2

Motion in One Dimension

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ2.1 Count spaces (intervals), not dots. Count 5, not 6. The first drop falls at time zero and the last drop at 5 × 5 s = 25 s. The average speed is 600 m/25 s = 24 m/s, answer (b).

OQ2.2 The initial velocity of the car is $v_0 = 0$ and the velocity at time t is v. The constant acceleration is therefore given by

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} = \frac{v - 0}{t} = \frac{v}{t}$$

and the average velocity of the car is

$$\bar{v} = \frac{(v + v_0)}{2} = \frac{(v + 0)}{2} = \frac{v}{2}$$

The distance traveled in time t is $\Delta x = \bar{v}t = vt/2.$ In the special case where $a = 0$ (and hence $v = v_0 = 0$), we see that statements (a), (b), (c), and (d) are all correct. However, in the general case ($a \neq 0$, and hence

$v \neq 0$) only statements (b) and (c) are true. Statement (e) is not true in either case.

OQ2.3 The bowling pin has a constant downward acceleration while in flight. The velocity of the pin is directed upward on the ascending part of its flight and is directed downward on the descending part of its flight. Thus, only (d) is a true statement.

OQ2.4 The derivation of the equations of kinematics for an object moving in one dimension was based on the assumption that the object had a constant acceleration. Thus, (b) is the correct answer. An object would have constant velocity if its acceleration were zero, so (a) applies to cases of zero acceleration only. The speed (magnitude of the velocity) will increase in time only in cases when the velocity is in the same direction as the constant acceleration, so (c) is not a correct response. An object projected straight upward into the air has a constant downward acceleration, yet its position (altitude) does not always increase in time (it eventually starts to fall back downward) nor is its velocity always directed downward (the direction of the constant acceleration). Thus, neither (d) nor (e) can be correct.

OQ2.5 The maximum height (where $v = 0$) reached by a freely falling object shot upward with an initial velocity $v_0 = +225$ m/s is found from $v_f^2 = v_i^2 + 2a(y_f - y_i) = v_i^2 + 2a\Delta y$, where we replace a with $-g$, the downward acceleration due to gravity. Solving for Δy then gives

$$\Delta y = \frac{\left(v_f^2 - v_i^2\right)}{2a} = \frac{-v_0^2}{2(-g)} = \frac{-(225\text{ m/s})^2}{2\left(-9.80\text{ m/s}^2\right)} = 2.58 \times 10^3\text{ m}$$

Thus, the projectile will be at the $\Delta y = 6.20 \times 10^2$ m level twice, once on the way upward and once coming back down.

The elapsed time when it passes this level coming downward can be found by using $v_f^2 = v_i^2 + 2a\Delta y$ again by substituting $a = -g$ and solving for the velocity of the object at height (displacement from original position) $\Delta y = +6.20 \times 10^2$ m.

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v^2 = (225\text{ m/s})^2 + 2\left(-9.80\text{ m/s}^2\right)\left(6.20 \times 10^2\text{ m}\right)$$
$$v = \pm 196\text{ m/s}$$

The velocity coming down is –196 m/s. Using $v_f = v_i + at$, we can solve for the time the velocity takes to change from +225 m/s to –196 m/s:

$$t = \frac{(v_f - v_i)}{a} = \frac{(-196\text{ m/s} - 225\text{ m/s})}{(-9.80\text{ m/s}^2)} = 43.0\text{ s}.$$

The correct choice is (e).

OQ2.6 Once the arrow has left the bow, it has a constant downward acceleration equal to the free-fall acceleration, g. Taking upward as the positive direction, the elapsed time required for the velocity to change from an initial value of 15.0 m/s upward (v_0 = +15.0 m/s) to a value of 8.00 m/s downward (v_f = –8.00 m/s) is given by

$$\Delta t = \frac{\Delta v}{a} = \frac{v_f - v_0}{-g} = \frac{-8.00\text{ m/s} - (+15.0\text{ m/s})}{-9.80\text{ m/s}^2} = 2.35\text{ s}$$

Thus, the correct choice is (d).

OQ2.7 (c) The object has an initial positive (northward) velocity and a negative (southward) acceleration; so, a graph of velocity versus time slopes down steadily from an original positive velocity. Eventually, the graph cuts through zero and goes through increasing-magnitude-negative values.

OQ2.8 (b) Using $v_f^2 = v_i^2 + 2a\Delta y$, with $v_i = -12$ m/s and $\Delta y = -40$ m:

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v^2 = (-12\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-40\text{ m})$$
$$v = -30\text{ m/s}$$

OQ2.9 With original velocity zero, displacement is proportional to the square of time in $(1/2)at^2$. Making the time one-third as large makes the displacement one-ninth as large, answer (c).

OQ2.10 We take downward as the positive direction with $y = 0$ and $t = 0$ at the top of the cliff. The freely falling marble then has $v_0 = 0$ and its displacement at $t = 1.00$ s is $\Delta y = 4.00$ m. To find its acceleration, we use

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \rightarrow (y - y_0) = \Delta y = \frac{1}{2}at^2 \rightarrow a = \frac{2\Delta y}{t^2}$$
$$a = \frac{2(4.00\text{ m})}{(1.00\text{ s})^2} = 8.00\text{ m/s}^2$$

The displacement of the marble (from its initial position) at $t = 2.00$ s is found from

$$\Delta y = \frac{1}{2}at^2$$

$$\Delta y = \frac{1}{2}\left(8.00\ \text{m/s}^2\right)\left(2.00\ \text{s}\right)^2 = 16.0\ \text{m}.$$

The distance the marble has fallen in the 1.00 s interval from $t = 1.00$ s to $t = 2.00$ s is then

$$\Delta y = 16.0\ \text{m} - 4.0\ \text{m} = 12.0\ \text{m}.$$

and the answer is (c).

OQ2.11 In a position vs. time graph, the velocity of the object at any point in time is the slope of the line tangent to the graph at that instant in time. The speed of the particle at this point in time is simply the magnitude (or absolute value) of the velocity at this instant in time. The displacement occurring during a time interval is equal to the difference in x coordinates at the final and initial times of the interval, $\Delta x = x_f - x_i$.

The average velocity during a time interval is the slope of the straight line connecting the points on the curve corresponding to the initial and final times of the interval,

$$\bar{v} = \Delta x/\Delta t$$

Thus, we see how the quantities in choices (a), (e), (c), and (d) can all be obtained from the graph. Only the acceleration, choice (b), *cannot be obtained* from the position vs. time graph.

OQ2.12 We take downward as the positive direction with $y = 0$ and $t = 0$ at the top of the cliff. The freely falling pebble then has $v_0 = 0$ and $a = g = +9.8\ \text{m/s}^2$. The displacement of the pebble at $t = 1.0$ s is given: $y_1 = 4.9$ m. The displacement of the pebble at $t = 3.0$ s is found from

$$y_3 = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(9.8\ \text{m/s}^2\right)\left(3.0\ \text{s}\right)^2 = 44\ \text{m}$$

The distance fallen in the 2.0-s interval from $t = 1.0$ s to $t = 3.0$ s is then

$$\Delta y = y_3 - y_1 = 44\ \text{m} - 4.9\ \text{m} = 39\ \text{m}$$

and choice (c) is seen to be the correct answer.

OQ2.13 (c) They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity of magnitude v_i. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

OQ2.14 (b) Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point. So your ball must travel a smaller distance to the passing point than the ball your friend throws.

OQ2.15 Take down as the positive direction. Since the pebble is released from rest, $v_f^2 = v_i^2 + 2a\Delta y$ becomes

$$v_f^2 = (4\text{ m/s})^2 = 0^2 + 2gh.$$

Next, when the pebble is thrown with speed 3.0 m/s from the same height h, we have

$$v_f^2 = (3\text{ m/s})^2 + 2gh = (3\text{ m/s})^2 + (4\text{ m/s})^2 \rightarrow v_f = 5\text{ m/s}$$

and the answer is (b). Note that we have used the result from the first equation above and replaced $2gh$ with $(4\text{ m/s})^2$ in the second equation.

OQ2.16 Once the ball has left the thrower's hand, it is a freely falling body with a constant, nonzero, acceleration of $a = -g$. Since the acceleration of the ball is not zero at any point on its trajectory, choices (a) through (d) are all false and the correct response is (e).

OQ2.17 (a) Its speed is zero at points B and D where the ball is reversing its direction of motion. Its speed is the same at A, C, and E because these points are at the same height. The assembled answer is A = C = E > B = D.

(b) The acceleration has a very large positive (upward) value at D. At all the other points it is -9.8 m/s^2. The answer is D > A = B = C = E.

OQ2.18 (i) (b) shows equal spacing, meaning constant nonzero velocity and constant zero acceleration. (ii) (c) shows positive acceleration throughout. (iii) (a) shows negative (leftward) acceleration in the first four images.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ2.1 The net displacement must be zero. The object could have moved away from its starting point and back again, but it is at its initial position again at the end of the time interval.

CQ2.2 Tramping hard on the brake at zero speed on a level road, you do not feel pushed around inside the car. The forces of rolling resistance and air resistance have dropped to zero as the car coasted to a stop, so the car's acceleration is zero at this moment and afterward.

Tramping hard on the brake at zero speed on an uphill slope, you feel thrown backward against your seat. Before, during, and after the zero-speed moment, the car is moving with a downhill acceleration if you do not tramp on the brake.

CQ2.3 Yes. If a car is travelling eastward and slowing down, its acceleration is opposite to the direction of travel: its acceleration is westward.

CQ2.4 Yes. Acceleration is the time rate of change of the velocity of a particle. If the velocity of a particle is zero at a given moment, and if the particle is not accelerating, the velocity will remain zero; if the particle is accelerating, the velocity will change from zero—the particle will begin to move. Velocity and acceleration are independent of each other.

CQ2.5 Yes. Acceleration is the time rate of change of the velocity of a particle. If the velocity of a particle is nonzero at a given moment, and the particle is not accelerating, the velocity will remain the same; if the particle is accelerating, the velocity will change. The velocity of a particle at a given moment and how the velocity is changing at that moment are independent of each other.

CQ2.6 Assuming no air resistance: (a) The ball reverses direction at its maximum altitude. For an object traveling along a straight line, its velocity is zero at the point of reversal. (b) Its acceleration is that of gravity: -9.80 m/s^2 (9.80 m/s^2, downward). (c) The velocity is -5.00 m/s^2. (d) The acceleration of the ball remains -9.80 m/s^2 as long as it does not touch anything. Its acceleration changes when the ball encounters the ground.

CQ2.7 (a) No. Constant acceleration only: the derivation of the equations assumes that d^2x/dt^2 is constant. (b) Yes. Zero is a constant.

CQ2.8 Yes. If the speed of the object varies at all over the interval, the instantaneous velocity will sometimes be greater than the average velocity and will sometimes be less.

CQ2.9 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past to give car B greater acceleration just then.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 2.1 Position, Velocity, and Speed

P2.1 The average velocity is the slope, not necessarily of the graph line itself, but of a secant line cutting across the graph between specified points. The slope of the graph line itself is the instantaneous velocity, found, for example, in Problem 6 part (b). On this graph, we can tell positions to two significant figures:

(a) $x = 0$ at $t = 0$ and $x = 10$ m at $t = 2$ s:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{10 \text{ m} - 0}{2 \text{ s} - 0} = \boxed{5.0 \text{ m/s}}$$

(b) $x = 5.0$ m at $t = 4$ s:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{5.0 \text{ m} - 0}{4 \text{ s} - 0} = \boxed{1.2 \text{ m/s}}$$

(c) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{5.0 \text{ m} - 10 \text{ m}}{4 \text{ s} - 2 \text{ s}} = \boxed{-2.5 \text{ m/s}}$$

(d) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{-5.0 \text{ m} - 5.0 \text{ m}}{7 \text{ s} - 4 \text{ s}} = \boxed{-3.3 \text{ m/s}}$$

(e) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{0.0 \text{ m} - 0.0 \text{ m}}{8 \text{ s} - 0 \text{ s}} = \boxed{0 \text{ m/s}}$$

P2.2 We assume that you are approximately 2 m tall and that the nerve impulse travels at uniform speed. The elapsed time is then

$$\Delta t = \frac{\Delta x}{v} = \frac{2 \text{ m}}{100 \text{ m/s}} = 2 \times 10^{-2} \text{ s} = \boxed{0.02 \text{ s}}$$

P2.3 Speed is positive whenever motion occurs, so the average speed must be positive. For the velocity, we take as positive for motion to the right and negative for motion to the left, so its average value can be positive, negative, or zero.

(a) The average speed during any time interval is equal to the total distance of travel divided by the total time:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{d_{AB} + d_{BA}}{t_{AB} + t_{BA}}$$

But $d_{\text{AB}} = d_{\text{BA}}$, $t_{\text{AB}} = d/v_{AB}$, and $t_{\text{BA}} = d/v_{\text{BA}}$

so $$\text{average speed} = \frac{d + d}{(d/v_{\text{AB}}) + (d/v_{\text{BA}})} = \frac{2(v_{\text{AB}})(v_{\text{BA}})}{v_{AB} + v_{BA}}$$

and

$$\text{average speed} = 2\left[\frac{(5.00 \text{ m/s})(3.00 \text{ m/s})}{5.00 \text{ m/s} + 3.00 \text{ m/s}}\right] = \boxed{3.75 \text{ m/s}}$$

(b) The average velocity during any time interval equals total displacement divided by elapsed time.

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t}$$

Since the walker returns to the starting point, $\Delta x = 0$ and $\boxed{v_{x,\text{avg}} = 0}$.

P2.4 We substitute for t in $x = 10t^2$, then use the definition of average velocity:

t (s)	2.00	2.10	3.00
x (m)	40.0	44.1	90.0

(a) $v_{\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{90.0 \text{ m} - 40.0 \text{ m}}{1.00 \text{ s}} = \dfrac{50.0 \text{ m}}{1.00 \text{ s}} = \boxed{50.0 \text{ m/s}}$

(b) $v_{\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{44.1 \text{ m} - 40.0 \text{ m}}{0.100 \text{ s}} = \dfrac{4.10 \text{ m}}{0.100 \text{ s}} = \boxed{41.0 \text{ m/s}}$

***P2.5** We read the data from the table provided, assume three significant figures of precision for all the numbers, and use Equation 2.2 for the definition of average velocity.

(a) $v_{x,\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{2.30 \text{ m} - 0 \text{ m}}{1.00 \text{ s}} = \boxed{2.30 \text{ m/s}}$

(b) $v_{x,\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5 \text{ m} - 9.20 \text{ m}}{3.00 \text{ s}} = \boxed{16.1 \text{ m/s}}$

(c) $v_{x,\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5 \text{ m} - 0 \text{ m}}{5.00 \text{ s}} = \boxed{11.5 \text{ m/s}}$

Section 2.2 Instantaneous Velocity and Speed

P2.6 (a) At any time, t, the position is given by $x = (3.00\text{ m/s}^2)t^2$.

Thus, at $t_i = 3.00$ s: $x_i = (3.00\text{ m/s}^2)(3.00\text{ s})^2 = \boxed{27.0\text{ m}}$.

(b) At $t_f = 3.00\text{ s} + \Delta t$: : $x_f = (3.00\text{ m/s}^2)(3.00\text{ s} + \Delta t)^2$, or

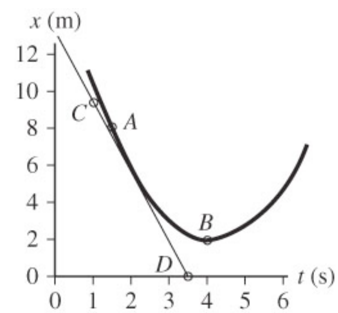
$$x_f = \boxed{27.0\text{ m} + (18.0\text{ m/s})\Delta t + \left(3.00\text{ m/s}^2\right)(\Delta t)^2}$$

(c) The instantaneous velocity at $t = 3.00$ s is:

$$\boxed{\begin{aligned}\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} &= \lim_{\Delta t\to 0}\frac{(18.0\text{ m/s})\Delta t + \left(3.00\text{ m/s}^2\right)(\Delta t)^2}{\Delta t}\\ &= \lim_{\Delta t\to 0}(18.0\text{ m/s}) + \left(3.00\text{ m/s}^2\right)(\Delta t) = \boxed{18.0\text{ m/s}}\end{aligned}}$$

P2.7 For average velocity, we find the slope of a secant line running across the graph between the 1.5-s and 4-s points. Then for instantaneous velocities we think of slopes of tangent lines, which means the slope of the graph itself at a point.

