



جامعة الطفيلة التقنية  
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QS



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# Wind Energy Systems

## Chapter 5: Type 3 Wind Turbine System Doubly Fed Induction Generator (DFIG)

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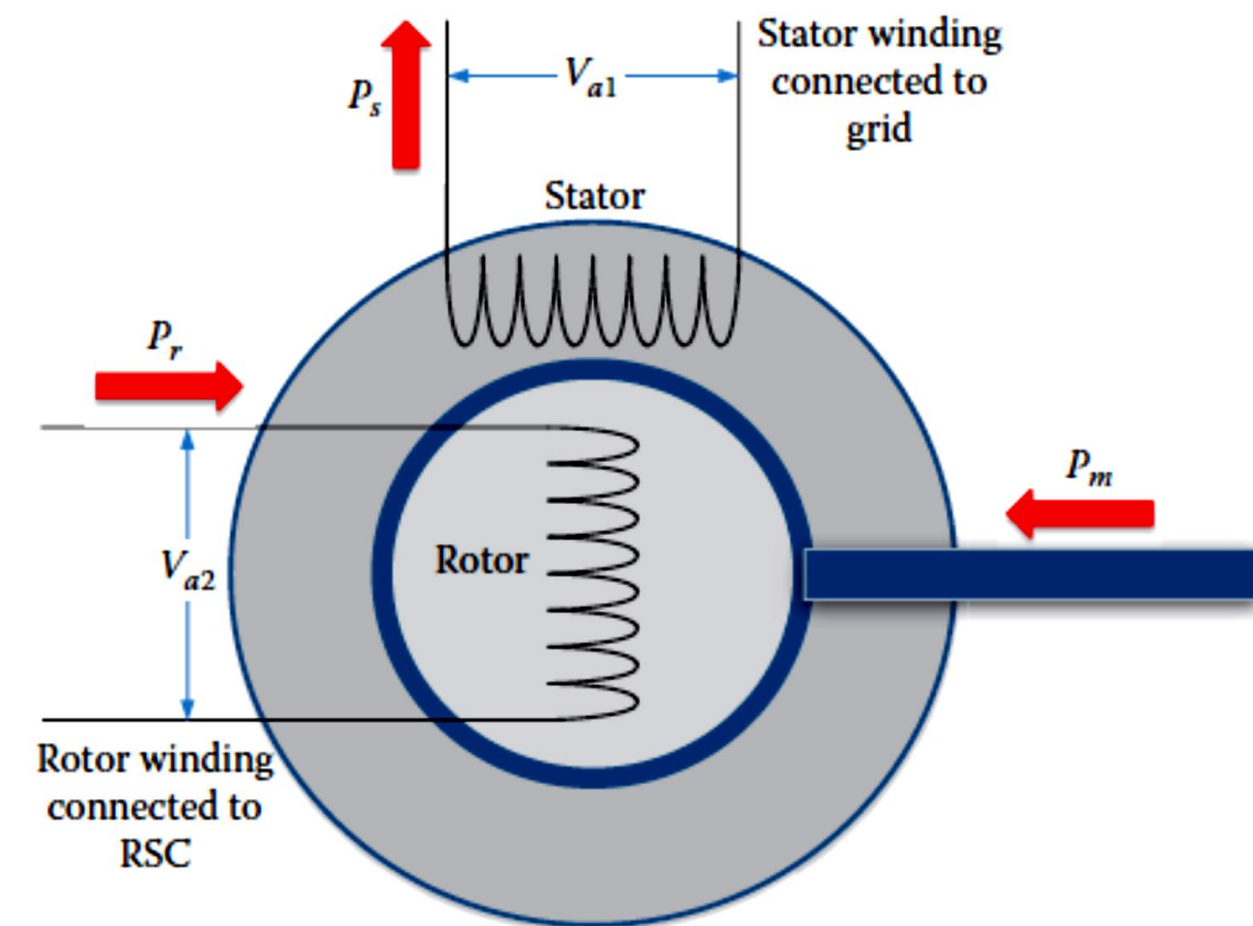
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# Doubly Fed Induction Generator (DFIG)

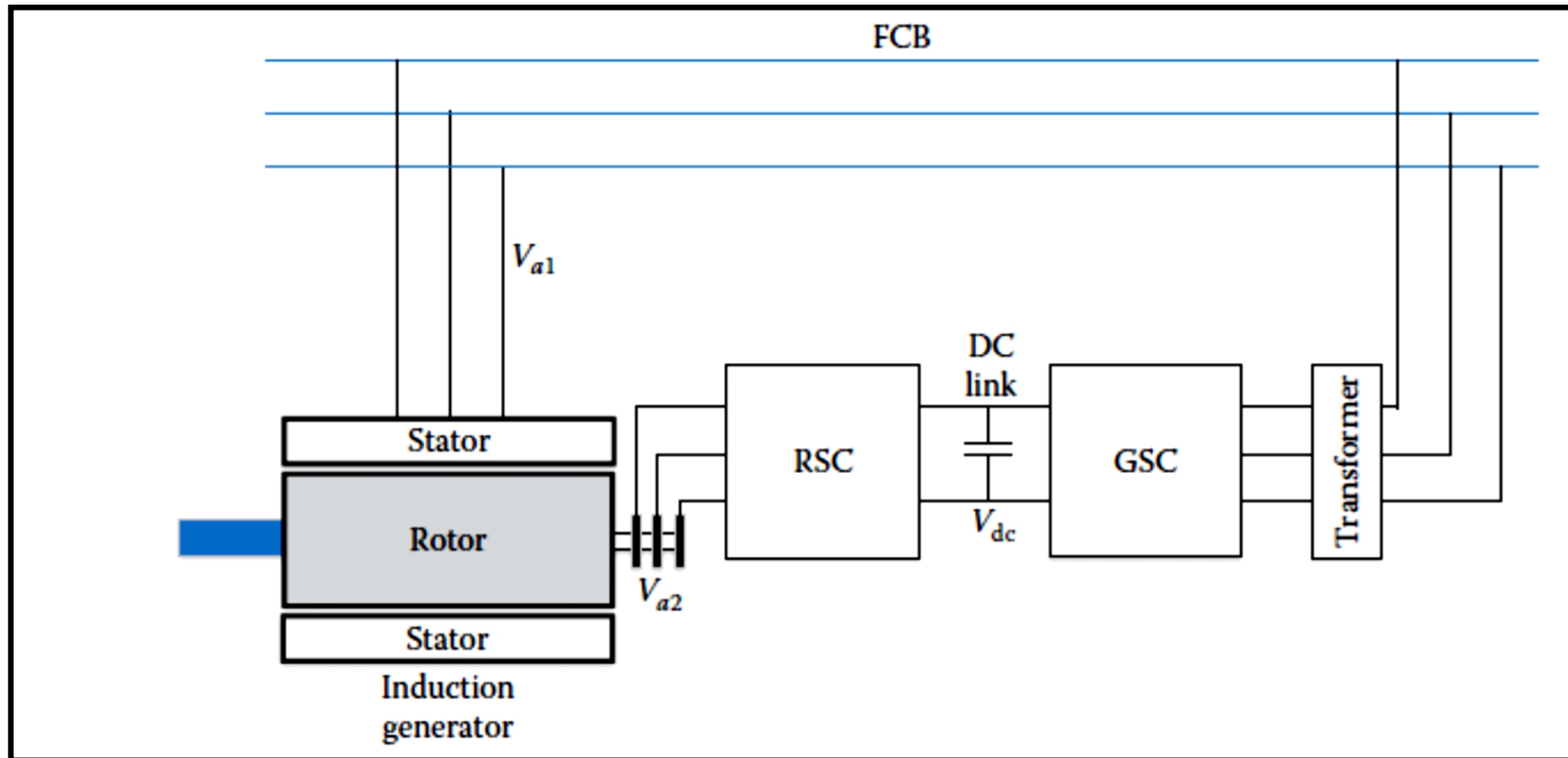
Type 3 system, known as the doubly fed induction generator (DFIG), is the most common type of wind turbines. Although more expensive than types 1 and 2, it offers excellent operational and control features that make their integration with power grids easy and effective. Type 3 generator is a slip-ring machine similar to that of type 2. The generator is fed from the stator as well as the rotor (thus, doubly fed).

The mechanical power from the turbine enters the rotor and is converted into electrical (developed power). The rotor injection circuit also inserts power into the rotor. The stator power is the developed power plus the rotor power minus the losses. When the rotor power is negative, the flow is reversed: power is extracted from the rotor and delivered to the grid.

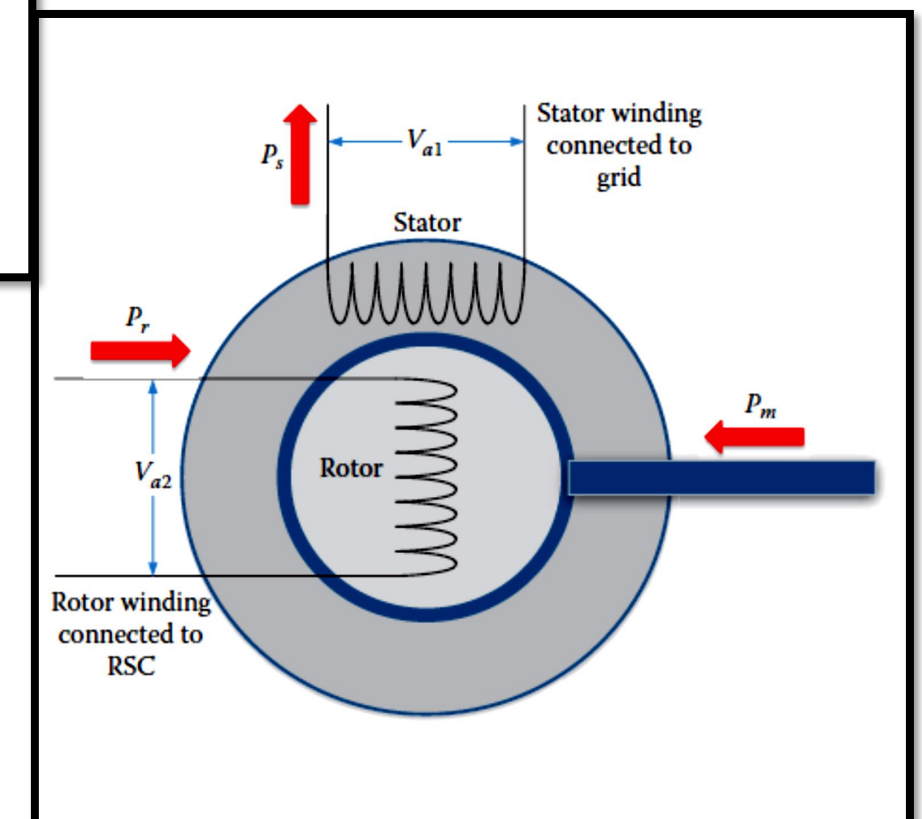


Concept of DFIG

# Components of DFIG



Main components of DFIG



# DFIG converters

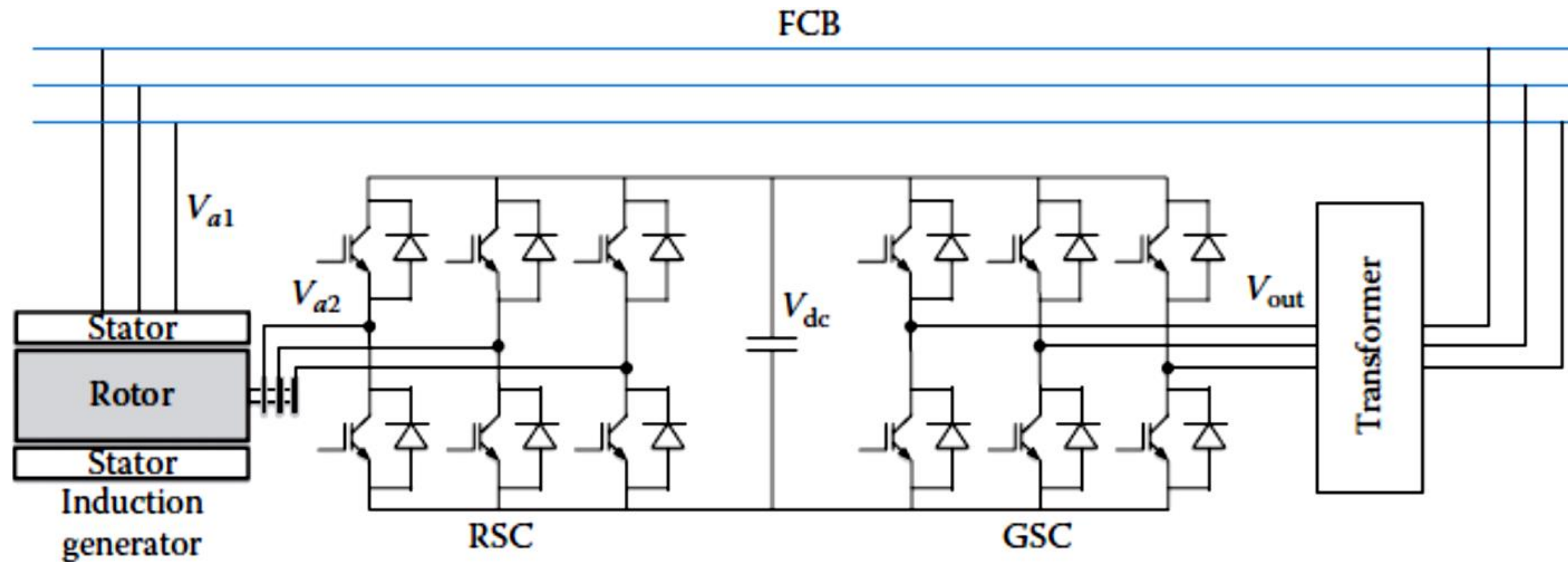


FIGURE 10.3  
DFIG converters.

- Developed (electric) torque
- Speed of rotation
- Reactive power flow and power factor at the terminals of the stator.
- Speed range of generation
- Direction of power flow in the converters

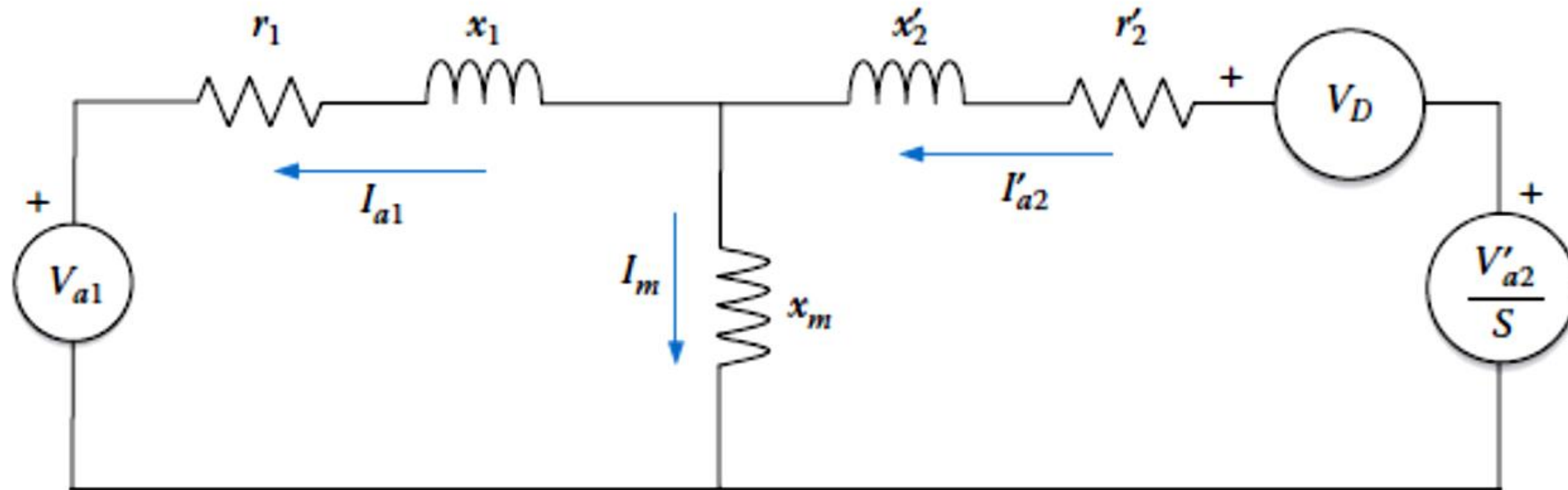
- Keeps the voltage of the dc-link at a specified value
- Allows bi-directional real power flow in the converters
- Any flow toward the grid is kept at the grid frequency.
- Produces or absorbs reactive power.



# Equivalent Circuit

# Equivalent Circuit for DFIG

$$\bar{V}_D = -\frac{r_2'}{s}(1-s)\bar{I}'_{a2}$$



**FIGURE 10.4**  
Model of the DFIG.

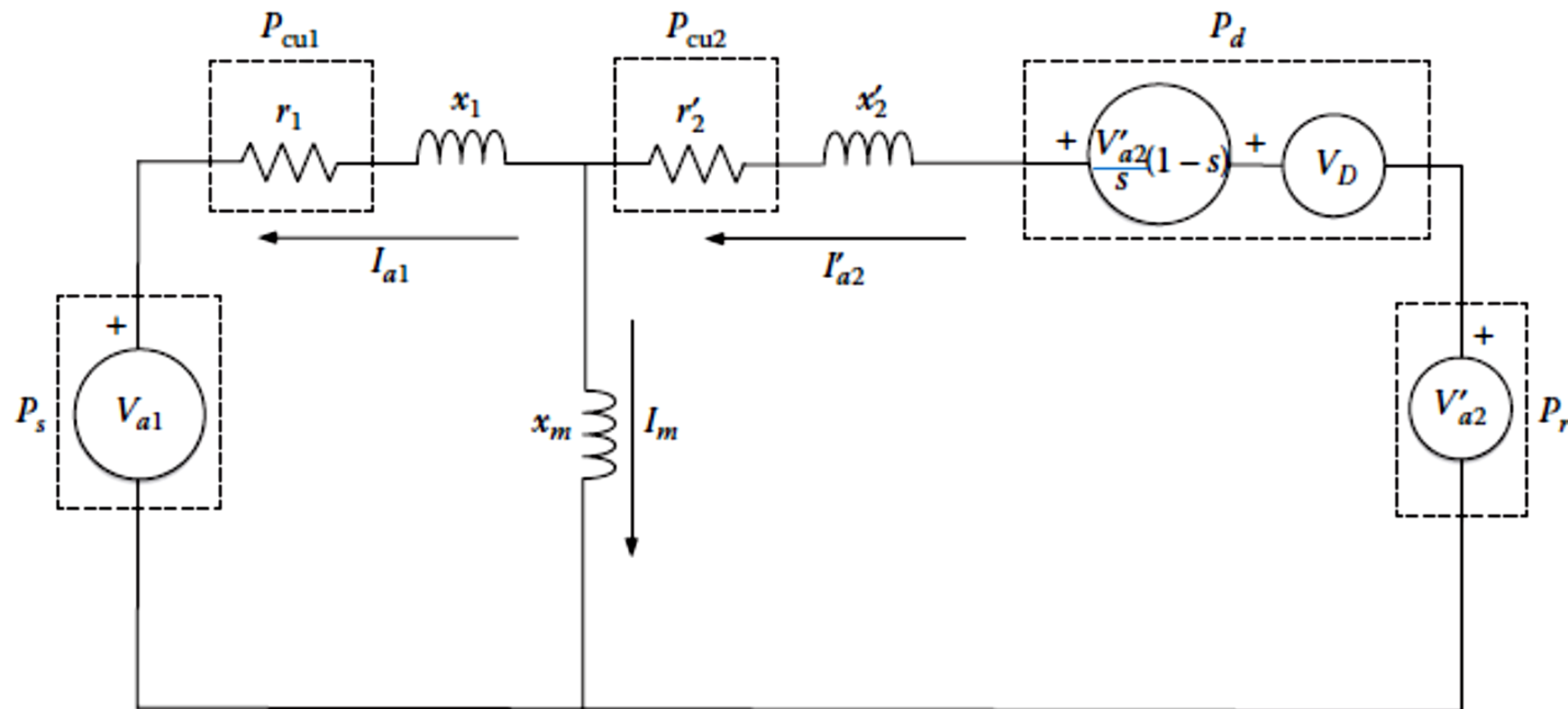
$$\frac{V'_{a2}}{s} = V'_{a2} + \frac{V'_{a2}}{s}(1-s)$$



