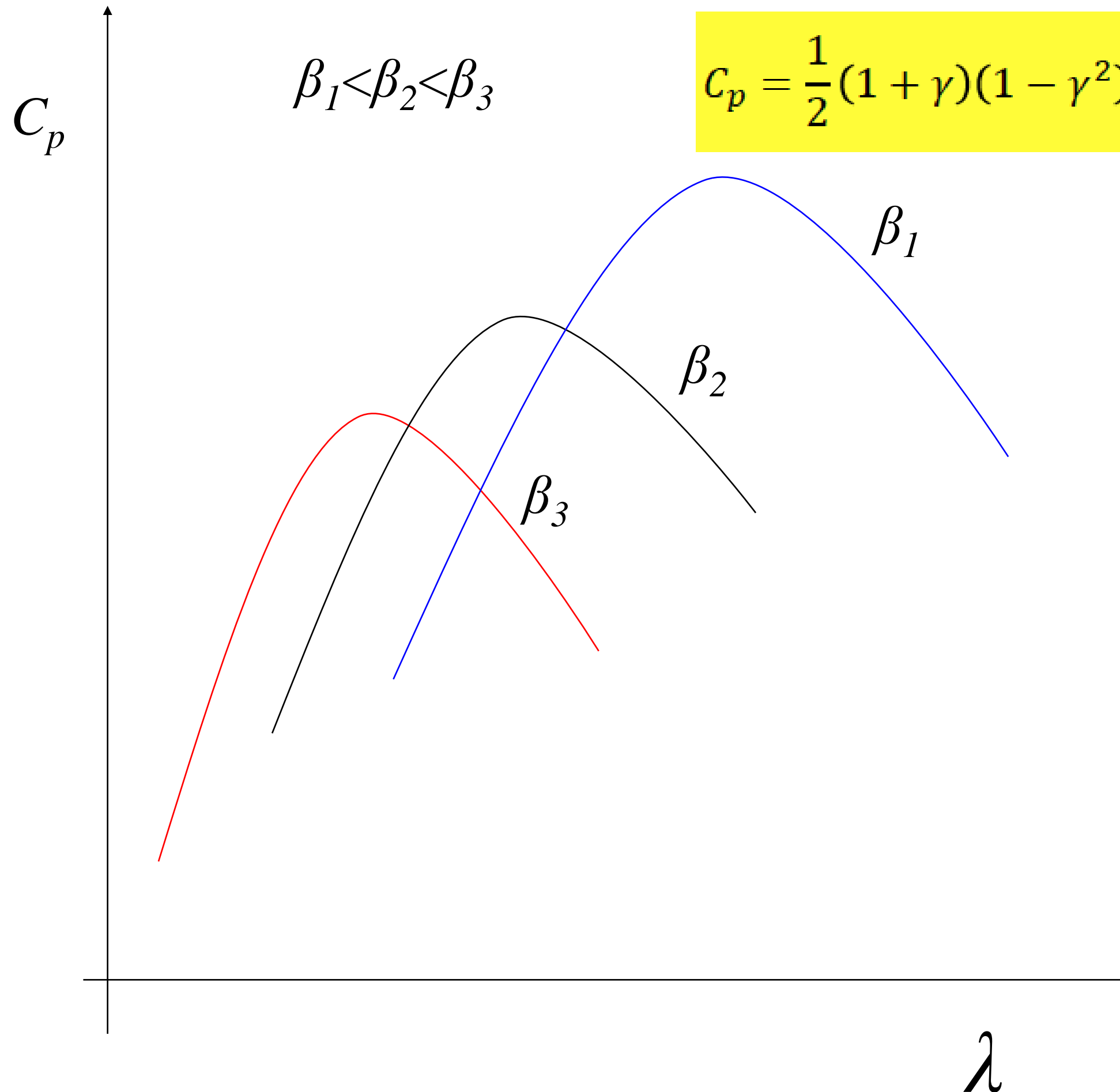


Variable Speed Turbine



- Newer generation of wind turbines allows the speed of the generator to vary.
 - To generate at lower than synchronous speeds
 - To track the maximum C_p operating points
- Variable speed can be achieved by
 - the electronics of the controller
 - the pitch angle
- For large inertia systems, the speed change by the electronic converter could be slow
 - May not be fast enough in highly variable wind applications
 - Pitch control is relatively faster

Effect of Pitch Angle



$$C_p = \frac{1}{2} (1 + \gamma)(1 - \gamma^2) = \lambda \frac{w}{V_{tip}} \left[1 - \left(2\lambda \frac{w}{V_{tip}} - 1 \right)^2 \right]$$