We place two points on the curve: Point A, at $t = 1.5$ s, and Point B, at $t = 4.0$ s, and read the corresponding values of x.



ANS. FIG. P2.7

(a) At $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

At $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\text{ m}}{(4.0 - 1.5)\text{ s}}$$
$$= -\frac{6.0\text{ m}}{2.5\text{ s}} = \boxed{-2.4\text{ m/s}}$$

(b) The slope of the tangent line can be found from points C and D. ($t_C = 1.0$ s, $x_C = 9.5$ m) and ($t_D = 3.5$ s, $x_D = 0$),

$$v \approx \boxed{-3.8\text{ m/s}}$$

The negative sign shows that the **direction** of v_x is along the negative x direction.

(c) The velocity will be zero when the slope of the tangent line is zero. This occurs for the point on the graph where x has its minimum value. This is at $t \approx \boxed{4.0\text{ s}}$.

P2.8 We use the definition of average velocity.

(a) $v_{1,x,\text{ave}} = \dfrac{(\Delta x)_1}{(\Delta t)_1} = \dfrac{L-0}{t_1} = \boxed{+L/t_1}$

(b) $v_{2,x,\text{ave}} = \dfrac{(\Delta x)_2}{(\Delta t)_2} = \dfrac{0-L}{t_2} = \boxed{-L/t_2}$

(c) To find the average velocity for the round trip, we add the displacement and time for each of the two halves of the swim:

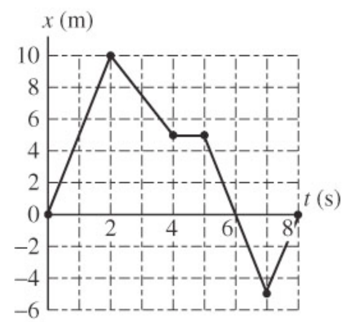
$$v_{x,\text{ave,total}} = \frac{(\Delta x)_{\text{total}}}{(\Delta t)_{\text{total}}} = \frac{(\Delta x)_1 + (\Delta x)_2}{t_1 + t_2} = \frac{+L - L}{t_1 + t_2} = \frac{0}{t_1 + t_2} = \boxed{0}$$

(d) The average speed of the round trip is the total distance the athlete travels divided by the total time for the trip:

$$v_{\text{ave,trip}} = \frac{\text{total distance traveled}}{(\Delta t)_{\text{total}}} = \frac{|(\Delta x)_1| + |(\Delta x)_2|}{t_1 + t_2}$$

$$= \frac{|+L| + |-L|}{t_1 + t_2} = \boxed{\frac{2L}{t_1 + t_2}}$$

P2.9 The instantaneous velocity is found by evaluating the slope of the $x - t$ curve at the indicated time. To find the slope, we choose two points for each of the times below.



ANS. FIG. P2.9

(a) $v = \dfrac{(5-0)\text{ m}}{(1-0)\text{ s}} = \boxed{5\text{ m/s}}$

(b) $v = \dfrac{(5-10)\text{ m}}{(4-2)\text{ s}} = \boxed{-2.5\text{ m/s}}$

(c) $v = \dfrac{(5-5)\text{ m}}{(5\text{ s} - 4\text{ s})} = \boxed{0}$

(d) $v = \dfrac{0-(-5\text{ m})}{(8\text{ s} - 7\text{ s})} = \boxed{+5\text{ m/s}}$

Section 2.3 Analysis Model: Particle Under Constant Velocity

P2.10 The plates spread apart distance d of 2.9×10^3 mi in the time interval Δt at the rate of 25 mm/year. Converting units:

$$\left(2.9\times10^3\ \text{mi}\right)\left(\frac{1609\ \text{m}}{1\ \text{mi}}\right)\left(\frac{10^3\ \text{mm}}{1\ \text{m}}\right)=4.7\times10^9\ \text{mm}$$

Use $d = v\Delta t$, and solve for Δt:

$$d = v\Delta t \rightarrow \Delta t = \frac{d}{v}$$

$$\Delta t = \frac{4.7\times10^9\ \text{mm}}{25\ \text{mm/year}} = \boxed{1.9\times10^8\ \text{years}}$$

P2.11 (a) The tortoise crawls through a distance D before the rabbit resumes the race. When the rabbit resumes the race, the rabbit must run through 200 m at 8.00 m/s while the tortoise crawls through the distance (1 000 m – D) at 0.200 m/s. Each takes the same time interval to finish the race:

$$\Delta t = \left(\frac{200\ \text{m}}{8.00\ \text{m/s}}\right) = \left(\frac{1\ 000\ \text{m} - D}{0.200\ \text{m/s}}\right)$$

Solving,

$$\rightarrow (0.200\ \text{m/s})(200\ \text{m}) = (8.00\ \text{m/s})(1\ 000\ \text{m} - D)$$

$$1\ 000\ \text{m} - D = \frac{(0.200\ \text{m/s})(200\ \text{m})}{8.00\ \text{m/s}}$$

$$\rightarrow D = 995\ \text{m}$$

So, the tortoise is 1 000 m – D = $\boxed{5.00\ \text{m}}$ from the finish line when the rabbit resumes running.

(b) Both begin the race at the same time: $t = 0$. The rabbit reaches the 800-m position at time t = 800 m/(8.00 m/s) = 100 s. The tortoise has crawled through 995 m when t = 995 m/(0.200 m/s) = 4 975 s. The rabbit has waited for the time interval Δt = 4 975 s – 100 s = $\boxed{4\ 875\ \text{s}}$.

P2.12 The trip has two parts: first the car travels at constant speed v_1 for distance d, then it travels at constant speed v_2 for distance d. The first part takes the time interval $\Delta t_1 = d/v_1$, and the second part takes the time interval $\Delta t_2 = d/v_2$.

(a) By definition, the average velocity for the entire trip is $v_{\text{avg}} = \Delta x / \Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = 2d$, and

$\Delta t = \Delta t_1 + \Delta t_2 = d / v_1 + d / v_2$. Putting these together, we have

$$v_{\text{avg}} = \left(\frac{\Delta d}{\Delta t}\right) = \left(\frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}\right) = \left(\frac{2d}{d/v_1 + d/v_2}\right) = \left(\frac{2v_1 v_2}{v_1 + v_2}\right)$$

We know v_{avg} = 30 mi/h and v_1 = 60 mi/h.

Solving for v_2 gives

$$(v_1 + v_2)v_{avg} = 2v_1 v_2 \rightarrow v_2 = \left(\frac{v_1 v_{avg}}{2v_1 - v_{avg}}\right).$$

$$v_2 = \left[\frac{(30 \text{ mi/h})(60 \text{ mi/h})}{2(60 \text{ mi/h}) - (30 \text{ mi/h})}\right] = \boxed{20 \text{ mi/h}}$$

(b) The average velocity for this trip is $v_{\text{avg}} = \Delta x / \Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = d + (-d) = 0$; so, $v_{\text{avg}} = \boxed{0}$.

(c) The average speed for this trip is $v_{\text{avg}} = d / \Delta t$, where $d = d_1 + d_2 = d + d = 2d$ and $\Delta t = \Delta t_1 + \Delta t_2 = d / v_1 + d / v_2$; so, the average speed is the same as in part (a): $v_{\text{avg}} = \boxed{30 \text{ mi/h.}}$

***2.13** (a) The total time for the trip is $t_{\text{total}} = t_1 + 22.0$ min $= t_1 + 0.367$ h, where t_1 is the time spent traveling at v_1 = 89.5 km/h. Thus, the distance traveled is $\Delta x = v_1 t_1 = v_{\text{avg}} t_{\text{total}}$, which gives

$$\begin{aligned}(89.5 \text{ km/h})t_1 &= (77.8 \text{ km/h})(t_1 + 0.367 \text{ h}) \\ &= (77.8 \text{ km/h})t_1 + 28.5 \text{ km}\end{aligned}$$

or $\quad (89.5 \text{ km/h} - 77.8 \text{ km/h})t_1 = 28.5 \text{ km}$

from which, t_1= 2.44 h, for a total time of

$$t_{\text{total}} = t_1 + 0.367 \text{ h} = \boxed{2.81 \text{ h}}$$

(b) The distance traveled during the trip is $\Delta x = v_1 t_1 = v_{\text{avg}} t_{\text{total}}$, giving

$$\Delta x = v_{\text{avg}} t_{\text{total}} = (77.8 \text{ km/h})(2.81 \text{ h}) = \boxed{219 \text{ km}}$$

Section 2.4 Acceleration

P2.14 The ball's motion is entirely in the horizontal direction. We choose the positive direction to be the outward direction, perpendicular to the wall. With outward positive, $v_i = -25.0$ m/s and $v_f = 22.0$ m/s. We use Equation 2.13 for one-dimensional motion with constant acceleration, $v_f = v_i + at$, and solve for the acceleration to obtain

$$a = \frac{\Delta v}{\Delta t} = \frac{22.0 \text{ m/s} - (-25.0 \text{ m/s})}{3.50 \times 10^{-3} \text{ s}} = \boxed{1.34 \times 10^4 \text{ m/s}^2}$$

P2.15 (a) Acceleration is the slope of the graph of v versus t.

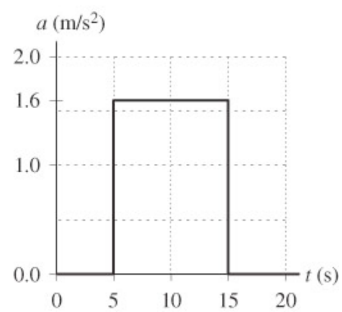
For $0 < t < 5.00$ s, $a = 0$.

For $15.0 \text{ s} < t < 20.0$ s, $a = 0$.

For $5.0 \text{ s} < t < 15.0$ s, $a = \frac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00 \text{ m/s} - (-8.00 \text{ m/s})}{15.0 \text{ s} - 5.00 \text{ s}} = \boxed{1.60 \text{ m/s}^2}$$

We can plot $a(t)$ as shown in ANS. FIG. P2.15 below.



ANS. FIG. P2.15

For (b) and (c) we use $a = \frac{v_f - v_i}{t_f - t_i}$.

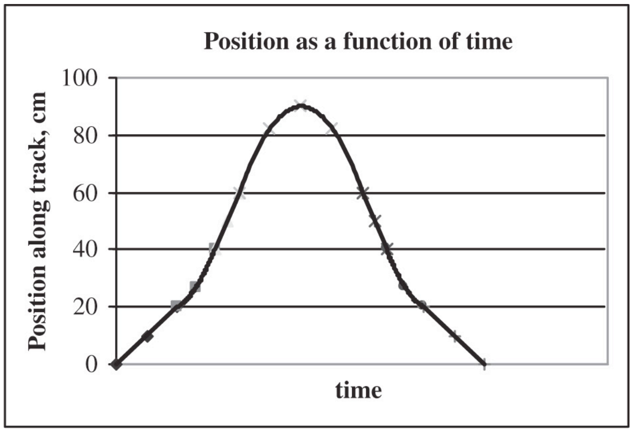
(b) For $5.00 \text{ s} < t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s, $t_f = 15.0$ s, and $v_f = 8.00$ m/s:

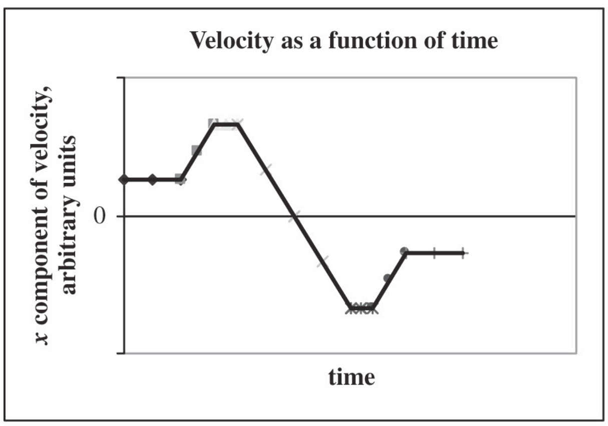
$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 \text{ m/s} - (-8.00 \text{ m/s})}{15.0 \text{ s} - 5.00 \text{ s}} = \boxed{1.60 \text{ m/s}^2}$$

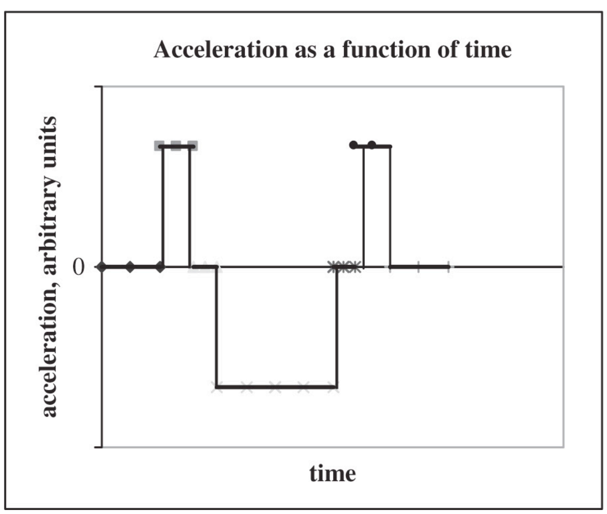
(c) We use $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, and $v_f = 8.00$ m/s:

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 \text{ m/s} - (-8.00 \text{ m/s})}{20.0 \text{ s} - 0} = \boxed{0.800 \text{ m/s}^2}$$

P2.16 The acceleration is zero whenever the marble is on a horizontal section. The acceleration has a constant positive value when the marble is rolling on the 20-to-40-cm section and has a constant negative value when it is rolling on the second sloping section. The position graph is a straight sloping line whenever the speed is constant and a section of a parabola when the speed changes.







P2.17 (a) In the interval $t_i = 0$ s and $t_f = 6.00$ s, the motorcyclist's velocity changes from $v_i = 0$ to $v_f = 8.00$ m/s. Then,

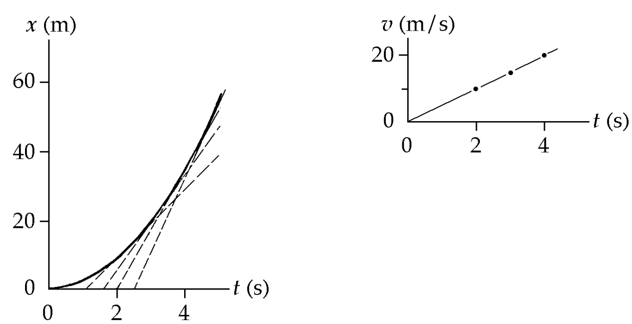
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{8.0\ \text{m/s} - 0}{6.0\ \text{s} - 0} = \boxed{1.3\ \text{m/s}^2}$$

(b) Maximum positive acceleration occurs when the slope of the velocity-time curve is greatest, at $t = 3$ s, and is equal to the slope of the graph, approximately (6 m/s – 2 m/s)/(4 s – 2 s) = $\boxed{2\ \text{m/s}^2}$.

(c) The acceleration $a = 0$ when the slope of the velocity-time graph is zero, which occurs at $\boxed{t = 6\ \text{s}}$, and also for $\boxed{t > 10\ \text{s}}$.

(d) Maximum negative acceleration occurs when the velocity-time graph has its maximum negative slope, at $t = 8$ s, and is equal to the slope of the graph, approximately $\boxed{-1.5\ \text{m/s}^2}$.

***P2.18** (a) The graph is shown in ANS. FIG. P2.18 below.



ANS. FIG. P2.18

(b) At $t = 5.0$ s, the slope is $v \approx \dfrac{58\ \text{m}}{2.5\ \text{s}} \approx \boxed{23\ \text{m/s}}$.

At $t = 4.0$ s, the slope is $v \approx \dfrac{54\ \text{m}}{3\ \text{s}} \approx \boxed{18\ \text{m/s}}$.

At $t = 3.0$ s, the slope is $v \approx \dfrac{49\ \text{m}}{3.4\ \text{s}} \approx \boxed{14\ \text{m/s}}$.

At $t = 2.0$ s, the slope is $v \approx \dfrac{36\ \text{m}}{4.0\ \text{s}} \approx \boxed{9.0\ \text{m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \approx \dfrac{23\ \text{m/s}}{5.0\ \text{s}} \approx \boxed{4.6\ \text{m/s}^2}$

(d) The initial velocity of the car was $\boxed{\text{zero}}$.

P2.19 (a) The area under a graph of *a* vs. *t* is equal to the change in velocity, Δv. We can use Figure P2.19 to find the change in velocity during specific time intervals.

