Wastewater Treatment Lecture 4

Wastewater Engineering – Population

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In the design of any waterworks project it's necessary to estimate the ***amount of water required***. حساب كميات المياه المطلوبة

This involves determining:

1- the number of people who will be served عدد السكان الذي ستتم خدمتهم

2- their per capita water consumption, نصيب الفرد من استهلاك المياه

together with an analysis of the factors that may operate to affect consumption.

Forecasting Population : توقع السكان

Design of water supply and sanitation scheme is based on the projected population of a particular city, estimated for the design period. Any underestimated value will make system inadequate for the purpose intended; similarly overestimated value will make it costly. Changes in the population of the city over the years occur, and the system should be designed taking into account of the population at the end of the design period.

أهمية تقدير عدد السكان من خلال الفترة التصميمية للمشروع

Raw Water source

Water Quantity Estimation

The quantity of water required for municipal uses for which the water supply scheme has to be designed requires following data:

1. **Water consumption rate (Per Capita Demand in litres per day per head)**
2. **Population to be served.**

Quantity= Per capita demand x Population

Forecasting Population :

It is usual to express water consumption in ***liters or gallons*** per capita per day, obtaining this figure by dividing the total number of people in the city into the total daily water consumption.

معدل استهلاك اليومي يحسب من خلال تقسيم عدد السكان لمدينة على اجمالي استهلاك المياه اليومي

For many purposes the average daily consumption is convenient. It is obtained by dividing the population into the total daily consumption averaged over one year.

وغالبا يتم اخذ متوسط الاستهلاك السنوي للمدينة

Forecasting Population :

It must be realized, however, that using the total population may , in some cases, result in serious inaccuracy, since a large proportion of the population may be served ***by privately owned wells.*** A more accurate figure would be the daily consumption per person served.

الابار

بعض السكان يستخدم ابار شخصية لذا ربما يكون عدد السكان غير دقيق لحساب معدل الاستهلاك اليومي

Forecasting Population :

Since population is always a relevant factor in estimating future water use, it is necessary to predict, in some manner, what the future population will be. ***The date in the future for which the projection*** is made depends on the particular component of the system which is being designed.

يعتمد توقع السكان في المستقبل على التاريخ المنوي تصميم المشروع المحدد له

Factors affecting population :

- Increase due to births زيادة المواليد
- Decrease due to deaths نقصان بسبب الوفيات
- Increase/ decrease due to migration الزيادة او النقصان بسبب الهجرات

Population Forecasting Methods :

1- Arithmetic Method : طريقة حسابية

للمدن الكبيرة والقديمة ذات التطور الكبير

This method is suitable for ***large and old city*** with considerable development. If it is used for small, average or comparatively new cities, it will give lower population estimate than actual value. In this method the average increase in population per decade is calculated from the past census reports. This increase is added to the present population to find out the population of the next decade. Thus, it is assumed that the population is increasing at constant rate. تفترض ان عدد السكان يزيد بشكل ثابت

Hence, $\frac{dP}{dt} = K \qquad , \Delta P = K * \Delta t$

In which $\frac{dP}{dT}$ is the rate of change of population with respect to time and K is constant.

Therefore, The Population is estimated from:

$$P_t = P_o + K\Delta t$$

$\frac{dP}{dt}$: rate of change of popualtion معدل الزيادة في السكان

Pt :population at some time in the future عدد السكان في المستقبل

P_0 **:** present or initial population عدد السكان الحالي

Δt **:** period of the projection in decades مقدار السنوات

K **:** population growth rate (constant) معدل الزيادة (رقم ثابت)

Example: 1

The population of a town is obtained from the following population data as follows, estimate the population of the town on 1992 by using Arithmetic method:

Year	1957	1967	1977	1987
Population	58000	65000	73000	81000

Solution :

Year	Population	K
1957	58000	-
1967	65000	700
1977	73000	800
1987	81000	800

$$K = \frac{\Delta P}{\Delta t} \rightarrow K = \frac{65000 - 58000}{10} = 700 \text{ capita}$$

$$K = \frac{73000 - 65000}{10} = 800 \text{ capita}$$

$$K = \frac{8100 - 73000}{10} = 800 \text{ capita}$$

$$k\ avg. = \frac{700 + 800 + 800}{3} = 767\ capita$$

Δt =1992-1987 = 5 years $\longrightarrow$ $P_t = P_0 + K\Delta t$

Population forecast for year 1992 is : P_{1992} = 81000 + 767 x 5 = 84835 capita

2- Uniform Percentage Method : طريقة النسبة المئوية الموحدة

This method assumes uniform rate of increase, that is the rate of increase is proportional to population.