$$V_{tip} = f^n(\beta)$$

Empirical Formula for C_p

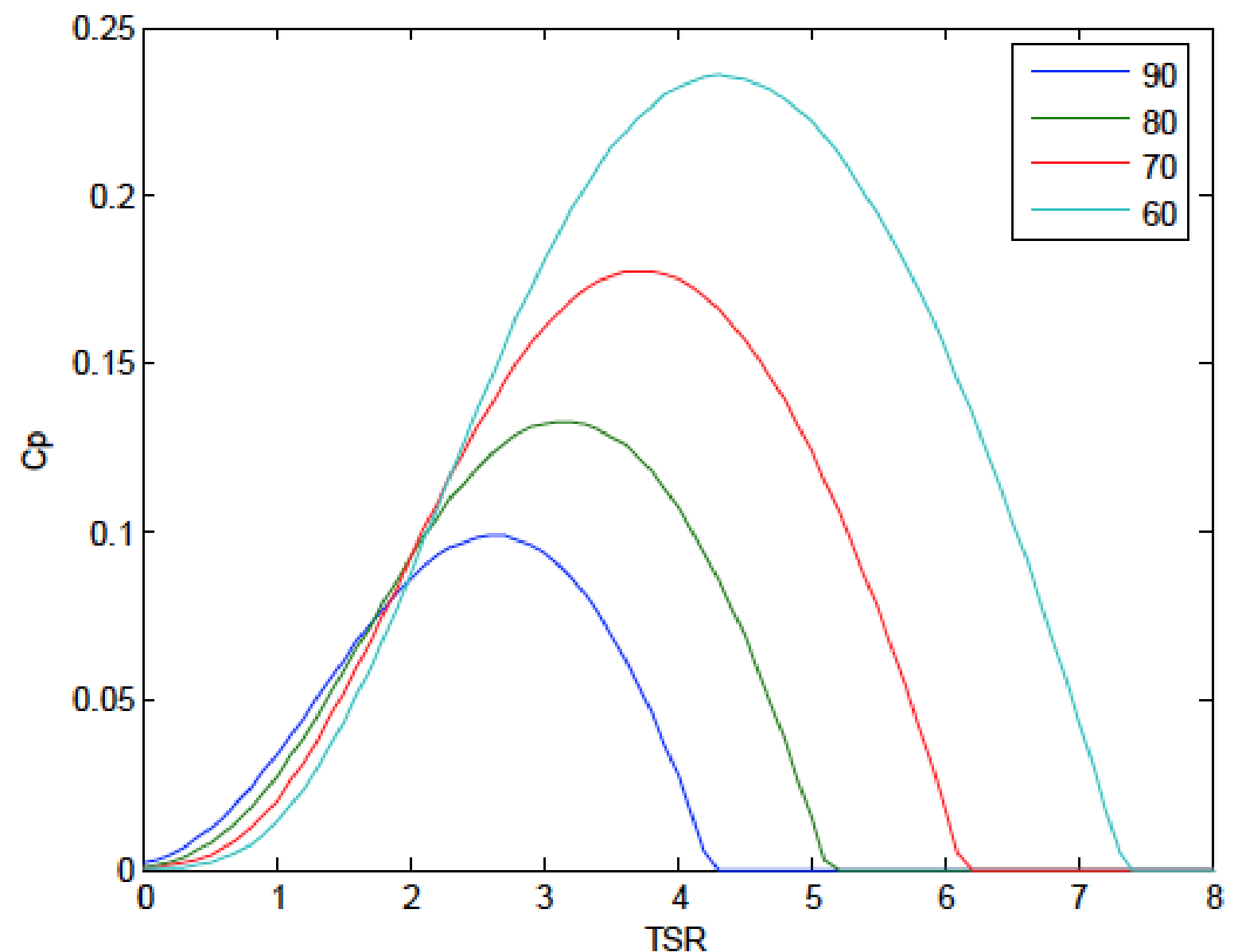


$$C_p = k_1(\Lambda - k_2\beta - k_3\beta^3 - k_4)e^{-\Lambda k_5}$$

$$\Lambda = \frac{1}{\lambda + k_6\beta} - \frac{k_7}{1 + \beta^3}$$

The figure is based on the following parameters

$$\begin{aligned}k_1 &= 50 \\k_2 &= 0.1 \\k_3 &= 0.002 \\k_4 &= 0.003 \\k_5 &= 15 \\k_6 &= 1 \\k_7 &= 0.02\end{aligned}$$



