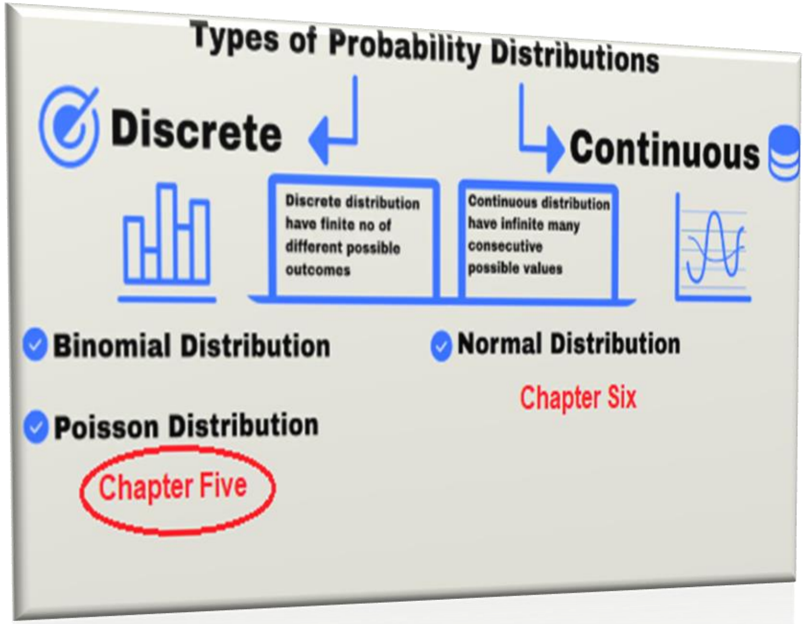
# Introduction to Probability and Statistics

## Topic 5: “ Several Useful Discrete Distributions”

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**Solution to the Exercises for the Fifth Topic**

**Exercise (1):** A student takes a 10-question, multiple-choice physics exam with four choices for each question and guesses on each question. Find the probability that the student will pass the exam (Knowing that the passing mark is 6).

$$x \sim Bin\left(10, \frac{1}{4}\right)$$

$$P(X \geq 6) = P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$$

$$= \binom{10}{6}\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^4 + \binom{10}{7}\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3 + \binom{10}{8}\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^2 + \binom{10}{9}\left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right)^1 + \binom{10}{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^0$$

$$= 0.0197$$

The probability that the student will pass the exam by guessing is approximately **0.0197**, or **1.97%.**

**Exercise (2):** If $x \sim Bin(n, p)$ with mean 5 and variance 4. Find the values of $n$ and $p$.

$\mu = np$

$5 = np \ldots\ldots(1)$

$\sigma^2 = np(1-p)$

$4 = np(1-p) \ldots\ldots(2)$

$\therefore \sigma^2 = \mu \times (1-p)$

$4 = 5 \times (1-p)$

$1 - p = 0.8$

$p = 0.2$

From (1): $np = 5$

$n\,(0.2) = 5$

$n = 25$

**Exercise (3):** If $x \sim Bin(n, 0.6)$ and $P(X < 1) = 0.0256$. Find the value of $n$.

$$P(X < 1) = P(X = 0)$$

$$P(X = 0) = 0.0256$$

$$\binom{n}{0}(0.6)^0(0.4)^n = 0.0256$$

$$(0.4)^n = 0.0256$$

$$\ln(0.4)^n = \ln(0.0256)$$

$$n\ln(0.4) = \ln(0.0256)$$

$$\therefore n = 4$$

**Exercise (4):** If $x \sim Bin(4, p)$ and $P(X = 4) = 0.0256$. Find the value of $p$.

$$P(X = 4) = 0.0256$$

$$\binom{4}{4}(p)^4(1-p)^0 = 0.0256$$

$$p^4 = 0.0256$$

$$p = \sqrt[4]{0.0256}$$

$$\therefore p = 0.4$$

**Exercise (5):** Let $x$ be a Poisson random variable with mean $\lambda = 2$. Calculate these probabilities:

$x \sim Po(2)$

$$P(X = x) = \frac{e^{-2}2^x}{x!}, \qquad x = 0,1,2,3,\dots$$

a. $P(X = 0) = \frac{e^{-2}2^0}{0!} = e^{-2} = 0.135$

b. $P(X = 1) = \frac{e^{-2}2^1}{1!} = 2e^{-2} = 0.271$

c. $P(X > 1) = 1 - P(X \leq 1) = 1 - P(X = 0) = 1 - 0.135 = 0.865$

d. $P(X = 5) = \frac{e^{-2}2^5}{5!} = \frac{32}{120}e^{-2} = 0.036$

**Exercise (6):** Let $x$ be a Poisson random variable with $\lambda = 3.8$. Find the mean and variance.

$x \sim Po(3.8)$

Mean: $E(x) = 3.8$

Variance: $Var(x) = 3.8$

**Exercise (7):** True or False.

1) In a binomial distribution, the number of trials must be fixed. **True**

2) The probability of success in a binomial distribution must change from trial to trial. **False**

3) A binomial random variable counts the number of successes in a fixed number of independent trials. **True**

4) The mean of a binomial distribution is given by $np(1-p)$. **False**

5) The Poisson distribution is used to model the number of occurrences of an event in a fixed interval of time or space. **True**

6) In a Poisson distribution, events must occur independently of each other. **True**

7) The mean and variance of a Poisson distribution are always equal. **True**