The area under the curve for the time interval 0 to 10 s has the shape of a rectangle. Its area is

$$\Delta v = (2 \text{ m/s}^2)(10 \text{ s}) = 20 \text{ m/s}$$

The particle starts from rest, $v_0 = 0$, so its velocity at the end of the 10-s time interval is

$$v = v_0 + \Delta v = 0 + 20 \text{ m/s} = \boxed{20 \text{ m/s}}$$

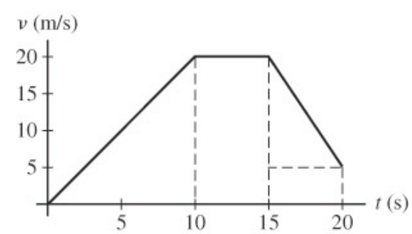
Between $t = 10$ s and $t = 15$ s, the area is zero: $\Delta v = 0$ m/s.

Between $t = 15$ s and $t = 20$ s, the area is a rectangle: $\Delta v = (-3 \text{ m/s}^2)(5 \text{ s}) = -15 \text{ m/s}$.

So, between $t = 0$ s and $t = 20$ s, the total area is $\Delta v = (20 \text{ m/s}) + (0 \text{ m/s}) + (-15 \text{ m/s}) = 5 \text{ m/s}$, and the velocity at $t = 20$ s is $\boxed{5 \text{ m/s.}}$

(b) We can use the information we derived in part (a) to construct a graph of *x* vs. *t*; the area under such a graph is equal to the displacement, Δx, of the particle.

From (a), we have these points (t, v) = (0 s, 0 m/s), (10 s, 20 m/s), (15 s, 20 m/s), and (20 s, 5 m/s). The graph appears below.



The displacements are:

0 to 10 s (area of triangle): $\Delta x = (1/2)(20 \text{ m/s})(10 \text{ s}) = 100 \text{ m}$

10 to 15 s (area of rectangle): $\Delta x = (20 \text{ m/s})(5 \text{ s}) = 100 \text{ m}$

15 to 20 s (area of triangle and rectangle):

$$\Delta x = (1/2)[(20 - 5) \text{ m/s}](5 \text{ s}) + (5 \text{ m/s})(5 \text{ s})$$
$$= 37.5 \text{ m} + 25 \text{ m} = 62.5 \text{ m}$$

Total displacement over the first 20.0 s:

$$\Delta x = 100 \text{ m} + 100 \text{ m} + 62.5 \text{ m} = 262.5 \text{ m} = \boxed{263 \text{ m}}$$

P2.20 (a) The average velocity is the change in position divided by the length of the time interval. We plug in to the given equation.

At $t = 2.00$ s, $x = [3.00(2.00)^2 - 2.00(2.00) + 3.00]$ m = 11.0 m.

At $t = 3.00$ s, $x = [3.00(3.00)^2 - 2.00(3.00) + 3.00]$ m = 24.0 m

so

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{24.0\text{ m} - 11.0\text{ m}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{13.0\text{ m/s}}$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = (6.00t - 2.00)\text{ m/s}$$

At $t = 2.00$ s, $v = [6.00(2.00) - 2.00]$ m/s = $\boxed{10.0\text{ m/s}}$.

At $t = 3.00$ s, $v = [6.00(3.00) - 2.00]$ m/s = $\boxed{16.0\text{ m/s}}$.

(c) $a_{\text{avg}} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0\text{ m/s} - 10.0\text{ m/s}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{6.00\text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00t - 2.00) = \boxed{6.00\text{ m/s}^2}$. This includes both $t = 2.00$ s and $t = 3.00$ s.

(e) From (b), $v = (6.00t - 2.00) = 0 \rightarrow t = (2.00)/(6.00) = \boxed{0.333\text{ s.}}$

P2.21 To find position we simply evaluate the given expression. To find velocity we differentiate it. To find acceleration we take a second derivative.

With the position given by $x = 2.00 + 3.00t - t^2$, we can use the rules for differentiation to write expressions for the velocity and acceleration as functions of time:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(2 + 3t - t^2\right) = 3 - 2t \quad \text{and} \quad a_x = \frac{dv}{dt} = \frac{d}{dt}(3 - 2t) = -2$$

Now we can evaluate x, v, and a at $t = 3.00$ s.

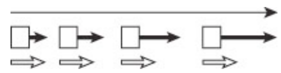
(a) $x = (2.00 + 9.00 - 9.00)$ m = $\boxed{2.00\text{ m}}$

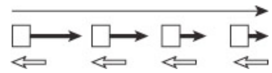
(b) $v = (3.00 - 6.00)$ m/s = $\boxed{-3.00\text{ m/s}}$

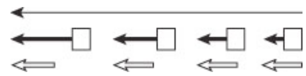
(c) $a = \boxed{-2.00\text{ m/s}^2}$

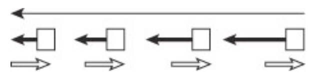
Section 2.5 Motion Diagrams

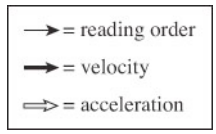
P2.22 (a) 

(b) 

(c) 

(d) 

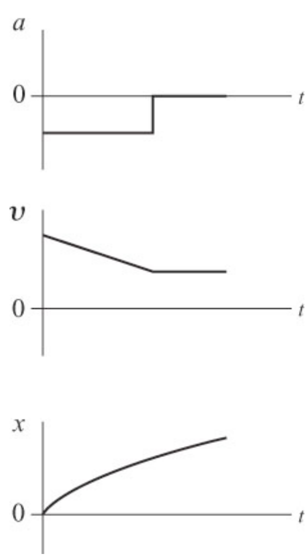
(e) 



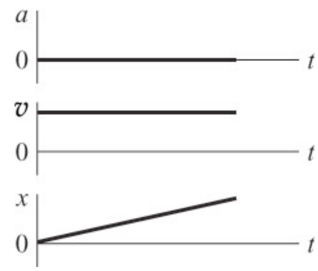
(f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.

Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the acceleration vectors would vary in magnitude and direction.

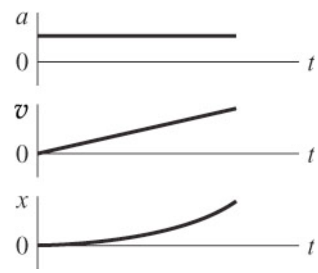
P2.23 (a) The motion is fast at first but slowing until the speed is constant. We assume the acceleration is constant as the object slows.



(b) The motion is constant in speed.



(c) The motion is speeding up, and we suppose the acceleration is constant.



Section 2.6 Analysis Model: Particle Under Constant Acceleration

***P2.24** Method One

Suppose the unknown acceleration is constant as a car moving at $v_{i1} = 35.0$ mi/h comes to a stop, $v_f = 0$ in $x_{f1} - x_i = 40.0$ ft. We find its acceleration from $v_{f1}^2 = v_{i1}^2 + 2a(x_{f1} - x_i)$:

$$a = \frac{v_{f1}^2 - v_i^2}{2(x_{f1} - x_i)} = \frac{0 - (35.0 \text{ mi/h})^2}{2(40.0 \text{ ft})}\left(\frac{5\,280 \text{ ft}}{\text{mi}}\right)^2\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)^2 = -32.9 \text{ ft/s}^2$$

Now consider a car moving at $v_{i2} = 70.0$ mi/h and stopping, $v_f = 0$, with $a = -32.9$ ft/s^2. From the same equation, its stopping distance is

$$x_{f2} - x_i = \frac{v_{f2}^2 - v_i^2}{2a} = \frac{0 - (70.0 \text{ mi/h})^2}{2(-32.9 \text{ ft/s}^2)}\left(\frac{5\,280 \text{ ft}}{1 \text{ mi}}\right)^2\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)^2$$

$$= \boxed{160 \text{ ft}}$$

Method Two

For the process of stopping from the lower speed v_{i1} we have $v_f^2 = v_{i1}^2 + 2a(x_{f1} - x_i)$, $0 = v_{i1}^2 + 2ax_{f1}$, and $v_{i1}^2 = -2ax_{f1}$. For stopping from $v_{i2} = 2v_{i1}$, similarly $0 = v_{i2}^2 + 2ax_{f2}$, and $v_{i2}^2 = -2ax_{f2}$. Dividing gives

$$\frac{v_{i2}^2}{v_{i1}^2} = \frac{x_{f2}}{x_{f1}};\ x_{f2} = 40\text{ ft} \times 2^2 = \boxed{160\text{ ft}}$$

***P2.25** We have $v_i = 2.00 \times 10^4\text{ m/s}$, $v_f = 6.00 \times 10^6\text{ m/s}$, and $x_f - x_i = 1.50 \times 10^{-2}\text{ m}$.

(a) $x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t$:

$$t = \frac{2\left(x_f - x_i\right)}{v_i + v_f} = \frac{2\left(1.50 \times 10^{-2}\text{ m}\right)}{2.00 \times 10^4\text{ m/s} + 6.00 \times 10^6\text{ m/s}}$$

$$= \boxed{4.98 \times 10^{-9}\text{ s}}$$

(b) $v_f^2 = v_i^2 + 2a_x\left(x_f - x_i\right)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{\left(6.00 \times 10^6\text{ m/s}\right)^2 - \left(2.00 \times 10^4\text{ m/s}\right)^2}{2(1.50 \times 10^{-2}\text{ m})}$$

$$= \boxed{1.20 \times 10^{15}\text{ m/s}^2}$$

***P2.26** (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy: $x_i = 0$, $x_f = 100\text{ m}$, $v_{xi} = 30\text{ m/s}$, $v_{xf} = ?$, $a_x = -3.5\text{ m/s}^2$, and $t = ?$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2:$$

$$100\text{ m} = 0 + (30\text{ m/s})t + \frac{1}{2}\left(-3.5\text{ m/s}^2\right)t^2$$

$$\left(1.75\text{ m/s}^2\right)t^2 - (30\text{ m/s})t + 100\text{ m} = 0$$

We use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{30\text{ m/s} \pm \sqrt{900\text{ m}^2/\text{s}^2 - 4\left(1.75\text{ m/s}^2\right)(100\text{ m})}}{2\left(1.75\text{ m/s}^2\right)}$$

$$= \frac{30\text{ m/s} \pm 14.1\text{ m/s}}{3.5\text{ m/s}^2} = 12.6\text{ s} \quad \text{or} \quad \boxed{4.53\text{ s}}$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30\ \text{m/s} - (3.5\ \text{m/s}^2)4.53\ \text{s} = \boxed{14.1\ \text{m/s}}$

P2.27 In parts (a) – (c), we use Equation 2.13 to determine the velocity at the times indicated.

(a) The time given is 1.00 s after 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(1.00\ \text{s}) = \boxed{9.00\ \text{m/s}}$$

(b) The time given is 4.00 s after 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(4.00\ \text{s}) = \boxed{-3.00\ \text{m/s}}$$

(c) The time given is 1.00 s before 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(-1.00\ \text{s}) = \boxed{17.0\ \text{m/s}}$$

(d) The graph of velocity versus time is a slanting straight line, having the value 13.0 m/s at 10:05:00 a.m. on the certain date, and sloping down by 4.00 m/s for every second thereafter.

(e) If we also know the velocity at any one instant, then knowing the value of the constant acceleration tells us the velocity at all other instants

P2.28 (a) We use Equation 2.15:

$$x_f - x_i = \frac{1}{2}(v_i + v_f)t \text{ becomes } 40.0\ \text{m} = \frac{1}{2}(v_i + 2.80\ \text{m/s})(8.50\ \text{s}),$$

which yields $v_i = \boxed{6.61\ \text{m/s}}$.

(b) From Equation 2.13,

$$a = \frac{v_f - v_i}{t} = \frac{2.80\ \text{m/s} - 6.61\ \text{m/s}}{8.50\ \text{s}} = \boxed{-0.448\ \text{m/s}^2}$$

P2.29 The velocity is always changing; there is always nonzero acceleration and the problem says it is constant. So we can use one of the set of equations describing constant-acceleration motion. Take the initial point to be the moment when $x_i = 3.00$ cm and $v_{xi} = 12.0$ cm/s. Also, at t = 2.00 s, $x_f = -5.00$ cm.

Once you have classified the object as a particle moving with constant acceleration and have the standard set of four equations in front of you, how do you choose which equation to use? Make a list of all of the six symbols in the equations: x_i, x_f, v_{xi}, v_{xf}, a_x, and t. On the list fill in values as above, showing that x_i, x_f, v_{xi}, and t are known. Identify a_x as the unknown. Choose an equation involving only one unknown and the knowns. That is, choose an equation *not* involving v_{xf}. Thus we choose the kinematic equation

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

and solve for a_x:

$$a_x = \frac{2\left[x_f - x_i - v_{xi}t\right]}{t^2}$$

We substitute:

$$a_x = \frac{2[-5.00\text{ cm} - 3.00\text{ cm} - (12.0\text{ cm/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= \boxed{-16.0\text{ cm/s}^2}$$

P2.30 We think of the plane moving with maximum-size backward acceleration throughout the landing, so the acceleration is constant, the stopping time a minimum, and the stopping distance as short as it can be. The negative acceleration of the plane as it lands can be called deceleration, but it is simpler to use the single general term *acceleration* for all rates of velocity change.

(a) The plane can be modeled as a particle under constant acceleration, with $a_x = -5.00\text{ m/s}^2$. Given $v_{xi} = 100\text{ m/s}$ and $v_{xf} = 0$, we use the equation $v_{xf} = v_{xi} + a_x t$ and solve for t:

$$t = \frac{v_{xf} - v_{xi}}{a_x} = \frac{0 - 100\text{ m/s}}{-5.00\text{ m/s}^2} = \boxed{20.0\text{ s}}$$

(b) Find the required stopping distance and compare this to the length of the runway. Taking x_i to be zero, we get

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

or $$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{0 - (100\text{ m/s})^2}{2(-5.00\text{ m/s}^2)} = \boxed{1\,000\text{ m}}$$

(c) The stopping distance is greater than the length of the runway; $\boxed{\text{the plane cannot land}}$.

P2.31 We assume the acceleration is constant. We choose the initial and final points 1.40 s apart, bracketing the slowing-down process. Then we have a straightforward problem about a particle under constant acceleration. The initial velocity is

$$v_{xi} = 632 \text{ mi/h} = 632 \text{ mi/h}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = 282 \text{ m/s}$$

(a) Taking $v_{xf} = v_{xi} + a_x t$ with $v_{xf} = 0$,

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{0 - 282 \text{ m/s}}{1.40 \text{ s}} = \boxed{-202 \text{ m/s}^2}$$

This has a magnitude of approximately $20g$.

(b) From Equation 2.15,

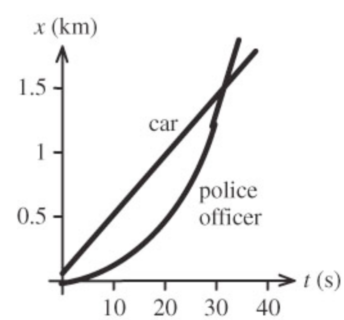
$$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t = \frac{1}{2}(282 \text{ m/s} + 0)(1.40 \text{ s}) = \boxed{198 \text{ m}}$$

P2.32 As in the algebraic solution to Example 2.8, we let t represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and $$x_{\text{trooper}} = 1.5t^2$$

They intersect at $t = \boxed{31 \text{ s}}$.



ANS. FIG. P2.32

***P2.33** (a) The time it takes the truck to reach 20.0 m/s is found from $v_f = v_i + at$. Solving for t yields

$$t = \frac{v_f - v_i}{a} = \frac{20.0 \text{ m/s} - 0 \text{ m/s}}{2.00 \text{ m/s}^2} = 10.0 \text{ s}$$

The total time is thus $10.0 \text{ s} + 20.0 \text{ s} + 5.00 \text{ s} = \boxed{35.0 \text{ s}}$.

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0+20.0}{2}\right)(10.0) = 100 \text{ m}$$

With $a = 0$ for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400 \text{ m}$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}t = \left(\frac{20.0+0}{2}\right)(5.00) = 50.0 \text{ m}$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550 \text{ m},$ and the average velocity is given by $\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7 \text{ m/s}}$.

P2.34 We ask whether the constant acceleration of the rhinoceros from rest over a period of 10.0 s can result in a final velocity of 8.00 m/s and a displacement of 50.0 m? To check, we solve for the acceleration in two ways.

