3

Vectors

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ3.1 Answer (e). The magnitude is $\sqrt{10^2 + 10^2}$ m/s.

OQ3.2 Answer (e). If the quantities x and y are positive, a vector with components $(-x, y)$ or $(x, -y)$ would lie in the second or fourth quadrant, respectively.

***OQ3.3** Answer (a). The vector $-2\vec{\mathbf{D}}_1$ will be twice as long as $\vec{\mathbf{D}}_1$ and in the opposite direction, namely northeast. Adding $\vec{\mathbf{D}}_2$, which is about equally long and southwest, we get a sum that is still longer and due east.

OQ3.4 The ranking is c = e > a > d > b. The magnitudes of the vectors being added are constant, and we are considering the magnitude only—not the direction—of the resultant. So we need look only at the angle between the vectors being added in each case. The smaller this angle, the larger the resultant magnitude.

OQ3.5 Answers (a), (b), and (c). The magnitude can range from the sum of the individual magnitudes, 8 + 6 = 14, to the difference of the individual magnitudes, 8 − 6 = 2. Because magnitude is the "length" of a vector, it is always positive.

OQ3.6 Answer (d). If we write vector $\vec{\mathbf{A}}$ as

$$\left(A_x, A_y\right) = \left(-\left|A_x\right|, \left|A_y\right|\right)$$

and vector $\vec{\mathbf{B}}$ as

$$\left(B_x, B_y\right) = \left(\left|B_x\right|, -\left|B_y\right|\right)$$

then

$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = \left(\left|B_x\right| - \left(-\left|A_x\right|\right), -\left|B_y\right| - \left|A_y\right|\right) = \left(\left|B_x\right| + \left|A_x\right|, -\left|B_y\right| - \left|A_y\right|\right)$$

which would be in the fourth quadrant.

OQ3.7 The answers are (a) yes (b) no (c) no (d) no (e) no (f) yes (g) no. Only force and velocity are vectors. None of the other quantities requires a direction to be described.

OQ3.8 Answer (c). The vector has no y component given. It is therefore 0.

OQ3.9 Answer (d). Take the difference of the x coordinates of the ends of the vector, head minus tail: –4 – 2 = –6 cm.

OQ3.10 Answer (a). Take the difference of the y coordinates of the ends of the vector, head minus tail: 1 – (–2) = 3 cm.

OQ3.11 Answer (c). The signs of the components of a vector are the same as the signs of the points in the quadrant into which it points. If a vector arrow is drawn to scale, the coordinates of the point of the arrow equal the components of the vector. All x and y values in the third quadrant are negative.

OQ3.12 Answer (c). The vertical component is opposite the 30° angle, so sin 30° = (*vertical component*)/50 m.

OQ3.13 Answer (c). A vector in the second quadrant has a negative x component and a positive y component.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ3.1 Addition of a vector to a scalar is not defined. Try adding the speed and velocity, 8.0 m/s + $(15.0\text{ m/s }\hat{\mathbf{i}})$: Should you consider the sum to be a vector or a scalar? What meaning would it have?

CQ3.2 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

CQ3.3 (a) The book's displacement is zero, as it ends up at the point from which it started. (b) The distance traveled is 6.0 meters.

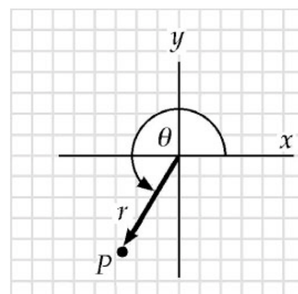
CQ3.4 Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other.