# Equivalent Circuit for DFIG



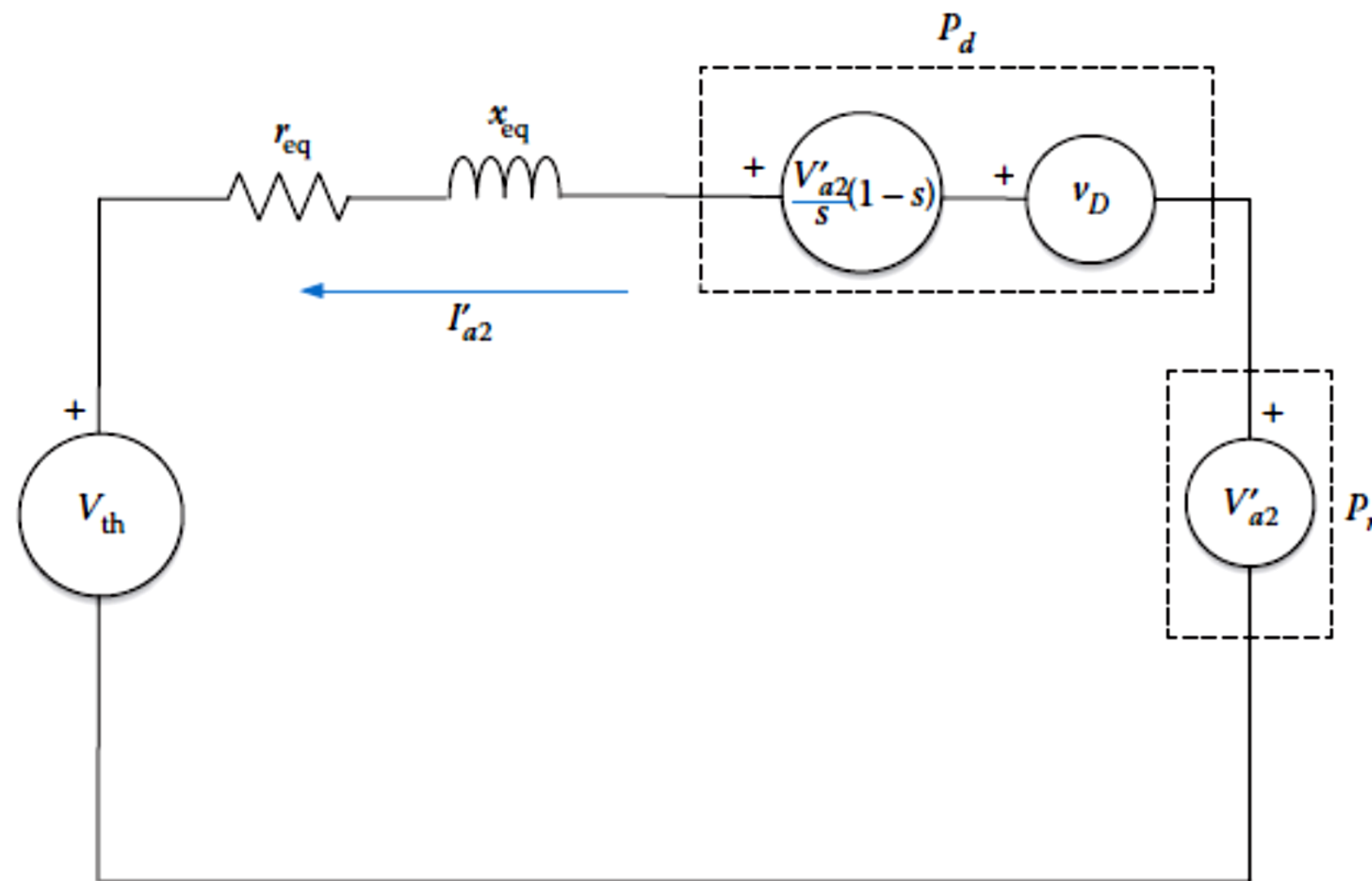
$$\bar{V}_D = -\frac{r'_2}{s} (1-s) \bar{I}'_{a2}$$



**FIGURE 10.5**  
Power components in the DFIG.

$$\frac{V'_{a2}}{s} = V'_{a2} + \frac{V'_{a2}}{s} (1-s)$$

# Simplified Model



$$\bar{I}'_{a2} = \frac{\frac{\bar{V}'_{a2}}{s} - \bar{V}_{a1}}{r_d + r'_2 + jx'_2}$$

$$\bar{I}_{a1} = \bar{I}'_{a2} - \frac{\bar{V}_{a1}}{jx_m}$$

**FIGURE 10.6**  
Thevenin's equivalent circuit for the DFIG.





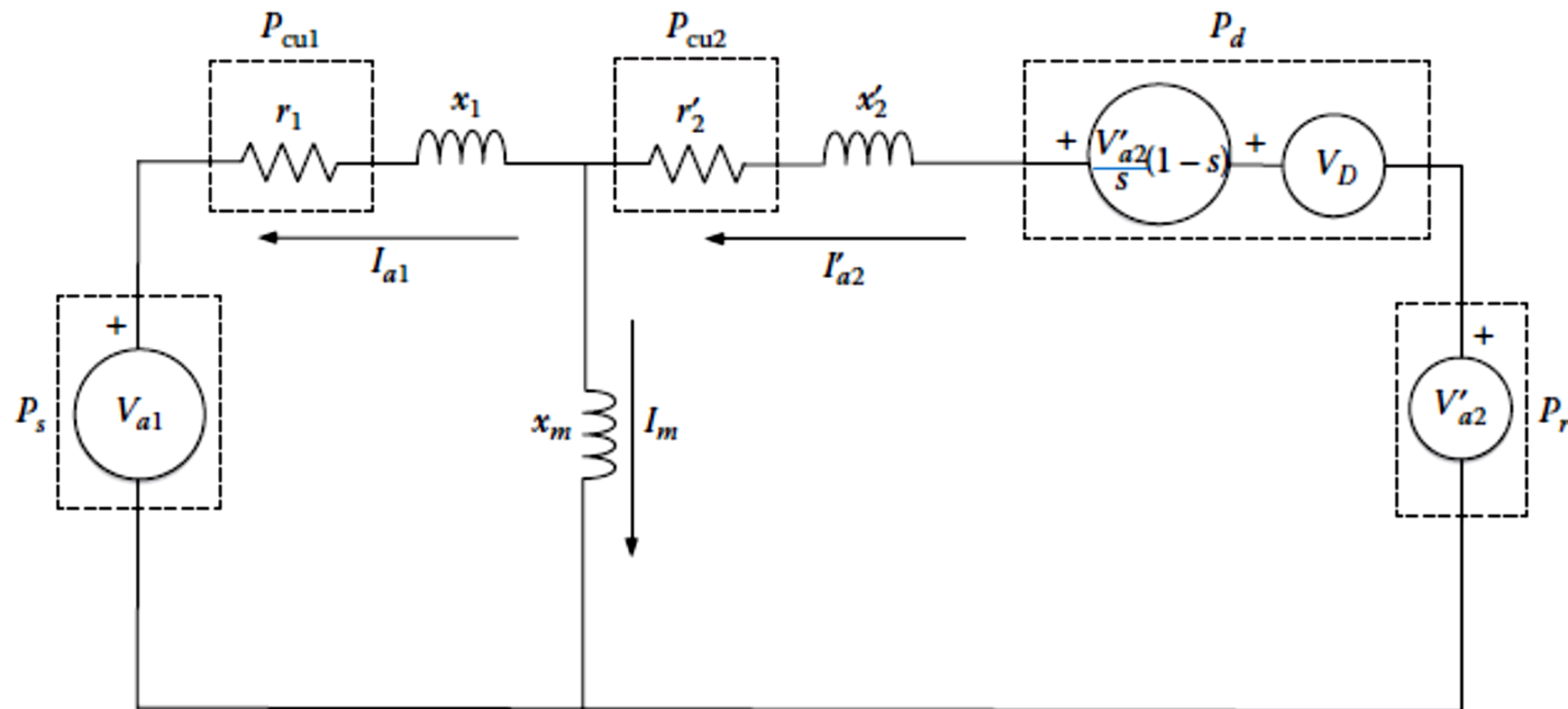
# Power Flow



# Equivalent Circuit for DFIG



$$\bar{V}_D = -\frac{r_2'}{s} (1-s) \bar{I}'_{a2}$$



**FIGURE 10.5**  
Power components in the DFIG.

# Power Flow

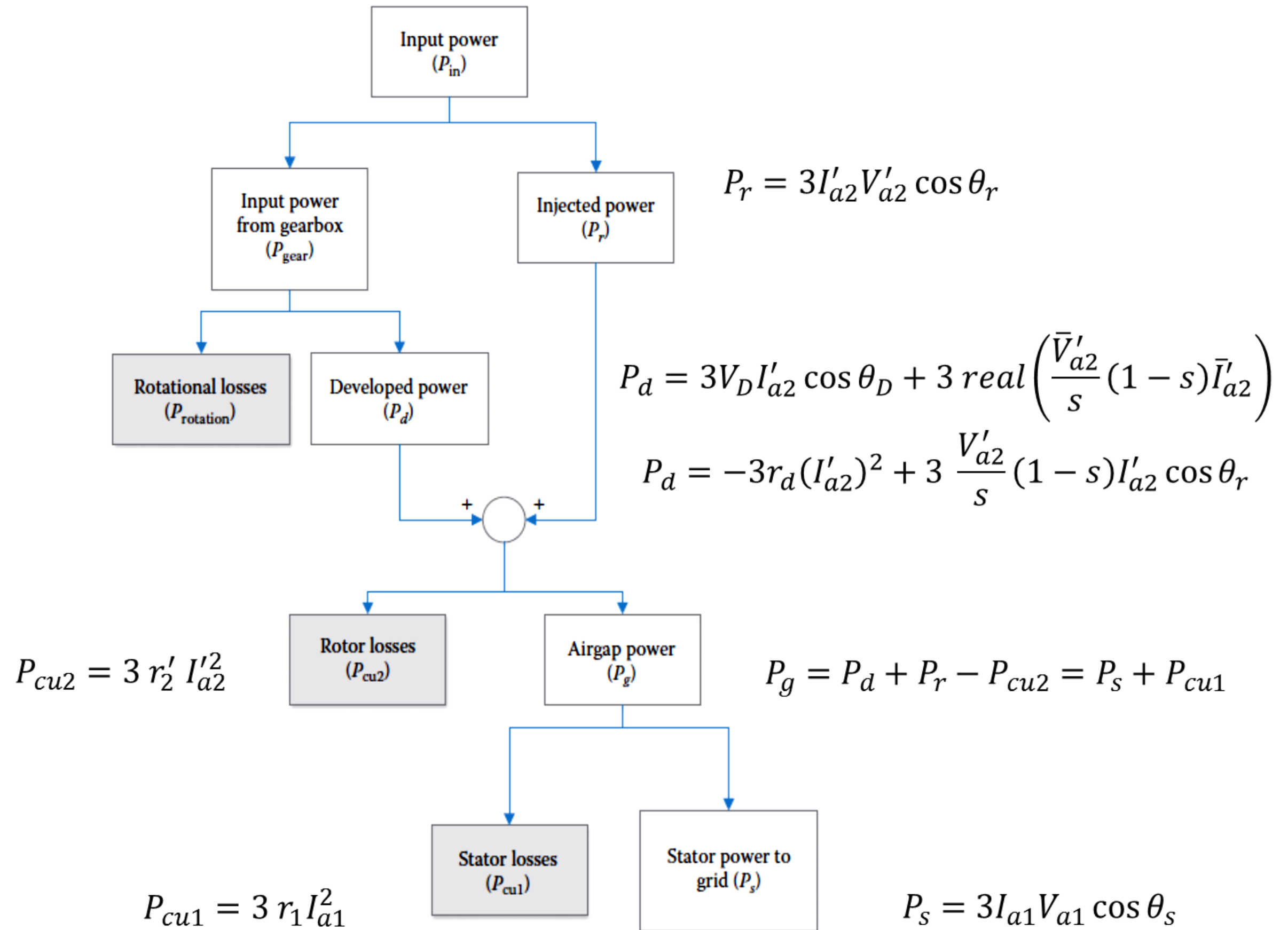


FIGURE 10.8 Power flow of wound rotor induction generator with rotor injection.

# Slip Power



$$P_g = P_d + P_r - P_{cu2} = P_s + P_{cu1} = T_d \omega_s$$

Commonly used term in industry

$$P_{slip} \equiv P_r - P_{cu2}$$

$$P_g = P_d + P_{slip} = T_d \omega_s$$

$$P_d = T_d \omega_2$$

$$P_{slip} = P_g - P_d = T_d \omega_s - T_d \omega_2 = T_d (\omega_s - \omega_2) = T_d s \omega_s$$

$$P_{slip} = s P_g$$



# Developed Power



$$P_{slip} = s P_g$$

$$P_g = P_d + P_{slip} = P_d + sP_g$$

$$P_d = P_g(1 - s)$$

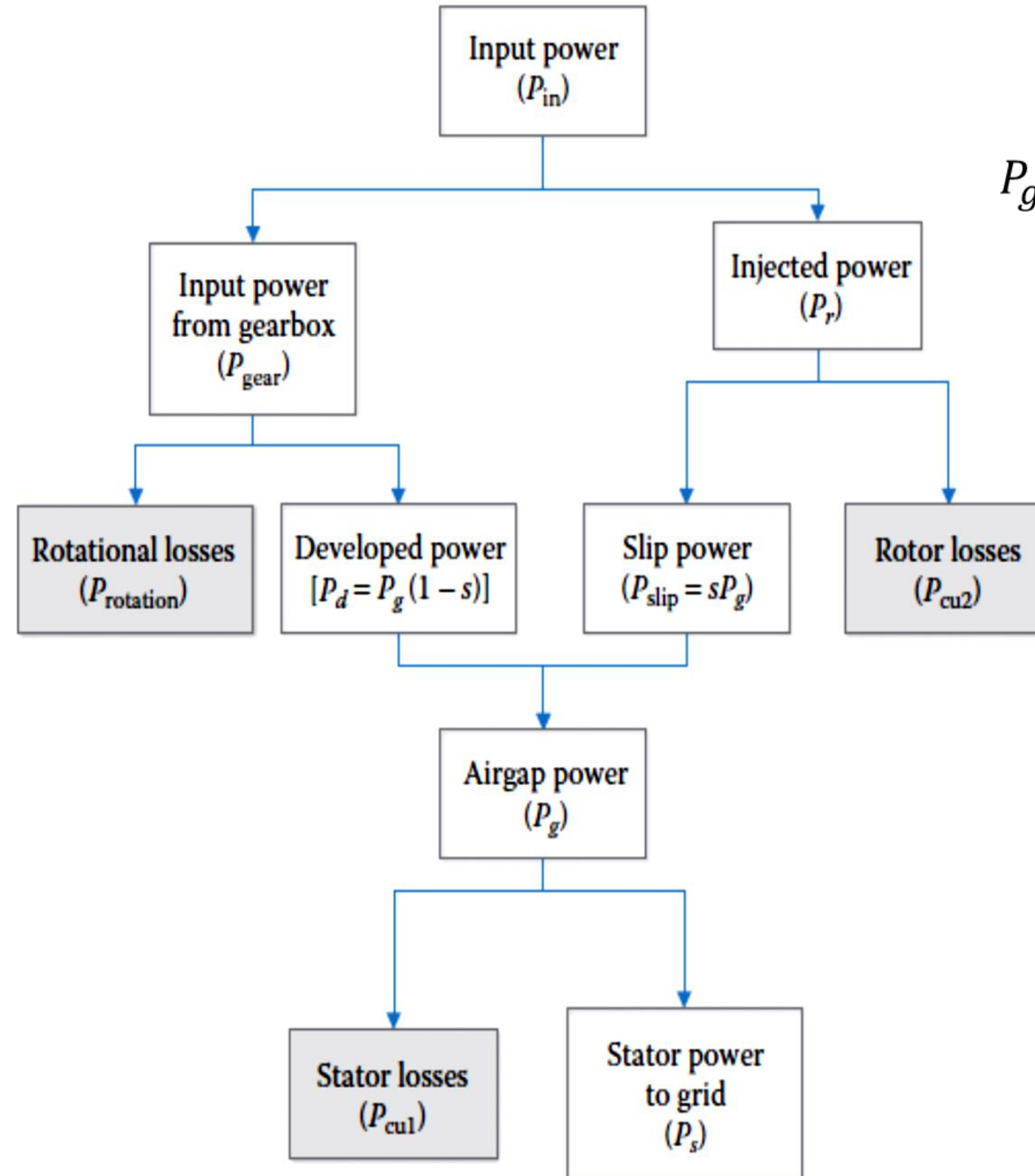
# Power Flow with Slip Power



$$P_{slip} = s P_g$$

$$P_g = P_d + P_{slip} = P_d + sP_g$$

$$P_d = P_g(1 - s)$$

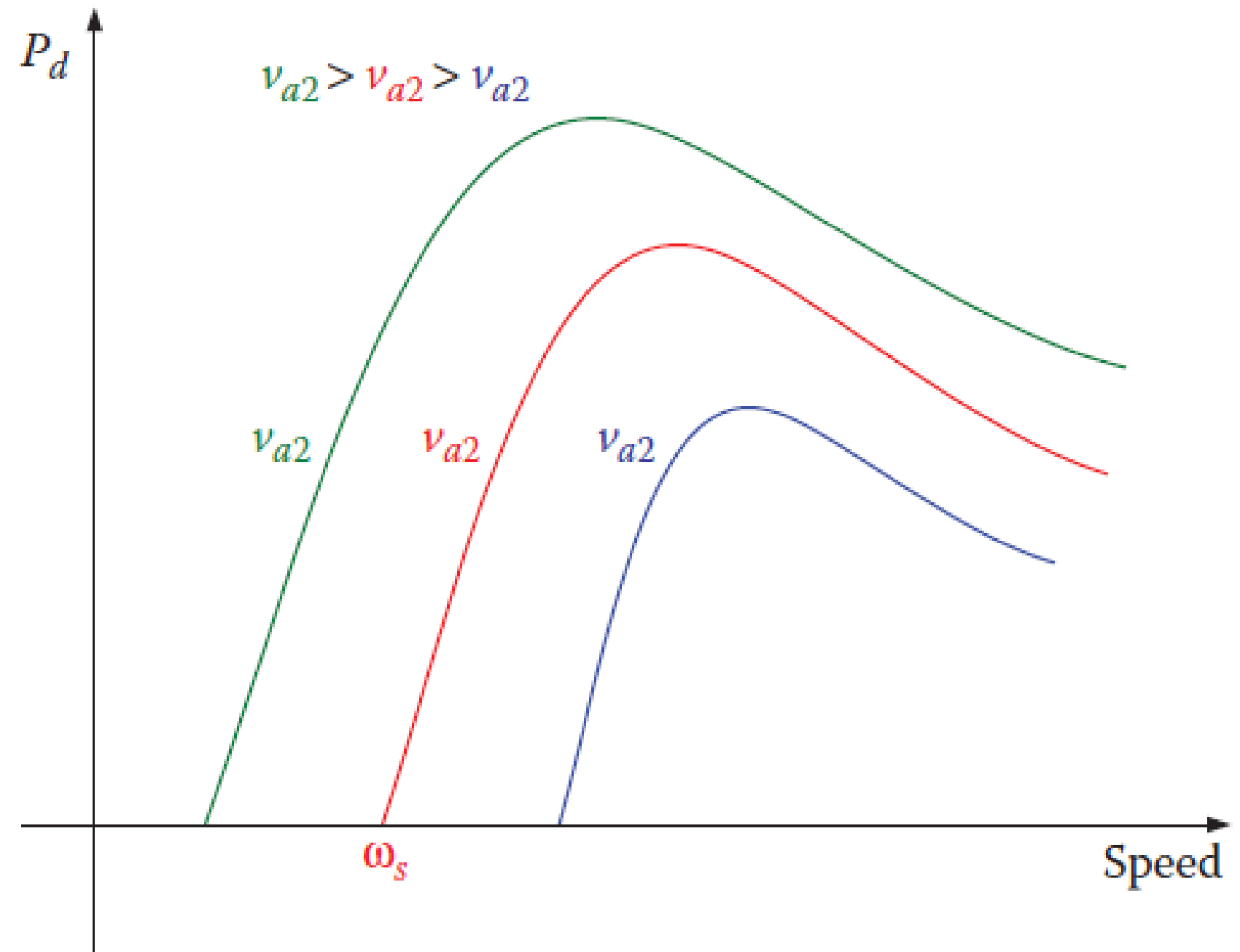


**FIGURE 10.9**  
Alternative power flow diagram of the DFIG with rotor injection.

# Power Characteristics

Note that

- Range of speed is wide (variable speed)
- Range of power is wide





# Example

- A DFIG wind turbine has 6-pole, 60Hz three-phase, Y-connected induction generator. The terminal voltage of the generator is 690 V. The effective turns ratio of the generator is 2. The rotor parameters of the machine are:

$$r_1 = r_2' = 5.0 \text{ m}\Omega; \quad x_1 = x_2' = 150 \text{ m}\Omega; \quad x_m = 5 \text{ }\Omega$$