1) $t_i = 0$, $v_i = 0$; $t = 10.0$ s, $v_f = 8.00$ m/s:

$$v_f = v_i + at \rightarrow a = \frac{v_f}{t}$$

$$a = \frac{8.00 \text{ m/s}}{10.0 \text{ s}} = 0.800 \text{ m/s}^2$$

2) $t_i = 0$, $x_i = 0$, $v_i = 0$; $t = 10.0$ s, $x_f = 50.0$ m:

$$x_f = x_i + v_i t + \frac{1}{2}at^2 \rightarrow x_f = \frac{1}{2}at^2$$

$$a = \frac{2x_f}{t^2} = \frac{2(50.0 \text{ m})}{(10.0 \text{ s})^2} = 1.00 \text{ m/s}^2$$

The accelerations do not match, therefore the situation is impossible.

P2.35 Since we don't know the initial and final velocities of the car, we will need to use two equations simultaneously to find the speed with which the car strikes the tree. From Equation 2.13, we have

$$v_{xf} = v_{xi} + a_x t = v_{xi} + (-5.60 \text{ m/s}^2)(4.20 \text{ s})$$

$$v_{xi} = v_{xf} + (5.60 \text{ m/s}^2)(4.20 \text{ s}) \qquad \textbf{[1]}$$

and from Equation 2.15,

$$x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

$$62.4\text{ m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20\text{ s}) \qquad \textbf{[2]}$$

Substituting for v_{xi} in [2] from [1] gives

$$62.4\text{ m} = \frac{1}{2}\left[v_{xf} + \left(5.60\text{ m/s}^2\right)(4.20\text{ s}) + v_{xf}\right](4.20\text{ s})$$

$$14.9\text{ m/s} = v_{xf} + \frac{1}{2}\left(5.60\text{ m/s}^2\right)(4.20\text{ s})$$

Thus, $v_{xf} = \boxed{3.10\text{ m/s}}$

P2.36 (a) Take any two of the standard four equations, such as

$$v_{xf} = v_{xi} + a_x t$$

$$x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

Solve one for v_{xi}, and substitute into the other:

$$v_{xi} = v_{xf} - a_x t$$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} - a_x t + v_{xf}\right)t$$

Thus

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}$$

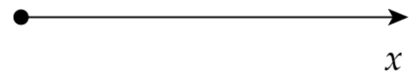
We note that the equation is dimensionally correct. The units are units of length in each term. Like the standard equation $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$, this equation represents that displacement is a quadratic function of time.

(b) Our newly derived equation gives us for the situation back in problem 35,

$$62.4\text{ m} = v_{xf}(4.20\text{ s}) - \frac{1}{2}\left(-5.60\text{ m/s}^2\right)(4.20\text{ s})^2$$

$$v_{xf} = \frac{62.4\text{ m} - 49.4\text{ m}}{4.20\text{ s}} = \boxed{3.10\text{ m/s}}$$

P2.37 (a) We choose a coordinate system with the x axis positive to the right, in the direction of motion of the speedboat, as shown on the right.



ANS. FIG. P2.37

(b) Since the speedboat is increasing its speed, the particle under constant acceleration model should be used here.

(c) Since the initial and final velocities are given along with the displacement of the speedboat, we use

$$v_{xf}^2 = v_{xi}^2 + 2a\Delta x$$

(d) Solving for the acceleration of the speedboat gives

$$a = \frac{v_{xf}^2 - v_{xi}^2}{2\Delta x}$$

(e) We have $v_i = 20.0$ m/s, $v_f = 30.0$ m/s, and $x_f - x_i = \Delta x = 200$ m:

$$a = \frac{v_{xf}^2 - v_{xi}^2}{2\Delta x} = \frac{(30.0\text{ m/s})^2 - (20.0\text{ m/s})^2}{2(200\text{ m})} = \boxed{1.25\text{ m/s}^2}$$

(f) To find the time interval, we use $v_f = v_i + at$, which gives

$$t = \frac{v_f - v_i}{a} = \frac{30.0\text{ m/s} - 20.0\text{ m/s}}{1.25\text{ m/s}^2} = \boxed{8.00\text{ s}}$$

P2.38 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00$ m, $v_i = 3.00$ m/s, and $a = -8.00$ m/s^2. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\text{ m/s} - (8.00\text{ m/s}^2)t$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}$ s. The position at this time is

$$x = 2.00\text{ m} + (3.00\text{ m/s})\left(\frac{3}{8}\text{ s}\right) - \left(4.00\text{ m/s}^2\right)\left(\frac{3}{8}\text{ s}\right)^2$$

$$= \boxed{2.56\text{ m}}$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00 \text{ m/s})}{-8.00 \text{ m/s}^2} = \frac{3}{4} \text{ s}$$

and the velocity is

$$v_f = 3.00 \text{ m/s} - (8.00 \text{ m/s}^2)\left(\frac{3}{4} \text{ s}\right) = \boxed{-3.00 \text{ m/s}}$$

P2.39 Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\ell = 0 + v_i \Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d \Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is $\boxed{\text{not equal to } v_d \text{ unless } a = 0}$.

(b) The speed halfway through the photogate in time is given by

$v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is $\boxed{\text{equal to } v_d}$ as determined above.

P2.40 (a) Let a stopwatch start from $t = 0$ as the front end of the glider passes point A. The average speed of the glider over the interval between $t = 0$ and $t = 0.628$ s is 12.4 cm/(0.628 s) = $\boxed{19.7 \text{ cm/s}}$, and this is the instantaneous speed halfway through the time interval, at $t = 0.314$ s.

(b) The average speed of the glider over the time interval between 0.628 + 1.39 = 2.02 s and 0.628 + 1.39 + 0.431 = 2.45 s is 12.4 cm/(0.431 s) = 28.8 cm/s and this is the instantaneous speed at the instant $t = (2.02 + 2.45)/2 = 2.23$ s.

Now we know the velocities at two instants, so the acceleration is found from

$$[(28.8 - 19.7) \text{ cm/s}]/[(2.23 - 0.314) \text{ s}] = \boxed{4.70 \text{ cm/s}^2}$$

(c) The distance between A and B is not used, but the length of the glider is used to find the average velocity during a known time interval.

P2.41 (a) What we know about the motion of an object is as follows: $a = 4.00\text{ m/s}^2$, $v_i = 6.00\text{ m/s}$, and $v_f = 12.0\text{ m/s}$.

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[(12.0\text{ m/s})^2 - (6.00\text{ m/s})^2\right]}{2\left(4.00\text{ m/s}^2\right)} = \boxed{13.5\text{ m}}$$

(b) From (a), the acceleration and velocity of the object are in the same (positive) direction, so the object speeds up. The distance is $\boxed{13.5\text{ m}}$ because the object always travels in the same direction.

(c) Given $a = 4.00\text{ m/s}^2$, $v_i = -6.00\text{ m/s}$, and $v_f = 12.0\text{ m/s}$. Following steps similar to those in (a) above, we will find the displacement to be the same: $\boxed{\Delta x = 13.5\text{ m.}}$ In this case, the object initially is moving in the negative direction but its acceleration is in the positive direction, so the object slows down, reverses direction, and then speeds up as it travels in the positive direction.

(d) We consider the motion in two parts.

(1) Calculate the displacement of the object as it slows down: $a = 4.00\text{ m/s}^2$, $v_i = -6.00\text{ m/s}$, and $v_f = 0\text{ m/s}$.

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[(0\text{ m/s})^2 - (-6.00\text{ m/s})^2\right]}{2\left(4.00\text{ m/s}^2\right)} = -4.50\text{ m}$$

The object travels 4.50 m in the negative direction.

(2) Calculate the displacement of the object after it has reversed direction: $a = 4.00\text{ m/s}^2$, $v_i = 0\text{ m/s}$, $v_f = 12.0\text{ m/s}$.

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[(12.0\text{ m/s})^2 - (0\text{ m/s})^2\right]}{2\left(4.00\text{ m/s}^2\right)} = 18.0\text{ m}$$

The object travels 18.0 m in the positive direction.

Total distance traveled: 4.5 m + 18.0 m = $\boxed{22.5\text{ m.}}$

P2.42 (a) For the first car, the speed as a function of time is

$$v_1 = v_{1i} + a_1 t = -3.50\text{ cm/s} + \left(2.40\text{ cm/s}^2\right)t$$

For the second car, the speed is

$$v_2 = v_{2i} + a_2 t = +5.5\text{ cm/s} + 0$$

Setting the two expressions equal gives

$$-3.50\text{ cm/s} + \left(2.40\text{ cm/s}^2\right)t = 5.5\text{ cm/s}$$

Solving for t gives

$$t = \frac{9.00\text{ cm/s}}{2.40\text{ cm/s}^2} = \boxed{3.75\text{ s}}$$

(b) The first car then has speed

$$v_1 = v_{1i} + a_1 t = -3.50\text{ cm/s} + \left(2.40\text{ cm/s}^2\right)(3.75\text{ s}) = \boxed{5.50\text{ cm/s}}$$

and this is also the constant speed of the second car.

(c) For the first car, the position as a function of time is

$$\begin{aligned} x_1 &= x_{1i} + v_{1i}t + \frac{1}{2}a_1 t^2 \\ &= 15.0\text{ cm} - (3.50\text{ cm/s})t + \frac{1}{2}\left(2.40\text{ cm/s}^2\right)t^2 \end{aligned}$$

For the second car, the position is

$$x_2 = 10.0\text{ cm} + (5.50\text{ cm/s})t$$

At the point where the cars pass one another, their positions are equal:

$$\begin{aligned} 15.0\text{ cm} - (3.50\text{ cm/s})t + \frac{1}{2}\left(2.40\text{ cm/s}^2\right)t^2 \\ = 10.0\text{ cm} + (5.50\text{ cm/s})t \end{aligned}$$

rearranging gives

$$\left(1.20\text{ cm/s}^2\right)t^2 - (9.00\text{ cm/s})t + 5.00\text{ cm} = 0$$

We solve this with the quadratic formula. Suppressing units,

$$t = \frac{9 \pm \sqrt{(9)^2 - 4(1.2)(5)}}{2(1.2)} = \frac{9 \pm \sqrt{57}}{2.4} = 6.90 \text{ s, or } \boxed{0.604 \text{ s}}$$

(d) At t = 0.604 s, the second and also the first car's position is

$$x_{1,2} = 10.0 \text{ cm} + (5.50 \text{ cm/s})(0.604 \text{ s}) = \boxed{13.3 \text{ cm}}$$

At t = 6.90 s, both are at position

$$x_{1,2} = 10.0 \text{ cm} + (5.50 \text{ cm/s})(6.90 \text{ s}) = \boxed{47.9 \text{ cm}}$$

(e) The cars are initially moving toward each other, so they soon arrive at the same position x when their speeds are quite different, giving one answer to (c) that is not an answer to (a). The first car slows down in its motion to the left, turns around, and starts to move toward the right, slowly at first and gaining speed steadily. At a particular moment its speed will be equal to the constant rightward speed of the second car, but at this time the accelerating car is far behind the steadily moving car; thus, the answer to (a) is not an answer to (c). Eventually the accelerating car will catch up to the steadily-coasting car, but passing it at higher speed, and giving another answer to (c) that is not an answer to (a).

P2.43 (a) Total displacement = area under the (v, t) curve from t = 0 to 50 s. Here, distance is the same as displacement because the motion is in one direction.

$$\Delta x = \frac{1}{2}(50 \text{ m/s})(15 \text{ s}) + (50 \text{ m/s})(40 - 15) \text{ s}$$

$$+ \frac{1}{2}(50 \text{ m/s})(10 \text{ s})$$

$$\Delta x = 1875 \text{ m} = \boxed{1.88 \text{ km}}$$

(b) From t = 10 s to t = 40 s, displacement is

$$\Delta x = \frac{1}{2}(50 \text{ m/s} + 33 \text{ m/s})(5 \text{ s}) + (50 \text{ m/s})(25 \text{ s}) = \boxed{1.46 \text{ km}}$$

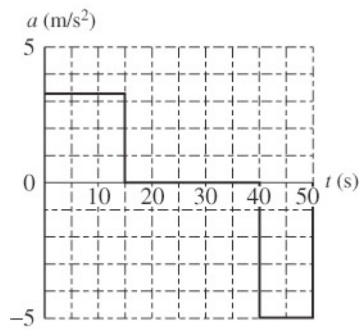
(c) We compute the acceleration for each of the three segments of the car's motion:

$$0 \le t \le 15 \text{ s:} \qquad a_1 = \frac{\Delta v}{\Delta t} = \frac{(50-0) \text{ m/s}}{15 \text{ s} - 0} = \boxed{3.3 \text{ m/s}^2}$$

$$15 \text{ s} < t < 40 \text{ s:} \qquad \boxed{a_2 = 0}$$

$$40 \text{ s} \le t \le 50 \text{ s:} \qquad a_3 = \frac{\Delta v}{\Delta t} = \frac{(0-50) \text{ m/s}}{50 \text{ s} - 40 \text{ s}} = \boxed{-5.0 \text{ m/s}^2}$$

ANS. FIG. P2.43 shows the graph of the acceleration during this interval.



ANS FIG. P2.43

(d) For segment *0a*,

$$x_1 = 0 + \frac{1}{2}a_1 t^2 = \frac{1}{2}\left(3.3 \text{ m/s}^2\right)t^2 \text{ or } \boxed{x_1 = \left(1.67 \text{ m/s}^2\right)t^2}$$

For segment *ab*,

$$x_2 = \frac{1}{2}(15\text{s})[50 \text{ m/s} - 0] + (50 \text{ m/s})(t - 15 \text{ s})$$

or $\boxed{x_2 = (50 \text{ m/s})t - 375 \text{ m}}$

For segment *bc*,

$$x_3 = \begin{pmatrix} \text{area under } v \text{ vs. } t \\ \text{from } t = 0 \text{ to } 40 \text{ s} \end{pmatrix} + \frac{1}{2}a_3(t - 40 \text{ s})^2 + (50 \text{ m/s})(t - 40 \text{ s})$$

or

$$x_3 = 375 \text{ m} + 1250 \text{ m} + \frac{1}{2}(-5.0 \text{ m/s}^2)(t - 40 \text{ s})^2 + (50 \text{ m/s})(t - 40 \text{ s})$$

which reduces to

$$\boxed{x_3 = (250 \text{ m/s})t - \left(2.5 \text{ m/s}^2\right)t^2 - 4\,375 \text{ m}}$$

(e) $\bar{v} = \dfrac{\text{total displacement}}{\text{total elapsed time}} = \dfrac{1\,875 \text{ m}}{50 \text{ s}} = \boxed{37.5 \text{ m/s}}$

2.44 (a) Take $t = 0$ at the time when the player starts to chase his opponent. At this time, the opponent is a distance $d = (12.0 \text{ m/s})(3.00 \text{ s}) = 36.0 \text{ m}$ in front of the player. At time $t > 0$, the displacements of the players from their initial positions are

$$\Delta x_{\text{player}} = v_{i,\text{player}} t + \frac{1}{2} a_{\text{player}} t^2 = 0 + \frac{1}{2}\left(4.00 \text{ m/s}^2\right) t^2 \qquad \textbf{[1]}$$

and

$$\Delta x_{\text{opponent}} = v_{i,\text{opponent}} t + \frac{1}{2} a_{\text{opponent}} t^2 = (12.0 \text{ m/s})t + 0 \qquad \textbf{[2]}$$

When the players are side-by-side, $\Delta x_{\text{player}} = \Delta x_{\text{opponent}} + 36.0 \text{ m}$. **[3]**

Substituting equations [1] and [2] into equation [3] gives

$$\frac{1}{2}\left(4.00 \text{ m/s}^2\right) t^2 = (12.0 \text{ m/s})t + 36.0 \text{ m}$$

or $\quad t^2 + (-6.00 \text{ s})t + \left(-18.0 \text{ s}^2\right) = 0$

Applying the quadratic formula to this equation gives

$$t = \frac{-(-6.00 \text{ s}) \pm \sqrt{(-6.00 \text{ s})^2 - 4(1)\left(-18.0 \text{ s}^2\right)}}{2(1)}$$

which has solutions of $t = -2.20$ s and $t = +8.20$ s. Since the time must be greater than zero, we must choose $t = \boxed{8.20 \text{ s}}$ as the proper answer.

