Introduction to Probability and Statistics

Topic (6): "The Normal Probability Distribution"

Types of Probability Distributions

Discrete / Continuous

Discrete distribution have finite no of different possible outcomes

Continuous distribution have infinite many consecutive possible values

Binomial Distribution

Normal Distribution

Chapter Six

Poisson Distribution

Chapter Five
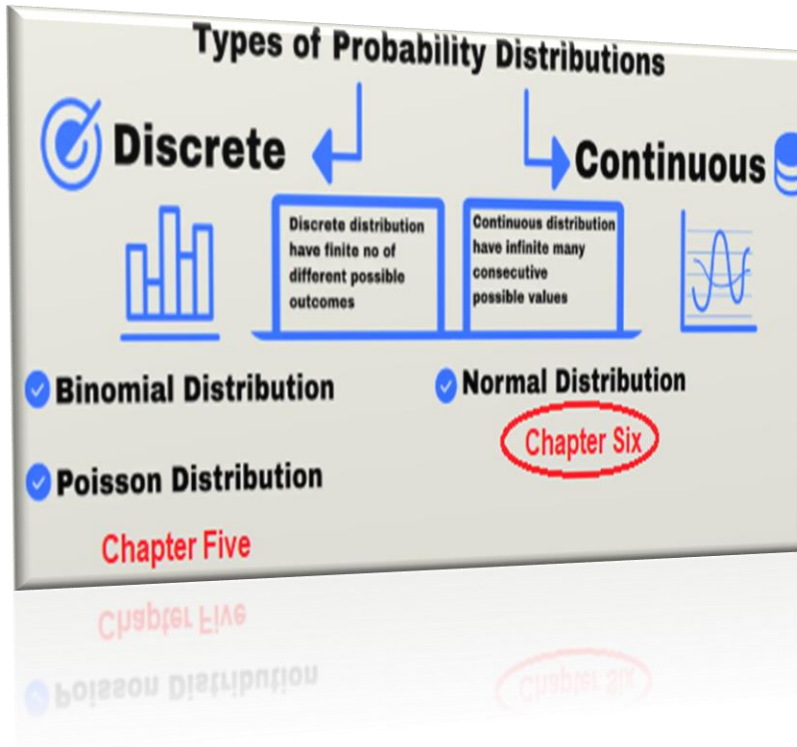
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Topic (6): "The Normal Probability Distribution"
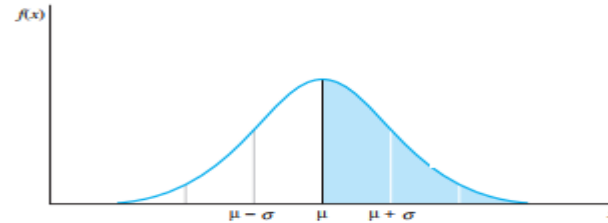
2   The Normal Probability Distribution

1) Continuous random variable.

2) Range $(-\infty, \infty)$

Definition: The random variable x is set to have a normal distribution with mean μ and variance σ^2, denoted by $x \sim N(\mu, \sigma^2)$, if its probability density function (pdf) given by:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \qquad -\infty < x < \infty$$
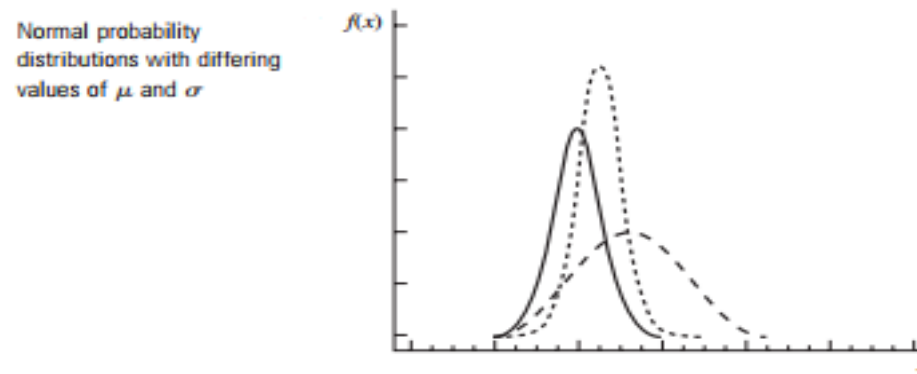
Properties of normal distribution:

1) The normal curve is symmetric about μ.

2) Area to the right of μ = Area to the left of μ

$$0.5 = 0.5$$

3) Small value of σ make the normal curve peak, while large value of σ make the normal curve flat.

Normal probability
distributions with differing
values of μ and σ

4) If x is continuous random variable, then $P(x = c) = 0$.

$$P(a < x < b) = P(a \leq x < b) = P(a < x \leq b) = P(a \leq x \leq b)$$

$$P(a < x < b) = \int_{a}^{b} f(x)dx$$

Example (1): Let $x \sim N(3, 25)$. Find:

1) The mean.

2) Ex^2.

3) $P(x = 4) =$

4) $P(-3 < x \leq 7) =$

3   Tabulated Areas of the Normal Probability Distribution

A standard normal distribution

<div align="center">

Denoted by: $z \sim N(0, 1)$

</div>

Rules: If

    1) $P(z \leq c) =$ From table.