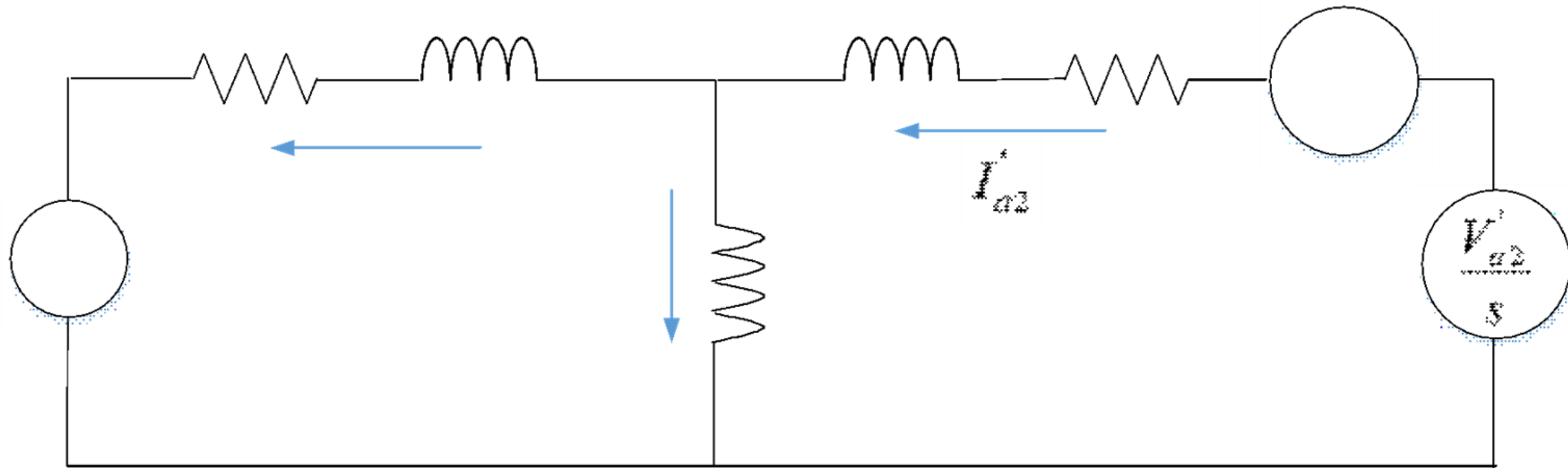
- Assuming the stator voltage is the reference phasor, the injected voltage seen from the stator side is adjusted to  $V_{a2}' = 5 \angle -120^\circ \text{ V}$  by a PWM technique. Compute the developed power, developed torque, rotor injected power, and stator power at 1230 rpm.



# Solution



$$s = \frac{n_s - n}{n_s} = \frac{1200 - 1230}{1200} = -0.025$$



$$\bar{V}_{th} = jx_m \frac{\bar{V}_{a1}}{r_1 + j(x_1 + x_m)} = j5 \frac{\frac{690}{\sqrt{3}}}{0.005 + j5.15} = 386.77 \angle 0.056^\circ$$

$$\bar{Z}_{th} = \frac{jx_m(r_1 + jx_1)}{r_1 + j(x_1 + x_m)} = \frac{j5 \times (0.005 + j0.15)}{0.005 + j5.15} = 0.0047 + j0.1456 = 0.1457 \angle 88.15^\circ$$

# Solution



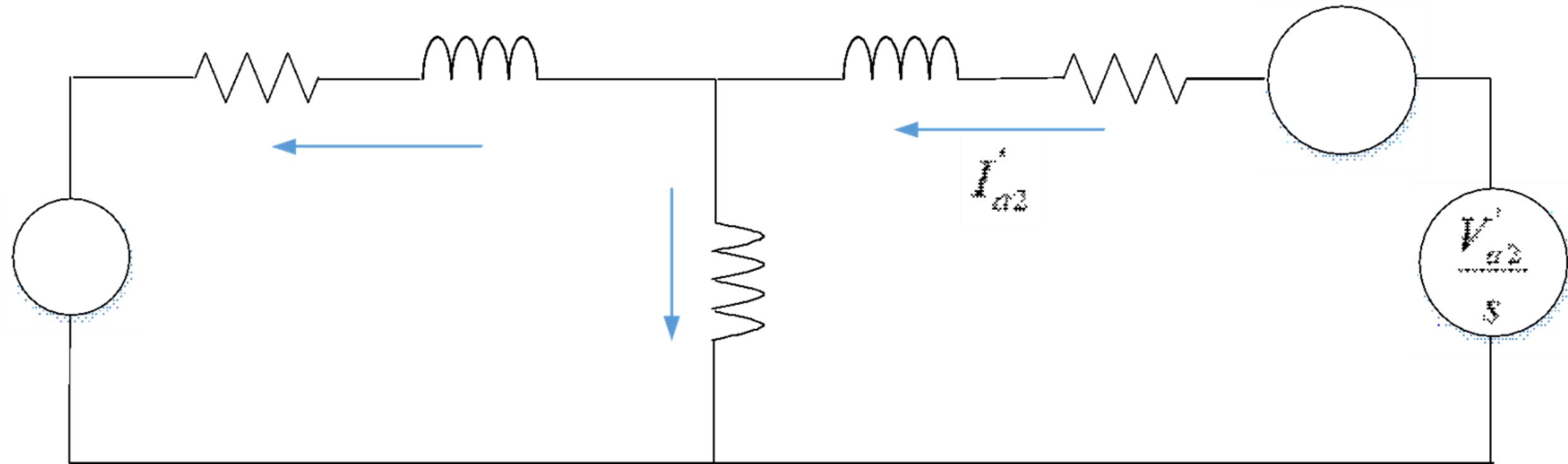
$$\bar{I}'_{a2} = \frac{\frac{\bar{V}'_{a2}}{s} - \bar{V}_{th}}{(r_{th} + (r'_2 + r_d)) + jx_{eq}} = \frac{5 \angle -120^\circ - 386.77 \angle 0.056^\circ}{\left(0.0047 + \frac{0.005}{-0.025}\right) + j0.2957} = 945 \angle 25.48^\circ \text{ A}$$

$$P_d = -3r_d(I'_{a2})^2 + 3 \frac{V'_{a2}}{s} (1 - s)I'_{a2} \cos \theta_r$$

$$P_d = -3 \frac{0.005}{-0.025} \times 1.025 \times 945^2 + 3 \frac{5}{-0.025} \times 1.025 \times 945 \cos(-120 - 25.48) = 1.028 \text{ MW}$$

$$T_d = \frac{P_d}{\omega_2} = 7.98 \text{ kNm}$$

# Solution

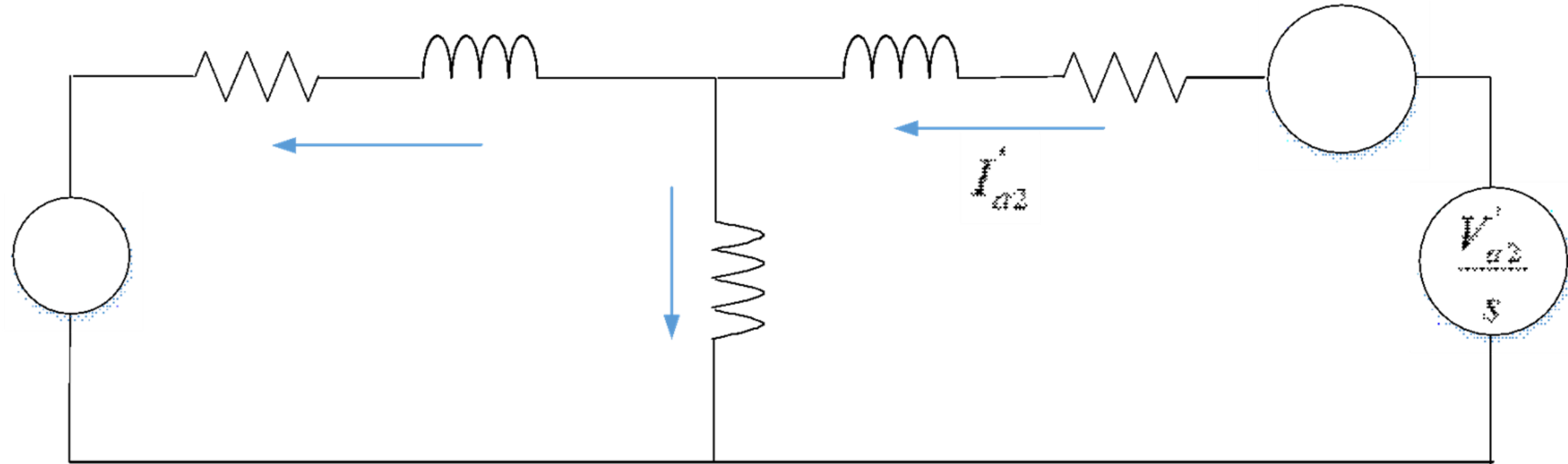


$$\bar{V}_{a1} = -(r_1 + jx_1)\bar{I}_{a1} + jx_m\bar{I}_m = -(r_1 + jx_1)\bar{I}_{a1} + jx_m(\bar{I}'_{a2} - \bar{I}_{a1})$$

$$\bar{I}_{a1} = \frac{jx_m\bar{I}'_{a2} - \bar{V}_{a1}}{(r_1 + j(x_1 + x_m))} = 953.3 \angle 29.7^\circ \text{ A}$$

$$P_s = 3I_{a1}V_{a1} \cos \theta_s = \sqrt{3} \times 953.3 \times 690 \times \cos(29.7^\circ) = 989.63 \text{ kW}$$

# Solution



$$P_{cu1} = 3r_1 I_{a1}^2 = 13.395 \text{ kW}$$

$$P_g = P_s + P_{cu1} = 1.003 \text{ MW}$$

$$P_{cu2} = 3r'_2 (I'_{a2})^2 = 13.4 \text{ kW}$$

$$P_{slip} = sP_g = -25.1 \text{ kW}$$

$$P_r = P_{slip} + P_{cu2} = -11.68 \text{ kW}$$

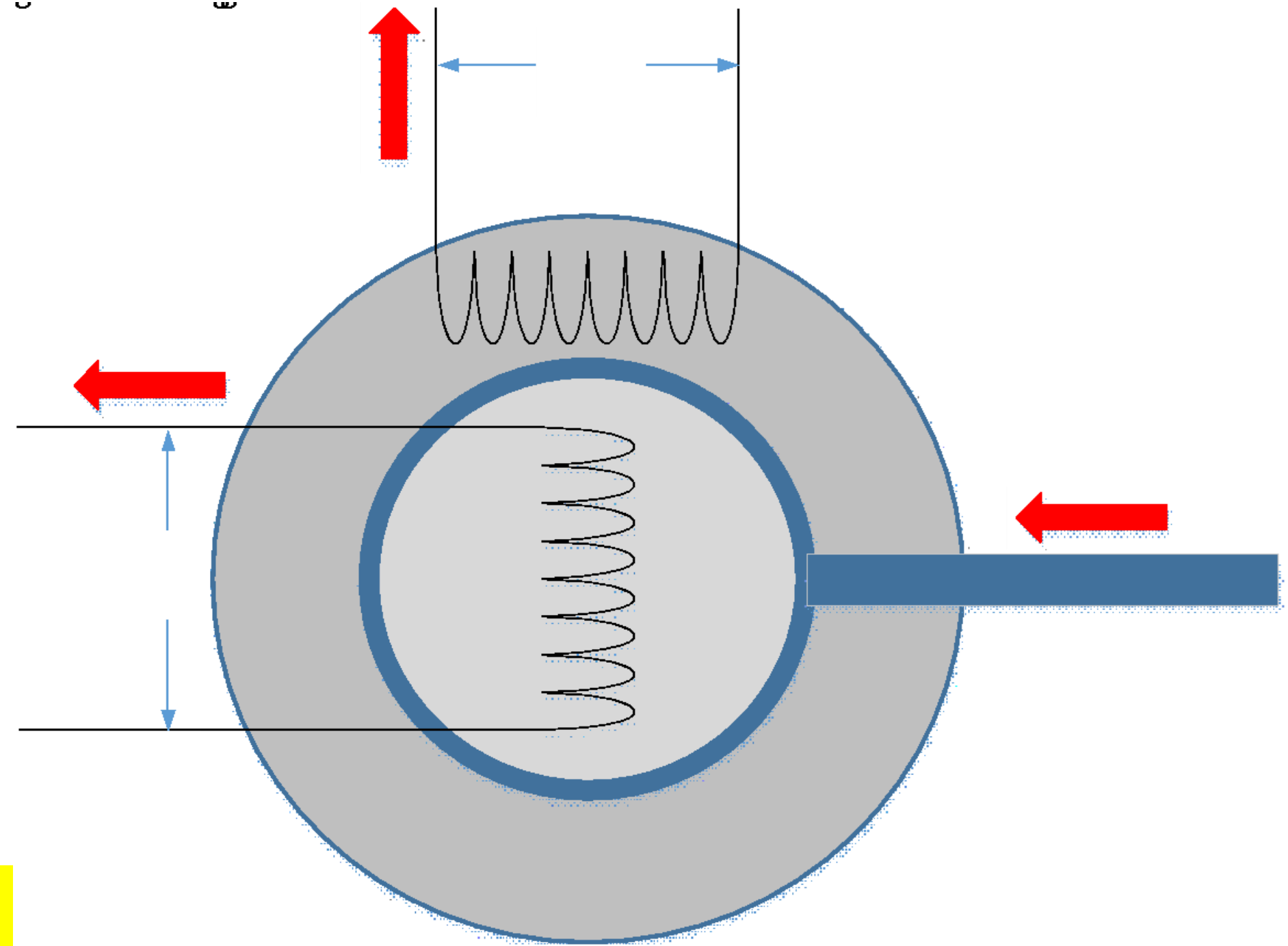
Negative sign: Power from rotor to grid)



# Direction of power Flow for Super-Synchronous Speed

$$S = \frac{n_s - n}{n_s} < 0$$

$$P_r \approx sP_s$$



Power flow from rotor to grid

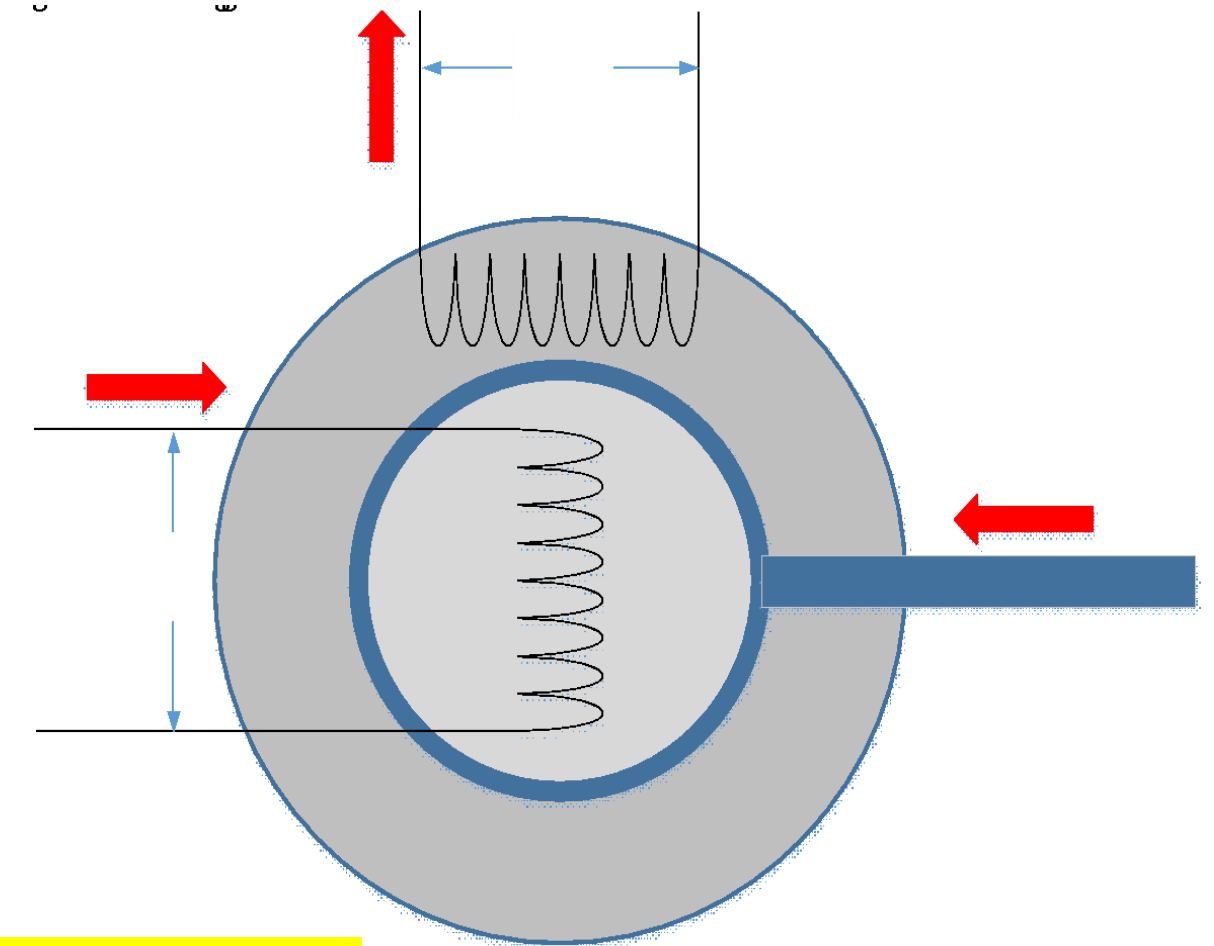
# Direction of power Flow for Sub-Synchronous Speed