(b) $\Delta x_{\text{player}} = v_{i,\text{player}} t + \frac{1}{2} a_{\text{player}} t^2 = 0 + \frac{1}{2}\left(4.00 \text{ m/s}^2\right)(8.20 \text{ s})^2 = \boxed{134 \text{ m}}$

Section 2.7 Freely Falling Objects

P2.45 This is motion with constant acceleration, in this case the acceleration of gravity. The equation of position as a function of time is

$$y_f = y_i + v_i t + \frac{1}{2} a t^2$$

Taking the positve y direction as up, the acceleration is $a = (9.80 \text{ m/s}^2$, downward) $= -g$; we also know that $y_i = 0$ and $v_i = 2.80$ m/s. The above equation becomes

$$y_f = v_i t - \frac{1}{2} g t^2$$

$$y_f = (2.80 \text{ m/s})t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

(a) At $t = 0.100$ s, $y_f = \boxed{0.231 \text{ m}}$

(b) At $t = 0.200$ s, $y_f = \boxed{0.364 \text{ m}}$

(c) At $t = 0.300$ s, $y_f = \boxed{0.399 \text{ m}}$

(d) At $t = 0.500$ s, $y_f = \boxed{0.175 \text{ m}}$

P2.46 We can solve (a) and (b) at the same time by assuming the rock passes the top of the wall and finding its speed there. If the speed comes out imaginary, the rock will not reach this elevation.

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$= (7.40 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(3.65 \text{ m} - 1.55 \text{ m})$$
$$= 13.6 \text{ m}^2/\text{s}^2$$

which gives $v_f = 3.69$ m/s.

$\boxed{\text{So the rock does reach the top of the wall with } v_f = 3.69 \text{ m/s}}$.

(c) The rock travels from $y_i = 3.65$ m to $y_f = 1.55$ m. We find the final speed of the rock thrown down:

$$v_f^2 = v_i^2 + 2a(y_f - y_i)$$
$$= (-7.40 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(1.55 \text{ m} - 3.65 \text{ m})$$
$$= 95.9 \text{ m}^2/\text{s}^2$$

which gives $v_f = -9.79$ m/s.

The change in speed of the rock thrown down is

$$|9.79 \text{ m/s} - 7.40 \text{ m/s}| = \boxed{2.39 \text{ m/s}}$$

(d) The magnitude of the speed change of the rock thrown up is $|7.40 \text{ m/s} - 3.69 \text{ m/s}| = 3.71$ m/s. This $\boxed{\text{does not agree}}$ with 2.39 m/s.

(e) The upward-moving rock spends more time in flight because its average speed is smaller than the downward-moving rock, so the rock has more time to change its speed.

P2.47 The bill starts from rest, $v_i = 0$, and falls with a downward acceleration of 9.80 m/s^2 (due to gravity). For an average human reaction time of about 0.20 s, we can find the distance the bill will fall:

$$y_f = y_i + v_i t + \frac{1}{2}at^2 \rightarrow \Delta y = v_i t - \frac{1}{2}gt^2$$

$$\Delta y = 0 - \frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.20 \text{ s})^2 = -0.20 \text{ m}$$

The bill falls about 20 cm—this distance is about twice the distance between the center of the bill and its top edge, about 8 cm. Thus David could not respond fast enough to catch the bill.

P2.48 Since the ball's motion is entirely vertical, we can use the equations for free fall to find the initial velocity and maximum height from the elapsed time. After leaving the bat, the ball is in free fall for $t = 3.00$ s and has constant acceleration $a_y = -g = -9.80$ m/s^2.

(a) The initial speed of the ball can be found from

$$v_f = v_i + at$$

$$0 = v_i - gt \rightarrow v_i = gt$$

$$v_i = \left(9.80 \text{ m/s}^2\right)(3.00 \text{ s}) = \boxed{29.4 \text{ m/s}}$$

(b) Find the vertical displacement Δy:

$$\Delta y = y_f - y_i = \frac{1}{2}\left(v_i + v_f\right)t$$

$$\Delta y = \frac{1}{2}(29.4 \text{ m/s} + 0)(3.00 \text{ s})$$

$$\Delta y = \boxed{44.1 \text{ m}}$$

***P2.49** (a) Consider the upward flight of the arrow.

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = (100 \text{ m/s})^2 + 2\left(-9.80 \text{ m/s}^2\right)\Delta y$$

$$\Delta y = \frac{10\,000 \text{ m}^2/\text{s}^2}{19.6 \text{ m/s}^2} = \boxed{510 \text{ m}}$$

(b) Consider the whole flight of the arrow.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (100 \text{ m/s})t + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

The root $t = 0$ refers to the starting point. The time of flight is given by

$$t = \frac{100 \text{ m/s}}{4.90 \text{ m/s}^2} = \boxed{20.4 \text{ s}}$$

P2.50 We are given the height of the helicopter: $y = h = 3.00t^3$.

At $t = 2.00$ s, $y = 3.00(2.00 \text{ s})^3 = 24.0$ m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s} \uparrow$$

If the helicopter releases a small mailbag at this time, the mailbag starts its free fall with velocity 36.0 m/s upward. The equation of motion of the mailbag is

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$

$$y_f = (24.0 \text{ m}) + (36.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$

Setting $y_f = 0$, dropping units, and rearranging the equation, we have

$$4.90t^2 - 36.0t - 24.0 = 0$$

We solve for t using the quadratic formula:

$$t = \frac{36.0 \pm \sqrt{(-36.0)^2 - 4(4.90)(-24.0)}}{2(4.90)}$$

Since only positive values of t count, we find $t = \boxed{7.96 \text{ s}}$.

P2.51 The equation for the height of the ball as a function of time is

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

$$0 = 30 \text{ m} + (-8.00 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$

Solving for t,

$$t = \frac{+8.00 \pm \sqrt{(-8.00)^2 - 4(-4.90)(30)}}{2(-4.90)} = \frac{+8.00 \pm \sqrt{64 + 588}}{-9.80}$$

$$t = \boxed{1.79\text{ s}}$$

***P2.52** The falling ball moves a distance of (15 m – h) before they meet, where h is the height above the ground where they meet. We apply

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

to the falling ball to obtain

$$-(15.0\text{ m} - h) = -\frac{1}{2}gt^2$$

or $$h = 15.0\text{ m} - \frac{1}{2}gt^2 \quad \textbf{[1]}$$

Applying $y_f = y_i + v_i t - \frac{1}{2}gt^2$ to the rising ball gives

$$h = (25\text{ m/s})t - \frac{1}{2}gt^2 \quad \textbf{[2]}$$

Combining equations [1] and [2] gives

$$(25\text{ m/s})t - \frac{1}{2}gt^2 = 15.0\text{ m} - \frac{1}{2}gt^2$$

or $$t = \frac{15\text{ m}}{25\text{ m/s}} = \boxed{0.60\text{ s}}$$

P2.53 We model the keys as a particle under the constant free-fall acceleration. Take the first student's position to be $y_i = 0$ and the second student's position to be $y_f = 4.00$ m. We are given that the time of flight of the keys is t = 1.50 s, and $a_y = -9.80\text{ m/s}^2$.

(a) We choose the equation $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$ to connect the data and the unknown.

We solve:

$$v_{yi} = \frac{y_f - y_i - \frac{1}{2}a_y t^2}{t}$$

and substitute:

$$v_{yi} = \frac{4.00\ \text{m} - \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)(1.50\ \text{s})^2}{1.50\ \text{s}} = \boxed{10.0\ \text{m/s}}$$

(b) The velocity at any time $t > 0$ is given by $v_{yf} = v_{yi} + a_y t$.

Therefore, at $t = 1.50$ s,

$$v_{yf} = 10.0\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)(1.50\ \text{s}) = \boxed{-4.68\ \text{m/s}}$$

The negative sign means that the keys are moving **downward** just before they are caught.

P2.54 (a) The keys, moving freely under the influence of gravity ($a = -g$), undergo a vertical displacement of $\Delta y = +h$ in time t. We use $\Delta y = v_i t + \frac{1}{2}at^2$ to find the initial velocity as

$$\Delta y = v_i t + \frac{1}{2}at^2 = h$$

$$\rightarrow h = v_i t - \frac{1}{2}gt^2$$

$$v_i = \frac{h + \frac{1}{2}gt^2}{t} = \boxed{\frac{h}{t} + \frac{gt}{2}}$$

(b) We find the velocity of the keys just before they were caught (at time t) using $v = v_i + at$:

$$v = v_i + at$$

$$v = \left(\frac{h}{t} + \frac{gt}{2}\right) - gt$$

$$v = \boxed{\frac{h}{t} - \frac{gt}{2}}$$

P2.55 Both horse and man have constant accelerations: they are g downward for the man and 0 for the horse. We choose to do part (b) first.

(b) Consider the vertical motion of the man after leaving the limb (with $v_i = 0$ at $y_i = 3.00$ m) until reaching the saddle (at $y_f = 0$).

Modeling the man as a particle under constant acceleration, we find his time of fall from $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$.

When $v_i = 0$,

$$t = \sqrt{\frac{2\left(y_f - y_i\right)}{a_y}} = \sqrt{\frac{2(0 - 3.00\ \text{m})}{-9.80\ \text{m/s}^2}} = \boxed{0.782\ \text{s}}$$

(a) During this time interval, the horse is modeled as a particle under constant velocity in the horizontal direction.

$$v_{xi} = v_{xf} = 10.0 \text{ m/s}$$

$$x_f - x_i = v_{xi}t = (10.0 \text{ m/s})(0.782 \text{ s}) = \boxed{7.82 \text{ m}}$$

and the ranch hand must let go when the horse is 7.82 m from the tree.

P2.56 (a) Let $t = 0$ be the instant the package leaves the helicopter. The package and the helicopter have a common initial velocity of $-v_i$ (choosing upward as positive). The helicopter has zero acceleration, and the package (in free-fall) has constant acceleration $a_y = -g$.

At times $t > 0$, the velocity of the package is

$$v_p = v_{yi} + a_y t \rightarrow v_p = -v_i - gt = -(v_i + gt)$$

so its speed is $|v_p| = \boxed{v_i + gt}$.

(b) Assume the helicopter is at height H when the package is released. Setting our clock to $t = 0$ at the moment the package is released, the position of the helicopter is

$$y_{hel} = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_{hel} = H + (-v_i)t$$

and the position of the package is

$$y_p = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_p = H + (-v_i)t - \frac{1}{2}gt^2$$

The vertical distance, d, between the helicopter and the package is

$$y_{hel} - y_p = \left[H + (-v_i)t\right] - \left[H + (-v_i)t - \frac{1}{2}gt^2\right]$$
$$d = \boxed{\frac{1}{2}gt^2}$$

The distance is independent of their common initial speed.

(c) Now, the package and the helicopter have a common initial velocity of $+v_i$ (choosing upward as positive). The helicopter has zero acceleration, and the package (in free-fall) has constant acceleration $a_y = -g$.

At times $t > 0$, the velocity of the package is

$$v_P = v_{yi} + a_y t \rightarrow v_p = +v_i - gt$$

Therefore, the speed of the package at time t is $v_p = \boxed{|v_i - gt|}$.

The position of the helicopter is

$$y_{hel} = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_{hel} = H + (+v_i)t$$

and the position of the package is

$$y_p = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_p = H + (+v_i)t - \frac{1}{2}gt^2$$

The vertical distance, d, between the helicopter and the package is

$$y_{hel} - y_p = \left[H + (+v_i)t\right] - \left[H + (+v_i)t - \frac{1}{2}gt^2\right]$$
$$d = \boxed{\frac{1}{2}gt^2}$$

As above, the distance is independent of their common initial speed.

Section 2.8 Kinematic Equations Derived from Calculus

P2.57 This is a derivation problem. We start from basic definitions. We are given $J = da_x/dt$ = constant, so we know that $da_x = Jdt$.

(a) Integrating from the 'initial' moment when we know the acceleration to any later moment,

$$\int_{a_{ix}}^{a_x} da = \int_0^t J\,dt \quad \rightarrow \quad a_x - a_{ix} = J(t - 0)$$

Therefore, $\boxed{a_x = Jt + a_{xi}}$.

From $a_x = dv_x/dt$, $dv_x = a_x\,dt$.

Integration between the same two points tells us the velocity as a function of time:

$$\int_{v_{xi}}^{v_x} dv_x = \int_0^t a_x\, dt = \int_0^t (a_{xi} + Jt)\, dt$$

$$v_x - v_{xi} = a_{xi}t + \frac{1}{2}Jt^2 \quad \text{or} \quad \boxed{v_x = v_{xi} + a_{xi}t + \frac{1}{2}Jt^2}$$

From $v_x = dx/dt$, $dx = v_x dt$. Integrating a third time gives us $x(t)$:

$$\int_{x_i}^{x} dx = \int_0^t v_x\, dt = \int_0^t \left(v_{xi} + a_{xi}t + \frac{1}{2}Jt^2\right)\, dt$$

$$x - x_i = v_{xi}t + \frac{1}{2}a_{xi}t^2 + \frac{1}{6}Jt^3$$

and $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_{xi}t^2 + v_{xi}t + x_i}$.

(b) Squaring the acceleration,

$$a_x^2 = (Jt + a_{xi})^2 = J^2t^2 + a_{xi}^2 + 2Ja_{xi}t$$

Rearranging,

$$a_x^2 = a_{xi}^2 + 2J\left(\frac{1}{2}Jt^2 + a_{xi}t\right)$$

The expression for v_x was

$$v_x = \frac{1}{2}Jt^2 + a_{xi}t + v_{xi}$$

So $\quad (v_x - v_{xi}) = \frac{1}{2}Jt^2 + a_{xi}t$

and by substitution

$$\boxed{a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})}$$

P2.58 (a) See the x vs. t graph on the top panel of ANS. FIG. P2.58, on the next page. Choose $x = 0$ at $t = 0$.

At $t = 3$ s, $x = \frac{1}{2}(8 \text{ m/s})(3 \text{ s}) = 12$ m.

At $t = 5$ s, $x = 12 \text{ m} + (8 \text{ m/s})(2 \text{ s}) = 28$ m.

At $t = 7$ s, $x = 28 \text{ m} + \frac{1}{2}(8 \text{ m/s})(2 \text{ s})$

$= 36$ m

(b) See the a vs. t graph at the bottom right.

For $0 < t < 3$ s, $a = \dfrac{8 \text{ m/s}}{3 \text{ s}} = 2.67 \text{ m/s}^2$.

For $3 < t < 5$ s, $a = 0$.

At the points of inflection, $t = 3$ and 5 s, the slope of the velocity curve changes abruptly, so the acceleration is not defined.

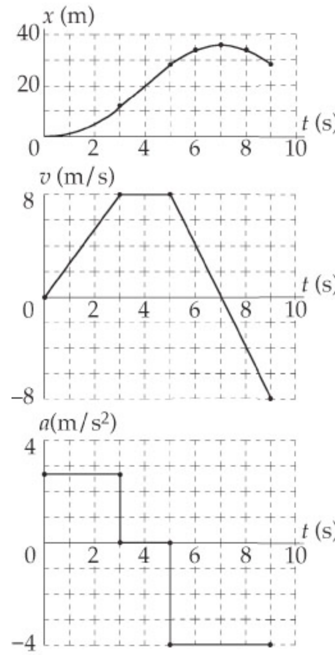
(c) For $5 \text{ s} < t < 9$ s,

$$a = -\frac{16 \text{ m/s}}{4 \text{ s}} = \boxed{-4 \text{ m/s}^2}$$

(d) The average velocity between $t = 5$ and 7 s is

$$v_{\text{avg}} = (8 \text{ m/s} + 0)/2 = 4 \text{ m/s}$$

At $t = 6$ s, $x = 28 \text{ m} + (4 \text{ m/s})(1 \text{ s}) = \boxed{32 \text{ m}}$



ANS. FIG. P2.58

(e) The average velocity between $t = 5$ and 9 s is

$$v_{\text{avg}} = [(8 \text{ m/s}) + (-8 \text{ m/s})]/2 = 0 \text{ m/s}$$

At $t = 9$ s, $x = 28 \text{ m} + (0 \text{ m/s})(1 \text{ s}) = \boxed{28 \text{ m}}$

P2.59 (a) To find the acceleration, we differentiate the velocity equation with respect to time:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[\left(-5.00 \times 10^7\right)t^2 + \left(3.00 \times 10^5\right)t\right]$$

$$\boxed{a = -\left(10.0 \times 10^7\right)t + 3.00 \times 10^5}$$

where a is in m/s^2 and t is in seconds.