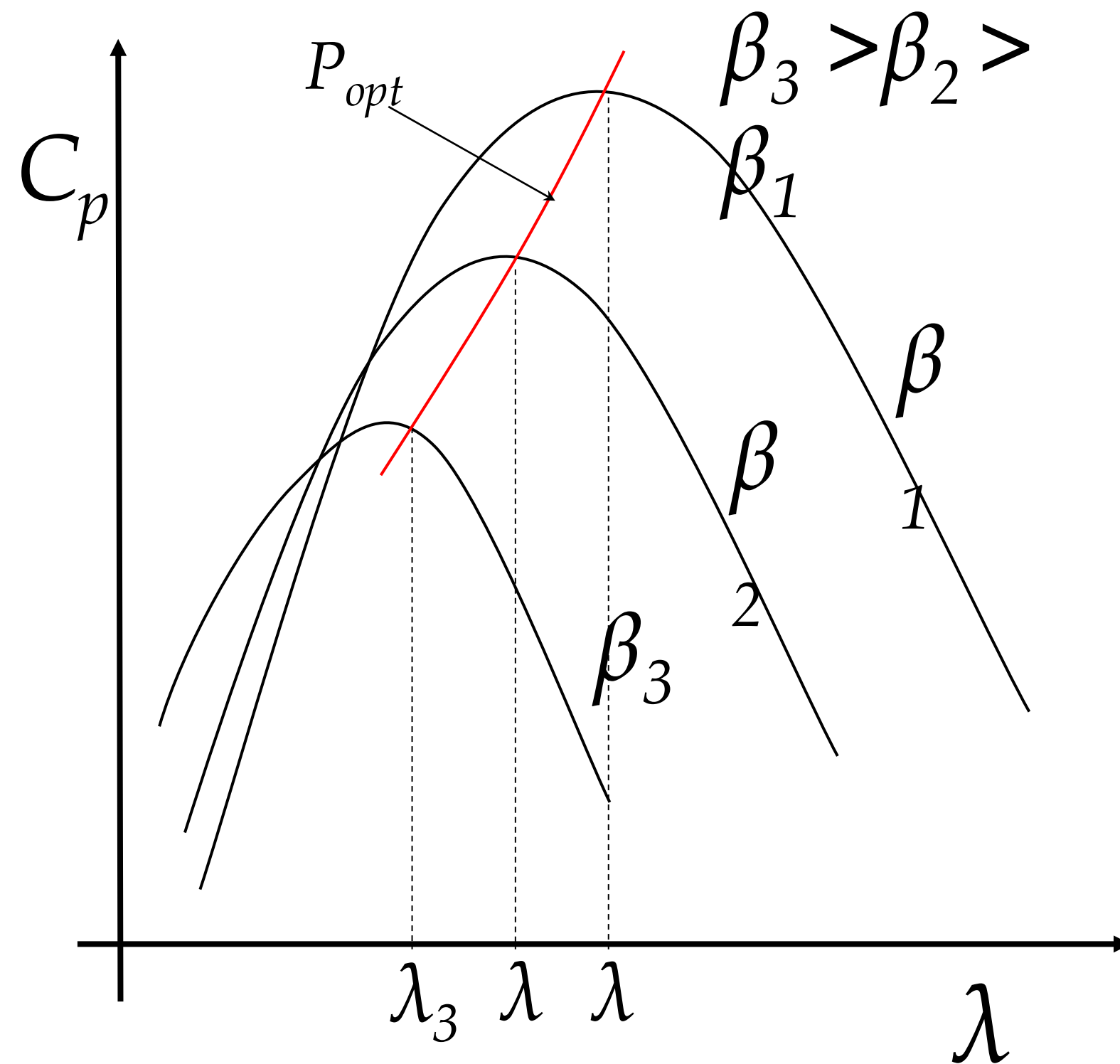


Maximum Power Tracking

$$P_{blade} = P_u C_p = \frac{1}{2} \rho A_b w_u^3 C_p$$

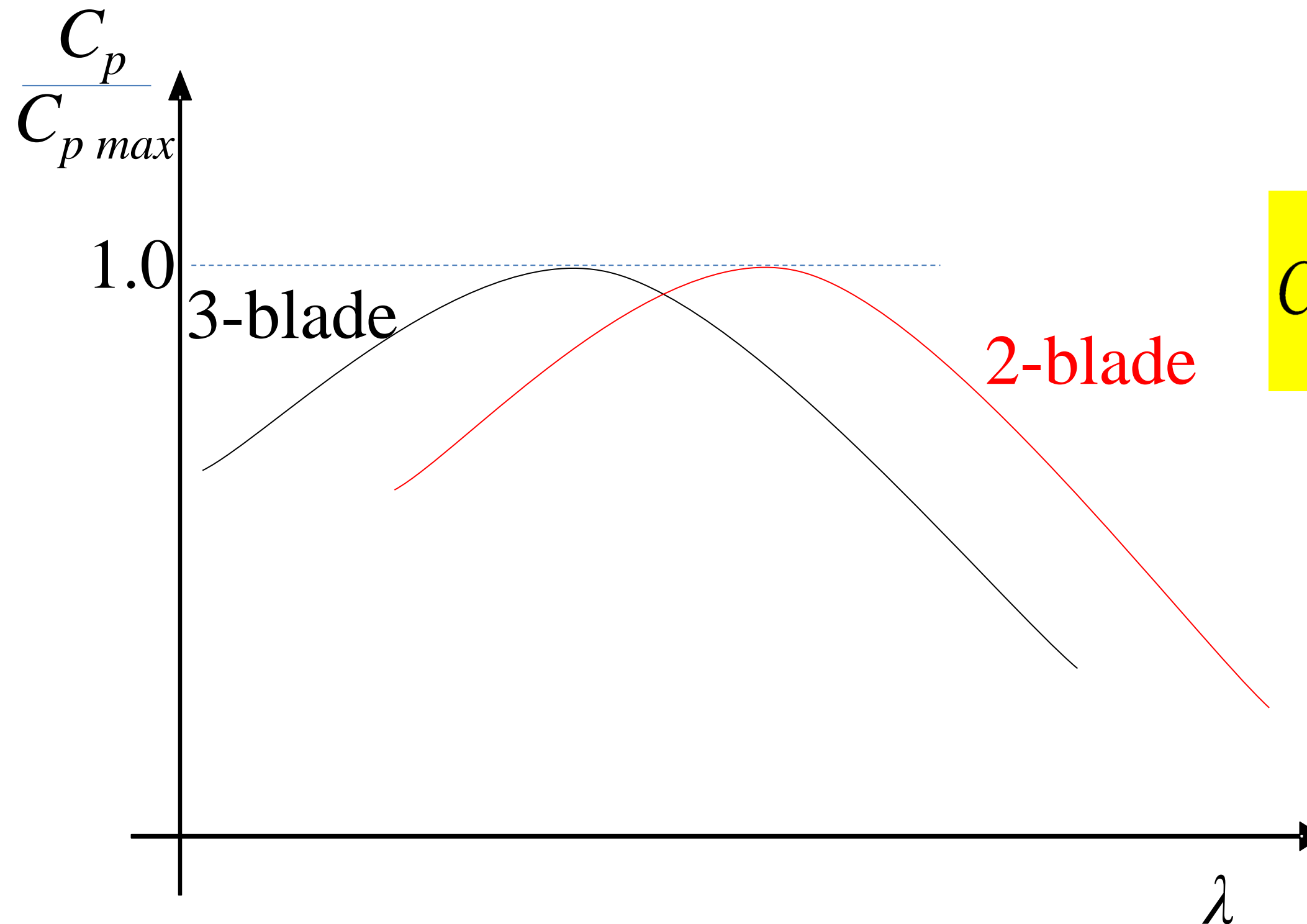
$$P_{blade} = K w_u^3$$

$$P_{blade-opt} = K_{opt} w_u^3$$



- P_{opt} is optimal extracted power