$$s = \frac{n_s - n}{n_s} > 0$$

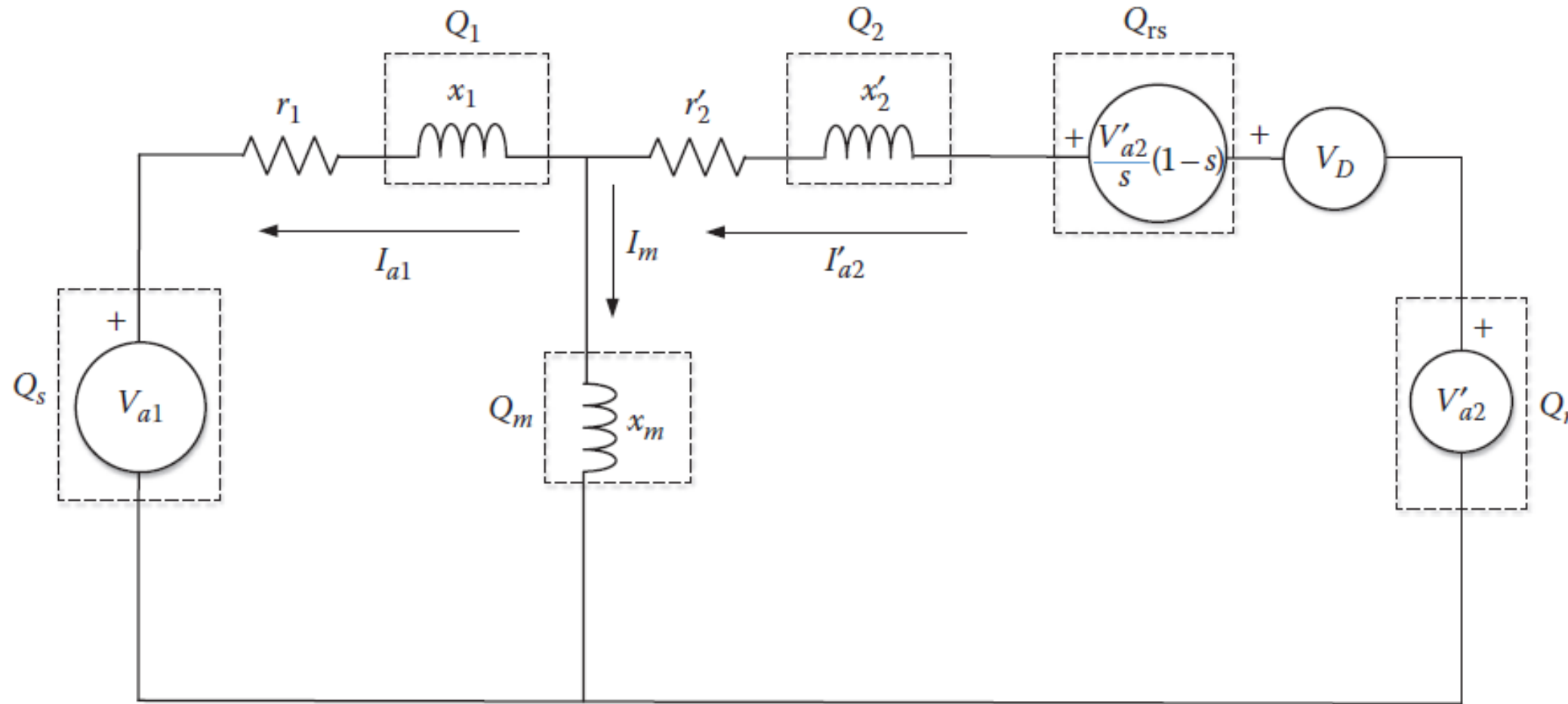
$$P_r \approx sP_s$$

Power flow from grid to rotor



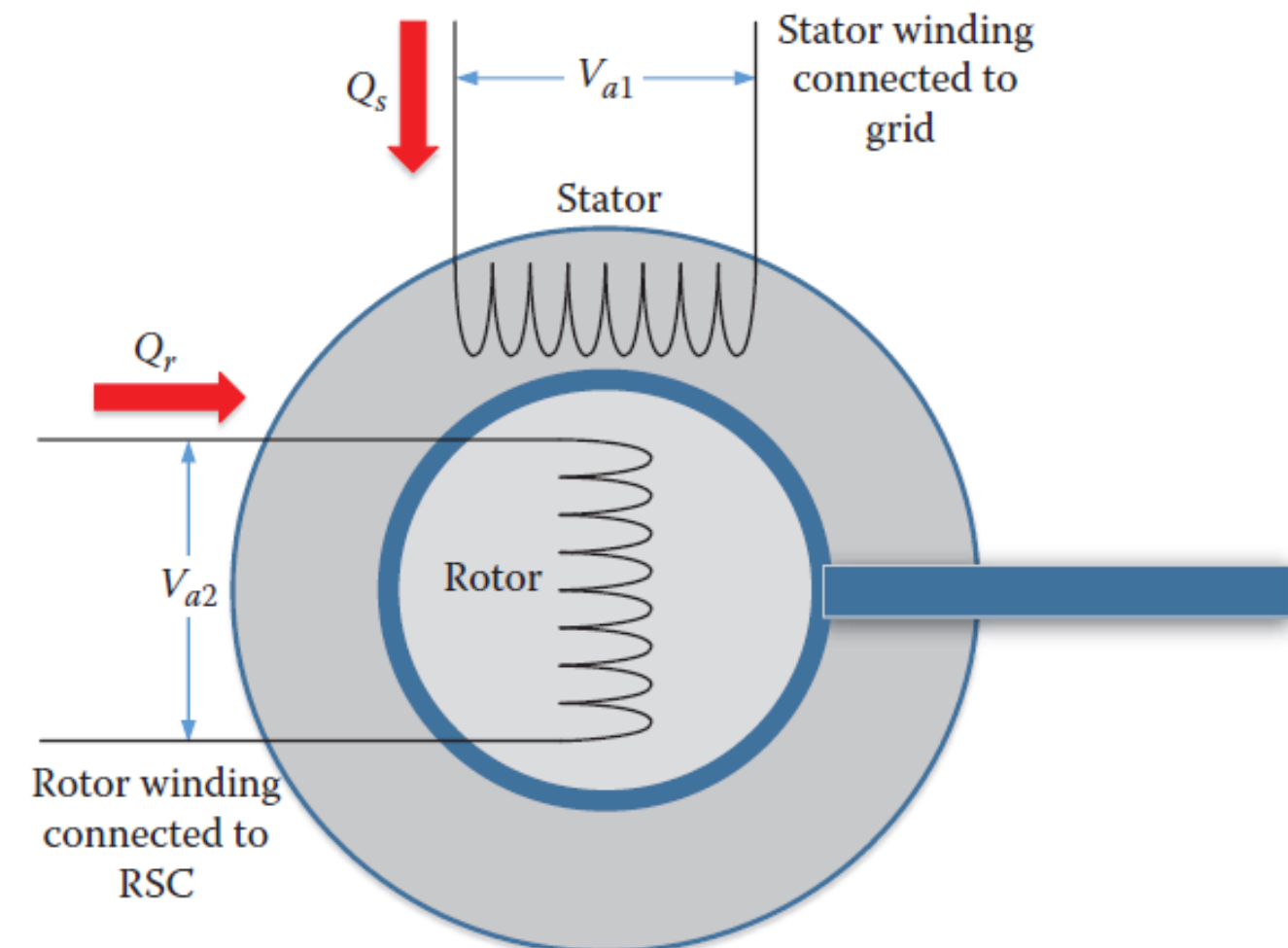
If  $s$  is maintained within  $\pm 30\%$ , the power passing through the converter is about 30% of the stator rated power. Therefore, the DFIG's converter is designed to handle about one-third of the rated power. This way, the DFIG system can produce 30% more power as compared with types 1 or 2 systems, and it can operate at wider speed range.

# Reactive Power



$$Q_s + Q_{r\text{-total}} = Q_1 + Q_2 + Q_m$$

$$Q_s + \frac{Q_r}{s} = Q_1 + Q_2 + Q_m$$

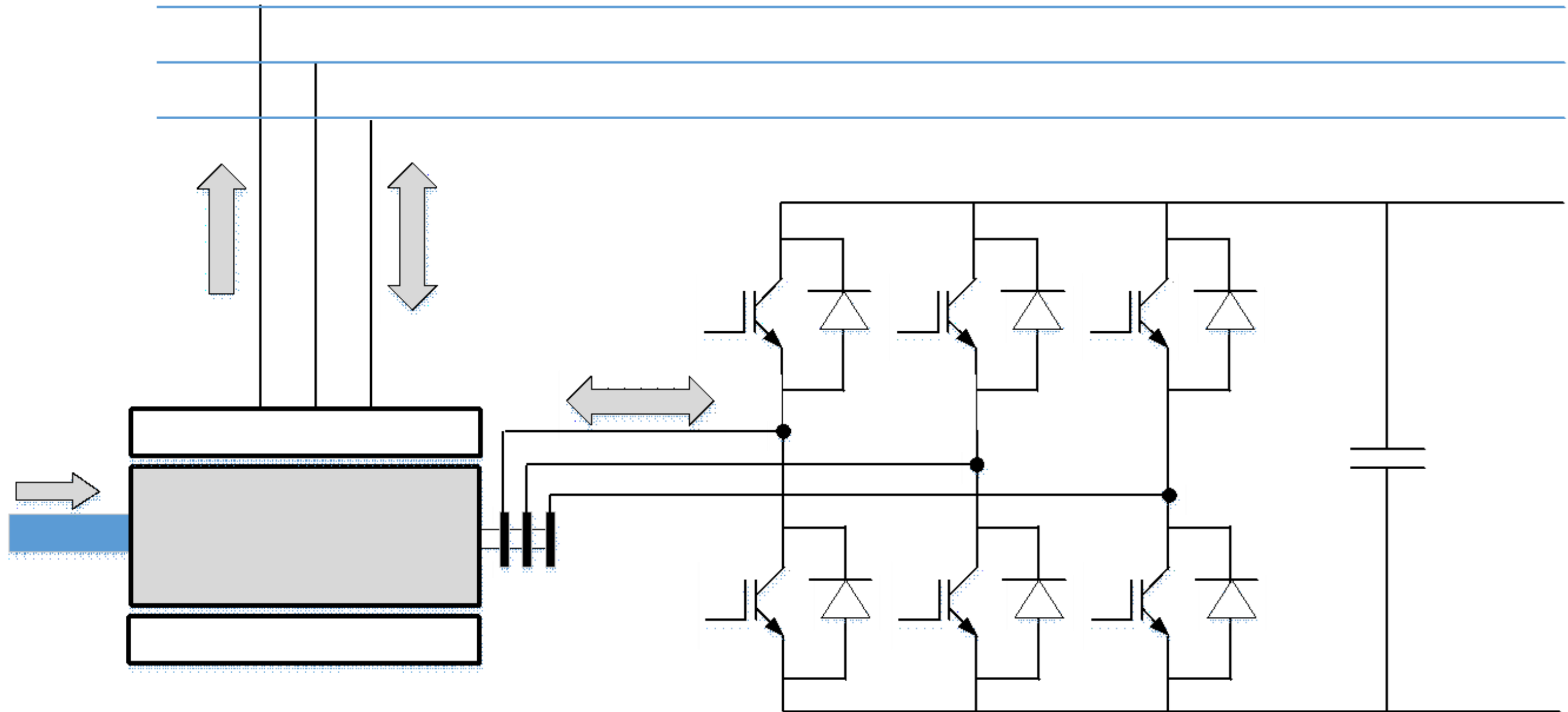




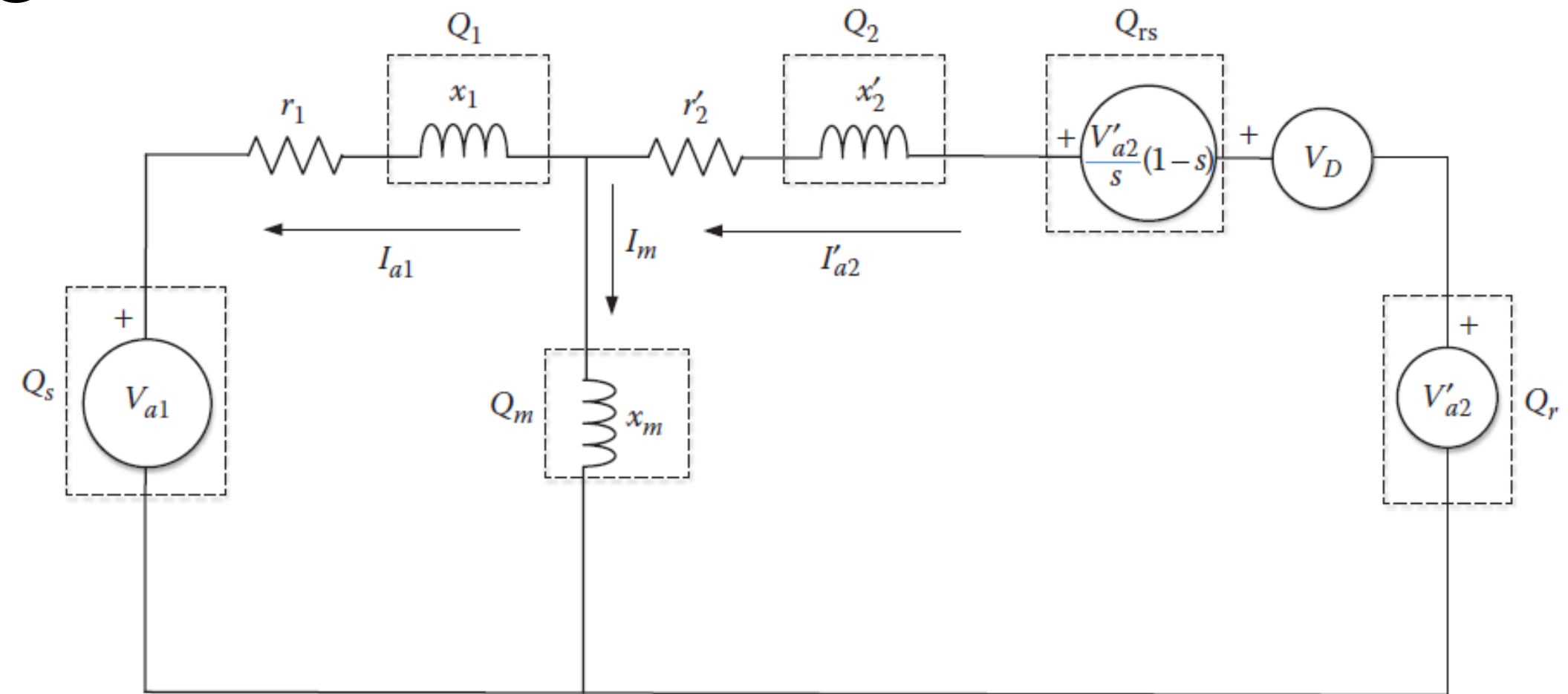
# Power Flow through RSC



# Power Flow through RSC



# Reactive Power Flow through RSC



$$\bar{S}_r = 3\bar{V}'_{a2}\bar{I}'_{a2*} = P_r + jQ_r$$

$$Q_{rs} = \text{imag} \left( 3 \frac{\bar{V}'_{a2}}{s} (1-s) \bar{I}'_{a2*} \right)$$

$$Q_{r-total} = Q_r + Q_{rs}$$

$$Q_{r-total} = \text{imag} \left( 3 \frac{\bar{V}'_{a2}}{s} \bar{I}'_{a2*} \right) = \frac{Q_r}{s}$$

$$Q_s + Q_{r-total} = Q_1 + Q_2 + Q_m$$

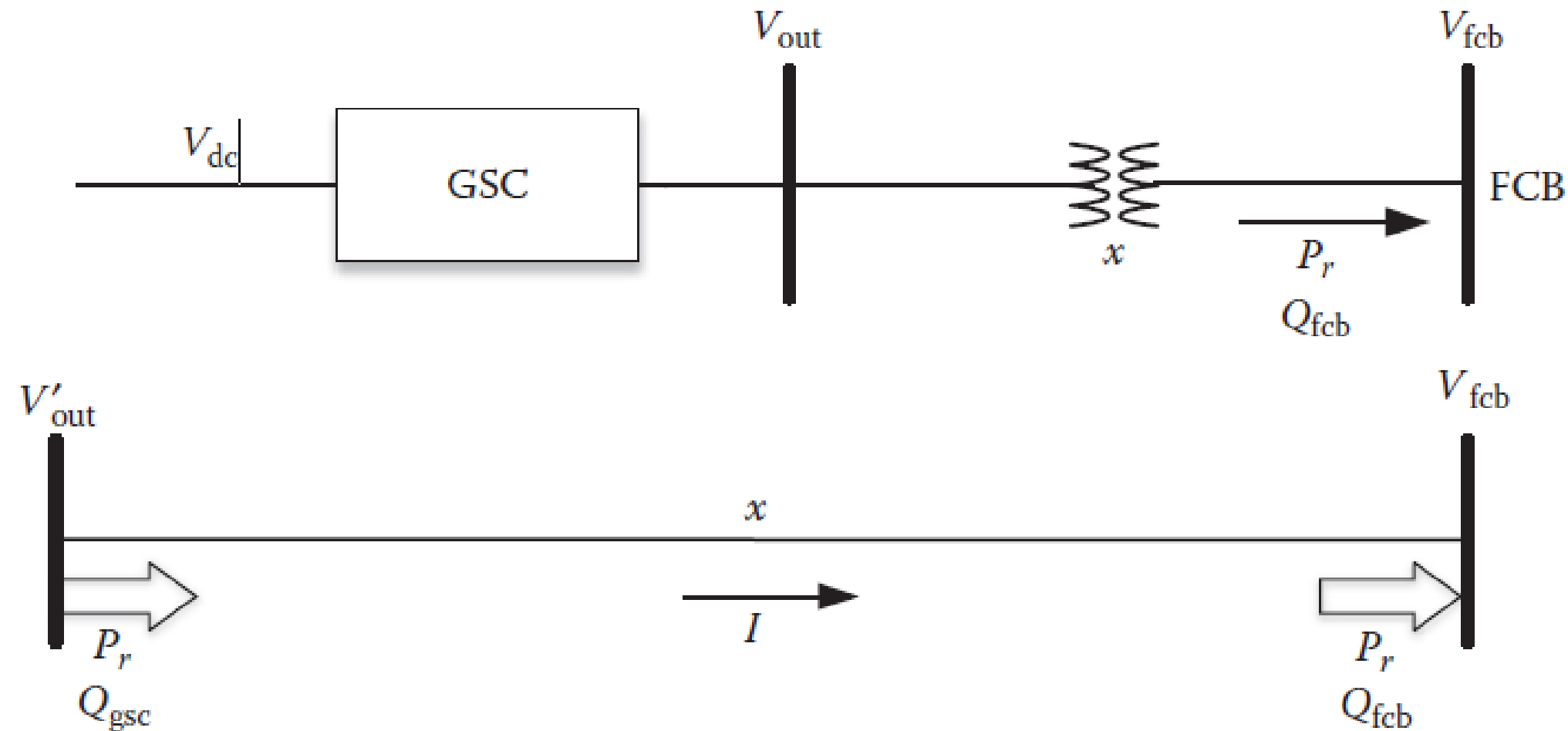
$$Q_s + \frac{Q_r}{s} = Q_1 + Q_2 + Q_m$$

$$Q_s = Q_1 + Q_2 + Q_m - \frac{Q_r}{s}$$



# Power Flow through GSC

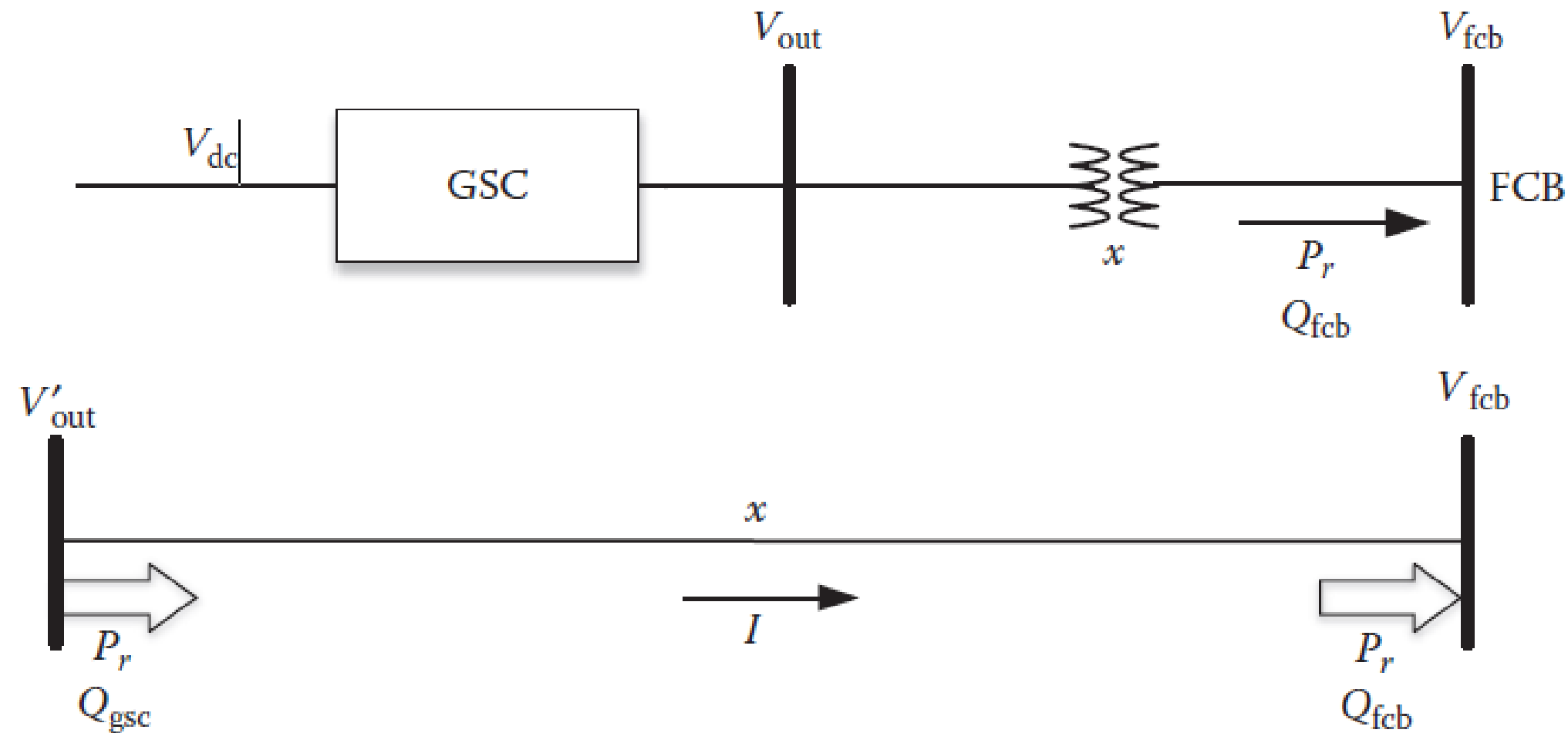
# Powers at the FCB



$$\bar{I} = \frac{V'_{out}(\cos \delta + j \sin \delta) - V_{fcb}}{jx} = \frac{V'_{out}}{x} \sin \delta + j \frac{V_{fcb} - V'_{out} \cos \delta}{x}$$