To find the position, take $x_i = 0$ at $t = 0$. Then, from $v = \dfrac{dx}{dt}$,

$$x - 0 = \int_0^t v\,dt = \int_0^t \left(-5.00 \times 10^7 t^2 + 3.00 \times 10^5 t\right)dt$$

$$x = -5.00 \times 10^7 \frac{t^3}{3} + 3.00 \times 10^5 \frac{t^2}{2}$$

which gives

$$\boxed{x = -\left(1.67 \times 10^7\right)t^3 + \left(1.50 \times 10^5\right)t^2}$$

where x is in meters and t is in seconds.

(b) The bullet escapes when $a = 0$:

$$a = -\left(10.0\times10^{7}\right)t + 3.00\times10^{5} = 0$$

$$t = \frac{3.00\times10^{5}\ \text{s}}{10.0\times10^{7}} = 3.00\times10^{-3}\ \text{s} = \boxed{3.00\ \text{ms}}$$

(c) Evaluate v when $t = 3.00 \times 10^{-3}$ s:

$$v = \left(-5.00\times10^{7}\right)\left(3.00\times10^{-3}\right)^{2} + \left(3.00\times10^{5}\right)\left(3.00\times10^{-3}\right)$$

$$v = -450 + 900 = \boxed{450\ \text{m/s}}$$

(d) Evaluate x when $t = 3.00 \times 10^{-3}$ s:

$$x = -\left(1.67\times10^{7}\right)\left(3.00\times10^{-3}\right)^{3} + \left(1.50\times10^{5}\right)\left(3.00\times10^{-3}\right)^{2}$$

$$x = -0.450 + 1.35 = \boxed{0.900\ \text{m}}$$

Additional Problems

***P2.60** (a) Assuming a constant acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{42.0\ \text{m/s}}{8.00\ \text{s}} = \boxed{5.25\ \text{m/s}^2}$$

(b) Taking the origin at the original position of the car,

$$x_f = \frac{1}{2}\left(v_i + v_f\right)t = \frac{1}{2}(42.0\ \text{m/s})(8.00\ \text{s}) = \boxed{168\ \text{m}}$$

(c) From $v_f = v_i + at$, the velocity 10.0 s after the car starts from rest is:

$$v_f = 0 + \left(5.25\ \text{m/s}^2\right)(10.0\ \text{s}) = \boxed{52.5\ \text{m/s}}$$

P2.61 (a) From $v^2 = v_i^2 + 2a\Delta y$, the insect's velocity after straightening its legs is

$$v = \sqrt{v_0^2 + 2a\left(\Delta y\right)}$$

$$= \sqrt{0 + 2\left(4\ 000\ \text{m/s}^2\right)\left(2.00\times10^{-3}\ \text{m}\right)} = \boxed{4.00\ \text{m/s}}$$

(b) The time to reach this velocity is

$$t = \frac{v - v_0}{a} = \frac{4.00\ \text{m/s} - 0}{4\ 000\ \text{m/s}^2} = 1.00\times10^{-3}\ \text{s} = \boxed{1.00\ \text{ms}}$$

(c) The upward displacement of the insect between when its feet leave the ground and its speed is momentarily zero is

$$\Delta y = \frac{v_f^2 - v_i^2}{2a}$$

$$\Delta y = \frac{0-(4.00\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = \boxed{0.816\text{ m}}$$

P2.62 (a) The velocity is constant between $t_i = 0$ and $t = 4$ s. Its acceleration is $\boxed{0}$.

(b) $a = (v_9 - v_4)/(9\text{ s} - 4\text{ s}) = (18 - [-12])\text{ (m/s)}/5\text{ s} = \boxed{6.0\text{ m/s}^2}$

(c) $a = (v_{18} - v_{13})/(18\text{ s} - 13\text{ s}) = (0 - 18)\text{ (m/s)}/5\text{ s} = \boxed{-3.6\text{ m/s}^2}$

(d) We read from the graph that the speed is zero $\boxed{\text{at } t = 6\text{ s and at }18\text{ s}}$.

(e) and (f) The object moves away from $x = 0$ into negative coordinates from $t = 0$ to $t = 6$ s, but then comes back again, crosses the origin and moves farther into positive coordinates until $\boxed{t = 18\text{ s}}$, then attaining its maximum distance, which is the cumulative distance under the graph line:

$$\Delta x = (-12\text{ m/s})(4\text{ s}) + \frac{1}{2}(-12\text{ m/s})(2\text{ s}) + \frac{1}{2}(18\text{ m/s})(3\text{ s})$$
$$+(18\text{ m/s})(4\text{ s})\frac{1}{2}(18\text{ m/s})(5\text{ s})$$
$$= \boxed{84\text{ m}}$$

(g) We consider the total distance, rather than the resultant displacement, by counting the contributions computed in part (f) as all positive:

$$d = +60\text{ m} + 144\text{ m} = \boxed{204\text{ m}}$$

P2.63 We set $y_i = 0$ at the top of the cliff, and find the time interval required for the first stone to reach the water using the particle under constant acceleration model:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

or in quadratic form,

$$-\frac{1}{2}a_y t^2 - v_{yi}t + y_f - y_i = 0$$

(a) If we take the direction downward to be negative,

$$y_f = -50.0\ \text{m}, \quad v_{yi} = -2.00\ \text{m/s}, \quad \text{and} \quad a_y = -9.80\ \text{m/s}^2$$

Substituting these values into the equation, we find

$$(4.90\ \text{m/s}^2)t^2 + (2.00\ \text{m/s})t - 50.0\ \text{m} = 0$$

We now use the quadratic formula. The stone reaches the pool after it is thrown, so time must be positive and only the positive root describes the physical situation:

$$t = \frac{-2.00\ \text{m/s} \pm \sqrt{(2.00\ \text{m/s})^2 - 4\left(4.90\ \text{m/s}^2\right)(-50.0\ \text{m})}}{2\left(4.90\ \text{m/s}^2\right)}$$

$$= \boxed{3.00\ \text{s}}$$

where we have taken the positive root.

(b) For the second stone, the time of travel is

$$t = 3.00\ \text{s} - 1.00\ \text{s} = 2.00\ \text{s}$$

Since $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$,

$$v_{yi} = \frac{\left(y_f - y_i\right) - \frac{1}{2}a_y t^2}{t}$$

$$= \frac{-50.0\ \text{m} - \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)(2.00\ \text{s})^2}{2.00\ \text{s}}$$

$$= \boxed{-15.3\ \text{m/s}}$$

The negative value indicates the downward direction of the initial velocity of the second stone.

(c) For the first stone,

$$v_{1f} = v_{1i} + a_1 t_1 = -2.00\ \text{m/s} + (-9.80\ \text{m/s}^2)(3.00\ \text{s})$$

$$v_{1f} = \boxed{-31.4\ \text{m/s}}$$

For the second stone,

$$v_{2f} = v_{2i} + a_2 t_2 = -15.3\ \text{m/s} + (-9.80\ \text{m/s}^2)(2.00\ \text{s})$$

$$v_{2f} = \boxed{-34.8\ \text{m/s}}$$

P2.64 (a) Area A_1 is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

Area A_2 is triangular. Therefore, $A_2 = \frac{1}{2}bh = \frac{1}{2}t\left(v_x - v_{xi}\right)$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{\left(v_x - v_{xi}\right)t}{2}$$

and since $v_x - v_{xi} = a_x t$,

$$\boxed{A = v_{xi}t + \frac{1}{2}a_x t^2}$$

(b) The displacement given by the equation is: $x = v_{xi}t + \frac{1}{2}a_x t^2$, the same result as above for the total area.

***P2.65** (a) Take initial and final points at top and bottom of the first incline, respectively. If the ball starts from rest, $v_i = 0$, $a = 0.500\ \text{m/s}^2$, and $x_f - x_i = 9.00$ m. Then

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 0^2 + 2\left(0.500\ \text{m/s}^2\right)(9.00\ \text{m})$$

$$v_f = \boxed{3.00\ \text{m/s}}$$

(b) To find the time interval, we use

$$x_f - x_i = v_i t + \frac{1}{2}at^2$$

Plugging in,

$$9.00 = 0 + \frac{1}{2}\left(0.500\ \text{m/s}^2\right)t^2$$

$$t = \boxed{6.00\ \text{s}}$$

(c) Take initial and final points at the bottom of the first plane and the top of the second plane, respectively: $v_i = 3.00$ m/s, $v_f = 0$, and $x_f - x_i = 15.0$ m. We use

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$$

which gives

$$a = \frac{v_f^2 - v_i^2}{2\left(x_f - x_i\right)} = \frac{0 - (3.00\ \text{m/s})^2}{2(15.0\ \text{m})} = \boxed{-0.300\ \text{m/s}^2}$$

(d) Take the initial point at the bottom of the first plane and the final point 8.00 m along the second plane:

$$v_i = 3.00 \text{ m/s}, \; x_f - x_i = 8.00 \text{ m}, \; a = -0.300 \text{ m/s}^2$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = (3.00 \text{ m/s})^2 + 2(-0.300 \text{ m/s}^2)(8.00 \text{ m})$$

$$= 4.20 \text{ m}^2/\text{s}^2$$

$$v_f = \boxed{2.05 \text{ m/s}}$$

***P2.66** Take downward as the positive y direction.

(a) While the woman was in free fall, $\Delta y = 144$ ft, $v_i = 0$, and we take $a = g = 32.0 \text{ ft/s}^2$. Thus,

$$\Delta y = v_i t + \frac{1}{2}at^2 \rightarrow 144 \text{ ft} = 0 + (16.0 \text{ ft/s}^2)t^2$$

giving $t_{\text{fall}} = 3.00$ s. Her velocity just before impact is:

$$v_f = v_i + gt = 0 + (32.0 \text{ ft/s}^2)(3.00 \text{ s}) = \boxed{96.0 \text{ ft/s}}$$

(b) While crushing the box, $v_i = 96.0 \text{ ft/s}$, $v_f = 0$, and $\Delta y = 18.0 \text{ in.} = 1.50 \text{ ft}$. Therefore,

$$a = \frac{v_f^2 - v_i^2}{2(\Delta y)} = \frac{0 - (96.0 \text{ ft/s})^2}{2(1.50 \text{ ft})} = -3.07 \times 10^3 \text{ ft/s}^2$$

or $\boxed{a = 3.07 \times 10^3 \text{ ft/s}^2 \text{ upward}} = 96.0g$.

(c) Time to crush box:

$$\Delta t = \frac{\Delta y}{\bar{v}} = \frac{\Delta y}{\frac{v_f + v_i}{2}} = \frac{2(1.50 \text{ ft})}{0 + 96.0 \text{ ft/s}}$$

or $\boxed{\Delta t = 3.13 \times 10^{-2} \text{ s}}$

P2.67 (a) The elevator, moving downward at the constant speed of 5.00 m/s has moved $d = v\Delta t = (5.00 \text{ m/s})(5.00 \text{ s}) = 25.0 \text{ m}$ below the position from which the bolt drops. Taking the positive direction to be downward, the initial position of the bolt to be $x_B = 0$, and setting $t = 0$ when the bolt drops, the position of the top of the elevator is

$$y_E = y_{Ei} + v_{Ei}t + \frac{1}{2}a_E t^2$$

$$y_E = 25.0 \text{ m} + (5.00 \text{ m/s})t$$

and the position of the bolt is

$$y_B = y_{Bi} + v_{Bi}t + \frac{1}{2}a_B t^2$$

$$y_B = \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Setting these expressions equal to each other gives

$$y_E = y_B$$

$$25.0 \text{ m} + (5.00 \text{ m/s})t = \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

$$4.90t^2 - 5.00t - 25.0 = 0$$

The (positive) solution to this is $t = \boxed{2.83 \text{ s}}$.

(b) Both problems have an object traveling at constant velocity being overtaken by an object starting from rest traveling in the same direction at a constant acceleration.

(c) The top of the elevator travels a total distance

$$d = (5.00 \text{ m/s})(5.00 \text{ s} + 2.83 \text{ s}) = 39.1 \text{ m}$$

from where the bolt drops to where the bolt strikes the top of the elevator. Assuming 1 floor $\cong$ 3 m, this distance is about

$$(39.1 \text{ m})(1 \text{ floor}/3 \text{ m}) \cong 13 \text{ floors}.$$

P2.68 For the collision not to occur, the front of the passenger train must not have a position that is equal to or greater than the position of the back of the freight train at any time. We can write expressions of position to see whether the front of the passenger car (P) meets the back of the freight car (F) at some time.

Assume at $t = 0$, the coordinate of the front of the passenger car is $x_{Pi} = 0$; and the coordinate of the back of the freight car is $x_{Fi} = 58.5$ m. At later time t, the coordinate of the front of the passenger car is

$$x_P = x_{Pi} + v_{Pi}t + \frac{1}{2}a_P t^2$$

$$x_P = (40.0 \text{ m/s})t + \frac{1}{2}(-3.00 \text{ m/s}^2)t^2$$

and the coordinate of the back of the freight car is

$$x_F = x_{Fi} + v_{Fi}t + \frac{1}{2}a_F t^2$$

$$x_F = 58.5 \text{ m} + (16.0 \text{ m/s})t$$

Setting these expression equal to each other gives

$$x_P = x_F$$

$$(40.0\ \text{m/s})t + \frac{1}{2}(-3.00\ \text{m/s}^2)t^2 = 58.5\ \text{m} + (16.0\ \text{m/s})t$$

or $$(1.50)t^2 + (-24.0)t + 58.5 = 0$$

after simplifying and suppressing units.

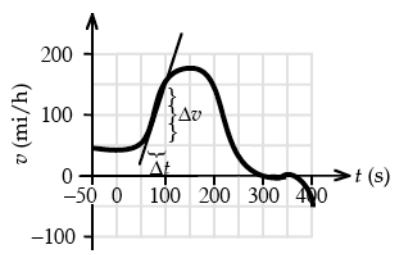
We do not have to solve this equation, we just want to check if a solution exists; if a solution does exist, then the trains collide. A solution does exist:

$$t = \frac{-(-24.0) \pm \sqrt{(-24.0)^2 - 4(1.50)(58.5)}}{2(1.50)}$$

$$t = \frac{24.0 \pm \sqrt{576 - 351}}{3.00} \rightarrow t = \frac{24.0 \pm \sqrt{225}}{3.00} = \frac{24.0 \pm 15}{3.00}$$

The situation is impossible since there is a finite time for which the front of the passenger train and the back of the freight train are at the same location.

P2.69 (a) As we see from the graph, from about –50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.



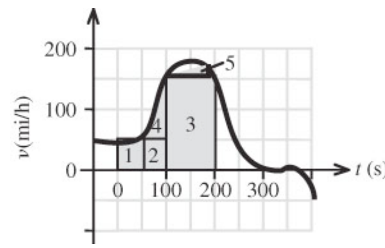
ANS. FIG. P2.69(a)

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155 - 45)\ \text{mi/h}}{(100 - 50)\ \text{s}}$$

$$= \boxed{2.2\ (\text{mi/h})/\text{s}} = 0.98\ \text{m/s}^2$$

(c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.



ANS. FIG. P2.69(c)

$$\begin{aligned}\Delta x_{0\to 200\text{ s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5 \\ &\approx (50\text{ mi/h})(50\text{ s}) + (50\text{ mi/h})(50\text{ s}) \\ &+(160\text{ mi/h})(100\text{ s}) \\ &+\frac{1}{2}(50\text{ s})(100\text{ mi/h}) \\ &+\frac{1}{2}(100\text{ s})(170\text{ mi/h} - 160\text{ mi/h}) \\ &= 24\ 000(\text{mi/h})(\text{s})\end{aligned}$$

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As 1 h = 3 600 s,

$$\Delta x_{0\to 200\text{ s}} = \left(\frac{24\ 000\text{ mi}}{3\ 600\text{ s}}\right)(\text{s}) = \boxed{6.7\text{ mi}}$$

P2.70 We use the relation $v_f^2 = v_i^2 + 2a(x_f - x_i)$, where $v_i = -8.00$ m/s and $v_f = 16.0$ m/s.

(a) The displacement of the first object is $\Delta x = +20.0$ m. Solving the above equation for the acceleration a, we obtain

$$a = \frac{v_f^2 - v_i^2}{2\Delta x}$$

$$a = \frac{(16.0\text{ m/s})^2 - (-8.00\text{ m/s})^2}{2(20.0\text{ m})}$$

$$a = \boxed{+4.80\text{ m/s}^2}$$

(b) Here, the total distance $d = 22.0$ m. The initial negative velocity and final positive velocity indicate that first the object travels through a negative displacement, slowing down until it reverses direction (where $v = 0$), then it returns to, and passes, its starting point, continuing to speed up until it reaches a speed of 16.0 m/s. We must consider the motion as comprising three displacements; the total distance d is the sum of the lengths of these displacements.