معدل الزيادة يتزايد مع عدد السكان

$$\frac{dP}{dt} = K' \quad , \quad \ln P_t = \ln P_o + K' * \Delta t, \quad K' = \frac{\ln P - \ln P_o}{\Delta t}$$

- $\frac{dP}{dt}$: rate of change of population
- P_t : population at some time in the future
- P_o : present or initial population
- Δt : period of the projection in decades
- K' : population growth rate

Example 2:

For the same data given in example 1 , estimate population on 1992 using logistic method :

$$K' = \frac{ln\ \Delta P}{\Delta t} \rightarrow K'_1 = \frac{\ln 65000 - \ln 58000}{10} = 0.0114$$

$\rightarrow K'_2 = \frac{\ln 73000 - \ln 65000}{10} = 0.0116$

$\rightarrow K'_3 \quad \frac{\ln 81000 - \ln 73000}{10} = 0.0104$ $\qquad \rightarrow K_{ave} = 0.0111$

$= \ln P_t = \ln P_o + K'\Delta t \quad \rightarrow \ln(P_{1992}) = \ln(81000) + 0.0111 \times 5 = 11.3577$

$\therefore P_{1992} = 85622$ Capita

Arithmetic Increase Method

The population of a town is shown below:

Year	Population
1960	32,000
1970	40,000
1980	49,000
1990	57,000

Estimate the population in the year 1995 using the arithmetic increase method.

Student Steps Expected:

1. Compute:

$$K_1 = (40,000 - 32,000)/10$$

$$K_2 = (49,000 - 40,000)/10$$

$$K_3 = (57,000 - 49,000)/10$$

2. Average K values
3. Compute Δt = 1995 − 1990
4. Multiply K × Δt
5. Add to 1990 population

Arithmetic Increase Method

Population data for a city:

Year	Population
1975	120,000
1985	133,000
1995	148,000
2005	162,000

Estimate the population in 2010 using the arithmetic increase method.

Expected Steps:

$$K_1 = (133,000 - 120,000)/10$$

$$K_2 = (148,000 - 133,000)/10$$

$$K_3 = (162,000 - 148,000)/10$$

Then:

- Average K
- Δt = 2010 − 2005 = 5 years
- Add the increase to 2005 population

Arithmetic Increase Method

A town has the following population records:

Year	Population
1980	75,000
1990	84,500
2000	94,000
2010	103,000

Predict the population in 2018 using the arithmetic increase method.

Expected Steps:

$$K_1 = (84,500 - 75,000)/10$$

$$K_2 = (94,000 - 84,500)/10$$

$$K_3 = (103,000 - 94,000)/10$$

Average K

Δt = 2018 − 2010 = 8 years

Population = P_{82010} + K × 8

Uniform Percentage Method

Population Data

Year	Population
1975	90,000
1985	102,000
1995	117,000
2005	130,000

Find the population in 2012.

Step 1 — Compute K′

$$K_1' = \frac{\ln(102,000) - \ln(90,000)}{10}$$

$$K_2' = \frac{\ln(117,000) - \ln(102,000)}{10}$$

$$K_3' = \frac{\ln(130,000) - \ln(117,000)}{10}$$

Step 2 — Average K′

$$K' = \frac{K'_1 + K'_2 + K'_3}{3}$$

Step 3 — Δt from 2005 to 2012

$$\Delta t = 7 \text{ years}$$

$$\Delta t = 7/10 = 0.7$$

Step 4 — Projection

$$\ln(P_{2012}) = \ln(130,000) + K' \times 0.7$$

Uniform Percentage Method

Population Data

Year	Population
1980	150,000
1990	165,000
2000	182,000
2010	198,000

Estimate population in 2018.