$$\bar{S}_{fcb} = \bar{V}_{fcb} \bar{I}^* = P_r + jQ_{fcb}$$

# Powers at the FCB



$$\bar{S}_{fcb} = \bar{V}_{fcb} \bar{I}^* = 3V_{fcb} \left[ \frac{V'_{out}}{x} \sin \delta - j \frac{V_{fcb} - V'_{out} \cos \delta}{x} \right]$$

$$P_r = 3 \frac{\bar{V}'_{out} V_{fcb}}{x} \sin \delta$$

$$Q_{fcb} = 3 \frac{V_{fcb}}{x} (V'_{out} \cos \delta - V_{fcb})$$



# Powers at the FCB



$$Q_{fcb} = 3 \frac{V_{fcb}}{x} (V'_{out} \cos \delta - V_{fcb})$$

*If  $V'_{out} \cos \delta > V_{fcb}$ ;  $Q_{fcb}$  is positive*

*If  $V'_{out} \cos \delta < V_{fcb}$ ;  $Q_{fcb}$  is negative*

*If  $V'_{out} \cos \delta = V_{fcb}$ ;  $Q_{fcb}$  is zero*



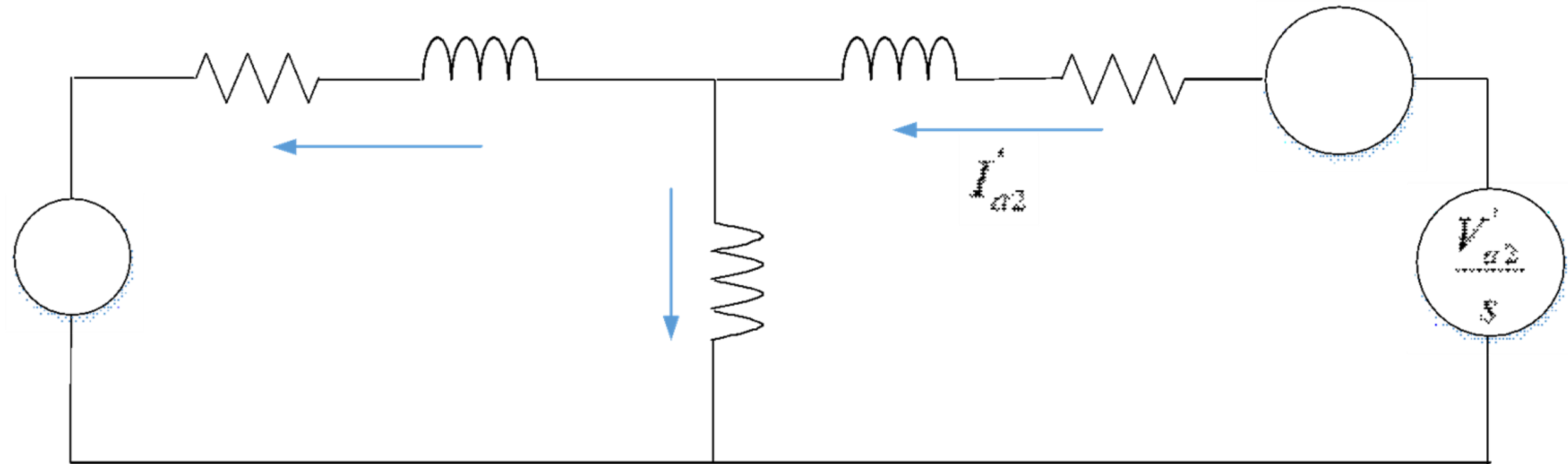
# Example

- A DFIG system has 6-pole, 60Hz, Y-connected induction generator. The terminal voltage of the generator is 690 V. The effective turns ratio of the generator is 5. The rotor parameters of the machine are:

$$r'_2 = 10 \text{ m}\Omega; x_1 = x'_2 = 100 \text{ m}\Omega; x_m = 5 \Omega$$

- Without injection, ignore all losses and compute the developed power at 1230 rpm.
- If the speed of the generator increases to 1260 rpm, compute the injected voltage that maintains the developed power constant. Assume the injected voltage is  $180^\circ$  out of phase with the stator voltage.

# Solution (Without injection)



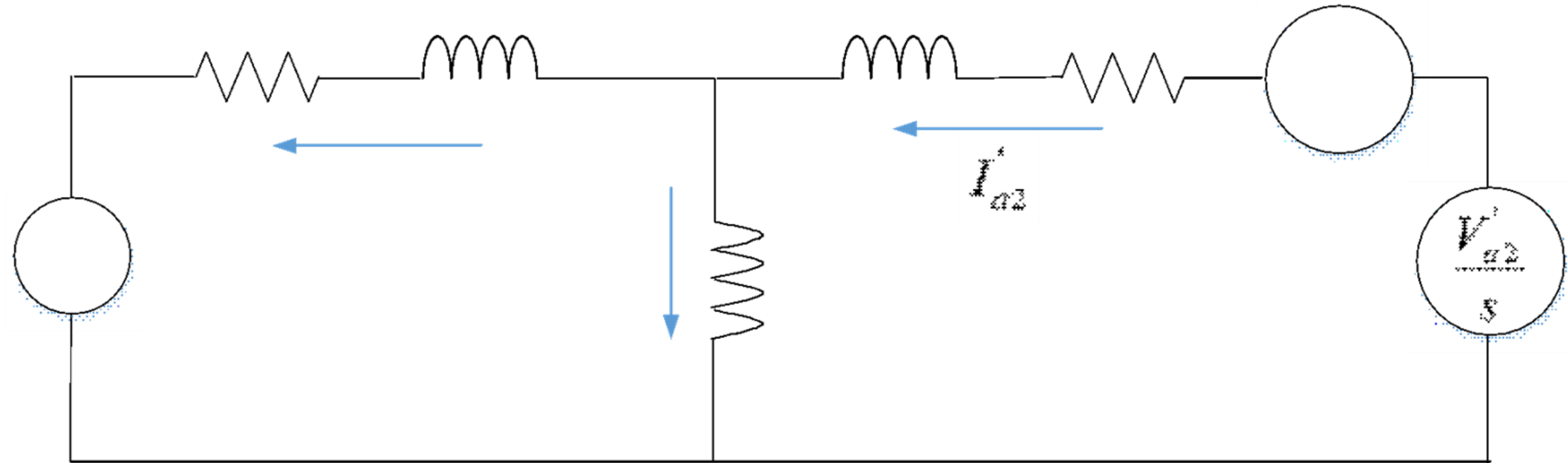
$$s = \frac{n_s - n}{n_s} = \frac{1200 - 1230}{1200} = -0.025$$

Since  $x_1 \ll x_m$

$$r_d = \frac{r'_2}{s} (1 - s) = \frac{0.01}{-0.025} (1.025) = -0.41 \Omega$$

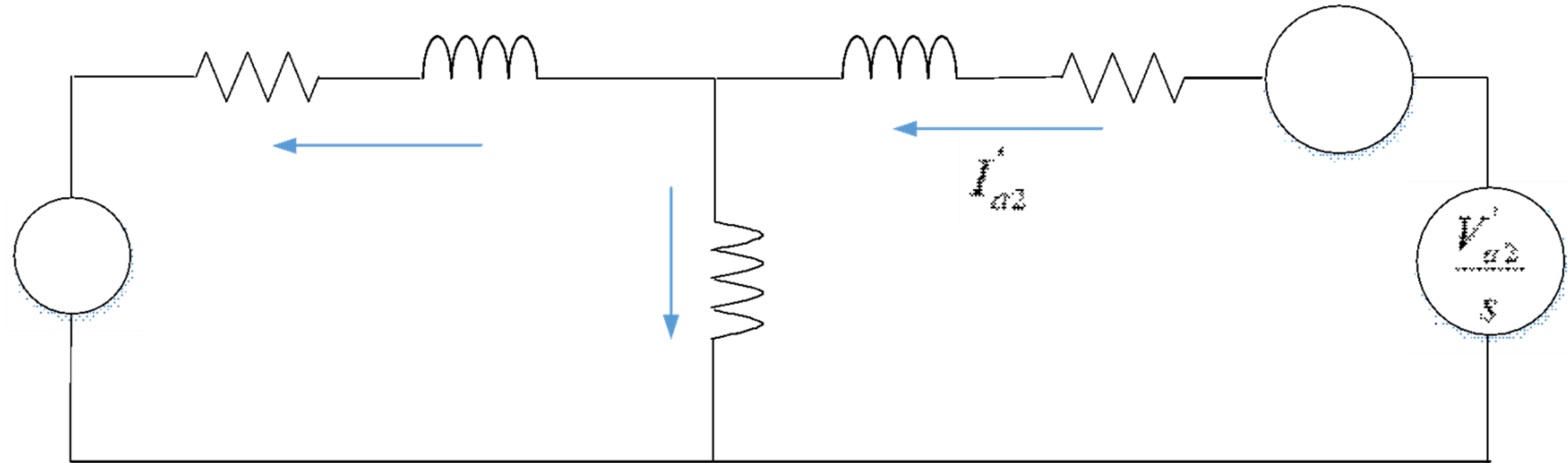
$$I'_{a2} = \frac{V_{a1}}{\sqrt{r_d^2 + (x_1 + x'_2)^2}}$$

# Solution (Without injection)



$$P_d = -3r_d(I'_{a2})^2 = -3r_d \frac{V_{a1}^2}{r_d^2 + (x_1 + x_2')^2} = 0.41 \frac{690^2}{(-0.41)^2 + (0.2)^2} = 938.015 \text{ kW}$$

# Solution (With injection)



$$s = \frac{n_s - n}{n_s} = \frac{1200 - 1360}{1200} = -0.05$$

Since  $x_1 \ll x_m$

$$r_d = \frac{r_2'}{s} (1 - s) = \frac{0.01}{-0.05} (1.05) = -0.21 \Omega$$

$$I_{a2}' = \frac{\frac{-V_{a2}'}{s} - V_{a1}}{\sqrt{r_d^2 + (x_1 + x_2')^2}}$$



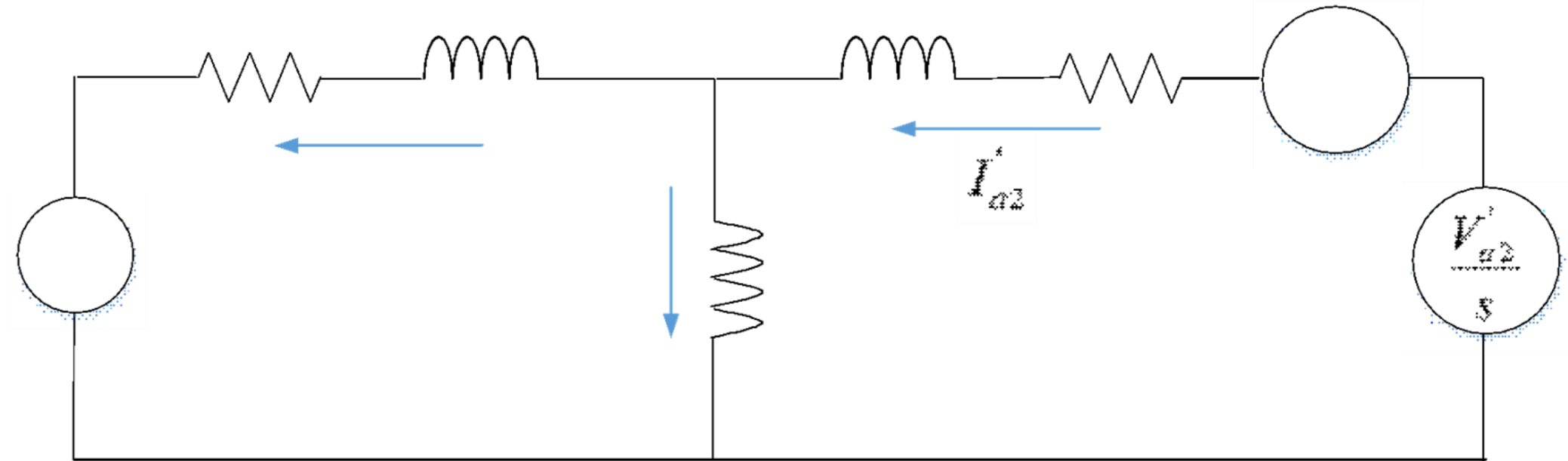
# Solution (With injection)

$$I'_{a2} = \frac{\frac{-V'_{a2}}{s} - V_{a1}}{\sqrt{r_d^2 + (x_1 + x'_2)^2}} = \frac{\frac{-V'_{a2}}{-0.05} - \frac{690}{\sqrt{3}}}{\sqrt{(-0.21)^2 + (0.2)^2}} = 68.97 \times V'_{a2} - 1373.7$$

$$P_d = -3r_d(I'_{a2})^2 + 3 \frac{V'_{a2}}{s} (1 - s)I'_{a2} \cos \theta_r$$

$$\theta_r = 180^\circ \quad P_d = -3r_d(I'_{a2})^2 - 3 \frac{V'_{a2}}{s} (1 - s)I'_{a2}$$

# Solution (With injection)



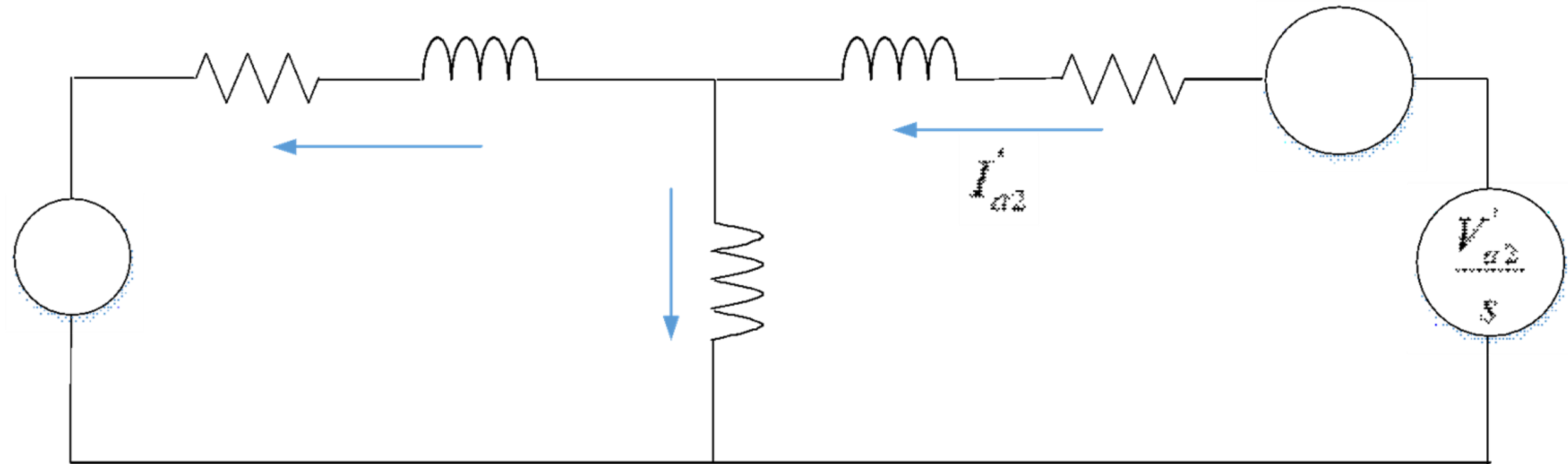
$$P_d = -3r_d(I'_{a2})^2 - 3 \frac{V'_{a2}}{s} (1 - s)I'_{a2}$$

$$P_d = 0.63 (I'_{a2})^2 - 3 \frac{V'_{a2}}{-0.05} (1.05)I'_{a2} = 0.63 (I'_{a2})^2 + 63 V'_{a2} I'_{a2} = 938.015 \times 10^3$$

$$I'_{a2} = 68.97 \times V'_{a2} - 1373.7$$

Two Equations with two unknowns

# Solution (With injection)



$$V'_{a2} = 12.85 \text{ V}$$

$$V_{a2} = V'_{a2} \frac{N_2}{N_1} = 12.85 \frac{1}{5} = 2.57 \text{ V}$$



# Speed Control of DFIG

# Dynamic Torque Equation

$$T_m - T_d = 2H \frac{d\omega_2}{dt}$$

$T_m$  is the input mechanical torque from the gearbox

$T_d$  is the developed electrical torque of the generator,  $T_d = \frac{P_d}{\omega_2}$

$H$  is the inertia time constant of the rotating mass (blades, generator, gearbox, etc.)

$\frac{d\omega_2}{dt}$  is the acceleration of the generator speed.





# Constant Speed Operation

- Any change in wind speed changes the lift force, and consequently increases the mechanical torque from the gearbox.
- To keep the rotation speed of the turbine constant even when wind speed changes,
  - The injected voltage is adjusted to change the electrical (developed) torque to match the change in the mechanical torque.
  - In this case the acceleration of the rotor is zero, and the speed is maintained constant.



# Variable Speed for Tracking Maximum $C_p$

- To track the maximum coefficient of performance, the tip speed ratio (TSR) must be controlled.
- If wind speed changes, the speed of the generator must change to achieve the optimum TSR.
- This is done by adjusting the injected voltage to produce a temporary change in the electrical torque until the desired speed is achieved.
- After reaching the new speed, the injected voltage is adjusted again to match the electrical torque to the mechanical torque, thus operating at the new constant speed.



# Protection of DFIG



# Solid State Operating Limits

- **Steady State Limit:** Below the voltage and current limit, the device can operate continuously without being damaged.
- **Junction Temperature:** Losses inside solid-state devices are due to impurities of their material as well as the operating conditions of their circuits. These losses are mainly occurring during
  - continuous operation
  - when the device toggles between the open and closed states.
- The electrical losses are converted into thermal energy that heats the junctions of the devices.
- The heat can be tolerated up to a critical level (125°C is a typical value).
  - excessive heat can cause permanent damage.



# Solid State Operating Limits

- **Surge Current:** It is the absolute maximum of the non-repetitive impulse current.
- **Critical Rate of Rise of Current** (or maximum  $\frac{di}{dt}$ ): Any solid-state device is damaged if the disturbance in the grid causes the current through the device to change rapidly (high  $\frac{di}{dt}$ ), even if the current is below the surge limit of the device.
- **Critical Rate of Rise of Voltage** (or maximum  $\frac{dv}{dt}$ ): When the rate of change in voltage across a device ( $\frac{dv}{dt}$ ) exceeds its limit, the device is forced to close. This is a form of false triggering and could lead to excessive current or excessive  $\frac{di}{dt}$ .





# Main Types of Protections

- **Electrical Protection:** Overcurrent and overvoltage
- **Electromechanical Protection:** when the mechanical power is higher than the delivered electrical power.



# Electrical Protection

- These events can cause current and voltage surges in the windings of the machine.
- Since the RSC is connected to the rotor of the generator, these surges could reach the RSC and could fail its power electronic devices.



# Electromechanical Protection

- Grid faults depress the voltage
  - output power of the generator  $P_s$  is reduced during faults.
  - Because the mechanical power of the machine cannot be reduced instantly, there will be a period of time when the mechanical power is higher than the delivered electrical power.
  - This difference in power causes the turbine to accelerate and it may reach a level that could damage its blades as well as various other mechanical parts.