- K is aerodynamic constant

Effect of Number of Blades

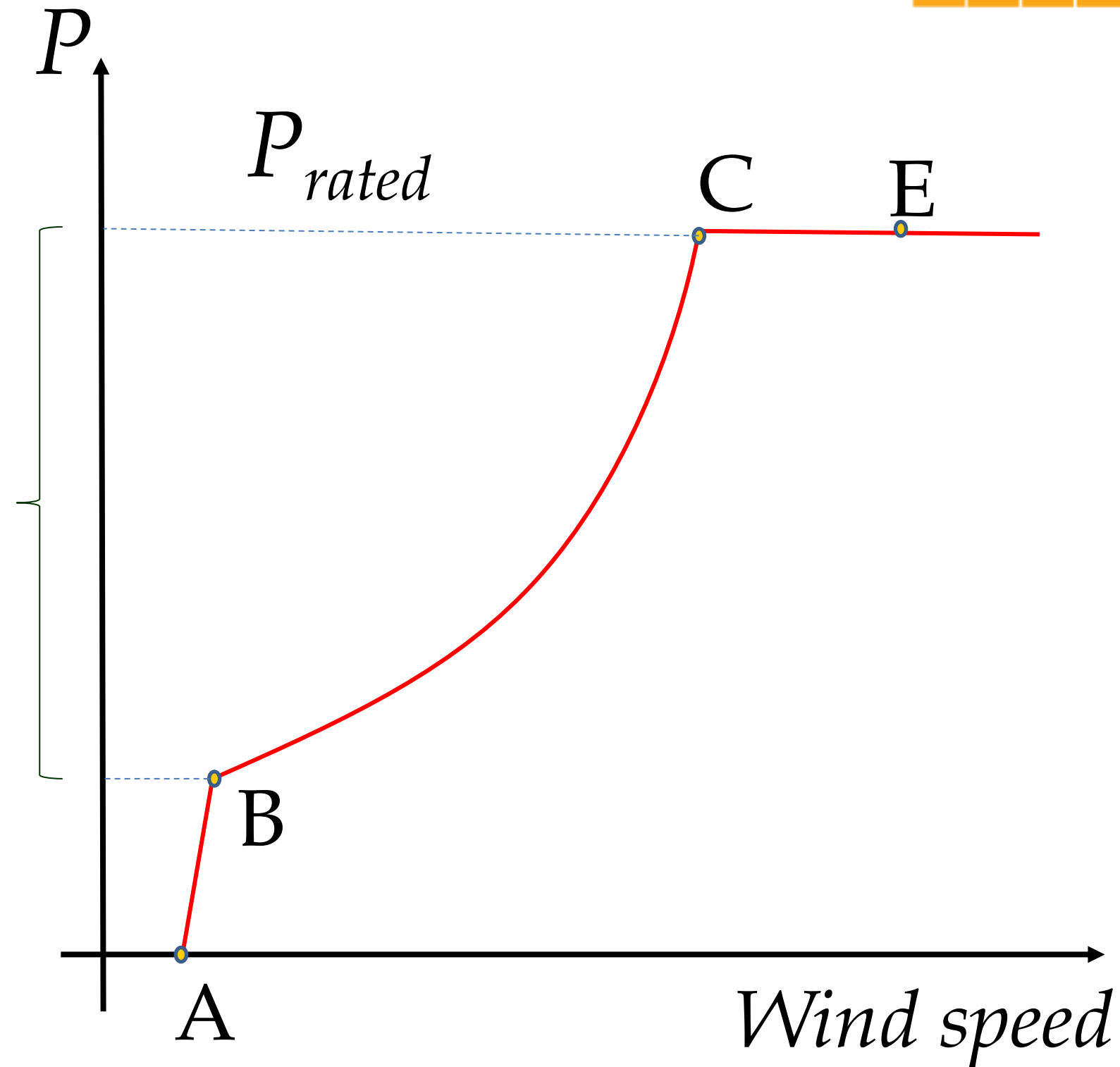


$$C_p = \frac{1}{2}(1 + \gamma)(1 - \gamma^2)$$

2-blade turbine rotates at higher speed to achieve the same C_p as the 3-blade turbine

