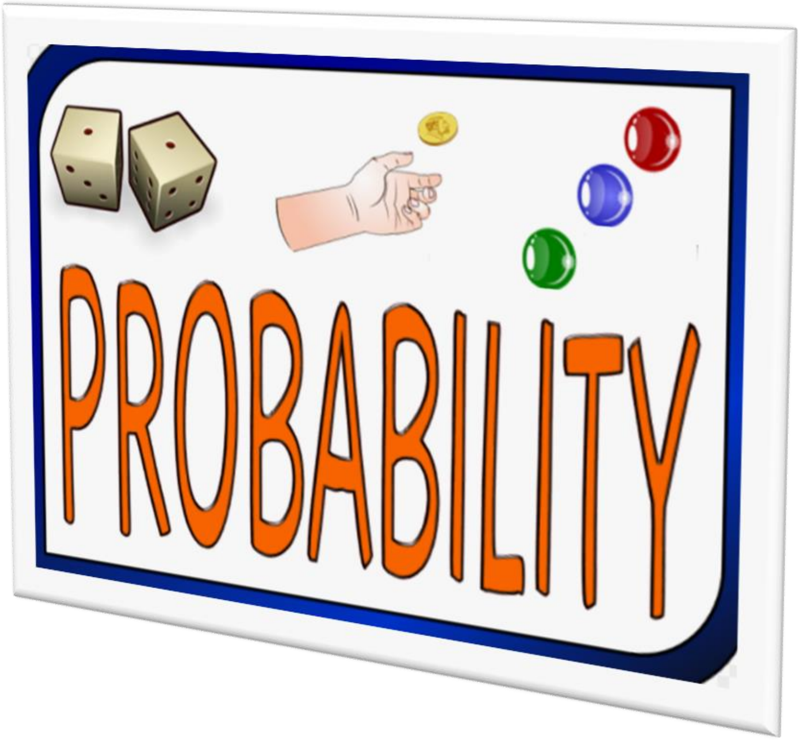
# Introduction to Probability and Statistics

## Topic (4): “ Probability and Probability distribution”




Dr. Heba Ayyoub

Philadelphia University

**Solution to the Exercises for the Fourth Topic**

**Exercise (1):** Find the sample space for the gender of the children if a family has three children. Use $B$ for boy and $G$ for girl.

$$S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$$

**Exercise (2):** Consider the experiment of choosing at random a digit from the digits $0, 1, 2, \cdots, 9$.

1. What is the sample space of this experiment?

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

1. Find the elements of each of the following events:

(a) $A$ = the number chosen is less than or equal to 3.

$$A = \{0, 1, 2, 3\}$$

(b) $B$ = the number chosen is between 4 and 6, inclusive.

$$B = \{4, 5, 6\}$$

(c) $C$ = the number chosen is greater than or equal to 7.

$$C = \{7, 8, 9\}$$

(d) $D$ = the number chosen is less than 4 or larger than 7.

$$D = \{0, 1, 2, 3, 8, 9\}$$

(e) $E$ = the number chosen is an even number.

$$E = \{0, 2, 4, 6, 8\}$$

**Exercise (3):** A sample space contains 10 simple events: $E_1, E_2, ..., E_{10}$. If $P(E_1) = 3P(E_2) = 0.45$ and the remaining simple events are equiprobable, find the probabilities of these remaining simple events.

$$P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) + P(E_7) + P(E_8) + P(E_9) + P(E_{10}) = 1$$

$$0.45 + 0.15 + 8P(E_3) = 1$$

Since, $P(E_3) = P(E_4) = P(E_5) = P(E_6) = P(E_7) = P(E_8) = P(E_9) = P(E_{10})$

$$8P(E_3) = 1 - 0.15 - 0.45$$

$$P(E_3) = \frac{0.4}{8} = 0.05$$

$\therefore P(E_4) = P(E_5) = P(E_6) = P(E_7) = P(E_8) = P(E_9) = P(E_{10}) = 0.05$

**Exercise (4):** The following table gives the two-way classification of 500 students based on gender and whether or not they suffer from math anxiety (تعاني من قلق في الرياضيات).

| Gender | Yes (Y) | No (N) | Σ |
|---|---|---|---|
| Male (M) | 145 | 95 | 240 |
| Female (F) | 190 | 70 | 260 |
| Σ | 335 | 165 | 500 |

If a student randomly selected.

1. What is the probability that this student is female given that she does not suffer from math anxiety?

$$P(F|N) = \frac{P(F \cap N)}{P(N)} = \frac{70/500}{165/500} = \frac{70}{165} = 0.42$$

2. What is the probability that this student suffers from math anxiety given that he is a male?

$$P(Y|M) = \frac{P(Y \cap M)}{P(M)} = \frac{145/500}{240/500} = \frac{145}{240} = 0.6$$

3. What is the probability that this student is a male?

$$P(M) = \frac{240}{500} = 0.48$$

4. What is the probability that this student does not suffer from math anxiety?

$$P(N) = \frac{335}{500} = 0.67$$

**Exercise (5):** An experiment can result in one of five equally likely simple events, $E_1, E_2, ..., E_5$. Events $A$, $B$, and $C$ are defined as follows:

$A: E_1, E_3.$ $P(A) = \frac{2}{5}$

$B: E_1, E_2, E_4, E_5.$ $P(B) = \frac{4}{5}$

$C: E_3, E_4.$ $P(C) = \frac{2}{5}$

Find the **probabilities** associated with these compound events by listing the simple events in each.

a) $A^c$ $\rightarrow P(A^c) = 1 - \frac{2}{5} = \frac{3}{5}$

b) $A \cap B$ $\rightarrow P(A \cap B) = \frac{1}{5}$

c) $B \cap C$ $\rightarrow P(B \cap C) = \frac{1}{5}$

d) $A \cup B$ $\rightarrow P(A \cup B) = \frac{5}{5} = 1$

e) $B|C$ $\rightarrow P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{1/5}{2/5} = \frac{1}{2}$

f) $A|B$ $\rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/5}{4/5} = \frac{1}{4}$

g) $A \cup B \cup C$ $\rightarrow P(A \cup B \cup C) = \frac{5}{5} = 1$

h) $(A \cap B)^c$ $\rightarrow P(A \cap B)^c = 1 - P(A \cap B) = 1 - \frac{1}{5} = \frac{4}{5}$

**Exercise (6):** Suppose that $P(A)$ = 0.4 and $P(B)$ = 0.2. If events $A$ and $B$ are **independent**, find these probabilities:

**a.** $P(A \cap B) = P(A)P(B) = (0.4)(0.2) = 0.08$

**b.** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.2 - 0.08 = 0.52$

**Exercise (7):** Suppose that $P(A) = 0.3$ and $P(B) = 0.5$. If events $A$ and $B$ are **mutually exclusive**, find these probabilities:

**a.** $P(A \cap B) = 0$

**b.** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.2 = 0.6$

**Exercise (8):** Suppose that $P(A) = 0.4$ and $P(A \cap B) = 0.12$.

**a.** Find $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.12}{0.4} = 0.3$$

**b.** Are events $A$ and $B$ mutually exclusive?

Events $A$ and $B$ are **not** mutually exclusive. Since $P(A \cap B) = 0.12$, which is **not** zero.

**c.** If $P(B)$ = 0.3, are events $A$ and $B$ independent?

$$P(A \cap B) \overset{??}{=} P(A)P(B)$$

$$0.12 \overset{??}{=} (0.4)(0.3)$$

$$0.12 = 0.12$$

∴ Events $A$ and $B$ **are independent.**

**Exercise (9):** An experiment can result in one or both of events $A$ and $B$ with the probabilities shown in this probability table:

| | $A$ | $A^c$ | |
|---|---|---|---|
| $B$ | 0.34 | 0.46 | 0.8 |
| $B^c$ | 0.15 | 0.05 | 0.2 |
| | 0.49 | 0.51 | 1 |

1) Find the following probabilities:

**a.** $P(A) = 0.49$

**b.** $P(B) = 0.8$

**c.** $P(A \cap B) = 0.34$

**d.** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.49 + 0.8 - 0.34 = 0.95$

**e.** $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.34}{0.8} = 0.425$

**f.** $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.34}{0.49} = 0.69$

2) Are $A$ and $B$ independent?

$$P(A \cap B) \overset{??}{=} P(A)P(B)$$

$$0.34 \overset{??}{=} (0.49)(0.8)$$

$$0.34 \neq 0.392$$

$\therefore$ Events $A$ and $B$ **are Not independent.**

**Exercise (10):** Construct a probability distribution for a family with 4 children. Let $X$ be the number of girls.

$$S = \{BBBB, BBBG, BBGB, BBGG, BGBB, BGBG, BGGB, BGGG, GBBB, GBBG, GBGB, GBGG, GGBB, GGBG, GGGB, GGGG\}$$

$$x = 0, 1, 2, 3, 4$$

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{1}{16}$ | $\frac{4}{16}$ | $\frac{6}{16}$ | $\frac{4}{16}$ | $\frac{1}{16}$ |

or

$$P(X = x) = \begin{cases} \frac{1}{16}, & x = 0, 4 \\ \frac{4}{16}, & x = 1, 3 \\ \frac{6}{16}, & x = 2 \\ 0, & o.w \end{cases}$$

**Exercise (11):** The discrete random variable $X$ has probability distribution given by the formula

$$P(X = x) = c(3 - x); \quad x = 0, 1, 2, 3.$$

Find the value of $c$, then evaluate $P(X \leq 2)$.

$$c(3-0) + c(3-1) + c(3-2) + c(3-3) = 1$$

$$3c + 2c + c = 1$$

$$6c = 1$$

$$c = \frac{1}{6}$$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | $\frac{3}{6}$ | $\frac{2}{6}$ | $\frac{1}{6}$ | 0 |

- $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = 1$

**Exercise (12):** True or False.

1) The probability of an event always lies between 0 and 1. **True**

2) The sum of probabilities of all possible outcomes in a sample space is always greater than 1. **False**

3) If two events are mutually exclusive, their intersection is zero. **True**

4) An independent event means that the occurrence of one event does not affect the probability of the other event. **True**

5) The total probability of all values in a probability distribution must sum to 1. **True**

6) The variance of a probability distribution measures the spread of data around the mean. **True**