# Electrical Protection

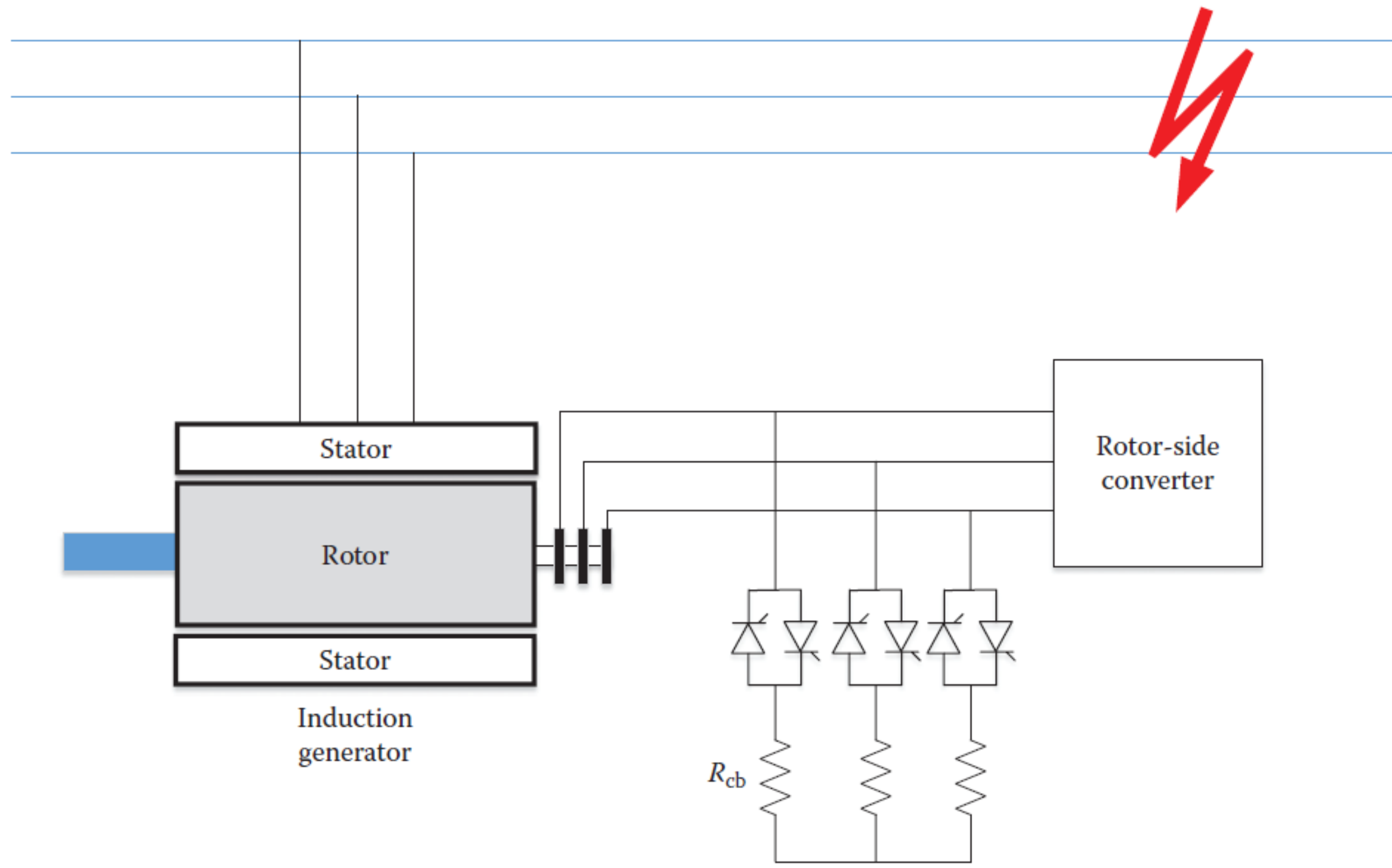


# Main Types of Electrical Protection

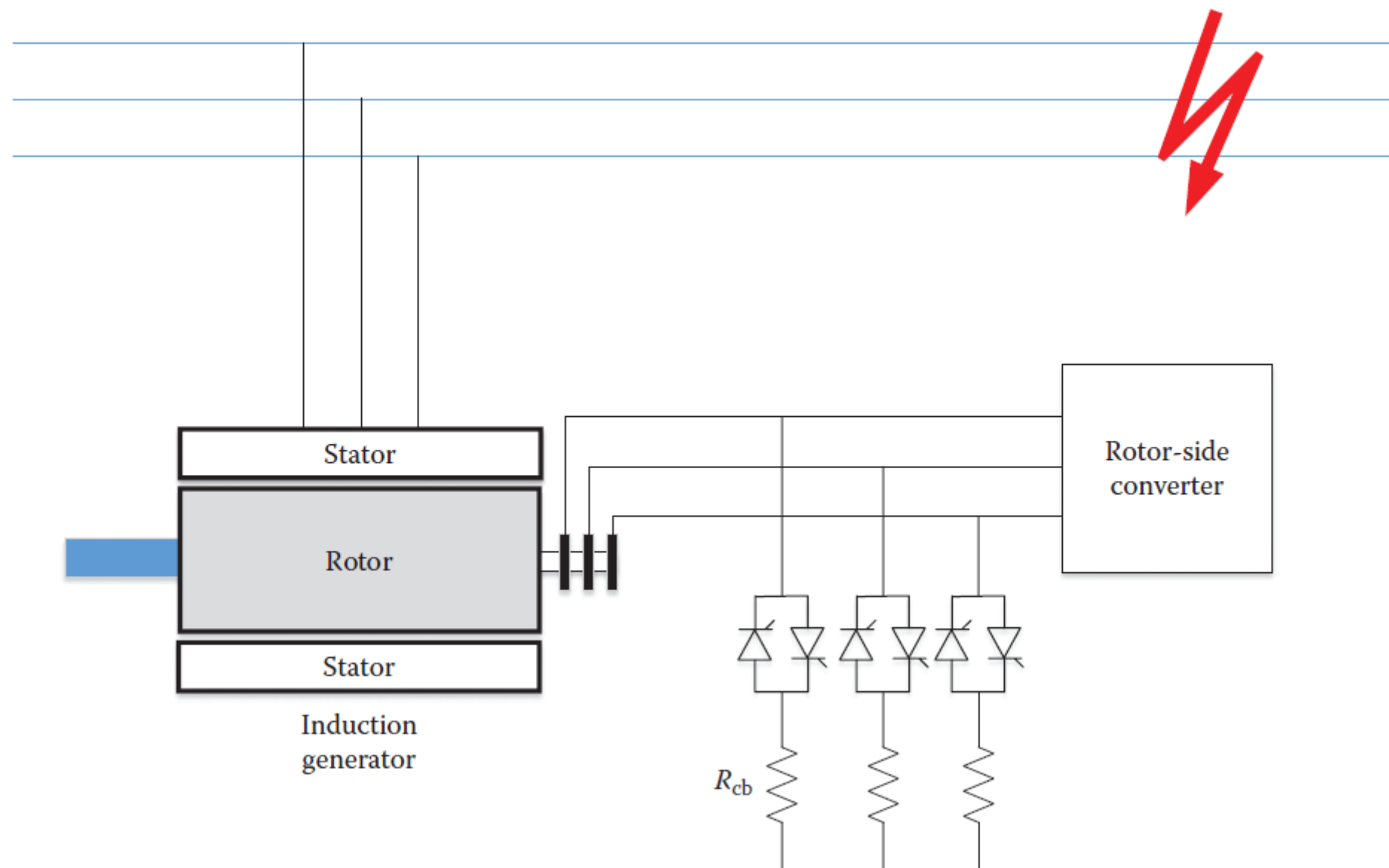
- Crowbar
- Chopper



# Crowbar System



# Crowbar System



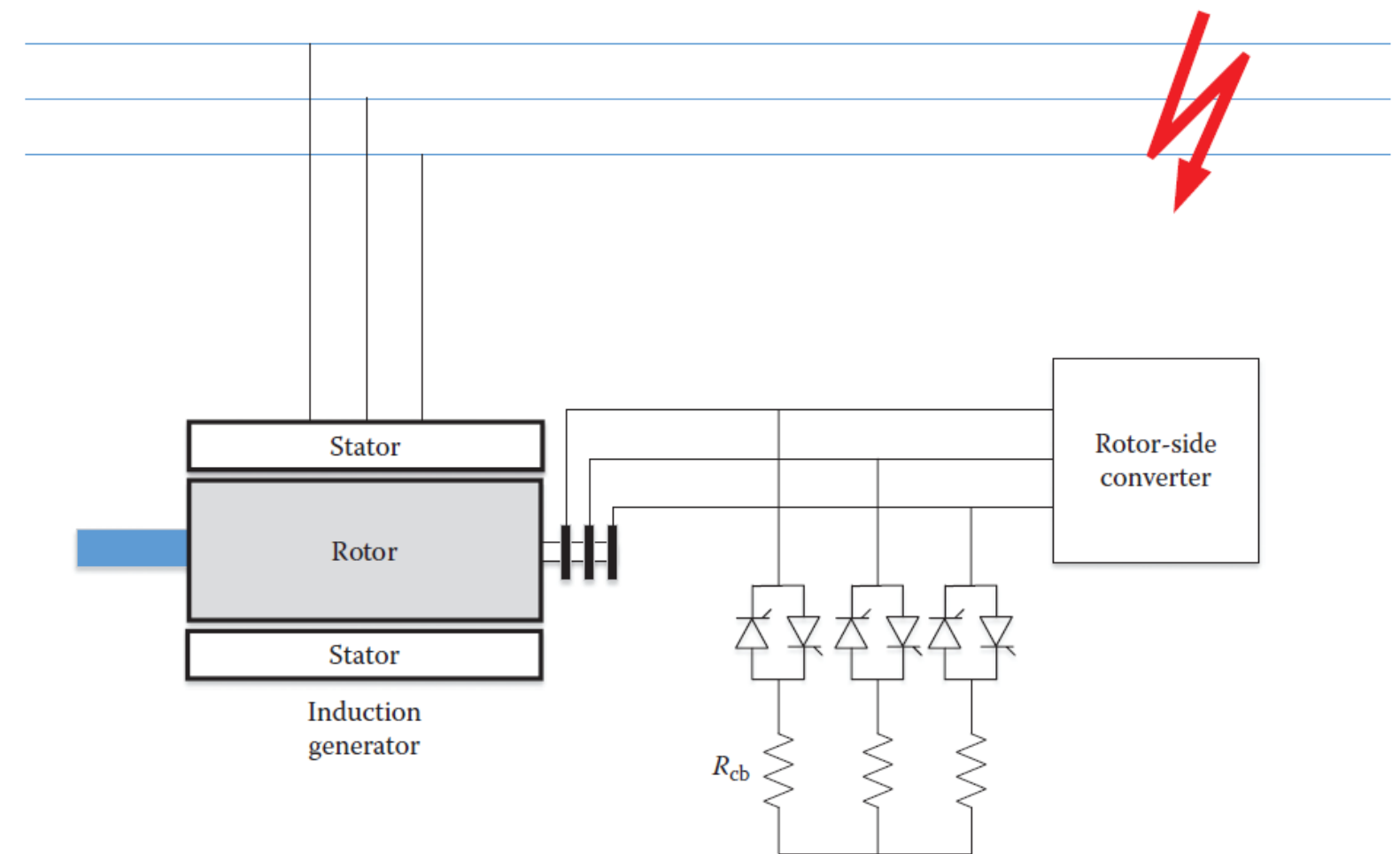
- $If v_{a2} > V_{dc}$
- RSC transistors could **falsely trigger**, or be damaged
- Capacitor of the DC link could be **overcharged**
- The dc bus voltage could be high enough to **damage other electronics**

To keep  $P_{dc}=0$ ,  $R_{cb}$  should be selected so that

$$R_{cb} i_{f-max} < V_{dc}$$

# Crowbar System

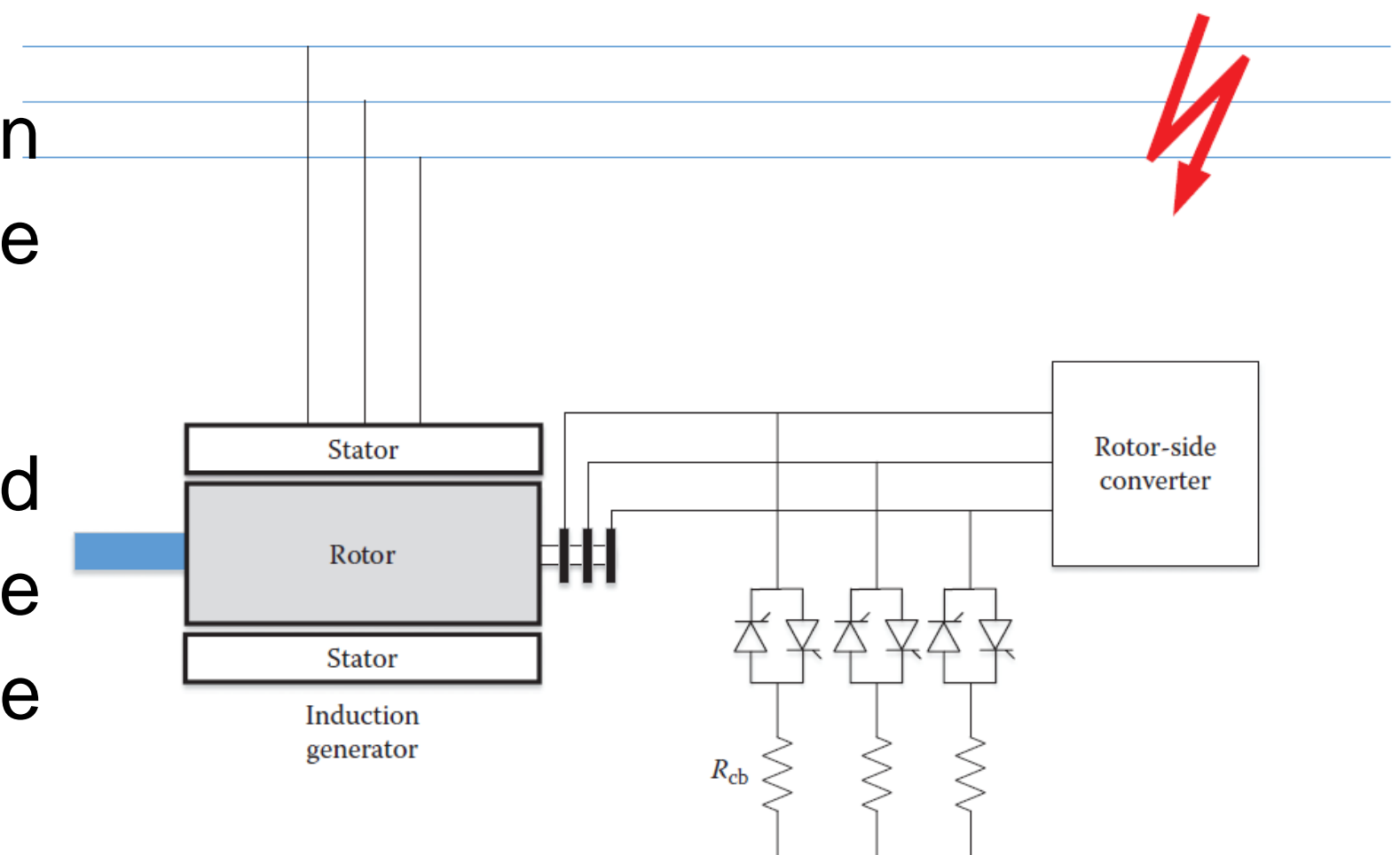
- When the absolute value of the **rotor current**, or the **dc link voltage**, reaches an upper threshold,
  - the crowbar circuit is **activated**
  - the triggering of the RSC transistors are **blocked**.
- The crowbar resistance prevents the damaging high current of the rotor from flowing through the converter.



# Crowbar System

The machine behaves like an induction machine with **dynamic braking** resistance that is directly coupled to the grid.

In most designs, the crowbar is activated for up to **2 seconds**, which is enough time to clear most temporary faults, or activate most circuit breakers.

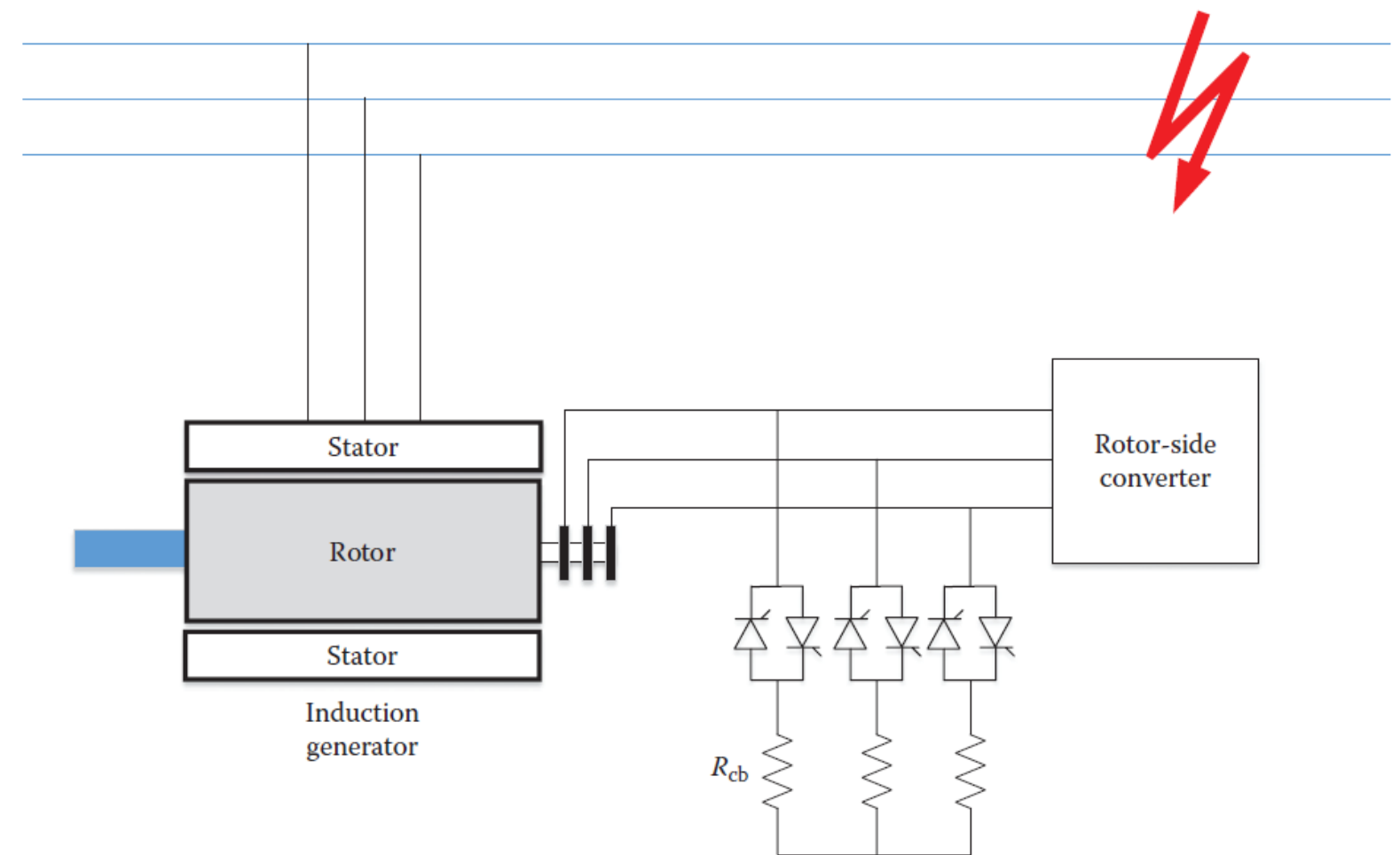


# Crowbar System

When the crowbar circuit is active, the reactive power of the generator will have to come from the grid.

This is a violation of some grid codes as it exacerbate the voltage dip.

In countries with reactive current injection requirement during fault, the crowbar system must actuate for a short time in order to enable some reactive power injection by the RSC and GSC.