We split the motion into three displacements in which the acceleration remains constant throughout. We can find each displacement using

$$\Delta x = \frac{v_f^2 - v_i^2}{2a}$$

Displacement $\Delta x_1 = -d_1$ for velocity change $-8.00 \rightarrow 0$ m/s:

$$\Delta x_1 = \frac{v_f^2 - v_i^2}{2a} = \frac{0 - (-8.00\text{ m/s})^2}{2a} = \frac{(-8)^2}{2a} \rightarrow d_1 = \frac{8^2}{2a}$$

Displacement $\Delta x_2 = +d_1$ for velocity change $0 \rightarrow +8.00$ m/s:

$$\Delta x_2 = \frac{v_f^2 - v_i^2}{2a} = \frac{(8.00\text{ m/s})^2 - 0}{2a} = \frac{8^2}{2a} \rightarrow d_2 = \frac{8^2}{2a}$$

Displacement $\Delta x_3 = +d_2$ for velocity change $+8.00 \rightarrow +16.0$ m/s:

$$\Delta x_3 = \frac{v_f^2 - v_i^2}{2a} = \frac{(16.0\text{ m/s})^2 - (8.00\text{ m/s})^2}{2a} = \frac{16^2 - 8^2}{2a}$$

$$\rightarrow d_3 = \frac{16^2 - 8^2}{2a}$$

Suppressing units, the total distance is $d = d_1 + d_2 + d_3$, or

$$d = d_1 + d_2 + d_3 = 2\left(\frac{8^2}{2a}\right) + \frac{16^2 - 8^2}{2a} = \frac{16^2 + 8^2}{2a}$$

Solving for the acceleration gives

$$a = \frac{v_f^2 - v_i^2}{2d} = \frac{(16\text{ m/s})^2 + (8\text{ m/s})^2}{2d} = \frac{(16\text{ m/s})^2 + (8\text{ m/s})^2}{2(22.0\text{ m})}$$

$$a = \boxed{7.27\text{ m/s}^2}$$

P2.71 (a) In order for the trailing athlete to be able to catch the leader, his speed (v_1) must be greater than that of the leading athlete (v_2), and the distance between the leading athlete and the finish line must be great enough to give the trailing athlete sufficient time to make up the deficient distance, d.

(b) During a time interval t the leading athlete will travel a distance $d_2 = v_2 t$ and the trailing athlete will travel a distance $d_1 = v_1 t$. Only when $d_1 = d_2 + d$ (where d is the initial distance the trailing athlete was behind the leader) will the trailing athlete have caught the leader. Requiring that this condition be satisfied gives the elapsed time required for the second athlete to overtake the first:

$$d_1 = d_2 + d \quad \text{or} \quad v_1 t = v_2 t + d$$

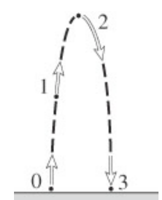
giving

$$v_1 t - v_2 t = d \quad \text{or} \quad t = \boxed{\frac{d}{(v_1 - v_2)}}$$

(c) In order for the trailing athlete to be able to at least tie for first place, the initial distance D between the leader and the finish line must be greater than or equal to the distance the leader can travel in the time t calculated above (i.e., the time required to overtake the leader). That is, we must require that

$$D \geq d_2 = v_2 t = v_2 \left[\frac{d}{(v_1 - v_2)}\right] \quad \text{or} \quad \boxed{d_2 = \frac{v_2 d}{v_1 - v_2}}$$

P2.72 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table below are found for each phase of the rocket's motion.



ANS. FIG. P2.72

(0 to 1): $v_f^2 - (80.0\text{ m/s})^2 = 2(4.00\text{ m/s}^2)(1\,000\text{ m})$

so $v_f = 120$ m/s. Then, $120\text{ m/s} = 80.0\text{ m/s} + (4.00\text{ m/s}^2)t$

giving $t = 10.0$ s.

(1 to 2) $0 - (120\text{ m/s})^2 = 2(-9.80\text{ m/s}^2)(y_f - y_i)$

giving $y_f - y_i = 735$ m,

$0 - 120\text{ m/s} = (-9.80\text{ m/s}^2)t$

giving $t = 12.2$ s.

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80\text{ m/s}^2)(-1\,735\text{ m})$ or $v_f = -184$ m/s

Then $v_f = -184\text{ m/s} = (-9.80\text{ m/s}^2)t$

giving $t = 18.8$ s.

(a) $t_{\text{total}} = 10\text{ s} + 12.2\text{ s} + 18.8\text{ s} = \boxed{41.0\text{ s}}$

(b) $(y_f - y_i)_{\text{total}} = \boxed{1.73\text{ km}}$

(c) $v_{\text{final}} = \boxed{-184\text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

P2.73 We have constant-acceleration equations to apply to the two cars separately.

(a) Let the times of travel for Kathy and Stan be t_K and t_S, where

$$t_S = t_K + 1.00\ \text{s}$$

Both start from rest ($v_{xi,K} = v_{xi,S} = 0$), so the expressions for the distances traveled are

$$x_K = \frac{1}{2}a_{x,K}t_K^2 = \frac{1}{2}(4.90\ \text{m/s}^2)t_K^2$$

and $$x_S = \frac{1}{2}a_{x,S}t_S^2 = \frac{1}{2}(3.50\ \text{m/s}^2)(t_K + 1.00\ \text{s})^2$$

When Kathy overtakes Stan, the two distances will be equal. Setting $x_K = x_S$ gives

$$\frac{1}{2}(4.90\ \text{m/s}^2)t_K^2 = \frac{1}{2}(3.50\ \text{m/s}^2)(t_K + 1.00\ \text{s})^2$$

This we simplify and write in the standard form of a quadratic as

$$t_K^2 - (5.00\ t_K)\text{s} - 2.50\ \text{s}^2 = 0$$

We solve using the quadratic formula $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, suppressing units, to find

$$t_K = \frac{5 \pm \sqrt{5^2 - 4(1)(-2.5)}}{2(1)} = \frac{5 + \sqrt{35}}{2} = \boxed{5.46\ \text{s}}$$

Only the positive root makes sense physically, because the overtake point must be after the starting point in time.

(b) Use the equation from part (a) for distance of travel,

$$x_K = \frac{1}{2}a_{x,K}t_K^2 = \frac{1}{2}(4.90\ \text{m/s}^2)(5.46\ \text{s})^2 = \boxed{73.0\ \text{m}}$$

(c) Remembering that $v_{xi,K} = v_{xi,S} = 0$, the final velocities will be:

$$v_{xf,K} = a_{x,K}t_K = (4.90\ \text{m/s}^2)(5.46\ \text{s}) = \boxed{26.7\ \text{m/s}}$$

$$v_{xf,S} = a_{x,S}t_S = (3.50\ \text{m/s}^2)(6.46\ \text{s}) = \boxed{22.6\ \text{m/s}}$$

P2.74 (a) While in the air, both balls have acceleration $a_1 = a_2 = -g$ (where upward is taken as positive). Ball 1 (thrown downward) has initial velocity $v_{01} = -v_0$, while ball 2 (thrown upward) has initial velocity $v_{02} = v_0$. Taking $y = 0$ at ground level, the initial y coordinate of each ball is $y_{01} = y_{02} = +h$. Applying $\Delta y = y - y_i = v_i t + \frac{1}{2}at^2$ to each ball gives their y coordinates at time t as

Ball 1: $y_1 - h = -v_0 t + \frac{1}{2}(-g)t^2$ or $\boxed{y_1 = h - v_0 t - \frac{1}{2}gt^2}$

Ball 2: $y_2 - h = +v_0 t + \frac{1}{2}(-g)t^2$ or $\boxed{y_2 = h + v_0 t - \frac{1}{2}gt^2}$

At ground level, $y = 0$. Thus, we equate each of the equations found above to zero and use the quadratic formula to solve for the times when each ball reaches the ground. This gives the following:

Ball 1: $0 = h - v_0 t_1 - \frac{1}{2}gt_1^2 \rightarrow gt_1^2 + (2v_0)t_1 + (-2h) = 0$

so $$t_1 = \frac{-2v_0 \pm \sqrt{(2v_0)^2 - 4(g)(-2h)}}{2g} = -\frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Using only the *positive* solution gives

$$t_1 = -\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Ball 2: $0 = h + v_0 t_2 - \frac{1}{2}gt_2^2 \rightarrow gt_2^2 + (-2v_0)t_2 + (-2h) = 0$

and $$t_2 = \frac{-(-2v_0) \pm \sqrt{(-2v_0)^2 - 4(g)(-2h)}}{2g} = +\frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Again, using only the *positive* solution,

$$t_2 = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Thus, the difference in the times of flight of the two balls is

$$\Delta t = t_2 - t_1$$
$$= \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}} - \left(-\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}\right) = \boxed{\frac{2v_0}{g}}$$

(b) Realizing that the balls are going *downward* ($v < 0$) as they near the ground, we use $v_f^2 = v_i^2 + 2a(\Delta y)$ with $\Delta y = -h$ to find the velocity of each ball just before it strikes the ground:

Ball 1:

$$v_{1f} = -\sqrt{v_{1i}^2 + 2a_1(-h)} = -\sqrt{(-v_0)^2 + 2(-g)(-h)} = \boxed{-\sqrt{v_0^2 + 2gh}}$$

Ball 2:

$$v_{2f} = -\sqrt{v_{2i}^2 + 2a_2(-h)} = -\sqrt{(+v_0)^2 + 2(-g)(-h)} = \boxed{-\sqrt{v_0^2 + 2gh}}$$

(c) While both balls are still in the air, the distance separating them is

$$d = y_2 - y_1 = \left(h + v_0 t - \frac{1}{2}gt^2\right) - \left(h - v_0 t - \frac{1}{2}gt^2\right) = \boxed{2v_0 t}$$

P2.75 We translate from a pictorial representation through a geometric model to a mathematical representation by observing that the distances x and y are always related by $x^2 + y^2 = L^2$.

(a) Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now the unknown velocity of B is $\frac{dy}{dt} = v_B$ and $\frac{dx}{dt} = -v$,

so the differentiated equation becomes

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\left(\frac{x}{y}\right)(-v) = v_B$$

But $\frac{y}{x} = \tan\theta$, so $\quad v_B = \boxed{\left(\frac{1}{\tan\theta}\right)v}$

(b) We assume that θ starts from zero. At this instant $1/\tan\theta$ is infinite, and the velocity of B is infinitely larger than that of A. As θ increases, the velocity of object B decreases, becoming equal to v when $\theta = 45°$. After that instant, B continues to slow down with non-constant acceleration, coming to rest as θ goes to 90°.

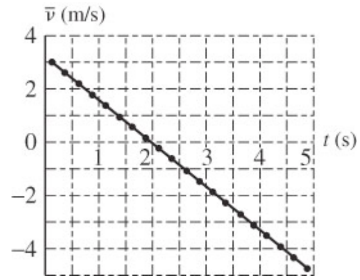
P2.76

Time t **(s)**	**Height** h **(m)**	Δh **(m)**	Δt **(s)**	$\bar{v}$ **(m/s)**	**midpoint time** t **(s)**
0.00	5.00	0.75	0.25	3.00	0.13
0.25	5.75	0.65	0.25	2.60	0.38
0.50	6.40	0.54	0.25	2.16	0.63
0.75	6.94	0.44	0.25	1.76	0.88
1.00	7.38	0.34	0.25	1.36	1.13
1.25	7.72	0.24	0.25	0.96	1.38
1.50	7.96	0.14	0.25	0.56	1.63
1.75	8.10	0.03	0.25	0.12	1.88
2.00	8.13	–0.06	0.25	–0.24	2.13
2.25	8.07	–0.17	0.25	–0.68	2.38
2.50	7.90	–0.28	0.25	–1.12	2.63
2.75	7.62	–0.37	0.25	–1.48	2.88
3.00	7.25	–0.48	0.25	–1.92	3.13
3.25	6.77	–0.57	0.25	–2.28	3.38
3.50	6.20	–0.68	0.25	–2.72	3.63
3.75	5.52	–0.79	0.25	–3.16	3.88
4.00	4.73	–0.88	0.25	–3.52	4.13
4.25	3.85	–0.99	0.25	–3.96	4.38
4.50	2.86	–1.09	0.25	–4.36	4.63
4.75	1.77	–1.19	0.25	–4.76	4.88
5.00	0.58				

TABLE P2.76

The very convincing fit of a single straight line to the points in the graph of velocity versus time indicates that the rock does fall with constant acceleration. The acceleration is the slope of line:

$$a_{\text{avg}} = -1.63 \text{ m/s}^2 = \boxed{1.63 \text{ m/s}^2 \text{ downward}}$$



***P2.77** Distance traveled by motorist = (15.0 m/s)t

Distance traveled by policeman $= \frac{1}{2}(2.00 \text{ m/s}^2)t^2$

(a) Intercept occurs when $15.0t = t^2$, or $t = \boxed{15.0 \text{ s}}$.

(b) $v(\text{officer}) = (2.00 \text{ m/s}^2)t = \boxed{30.0 \text{ m/s}}$

(c) $x(\text{officer}) = \frac{1}{2}(2.00 \text{ m/s}^2)t^2 = \boxed{225 \text{ m}}$

***P2.78** The train accelerates with $a_1 = 0.100 \text{ m/s}^2$ then decelerates with $a_2 = -0.500 \text{ m/s}^2$. We can write the 1.00-km displacement of the train as

$$x = 1\,000 \text{ m} = \frac{1}{2}a_1\Delta t_1^2 + v_{1f}\Delta t_2 + \frac{1}{2}a_2\Delta t_2^2$$

with $t = t_1 + t_2$. Now, $v_{1f} = a_1\Delta t_1 = -a_2\Delta t_2$; therefore

$$1\,000 \text{ m} = \frac{1}{2}a_1\Delta t_1^2 + a_1\Delta t_1\left(-\frac{a_1\Delta t_1}{a_2}\right) + \frac{1}{2}a_2\left(\frac{a_1\Delta t_1}{a_2}\right)^2$$

$$1\,000 \text{ m} = \frac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)\Delta t_1^2$$

$$1\,000 \text{ m} = \frac{1}{2}(0.100 \text{ m/s}^2)\left(1 - \frac{0.100 \text{ m/s}^2}{-0.500 \text{ m/s}^2}\right)\Delta t_1^2$$

$$\Delta t_1 = \sqrt{\frac{20\,000}{1.20}} \text{ s} = 129 \text{ s}$$

$$\Delta t_2 = \frac{a_1\Delta t_1}{-a_2} = \frac{12.9}{0.500} \text{ s} \approx 26 \text{ s}$$

Total time $= \Delta t = \Delta t_1 + \Delta t_2 = 129 \text{ s} + 26 \text{ s} = \boxed{155 \text{ s}}$

***P2.79** The average speed of every point on the train as the first car passes Liz is given by:

$$\frac{\Delta x}{\Delta t} = \frac{8.60 \text{ m}}{1.50 \text{ s}} = 5.73 \text{ m/s}$$

The train has this as its instantaneous speed halfway through the 1.50-s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\frac{8.60 \text{ m}}{1.10 \text{ s}} = 7.82 \text{ m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50 \text{ s}) + \frac{1}{2}(1.10 \text{ s}) = 1.30 \text{ s}$$

so the acceleration is: $a_x = \frac{\Delta v_x}{\Delta t} = \frac{7.82 \text{ m/s} - 5.73 \text{ m/s}}{1.30 \text{ s}} = \boxed{1.60 \text{ m/s}^2}$

P2.80 Let the ball fall freely for 1.50 m after starting from rest. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i)$$

$$v_{xf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(-1.50 \text{ m})$$

$$v_{xf} = -5.42 \text{ m/s}$$

If its acceleration were constant, its stopping would be described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = (-5.42 \text{ m/s})^2 + 2a_x(-10^{-2} \text{ m})$$

$$a_x = \frac{-29.4 \text{ m}^2/\text{s}^2}{-2.00 \times 10^{-2} \text{ m}} = +1.47 \times 10^3 \text{ m/s}^2$$

Upward acceleration of this same order of magnitude will continue for some additional time after the dent is at its maximum depth, to give the ball the speed with which it rebounds from the pavement. The ball's maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3 \text{m/s}^2}$.