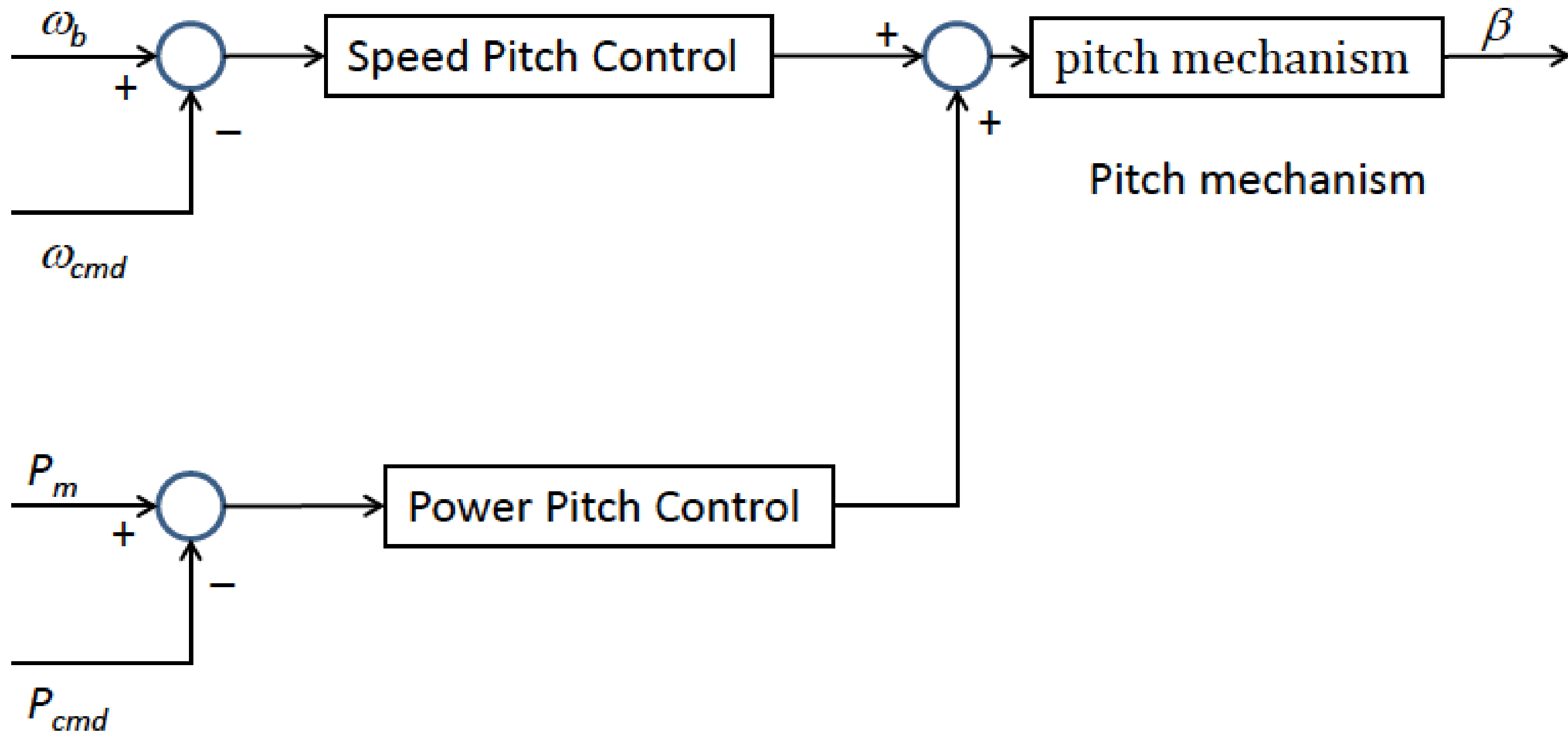
Maximum Power Tracking

$$P_{opt} = K_{opt} w_u^3$$

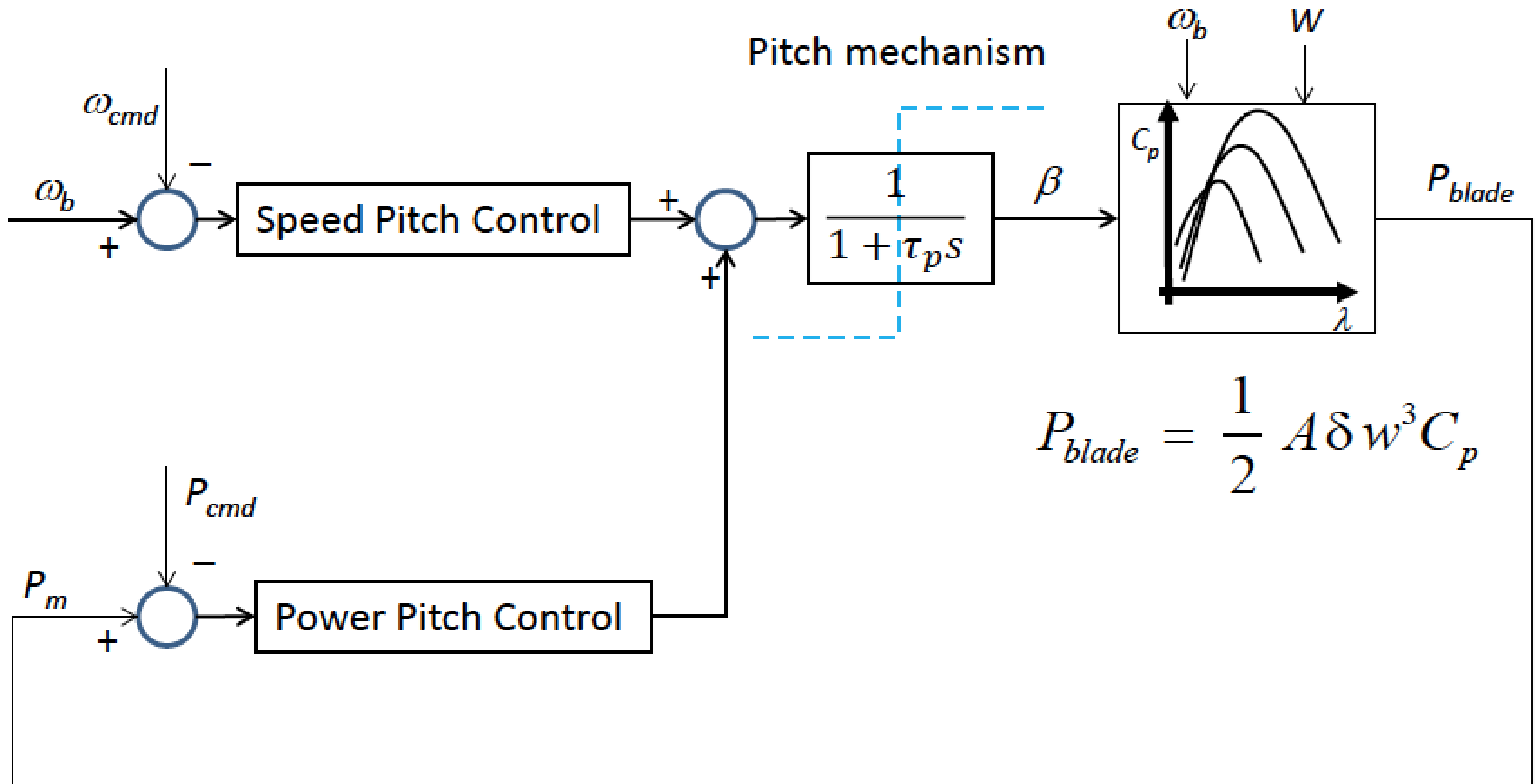


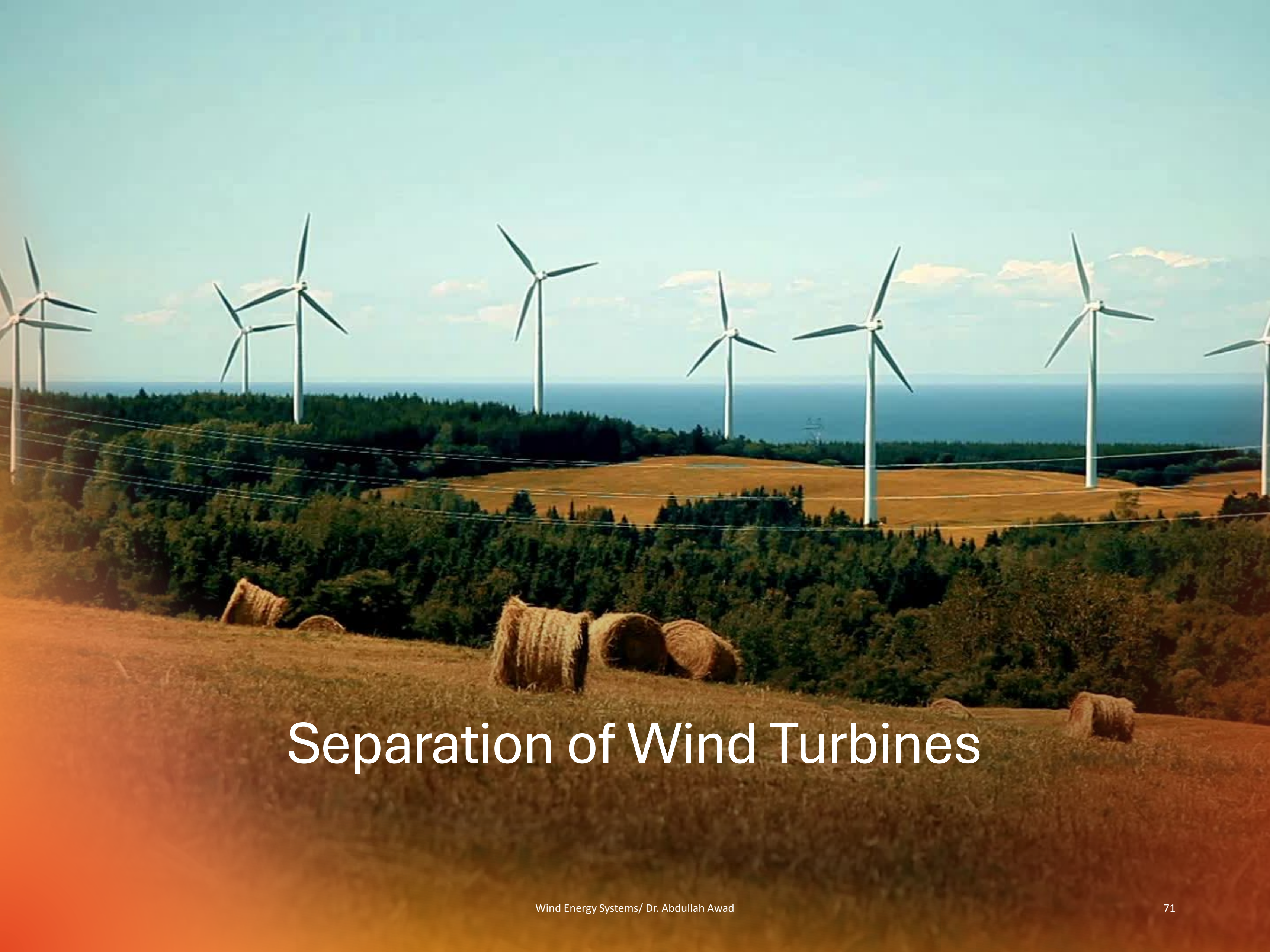
- At low speed, no control (A-B)
- Optimal power extraction (B-C). This is the P_{opt} region
- Turbine limit (C-E)