Step 1 — Compute K′ per decade

$$K_1' = \frac{\ln(165,000) - \ln(150,000)}{10}$$

$$K_2' = \frac{\ln(182,000) - \ln(165,000)}{10}$$

$$K_3' = \frac{\ln(198,000) - \ln(182,000)}{10}$$

tep 2 — Average K′

$$K' = \frac{K'_1 + K'_2 + K'_3}{3}$$

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tep 3 — Δt from 2010 → 2018

$$\Delta t = 8 \text{ years}$$

$$\Delta t = 0.8 \text{ decades}$$

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tep 4 — Apply formula

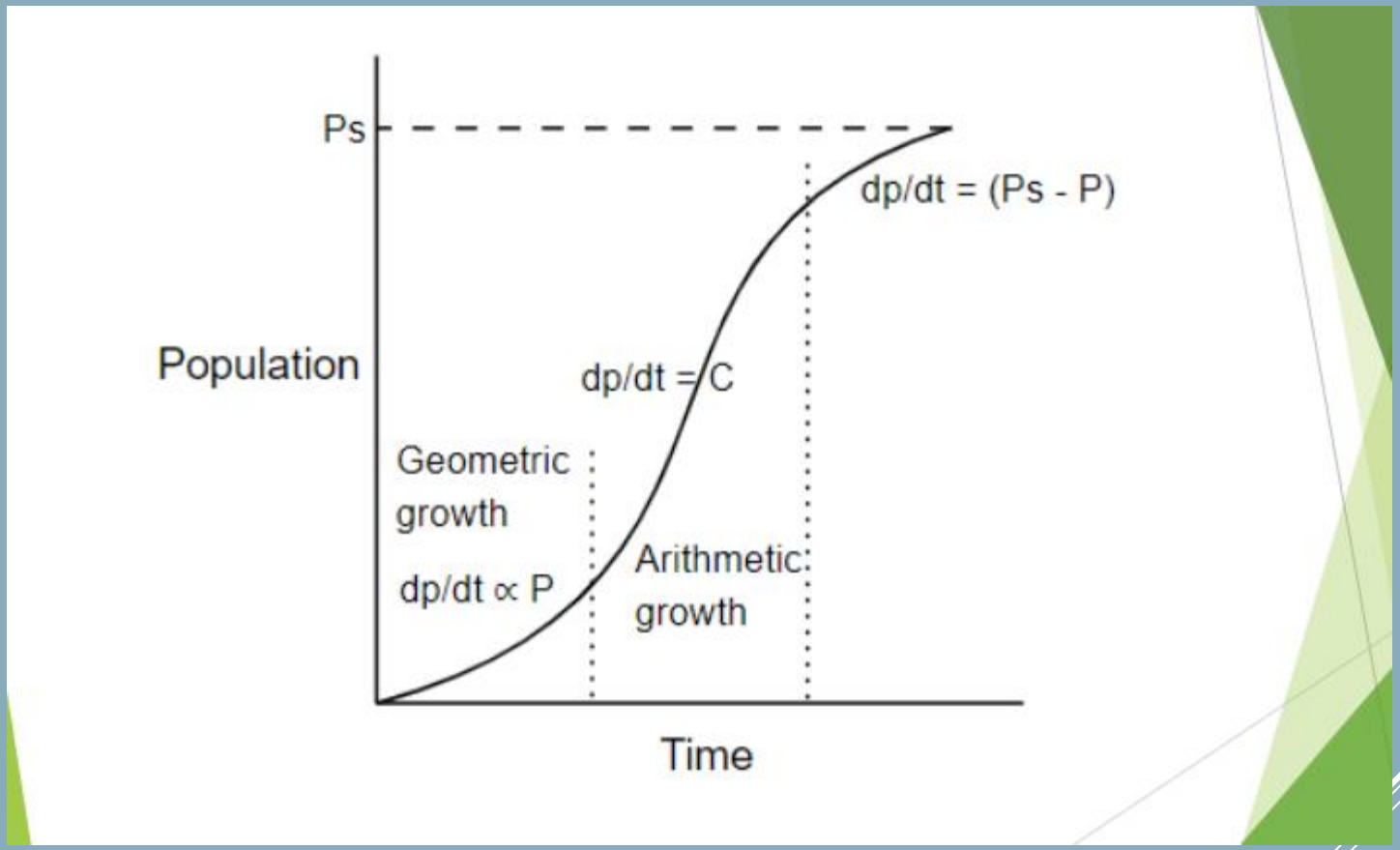
$$\ln(P_{2018}) = \ln(198,000) + K' \times 0.8$$

3- Logistic method : الطريقة لوجستية

This method has an S-shape combining a geometric rate of growth at low population with a declining growth rate as the city approaches some limiting population. A logistic projection can be based on the equation:

$$P = \frac{P_{sat}}{1 + e^{a+b\Delta t}}$$

$$a = \ln \frac{P_{sat} - P_o}{P_o} \quad , \quad P_{sat} = \frac{2P_o P_1 P_2 - P_1^2 (P_o + P_2)}{P_o P_2 - P_1^2} \quad , b = \frac{1}{n} \ln \frac{P_o (P_{sat} - P_1)}{P_1 (P_{sat} - P_o)}$$


Ps
Population
dp/dt = (Ps - P)
dp/dt = C
Geometric
growth
dp/dt ∝ P
Arithmetic
growth
Time

Where $\frac{dP}{dt}$: rate of change of population

P_t : population at some time in the future

P_{sat} : population at saturation level .

P_o : Initial population.

P_1, P_2 : population at time periods.

