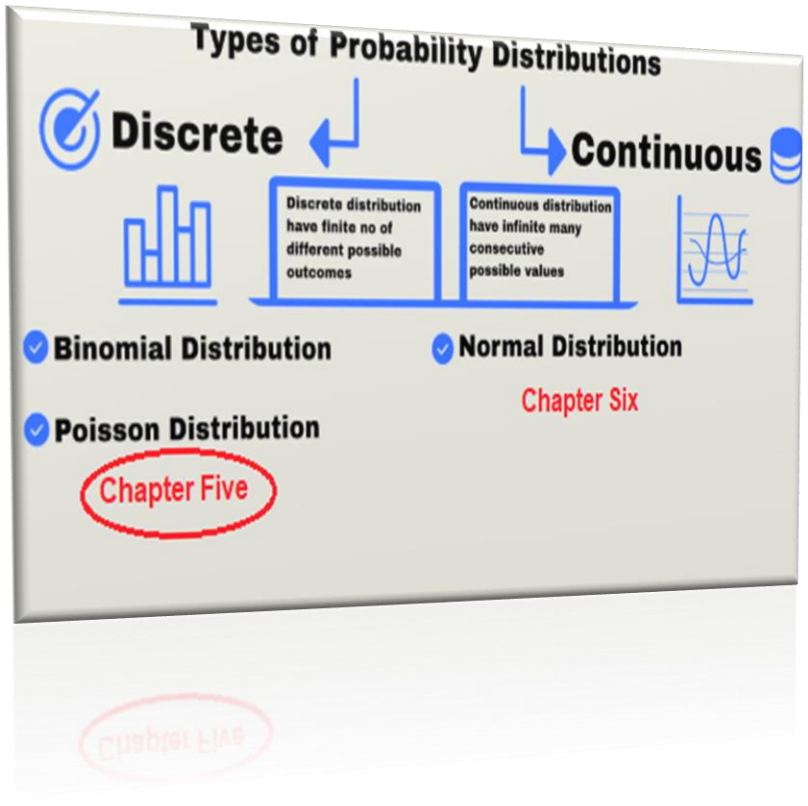
Introduction to Probability and Statistics

Topic 5: " Several Useful Discrete Distributions"



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Topic (5): “Several Useful Discrete Distributions”

1 Introduction

Discrete random variables are used in many practical applications. Two important discrete random variables: binomial and Poisson distributions.

2 The Binomial Probability Distribution

1. The experiment consists of n identical trial.
2. The outcome of each trail can be classified as success or failure.
3. The probability of success p remains the same from trail to trail.
4. The trails are independent.
5. The random variable x being studied is the number of successes obtain in n trails.

Definition: The random variable has a binomial distribution with n trails and probability of success p if its probability mass function (pmf) is given by:

$$P(X) = P(X = x) = \binom{n}{x} (p)^x (1-p)^{n-x}, \qquad x = 0,1,2,3, \dots, n$$

where

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

$$x \sim Bin(n, p)$$

If x has a binomial distribution with n and p, then:

Mean = $E(x) = np$

Variance = $Var(x) = \sigma^2 = np(1-p)$

Example (1): Let x has binomial distribution with n equal 2 and p equal 0.5.

1) Write the probability function of x.

2) Find: $P(X = 0), P(X = 1), P(X = 2)$.

3) Find: $P(X \leq 1) =$

4) Find the probability that we have at least one success.

5) Find the probability that we have at most one success.

6) Find the mean of x.

7) Find the standard deviation.

8) $P(X = 3) =$

Example (2): Let x has binomial distribution with n = 2 and p = 0.5. Find Ex^2.

Example (3): Toss a coin 10 times. Find the probability we get 3 heads.

Example (4): The student takes five question multiple choice exam; each question has 4 choices only one is correct. Student attempt to quest the answer.

1) Find the probability that he gets exactly 3 correct answers.

2) Find the probability that he gets at least one correct answer.

3) Find the expected value of number of questions that answered correctly.

4) Find the variance of number of questions that answered correctly.

Note: If x is a binomial distribution $Bin(n, p)$, then

1) If $p = 0.5$ the shape of the distribution is symmetric.
2) If $p > 0.5$ the shape of the distribution is skewed to the left.
3) If $p < 0.5$ the shape of the distribution is skewed to the right.

Example (5): Let x has a binomial distribution with $n = 10$ and $p = 0.3$. What is the shape of the distribution?

Example (6): The proportion of the defective in a factory is 0.01 we select 10 items randomly. Find the probability that we get 3 defective items.

Example (7): Roll a dice 10 times. Find the probability that number 3 occurs 4 times.

Example (8): Let $x \sim Bin(n, p)$, such that the mean of x is 2 and the variance is 1.2. Find values of n and p.

Exercise (1): A student takes a 10-question, multiple-choice physics exam with four choices for each question and guesses on each question. Find the probability that the student will pass the exam.

Exercise (2): If $x \sim Bin(n, p)$ with mean 5 and variance 4. Find the values of n and p.

Exercise (3): If $x \sim Bin(n, 0.6)$ and $P(X < 1) = 0.0256$. Find the value of n.

Exercise (4): If $x \sim Bin(4, p)$ and $P(X = 4) = 0.0256$. Find the value of p.

3 The Poisson Probability Distribution

Definition: Let λ be the average number of times that an event occurs in a certain period of time or space. Then the random variable x has a Poisson distribution if its probability mass function (pmf) is given by:

$$P(X) = P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \qquad x = 0,1,2,3, \dots$$

$$x \sim Po(\lambda)$$

If x has a Poisson distribution, then:

Mean = $E(x) = \lambda$

Variance = $Var(x) = \lambda$

Example (9): The average number of traffic accidents on a certain section of highway is two per week.

Assume that the number of accidents follows a Poisson distribution with $\lambda = 2$.

1. Find the probability of no accidents on this section of highway during a 1-week period.

2. Find the probability of at most three accidents on this section of highway during a 1-week period.

Exercise (5): Let x be a Poisson random variable with mean 2. Calculate these probabilities:

a. $P(x = 0)$ b. $P(x = 1)$

c. $P(x > 1)$ d. $P(x = 5)$

Exercise (6): Let x be a Poisson random variable with $\lambda = 3.8$. Find the mean and variance.

Exercise (7): True or False.

1) In a binomial distribution, the number of trials must be fixed.

2) The probability of success in a binomial distribution must change from trial to trial.

3) A binomial random variable counts the number of successes in a fixed number of independent trials.

4) The mean of a binomial distribution is given by $np(1-p)$.

5) The Poisson distribution is used to model the number of occurrences of an event in a fixed interval of time or space.

6) In a Poisson distribution, events must occur independently of each other.

7) The mean and variance of a Poisson distribution are always equal.