Pitch Controller Mechanism



Pitch Controller Model





Separation of Wind Turbines

Wind Clusters (Farms)

- Clusters of wind turbines makes engineering and economical sense
- Advantages of wind farms:
 - Reduced installation costs.
 - Reduced operation costs.
 - Reduced maintenance costs.
 - Simplified grid connection.



Main Wind Systems

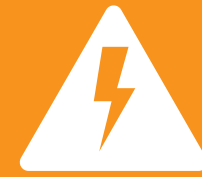


On-Shore



Off-Shore

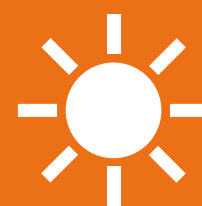
Separation in Wind Farms



Wind slows down as it passes through the blades



Available power to downwind machines is reduced

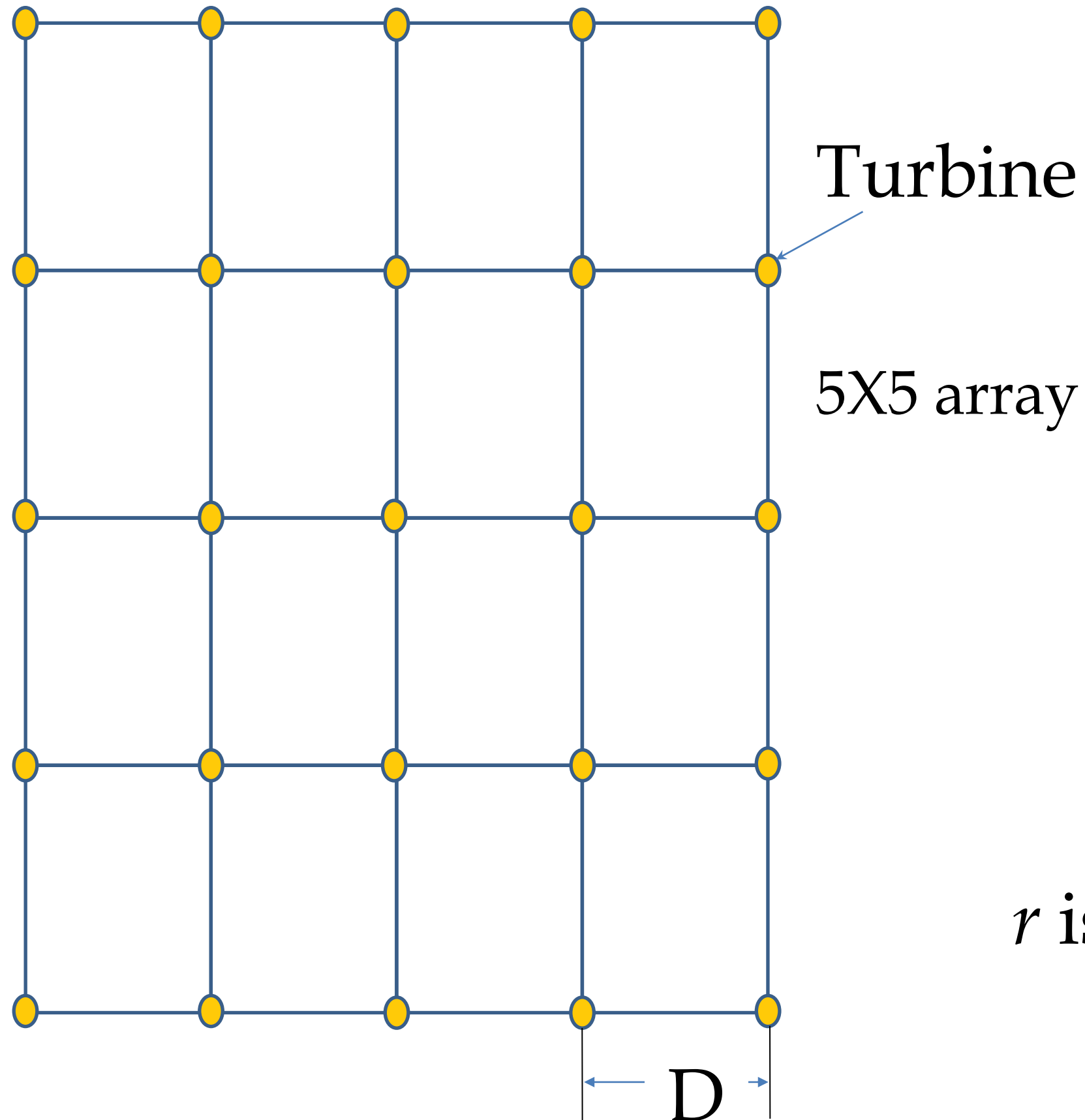


Turbine cannot efficiently capture energy from turbulent wind



Wind turbines must be separated to allow the wind to pick up speed before it reaches the next turbine