    2) $P(z \geq c) = 1 - P(z < c)$

    3) $P(a < z < b) = P(z < b) - P(z < a)$

    4) $P(z \leq -c) =$ From table   or $P(z \leq -c) = 1 - P(z < c)$

Example (2): Let z has a standard normal distribution, $z \sim N(0, 1)$. Find:

1) $P(z < 1) =$

2) $P(z \le 1.9) =$

3) $P(z < 1.96) =$

4) $P(z \le -1.96) =$

5) $P(z \ge 1.45) =$

6) $P(z = 1.45) =$

7) $P(1.41 < z < 2.32) =$

Exercise (1): Let z has a standard normal distribution. Find:

1) $P(0 < z \le 1)$.

2) $P(-1.53 < z < 1.62)$.

3) $P(z \le 1.92)$.

Example (3): Let $z \sim N(0, 1)$. Find c such that:

1) $P(z < c) = 0.975$

2) $P(z > c) = 0.975$

Example (4): Let $z \sim N(0, 1)$. Find c such that:

$$P(z \leq c) = 0.95$$

Example (5): Let $z \sim N(0, 1)$. Find A such that:

$$P(z \leq A) = 0.2946$$

Example (6): Let $z \sim N(0, 1)$. Find c such that:
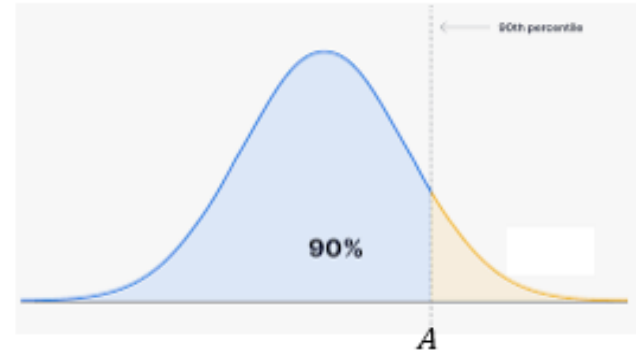
$$P(0 < z < c) = 0.1915$$

Example (7): Let $z \sim N(0, 1)$. Find A such that:

$$P(|z| < A) = 0.8$$

Example (8): Let $z \sim N(0, 1)$. Find the 90th percentile of z.

90th percentile

90%

A

Example (9): Let $z \sim N(0, 1)$. Find the 97.5^{th} percentile of z.

Exercise (2): Let $z \sim N(0, 1)$. Find the 95th, 98th and 99th percentiles of z.

Example (10): Find the probability that standard normal distribution random variable falls within this range:

1) One standard deviation of its mean.

2) Two standard deviations of its mean.

3) Three standard deviations of its mean. <mark>**Exercise (3)**</mark>

Example (11): Let $z \sim N(0, 1)$.Find:

1) $P(z < 3.59) =$

2) $P(z < 4) =$

3) $P(z < -4) =$

To transfer $x \rightarrow z$

Let $x \sim N(\mu, \sigma^2)$, then $z = \frac{x-\mu}{\sigma} \sim N(0, 1)$.

Example (12): Let $x \sim N(60, 16)$ is a random variable has a normal distribution with mean 60 and variance 16. Find:

1) $P(x > 65)$.

2) $P(55 < x < 67)$.

Example (13): Let $x \sim N(2, 16)$. Find A such that $P(x < A) = 0.975$.

Example (14): Let x has a normal distribution with mean 90 and standard deviation 10. Find 95th percentile of x.

Example (15): Assume that the random variable x which is the age of the mother at the birth of her first child is normally distributed with mean 20 years and variance 9.

Find the probability that the normally selected mother has her first child:

1) Before age 16.

2) After age 30.

3) Between age 16 and 24.

==Exercise (4)==

Example (16): Let $z \sim N(0, 1)$. Find c such that has area 0.9505 to its left.

Example (17): A normal random variable x has an unknown mean μ and standard deviation $\sigma = 2$. If the probability that x exceeds 7.5 is 0.8023, find μ.

Example (18): A normal random variable x has an unknown mean and standard deviation. The probability that x exceeds 4 is 0.9772, and the probability that x exceeds 5 is 0.9332. Find μ and σ.

Exercise (5): True or False:

1) A normal distribution is always symmetric about its mean.

2) The total area under the normal curve is equal to 0.5.

3) In a normal distribution, the mean, median, and mode are always equal.

4) A standard normal distribution has a mean of 1 and a standard deviation of 1.

5) The normal curve extends infinitely in both directions without touching the x-axis.

6) Any normal distribution can be converted into a standard normal distribution using a Z-score transformation.

7) The z-score represents the number of standard deviations a value is from the mean.

8) A z-score of 0 corresponds to a value equal to the mean of the distribution.

9) Approximately 95% of the data in a normal distribution falls within 1 standard deviation of the mean.

Table —Standard Normal Distribution
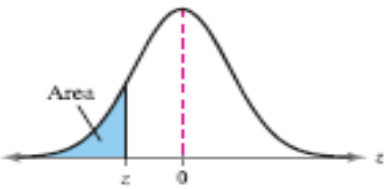
z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
−2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
−2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
−2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
−2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
−2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
−2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
−2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
−2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
−2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
−1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
−1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
−1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
−1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
−1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
−1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
−1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
−1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
−1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
−1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
−0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
−0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
−0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
−0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

Table —Standard Normal Distribution (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998