Challenge Problems

P2.81 (a) From the information in the problem, we model the blue car as a particle under constant acceleration. The important "particle" for this part of the problem is the nose of the car. We use the position equation from the particle under constant acceleration model to find the velocity v_0 of the particle as it enters the intersection

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\rightarrow 28.0 \text{ m} = 0 + v_0(3.10 \text{ s}) + \frac{1}{2}(-2.10 \text{ m/s}^2)(3.10 \text{ s})^2$$

$$\rightarrow v_0 = 12.3 \text{ m/s}$$

Now we use the velocity-position equation in the particle under constant acceleration model to find the displacement of the particle from the first edge of the intersection when the blue car stops:

$$v^2 = v_0^2 + 2a(x - x_0)$$

or $$x - x_0 = \Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12.3 \text{ m/s})^2}{2(-2.10 \text{ m/s}^2)} = \boxed{35.9 \text{ m}}$$

(b) The time interval during which any part of the blue car is in the intersection is that time interval between the instant at which the nose enters the intersection and the instant when the tail leaves the intersection. Thus, the change in position of the nose of the blue car is 4.52 m + 28.0 m = 32.52 m. We find the time at which the car is at position x = 32.52 m if it is at x = 0 and moving at 12.3 m/s at t = 0:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\rightarrow 32.52 \text{ m} = 0 + (12.3 \text{ m/s})t + \frac{1}{2}(-2.10 \text{ m/s}^2)t^2$$

$$\rightarrow -1.05t^2 + 12.3t - 32.52 = 0$$

The solutions to this quadratic equation are t = 4.04 s and 7.66 s. Our desired solution is the lower of two, so $t = \boxed{4.04 \text{ s}}$, (The later time corresponds to the blue car stopping and reversing, which it must do if the acceleration truly remains constant, and arriving again at the position x = 32.52 m.)

(c) We again define t = 0 as the time at which the nose of the blue car enters the intersection. Then at time t = 4.04 s, the tail of the blue car leaves the intersection. Therefore, to find the minimum distance from the intersection for the silver car, its nose must enter the intersection at t = 4.04 s. We calculate this distance from the position equation:

$$x - x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(5.60\text{ m/s}^2\right)(4.04\text{ s})^2 = \boxed{45.8\text{ m}}$$

(d) We use the velocity equation:

$$v = v_0 + at = 0 + \left(5.60\text{ m/s}^2\right)(4.04\text{ s}) = \boxed{22.6\text{ m/s}}$$

P2.82 (a) Starting from rest and accelerating at a_b = 13.0 mi/h · s, the bicycle reaches its maximum speed of $v_{b,\max}$ = 20.0 mi/h in a time

$$t_{b,1} = \frac{v_{b,\max} - 0}{a_b} = \frac{20.0\text{ mi/h}}{13.0\text{ mi/h}\cdot\text{s}} = 1.54\text{ s}$$

Since the acceleration a_c of the car is less than that of the bicycle, the car cannot catch the bicycle until some time $t > t_{b,1}$ (that is, until the bicycle is at its maximum speed and coasting). The total displacement of the bicycle at time t is

$$\begin{aligned}\Delta x_b &= \frac{1}{2}a_b t_{b,1}^2 + v_{b,\max}\left(t - t_{b,1}\right)\\ &= \left(\frac{1.47\text{ ft/s}}{1\text{ mi/h}}\right)\times\\ &\quad\left[\frac{1}{2}\left(13.0\frac{\text{mi/h}}{\text{s}}\right)(1.54\text{ s})^2 + (20.0\text{ mi/h})(t - 1.54\text{ s})\right]\\ &= (29.4\text{ ft/s})t - 22.6\text{ ft}\end{aligned}$$

The total displacement of the car at this time is

$$\Delta x_c = \frac{1}{2}a_c t^2 = \left(\frac{1.47\text{ ft/s}}{1\text{ mi/h}}\right)\left[\frac{1}{2}\left(9.00\frac{\text{mi/h}}{\text{s}}\right)t^2\right] = \left(6.62\text{ ft/s}^2\right)t^2$$

At the time the car catches the bicycle, $\Delta x_c = \Delta x_b$. This gives

$$\left(6.62\text{ ft/s}^2\right)t^2 = (29.4\text{ ft/s})t - 22.6\text{ ft}$$

or $\quad t^2 - (4.44\text{ s})t + 3.42\text{ s}^2 = 0$

that has only one physically meaningful solution $t > t_{b,1}$. This solution gives the total time the bicycle leads the car and is $t = \boxed{3.45\text{ s}}$.

(b) The lead the bicycle has over the car continues to increase as long as the bicycle is moving faster than the car. This means until the car attains a speed of $v_c = v_{b,\max} = 20.0$ mi/h. Thus, the elapsed time when the bicycle's lead ceases to increase is

$$t = \frac{v_{b,\max}}{a_c} = \frac{20.0 \text{ mi/h}}{9.00 \text{ mi/h} \cdot \text{s}} = 2.22 \text{ s}$$

At this time, the lead is

$$\begin{aligned}(\Delta x_b - \Delta x_c)_{\max} &= (\Delta x_b - \Delta x_c)\big|_{t=2.22\text{ s}} \\ &= \left[(29.4 \text{ ft/s})(2.22 \text{ s}) - 22.6 \text{ ft}\right] \\ &\quad - \left[\left(6.62 \text{ ft/s}^2\right)(2.22 \text{ s})^2\right]\end{aligned}$$

or $\quad (\Delta x_b - \Delta x_c)_{\max} = \boxed{10.0 \text{ ft}}$

P2.83 Consider the runners in general. Each completes the race in a total time interval T. Each runs at constant acceleration a for a time interval Δt, so each covers a distance (displacement) $\Delta x_a = \frac{1}{2} a \Delta t^2$ where they eventually reach a final speed (velocity) $v = a\Delta t$, after which they run at this constant speed for the remaining time $(T - \Delta t)$ until the end of the race, covering distance $\Delta x_v = v(T - \Delta t) = a\Delta t(T - \Delta t)$. The total distance (displacement) each covers is the same:

$$\begin{aligned}\Delta x &= \Delta x_a + \Delta x_v \\ &= \frac{1}{2} a \Delta t^2 + a\Delta t(T - \Delta t) \\ &= a\left[\frac{1}{2}\Delta t^2 + \Delta t(T - \Delta t)\right]\end{aligned}$$

so
$$a = \frac{\Delta x}{\frac{1}{2}\Delta t^2 + \Delta t(T - \Delta t)}$$

where $\Delta x = 100$ m and $T = 10.4$ s.

(a) For Laura (runner 1), $\Delta t_1 = 2.00$ s:

$$a_1 = (100 \text{ m})/(18.8 \text{ s}^2) = \boxed{5.32 \text{ m/s}^2}$$

For Healan (runner 2), $\Delta t_2 = 3.00$ s:

$$a_2 = (100 \text{ m})/(26.7 \text{ s}^2) = \boxed{3.75 \text{ m/s}^2}$$

(b) Laura (runner 1): $v_1 = a_1 \Delta t_1 = \boxed{10.6 \text{ m/s}}$

Healan (runner 2): $v_2 = a_2 \Delta t_2 = \boxed{11.2 \text{ m/s}}$

(c) The 6.00-s mark occurs after either time interval Δt. From the reasoning above, each has covered the distance

$$\Delta x = a\left[\frac{1}{2}\Delta t^2 + \Delta t(t-\Delta t)\right]$$

where t = 6.00 s.

Laura (runner 1): Δx_1 = 53.19 m

Healan (runner 2): Δx_2 = 50.56 m

So, Laura is ahead by (53.19 m – 50.56 m) = 2.63 m.

(d) Laura accelerates at the greater rate, so she will be ahead of Healen at, and immediately after, the 2.00-s mark. After the 3.00-s mark, Healan is travelling faster than Laura, so the distance between them will shrink. In the time interval

from the 2.00-s mark to the 3.00-s mark, the distance between them will be the greatest.

During that time interval, the distance between them (the position of Laura relative to Healan) is

$$D = \Delta x_1 - \Delta x_2 = a_1\left[\frac{1}{2}\Delta t_1^2 + \Delta t_1(t-\Delta t_1)\right] - \frac{1}{2}a_2 t^2$$

because Laura has ceased to accelerate but Healan is still accelerating. Differentiating with respect to time, (and doing some simplification), we can solve for the time t when D is an maximum:

$$\frac{dD}{dt} = a_1\Delta t_1 - a_2 t = 0$$

which gives

$$t = \Delta t_1\left(\frac{a_1}{a_2}\right) = (2.00\text{ s})\left(\frac{5.32\text{ m/s}^2}{3.75\text{ m/s}^2}\right) = 2.84\text{ s}$$

Substituting this time back into the expression for D, we find that D = 4.47 m, that is, Laura ahead of Healan by 4.47 m.

P2.84 (a) The factors to consider are as follows. The red bead falls through a greater distance with a downward acceleration of g. The blue bead travels a shorter distance, but with acceleration of $g\sin\theta$. A first guess would be that the blue bead "wins," but not by much. We do note, however, that points Ⓐ, Ⓑ, and Ⓒ are the vertices of a right triangle with Ⓐ Ⓒ as the hypotenuse.

(b) The red bead is a particle under constant acceleration. Taking downward as the positive direction, we can write

$$\Delta y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

as $$D = \frac{1}{2}gt_R^2$$

which gives $\boxed{t_R = \sqrt{\dfrac{2D}{g}}}$.

(c) The blue bead is a particle under constant acceleration, with $a = g\sin\theta$. Taking the direction along L as the positive direction, we can write

$$\Delta y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

as $$L = \frac{1}{2}(g\sin\theta)t_B^2$$

which gives $\boxed{t_B = \sqrt{\dfrac{2L}{g\sin\theta}}}$.

(d) For the two beads to reach point Ⓒ simultaneously, $t_R = t_B$. Then,

$$\sqrt{\frac{2D}{g}} = \sqrt{\frac{2L}{g\sin\theta}}$$

Squaring both sides and cross-multiplying gives

$$2gD\sin\theta = 2gL$$

or $$\sin\theta = \frac{L}{D}.$$

We note that the angle between chords Ⓐ Ⓒ and Ⓑ Ⓒ is $90° - \theta$, so that the angle between chords Ⓐ Ⓒ and Ⓐ Ⓑ is

θ. Then, $\sin\theta = \frac{L}{D}$, and the beads arrive at point Ⓒ simultaneously.

(e) Once we recognize that the two rods form one side and the hypotenuse of a right triangle with θ as its smallest angle, then the result becomes obvious.

P2.85 The rock falls a distance d for a time interval Δt_1 and the sound of the splash travels upward through the same distance d for a time interval Δt_2 before the man hears it. The total time interval $\Delta t = \Delta t_1 + \Delta t_2 = 2.40$ s.

(a) Relationship between distance the rock falls and time interval Δt_1:

$$d = \frac{1}{2} g \Delta t_1^2$$

Relationship between distance the sound travels and time interval Δt_2: $d = v_s \Delta t_2$, where $v_s = 336$ m/s.

$$d = v_s \Delta t_2 = \frac{1}{2} g \Delta t_1^2$$

Substituting $\Delta t_1 = \Delta t - \Delta t_2$ gives

$$2\frac{v_s \Delta t_2}{g} = (\Delta t - \Delta t_2)^2$$

$$(\Delta t_2)^2 - 2\left(\Delta t + \frac{v_s}{g}\right)\Delta t_2 + \Delta t^2 = 0$$

$$(\Delta t_2)^2 - 2\left(2.40 \text{ s} + \frac{336 \text{ m/s}}{9.80 \text{ m/s}^2}\right)\Delta t_2 + (2.40 \text{ s})^2 = 0$$

$$(\Delta t_2)^2 - (73.37)\Delta t_2 + 5.76 = 0$$

Solving the quadratic equation gives

$$\Delta t_2 = 0.078\ 6 \text{ s} \quad \rightarrow \quad d = v_s \Delta t_2 = \boxed{26.4 \text{ m}}$$

(b) Ignoring the sound travel time,
$d = \frac{1}{2}(9.80 \text{ m/s}^2)(2.40 \text{ s})^2 = 28.2 \text{ m}$, an error of $\boxed{6.82\%}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P2.2 0.02 s

P2.4 (a) 50.0 m/s; (b) 41.0 m/s

P2.6 (a) 27.0 m; (b) 27.0 m + (18.0 m/s) Δt + (3.00 m/s^2)(Δt^2); (c) 18.0 m/s

P2.8 (a) $+L/t_1$; (b) $-L/t_2$; (c) 0; (d) $2L/t_1 + t_2$

P2.10 1.9×10^8 years

P2.12 (a) 20 mi/h; (b) 0; (c) 30 mi/h

P2.14 1.34×10^4 m/s^2

P2.16 See graphs in P2.16.

P2.18 (a) See ANS. FIG. P2.18; (b) 23 m/s, 18 m/s, 14 m/s, and 9.0 m/s; (c) 4.6 m/s^2; (d) zero

P2.20 (a) 13.0 m/s; (b) 10.0 m/s, 16.0 m/s; (c) 6.00 m/s^2; (d) 6.00 m/s^2; (e) 0.333 s

P2.22 (a–e) See graphs in P2.22; (f) with less regularity

P2.24 160 ft.

P2.26 4.53 s

P2.28 (a) 6.61 m/s; (b) −0.448 m/s^2

P2.30 (a) 20.0 s; (b) No; (c) The plane would overshoot the runway.

P2.32 31 s

P2.34 The accelerations do not match.

P2.36 (a) $x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2$; (b) 3.10 m/s

P2.38 (a) 2.56 m; (b) −3.00 m/s

P2.40 19.7 cm/s; (b) 4.70 cm/s^2; (c) The length of the glider is used to find the average velocity during a known time interval.

P2.42 (a) 3.75 s; (b) 5.50 cm/s; (c) 0.604 s; (d) 13.3 cm, 47.9 cm; (e) See P2.42 part (e) for full explanation.

P2.44 (a) 8.20 s; (b) 134 m

P2.46 (a and b) The rock does not reach the top of the wall with v_f = 3.69 m/s; (c) 2.39 m/s; (d) does not agree; (e) The average speed of the upward-moving rock is smaller than the downward moving rock.

P2.48 (a) 29.4 m/s; (b) 44.1 m

P2.50 7.96 s

P2.52 0.60 s

P2.54 (a) $\frac{h}{t}+\frac{gt}{2}$; (b) $\frac{h}{t}-\frac{gt}{2}$

P2.56 (a) $(v_i + gt)$; (b) $\frac{1}{2}gt^2$; (c) $|v_i - gt|$; (d) $\frac{1}{2}gt^2$

P2.58 (a) See graphs in P2.58; (b) See graph in P2.58; (c) −4 m/s^2; (d) 32 m; (e) 28 m

P2.60 (a) 5.25 m/s^2; (b) 168 m; (c) 52.5 m/s

P2.62 (a) 0; (b) 6.0 m/s^2; (c) −3.6 m/s^2; (d) at t = 6 s and at 18 s; (e and f) t = 18 s; (g) 204 m

P2.64 (a) $A = v_{xi}t + \frac{1}{2}a_x t^2$; (b) The displacement is the same result for the total area.

P2.66 (a) 96.0 ft/s; (b) 3.07×10^3 ft/s^2 upward; (c) 3.13×10^{-2} s

P2.68 The trains do collide.

P2.70 (a) +4.8 m/s^2; (b) 7.27 m/s^2

P2.72 (a) 41.0 s; (b) 1.73 km; (c) −184 m/s

P2.74 (a) Ball 1: $y_1 = h - v_0 t - \frac{1}{2}gt^2$, Ball 2: $y_2 = h + v_0 t - \frac{1}{2}gt^2, \frac{2v_0}{g}$; (b) Ball 1: $-\sqrt{v_0^2 + 2gh}$, Ball 2: $-\sqrt{v_0^2 + 2gh}$; (c) $2v_0 t$

P2.76 (a and b) See TABLE P2.76; (c) 1.63 m/s^2 downward and see graph in P2.76

P2.78 155 s

P2.80 ~10^3 m/s^2

P2.82 (a) 3.45 s; (b) 10.0 ft.

P2.84 (a) The red bead falls through a greater distance with a downward acceleration of g. The blue bead travels a shorter distance, but with acceleration of $g\sin\theta$. A first guess would be that the blue bead "wins," but not by much. (b) $\sqrt{\frac{2D}{g}}$; (c) $\sqrt{\frac{2L}{g\sin\theta}}$; (d) the beads arrive at point Ⓒ simultaneously; (e) Once we recognize that the two rods form one side and the hypotenuse of a right triangle with θ as its smallest angle, then the result becomes obvious.