Δt : number of years after base year

n : period time (Time interval)

K' : population growth rate

Example 3:

In two periods each of 20 years a city has grew from 18000 to 58000 and then to 75800, Determine the extended population for the next 20 years :

Solution:

$$P_{sat} = \frac{2(18000)\,(58000)(75800) - (58000)^2\,(18000 + 75800)}{(18000)(75800) - (58000)^2} = 77582 \text{ capita}$$

$$a = \ln \frac{77582 - 18000}{18000} = 1.197 \quad , \qquad b = \frac{1}{20} \ln \frac{18000\,(77582 - 58000)}{58000\,(77582 - 18000)} = -0.114$$

$$P = \frac{77582}{1 + e^{1.197 - 0.114(60)}} = \mathbf{77308\ capita}$$

4- Declining growth method : طريقة النمو المتناقص

This technique, like the logistic method, assumes that the city has some limiting saturation population, and that its rate of growth is a function of its population deficit:

تفترض أن المدينة لديها بعض التشبع السكاني المحدود، وأن معدل نموها يعتمد على العجز السكاني:

$$\frac{dP}{dt} = K''\,(P_{sat} - P) \quad , \quad K'' = \frac{1}{n} \ln \frac{P_{sat} - P}{P_{sat} - P_o}$$

$$P = P_o + (P_{sat} - P_o)(1 - e^{K''\Delta t})$$

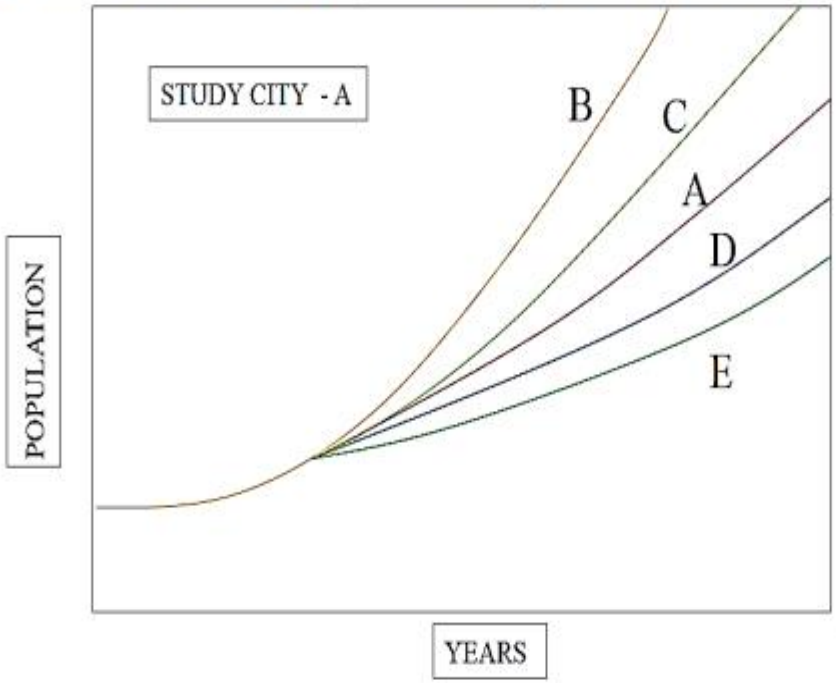
5- Curvilinear method (Comparative graphical extension method) :

This technique, involves the graphical projection of the past population growth curve, continuing whatever trends the historical data indicate. This method includes comparison of the projected growth to the recorded growth of other cities of larger size. The cities chosen for the comparison should be as similar as possible to the city being studied.

مقارنة النمو المتوقع بالنمو المسجل للمدن الأخرى الكبيرة

The curve is extended carefully by comparing with the population curve of some similar cities having the similar condition of growth. The advantage of this method is that the future population can be predicted from the present population even in the absence of some of the past census report.

يتم تمديد المنحنى من خلال المقارنة مع المنحنى السكاني لبعض المدن المماثلة التي تتمتع بحالة نمو مماثلة. وميزة هذه الطريقة هي أنه يمكن التنبؤ بالسكان في المستقبل من السكان الحاليين حتى في حالة عدم وجود بعض من تقرير التعداد السابق



6- Ratio method:

Ratio method of forecasting is based on the assumption that the population of a certain area or a city will increase in the same manner to a larger entity like a province, or a country. It requires calculation of ratio of locals to required population in a series of census years. Projection of the trend line using any of the technique and application of projected ratio to the estimated required population of projected ratio to the estimated required population in the year of interest.

تعتمد طريقة التنبؤ النسبية على افتراض أن عدد سكان منطقة معينة أو مدينة معينة سيزداد بنفس الطريقة إلى كيان أكبر مثل مقاطعة أو بلد.

Example 3:

Estimate the population of a city using ratio method, the design year is 2000, the estimated population of the region in the year 2000 is 988000?

Year	Region ($*10^3$)	City ($*10^3$)	
1960	455	50	0.11
1970	623	61	0.098
1980	766	72	0.094
1990	850	77	0.091
2000	988	??	0.088

نلاحظ ان نسبة النقصان في النسب هي 0.003 لذا فان
النسبة عند سنة 2000 هي 0.088

It is clear that the rate of decrease over 10 years is a constant 0.003

Pop. City = 0.088 * 988000

Pop. City = 86944 capita