# Crowbar System

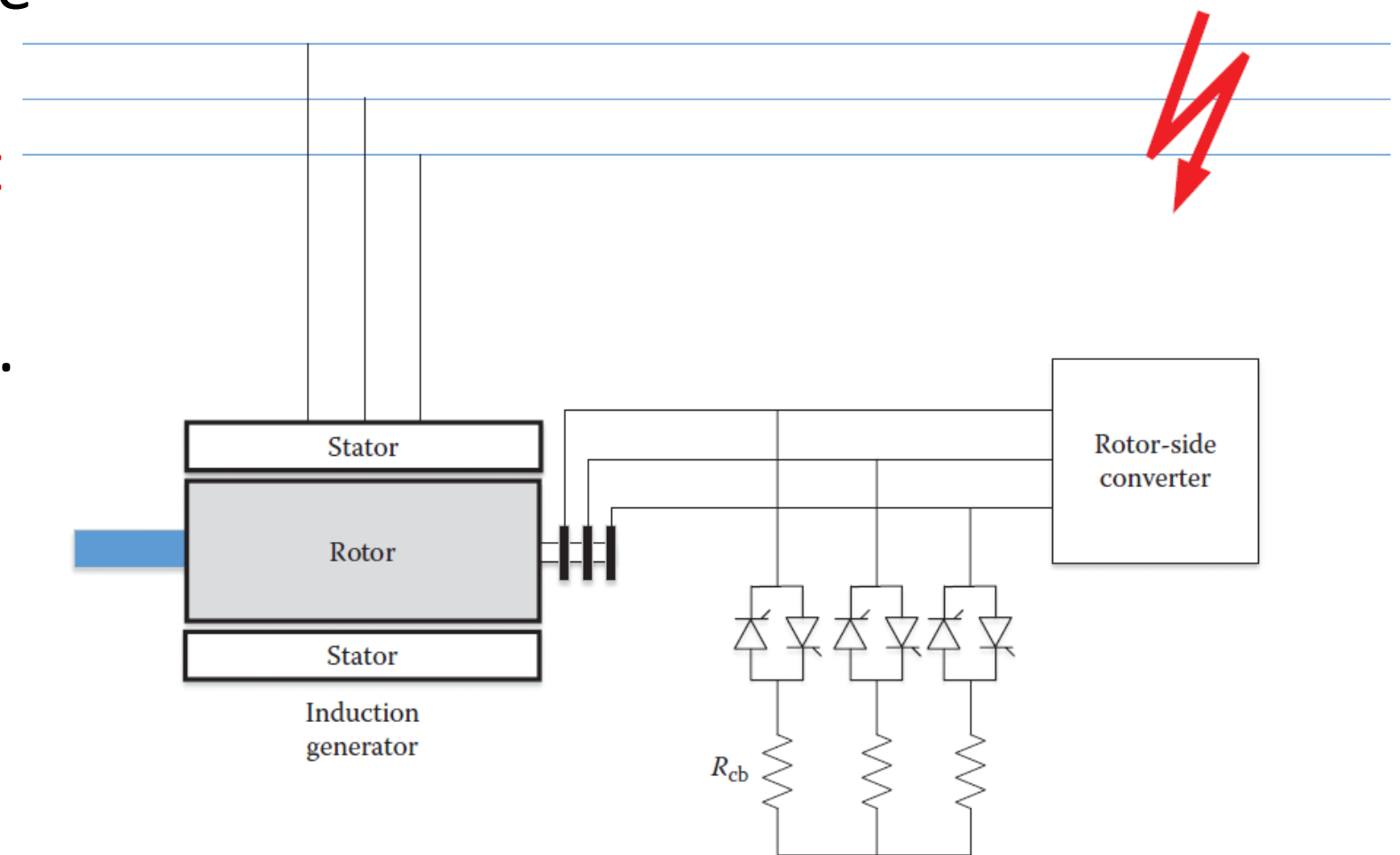
The value of the crowbar resistance should be chosen carefully.

- high enough to limit and damp the current and torque transients
- low enough to reduce the voltage across it.

If resistance is too large

- transient voltage across the crowbar terminals may become higher than the dc link voltage.
- high transient currents could flow through the antiparallel diodes

This could damage the capacitor due to overcharging and power electronic devices due to overvoltage.



# Crowbar System



Why not just short the windings of the rotor during faults?

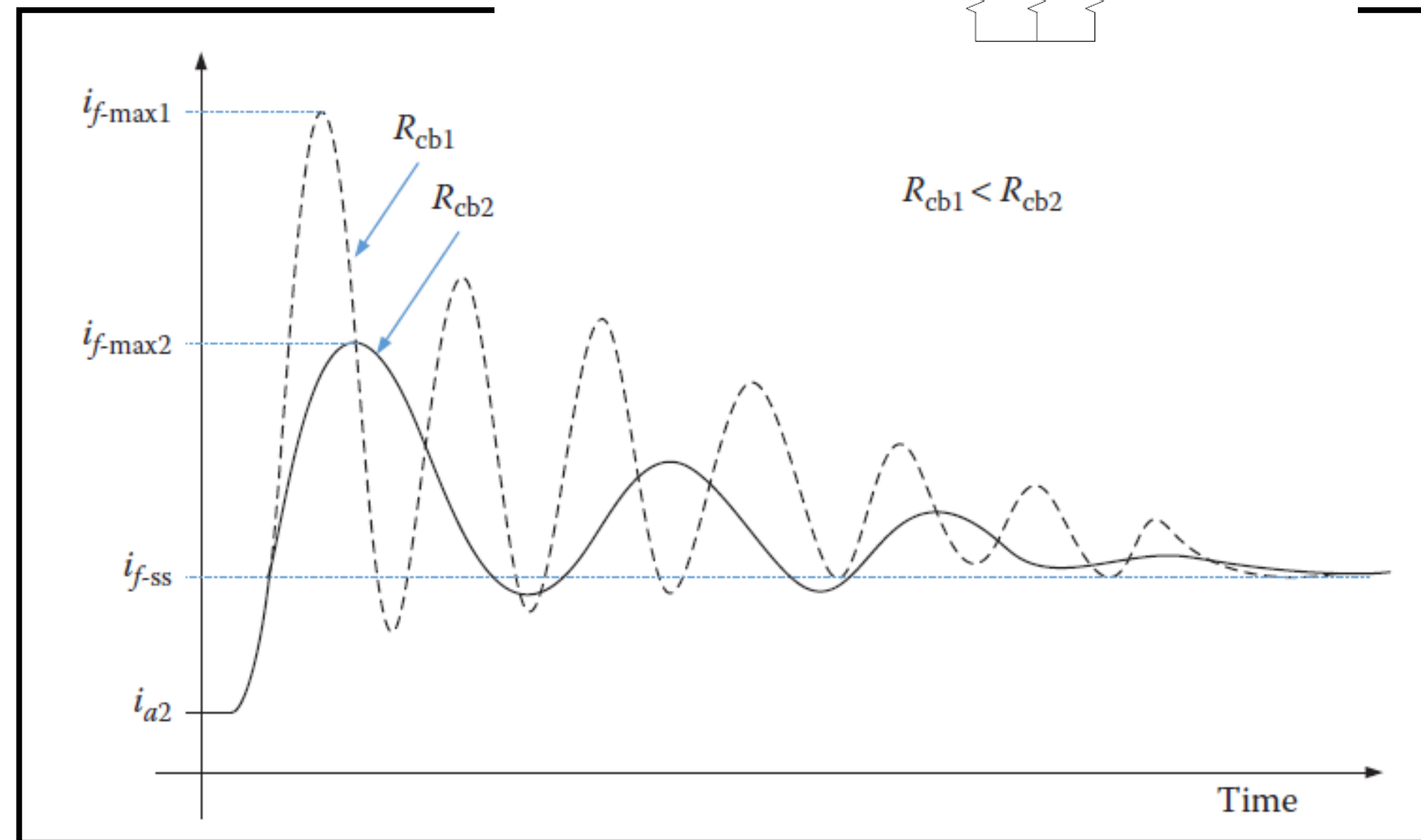
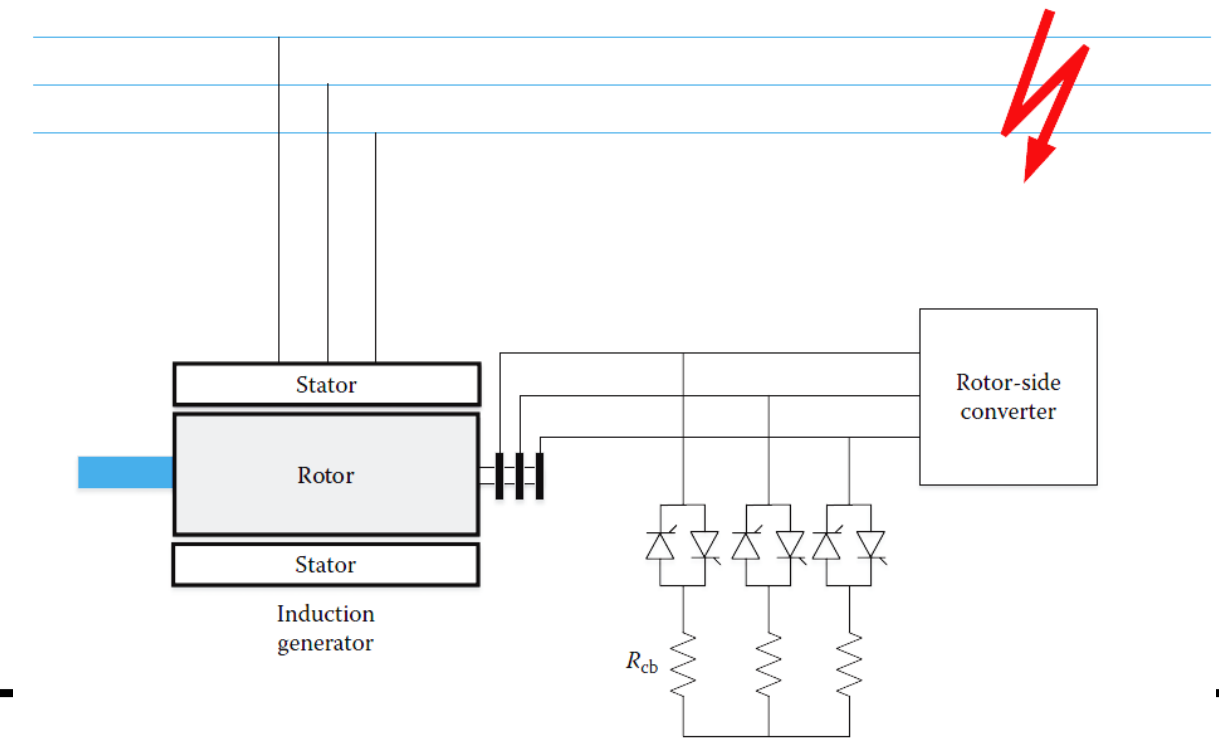
Because the generator has small winding resistances that provide poor damping of current transients.

$$\frac{d}{dt} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} = \frac{1}{L} \begin{bmatrix} r_1 L_{22} & -L_{11} L_{22} \omega + L_{12}^2 s \omega & -(r_2 + R_{cb}) L_{12} & -L_{12} L_{22} \omega_2 \\ L_{11} L_{22} \omega - L_{12}^2 s \omega & r_1 L_{22} & L_{12} L_{22} \omega_2 & -(r_2 + R_{cb}) L_{12} \\ -r_1 L_{12} & L_{11} L_{12} \omega_2 & (r_2 + R_{cb}) L_{11} & L_{12}^2 \omega - L_{11} L_{22} s \omega \\ -L_{11} L_{12} \omega_2 & -r_1 L_{12} & -L_{12}^2 \omega + L_{11} L_{22} s \omega & (r_2 + R_{cb}) L_{11} \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} + \frac{1}{L} \begin{bmatrix} -L_{22} & 0 & L_{12} & 0 \\ 0 & -L_{22} & 0 & L_{12} \\ L_{12} & 0 & -L_{11} & 0 \\ 0 & L_{12} & 0 & -L_{11} \end{bmatrix} \begin{bmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{bmatrix}$$

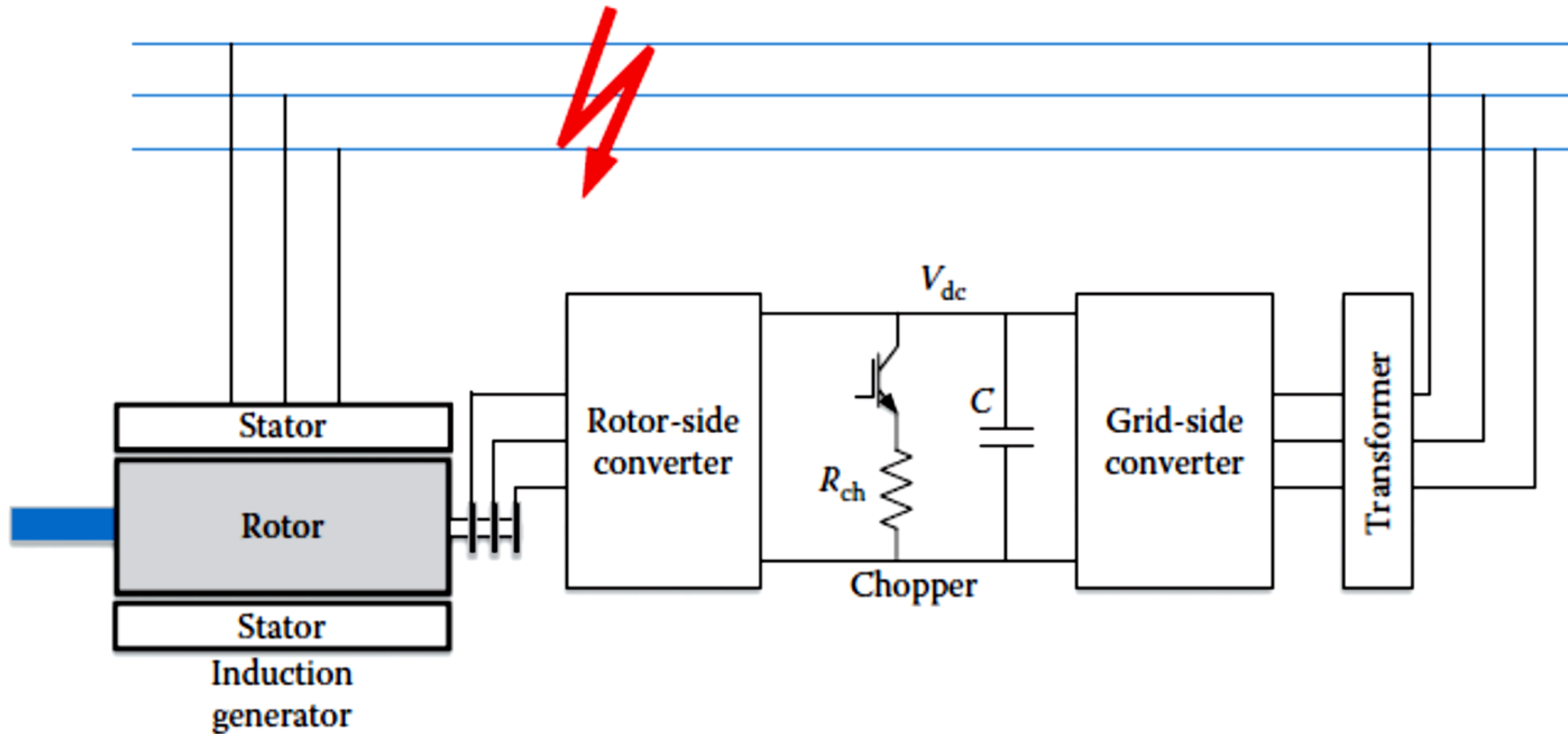
# Transient Protection: Crowbar System

$$\frac{d}{dt} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} = \hat{\mathbf{A}}_{\omega} \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{d2} \\ i_{q2} \end{bmatrix} + \mathbf{B} \begin{bmatrix} v_{d1} \\ v_{q1} \\ v_{d2} \\ v_{q2} \end{bmatrix}$$

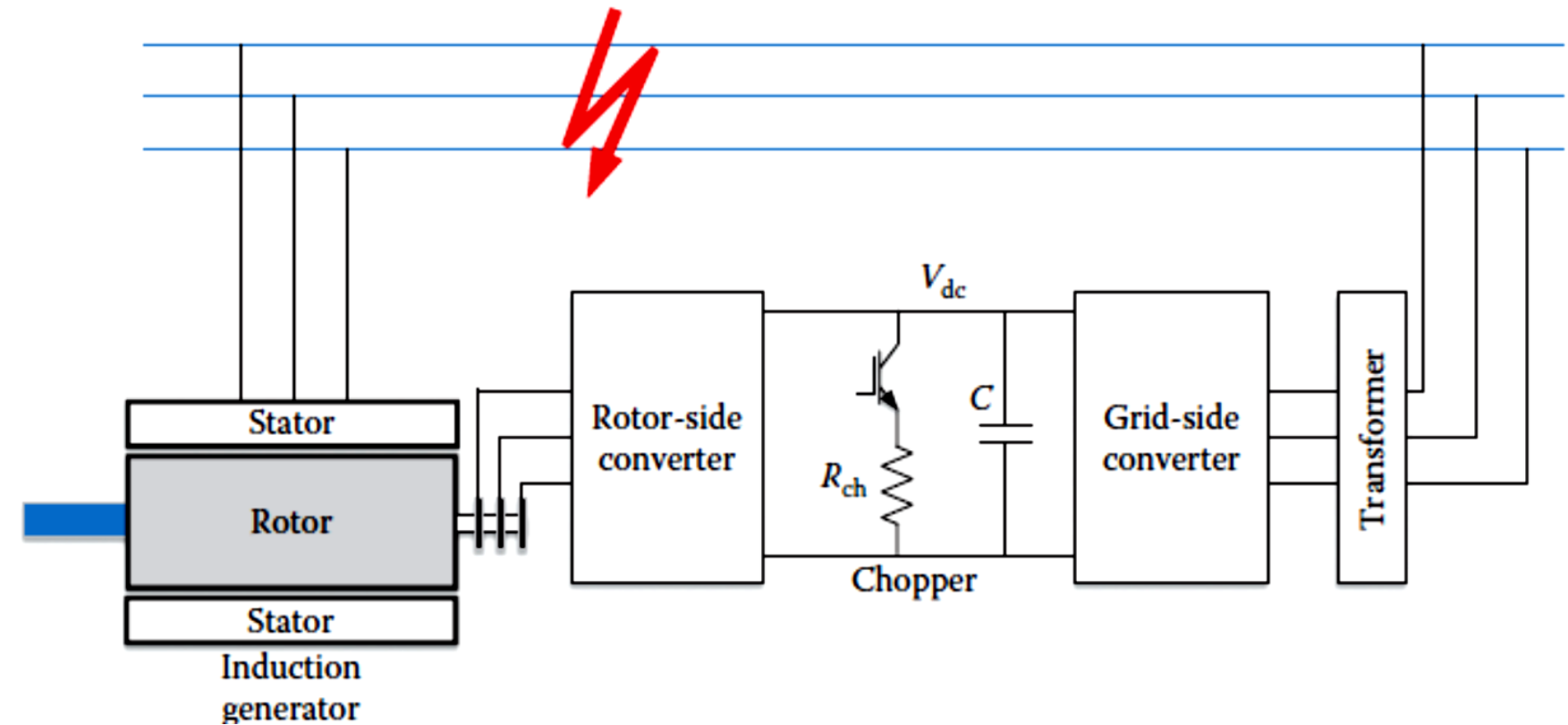
Eigenevalues of  $\hat{\mathbf{A}}_{\omega}$  are enhanced



# Chopper System



# Chopper System



- If the current transient during fault reaches the dc link, it could elevate its voltage due to the acquired extra energy by the capacitor.
- To address this problem, the extra energy is dissipated in a resistance located in the dc link and is in parallel with the capacitor.
- The resistance is switched in when needed and the duty ratio of its transistor is adjusted to consume the need amount of energy.
- Note that for the extra power to reach the chopper, it must pass through the RSC. This is the drawback of this method as the RSC components must be designed to handle high currents.