Square Array

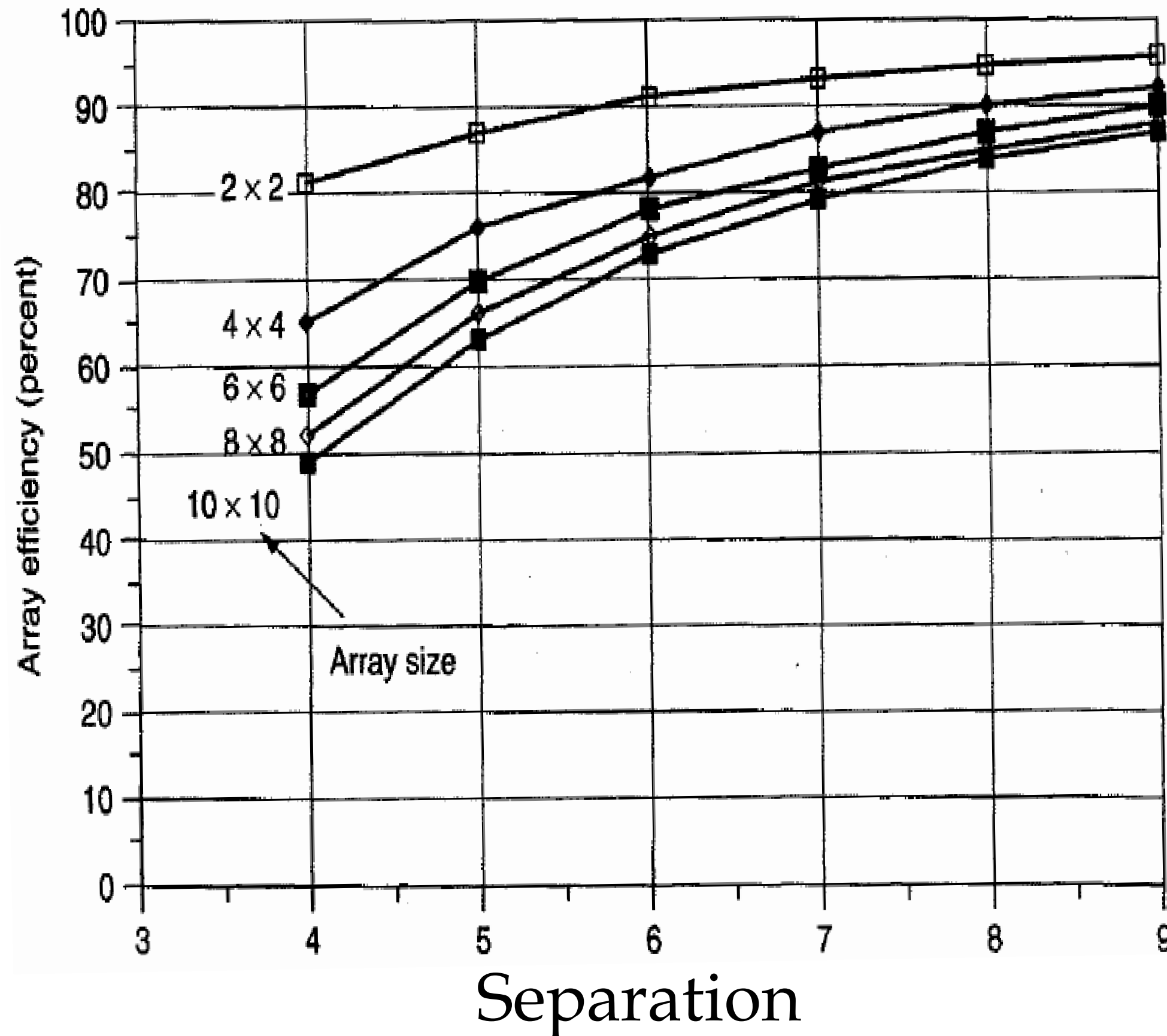


Separation S

$$S = \frac{D}{2r}$$

r is the length of the blade

Array Efficiency



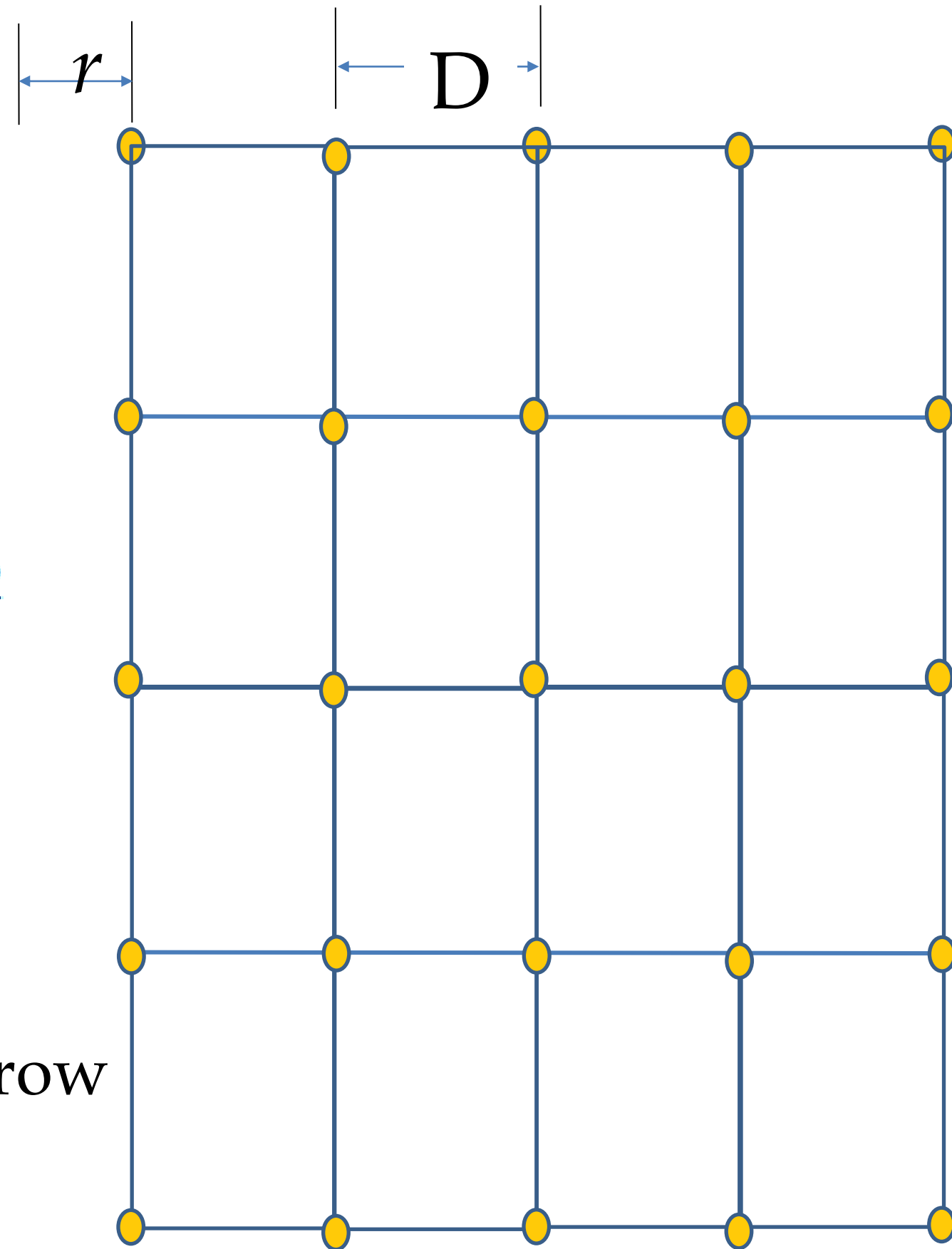
- Array Efficiency determines how much of the wind energy will be available to the turbines in the array
- The smaller the separation, the less efficient is the array