# Chopper System

The design limit

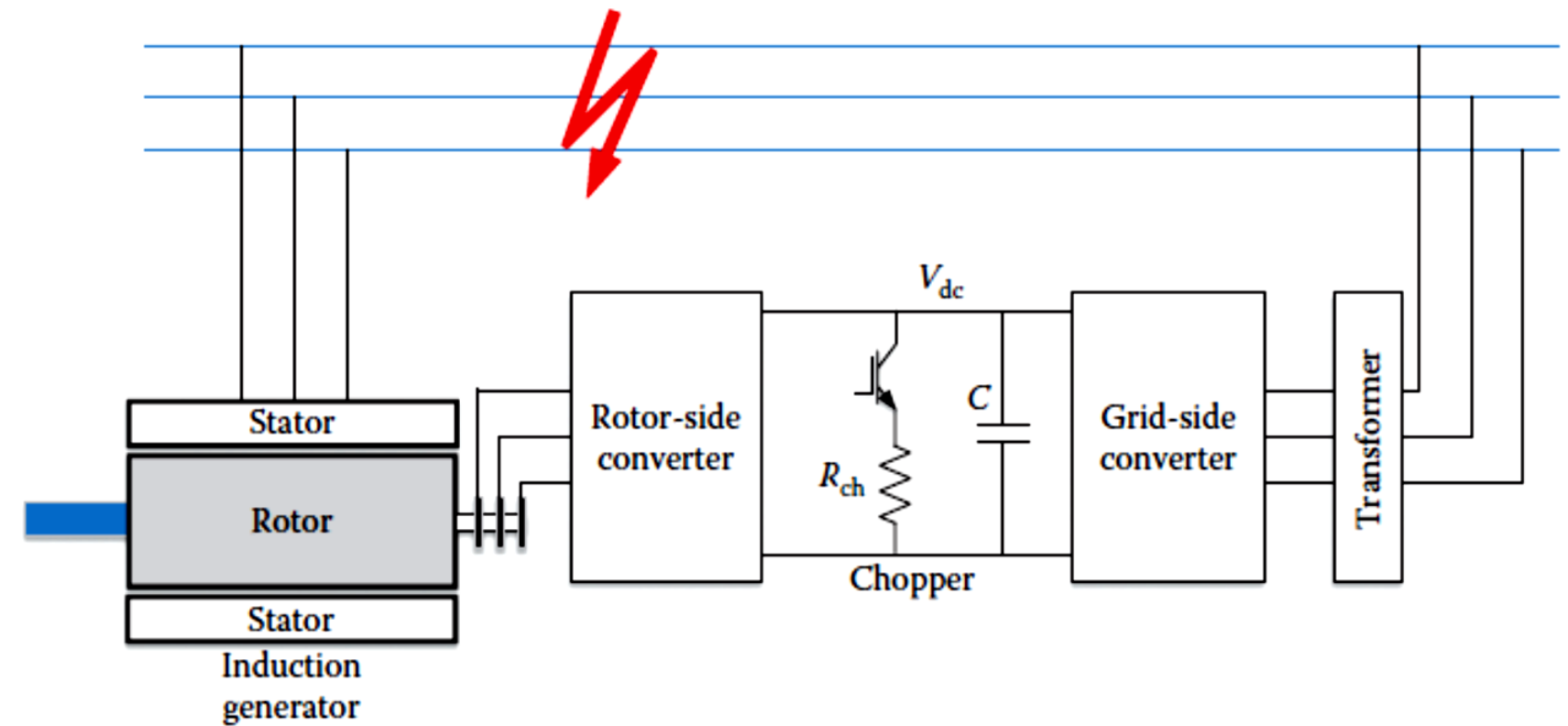
$$E_{co} = \frac{1}{2} C V_{dco}^2$$

When voltage increases

$$E_c = \frac{1}{2} C V_{dc}^2$$

The extra energy

$$\Delta E_c = E_c - E_{co} = \frac{1}{2} C (V_{dc}^2 - V_{dco}^2)$$



where:

$E_{co}$  is the stored energy in the capacitor during steady-state operation

$V_{dco}$  is the steady-state voltage of the dc link

$C$  is the capacitance of the capacitor

where:

$E_c$  is the stored energy in the capacitor after the current surge

$V_{dc}$  is the new voltage of the dc link,  $V_{dc} > V_{dco}$

# Chopper System

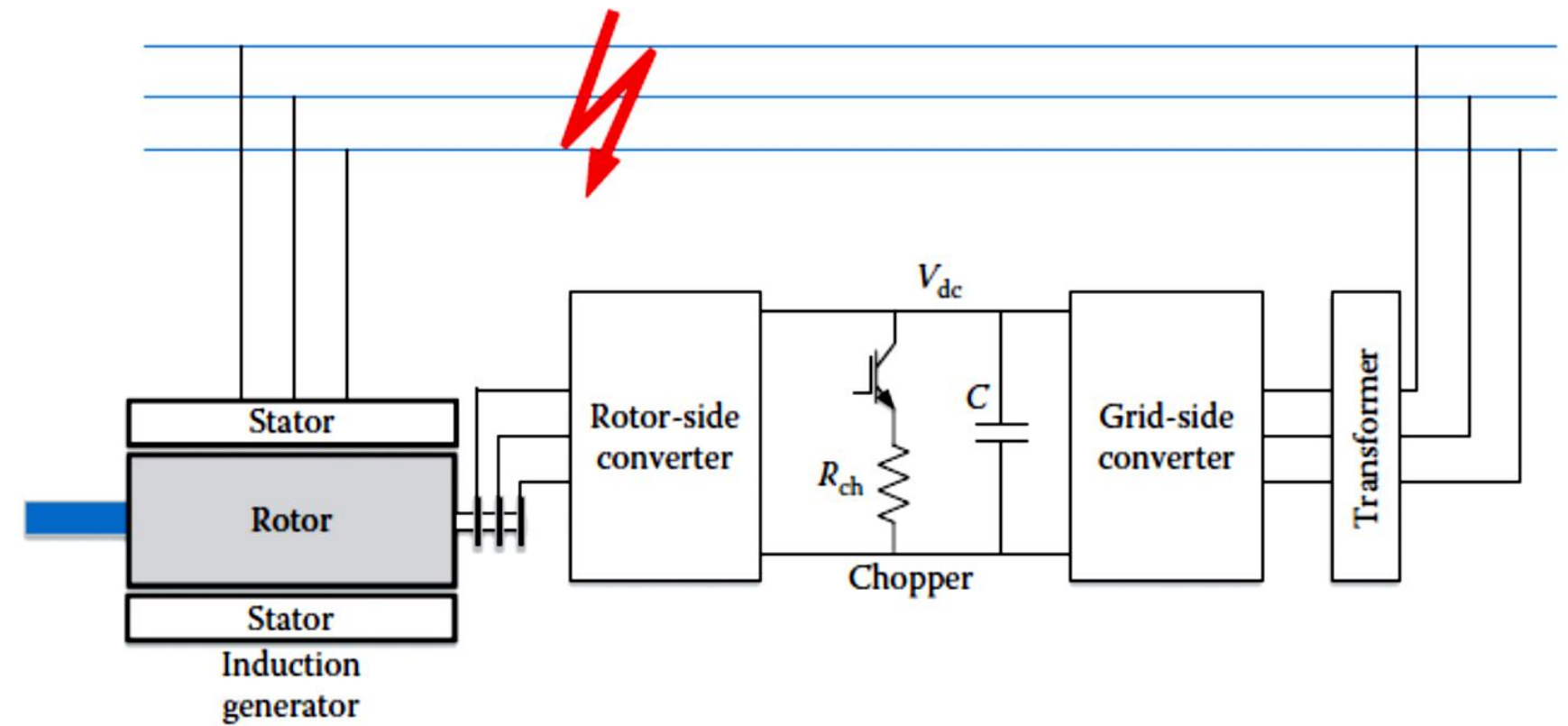
Voltage across the chopper resistance

$$V_R = V_{dc} k \quad k = \frac{t_{on}}{\tau}$$

Power consume by the resistance

$$P_R = \frac{V_R^2}{R_{ch}} = \frac{V_{dc}^2}{R_{ch}} k^2$$

$$E_R = P_R t = \frac{V_{dc}^2}{R_{ch}} k^2 t = \Delta E_c$$



The duty ratio of the chopper can be computed

$$k = \sqrt{\frac{C R_{ch}}{2t} \left( 1 - \left( \frac{V_{dco}}{V_{dc}} \right)^2 \right)}$$

# Electromechanical Protection

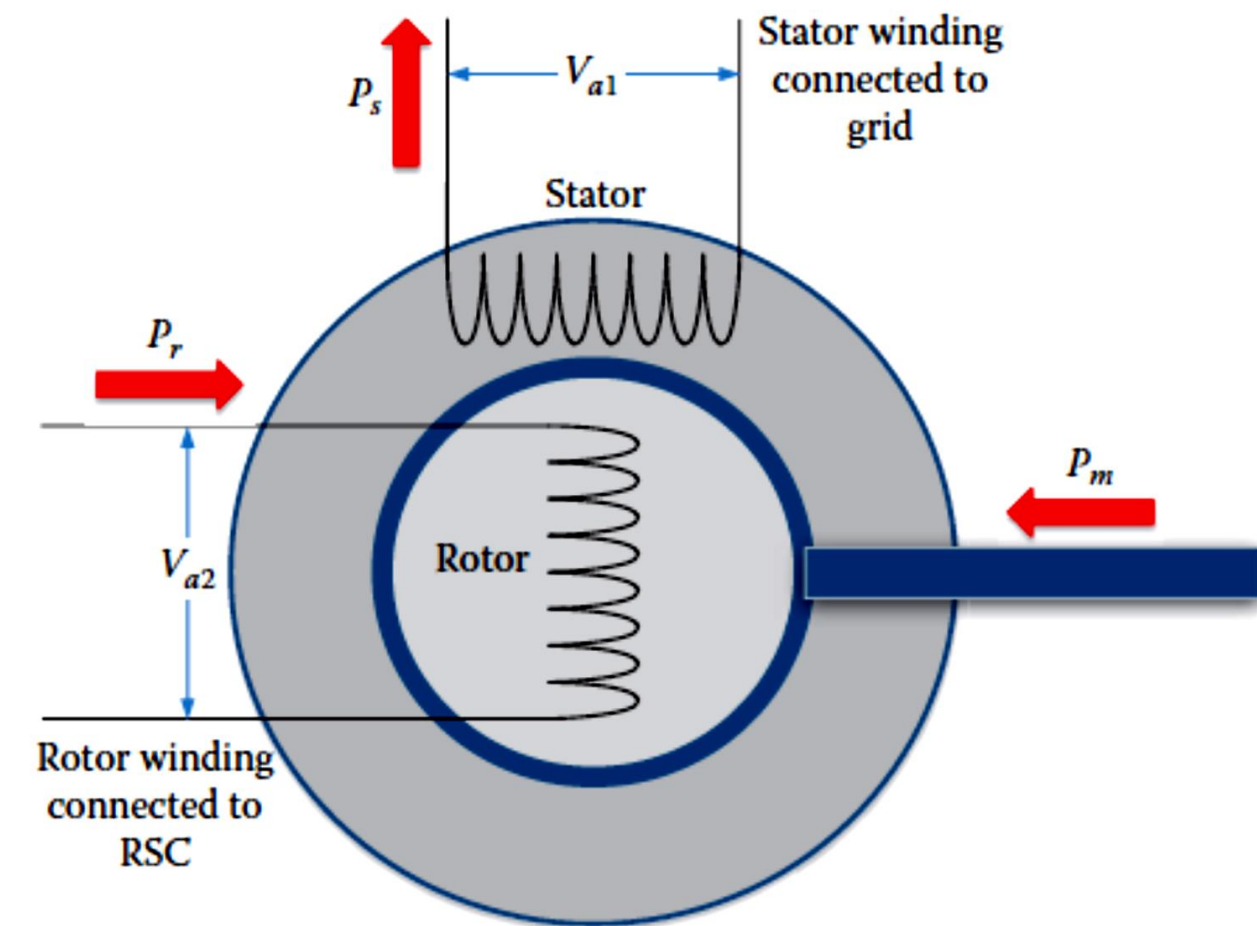
Ignore losses

Normal operation  $P_m = (P_s + P_r)$

During fault  $P_m > (P_s + P_r)$

Over time  $E_m > (E_s + E_r)$

$$E_m - (E_s + E_r) = KE = \frac{1}{2} J \frac{d\omega_2}{dt}$$



- Speed continuously increasing and could result in the blades speeding up to a level that could damage the turbine.
- To protect the turbine from over speeding, we have essentially two options:
  1. Store the extra energy somewhere outside the rotating mass
  2. Dissipate the extra energy in external circuits

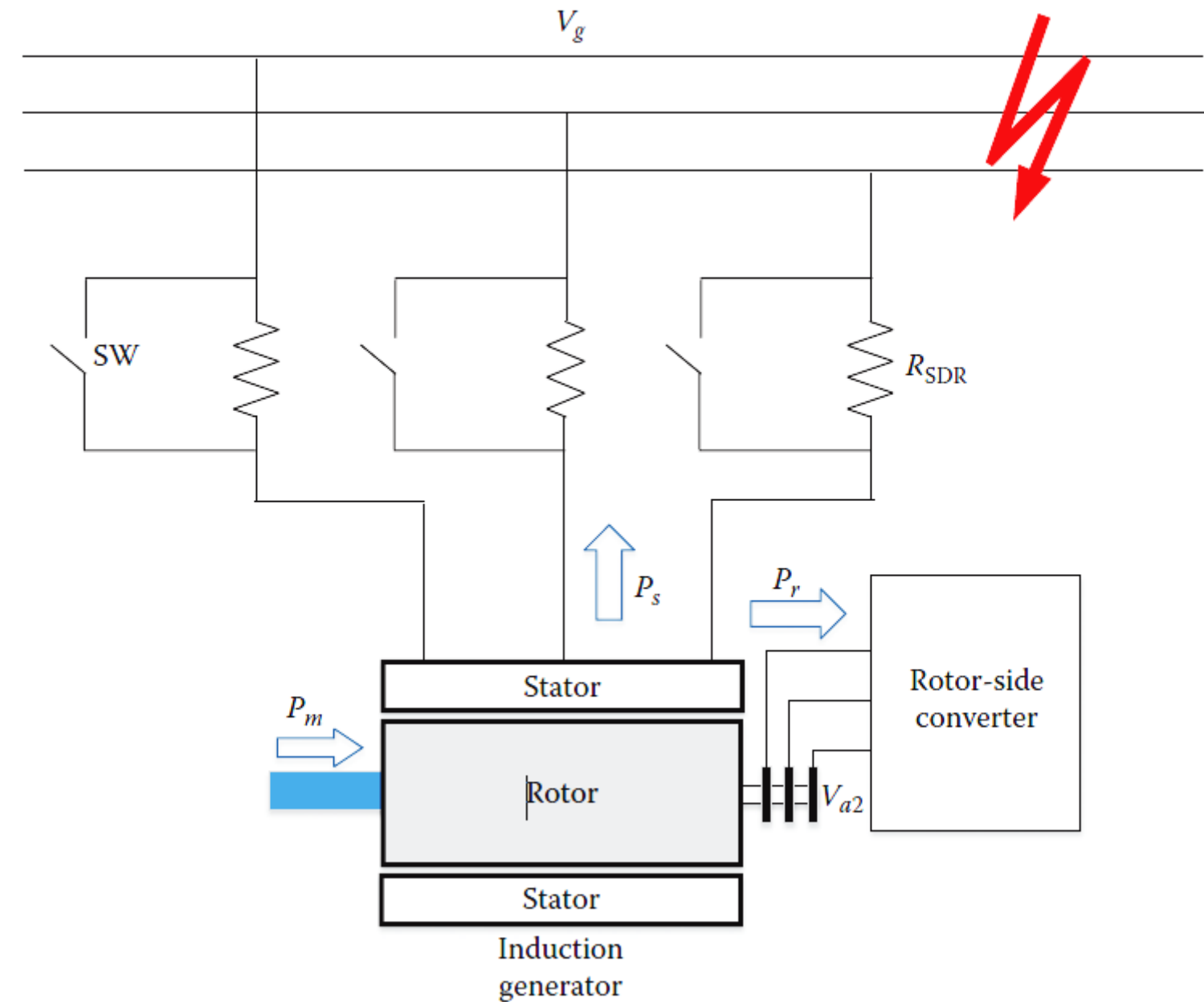
# Electromechanical Protection: Stator Dynamic Resistance

$$p_R = 3 i^2 R_s$$

$$P_R = \frac{3}{\pi} \left[ \int_0^\alpha i^2 R_s d\omega t + \int_\beta^\pi i^2 R_s d\omega t \right]$$

$$P_R = \frac{3R_s}{\pi} \left[ \int_0^\alpha I_{max}^2 \sin^2 \omega t d\omega t + \int_\beta^\pi I_{max}^2 \sin^2 \omega t d\omega t \right]$$

$$P_R = \frac{3 I_{a1}^2 R_s}{\pi} \left( \pi - \gamma - \frac{\sin 2\alpha}{2} + \frac{\sin 2\beta}{2} \right)$$



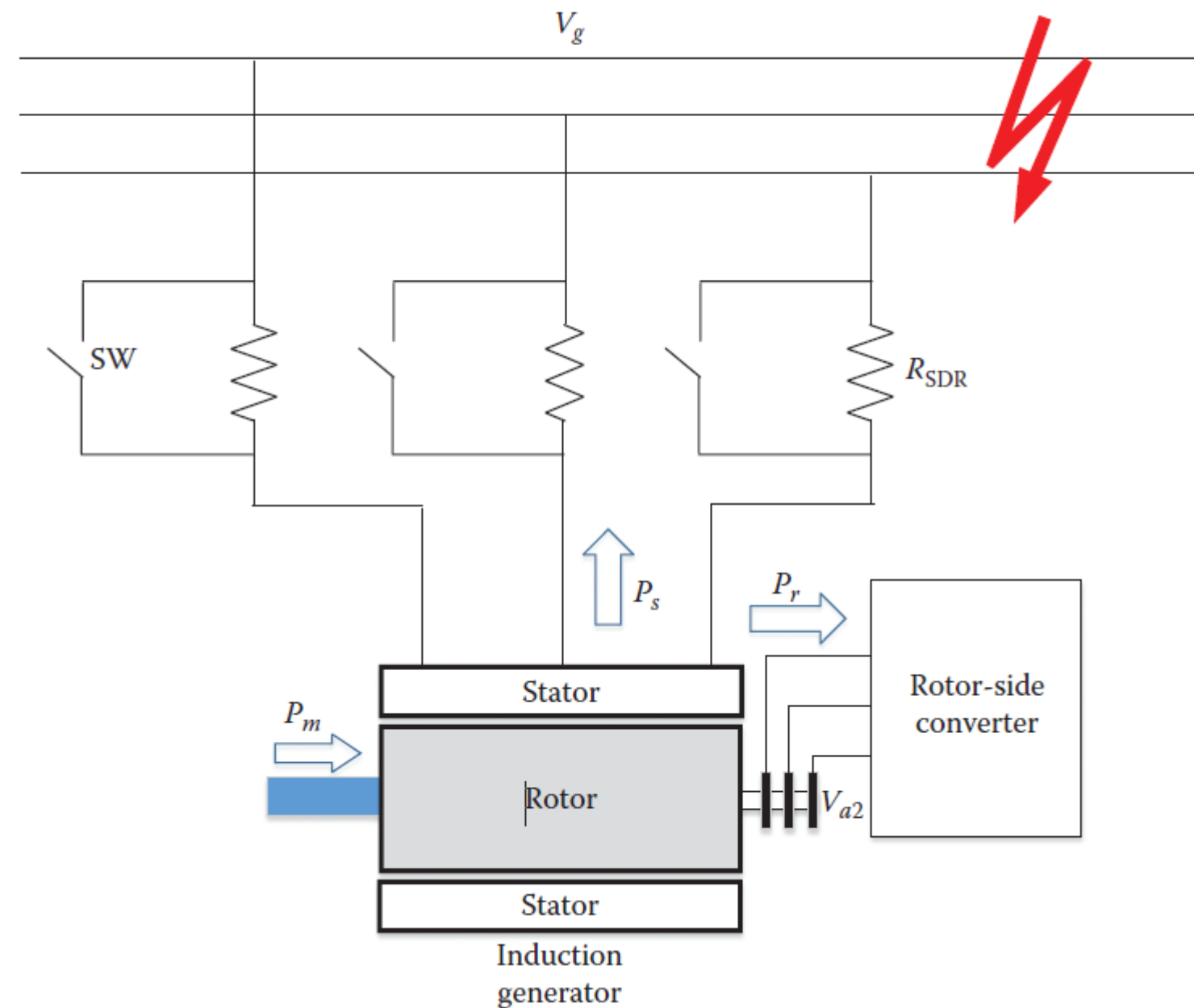
# Electromechanical Protection: Stator Dynamic Resistance

$$P_R = \frac{3 I_{a1}^2 R_s}{\pi} \left( \pi - \gamma - \frac{\sin 2\alpha}{2} + \frac{\sin 2\beta}{2} \right)$$

Define effective resistance  $R_e$

$$R_e = \frac{R_s}{\pi} \left( \pi + \gamma - \frac{\sin 2\alpha}{2} + \frac{\sin 2\beta}{2} \right)$$

$$P_R = 3 I_{a1}^2 R_e = \Delta P_s$$





# Website

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Use the	To check for announcements
course	
website:	To get copies of the lecture slides and other material
	To get the homework and project assignments

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<https://lms.ttu.edu.jo/>