Utilized Land for Wind Farm

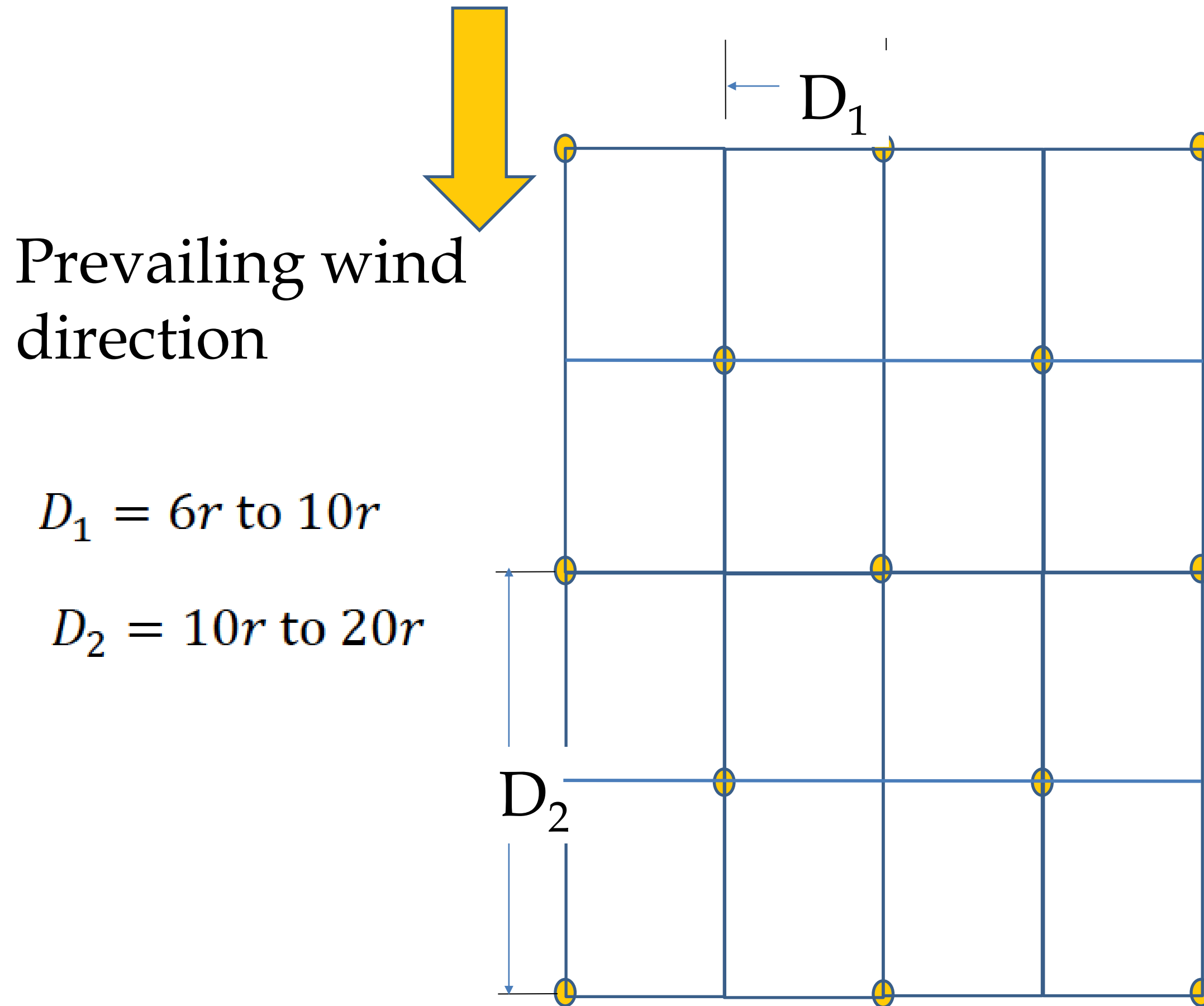
Farm area

$$A_{land} = [(X - 1)D + 2r]^2$$

X is the number of turbine in one row



Industry Practice





Example

- A 10X10 wind farm arranged in square array with a separation factor of 7.
- Assume the wind turbine efficiency is 85% , $C_p=0.4$, and the wind power density at the hub height is 1 kW/m²
- Compute the power production per unit area of land



Solution

$$D = 2r s = 14r$$

$$A_{land} = (9D + 2r)(9D + 2r) = 16,384 r^2$$

$$\text{Number of turbines } n = 10 \times 10 = 100$$

Using the graph, for $S=7$, the array efficiency is 80%

$$\text{Power production of farm } P = n * \rho A \eta C_p$$

$$P = 100 * 1000 (\pi r^2) * 0.85 * 0.8 * 0.4 = 85.45 r^2 \text{ kW}$$

$$\text{Power production per unit area of land} = \frac{85450 r^2}{16384 r^2} = 5.2 \text{ W/m}^2$$

Website

Use the	To check for announcements
course	
website:	To get copies of the lecture slides and other material
	To get the homework and project assignments

<https://lms.ttu.edu.jo/>