CQ3.5 The inverse tangent function gives the correct angle, relative to the $+x$ axis, for vectors in the first or fourth quadrant, and it gives an incorrect answer for vectors in the second or third quadrant. If the x and y components are both positive, their ratio y/x is positive and the vector lies in the first quadrant; if the x component is positive and the y component negative, their ratio y/x is negative and the vector lies in the fourth quadrant. If the x and y components are both negative, their ratio y/x is positive but the vector lies in the third quadrant; if the x component is negative and the y component positive, their ratio y/x is negative but the vector lies in the second quadrant.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 3.1 Coordinate Systems

P3.1 ANS. FIG. P3.1 helps to visualize the x and y coordinates, and trigonometric functions will tell us the coordinates directly. When the polar coordinates (r, θ) of a point P are known, the Cartesian coordinates are found as



$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

Then,

$$x = r\cos\theta = (5.50\text{ m})\cos 240°$$
$$= (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$$
$$y = r\sin\theta = (5.50\text{ m})\sin 240°$$
$$= (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$$

P3.2 (a) We use $x = r\cos\theta$. Substituting, we have $2.00 = r\cos 30.0°$, so

$$r = \frac{2.00}{\cos 30.0°} = \boxed{2.31}$$

(b) From $y = r\sin\theta$, we have $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$.

***P3.3** (a) The distance between the points is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2.00 - [-3.00])^2 + (-4.00 - 3.00)^2}$$

$$d = \sqrt{25.0 + 49.0} = \boxed{8.60 \text{ m}}$$

(b) To find the polar coordinates of each point, we measure the radial distance to that point and the angle it makes with the $+x$ axis:

$$r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47 \text{ m}}$$

$$\theta_1 = \tan^{-1}\left(-\frac{4.00}{2.00}\right) = \boxed{-63.4°}$$

$$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24 \text{ m}}$$

$\theta_2 = \boxed{135°}$ measured from the $+x$ axis.

P3.4 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore,

$x_1 = (2.50 \text{ m}) \cos 30.0°$, $y_1 = (2.50 \text{ m}) \sin 30.0°$, and

$(x_1, y_1) = \boxed{(2.17, 1.25) \text{ m}}$

$x_2 = (3.80 \text{ m}) \cos 120°$, $y_2 = (3.80 \text{ m}) \sin 120°$, and

$(x_2, y_2) = \boxed{(-1.90, 3.29) \text{ m}}$

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4.07^2 + 2.04^2} \text{ m} = \boxed{4.55 \text{ m}}$

P3.5 For polar coordinates (r, θ), the Cartesian coordinates are $(x = r\cos\theta, y = r\sin\theta)$, if the angle is measured relative to the $+x$ axis.

(a) $\boxed{(-3.56 \text{ cm}, -2.40 \text{ cm})}$

(b) $(+3.56 \text{ cm}, -2.40 \text{ cm}) \rightarrow \boxed{(4.30 \text{ cm}, -34.0°)}$

(c) $(7.12 \text{ cm}, 4.80 \text{ cm}) \rightarrow \boxed{(8.60 \text{ cm}, 34.0°)}$

(d) $(-10.7 \text{ cm}, 7.21 \text{ cm}) \rightarrow \boxed{(12.9 \text{ cm}, 146°)}$

P3.6 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}$$

(b) $\sqrt{(-2x)^2+(-2y)^2}=\boxed{2r}$. This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180°+\theta}$.

(c) $\sqrt{(3x)^2+(-3y)^2}=\boxed{3r}$. This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta \text{ or } 360-\theta}$.

Section 3.2 Vector and Scalar Quantities

Section 3.3 Some Properties of Vectors

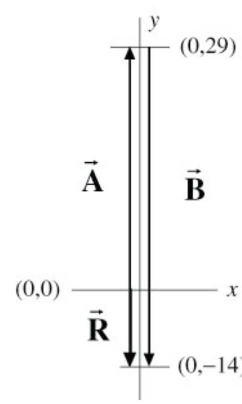
P3.7 Figure P3.7 suggests a right triangle where, relative to angle θ, its adjacent side has length d and its opposite side is equal to width of the river, y; thus,

$$\tan\theta=\frac{y}{d}\rightarrow y=d\tan\theta$$

$$y = (100 \text{ m})\tan(35.0°) = 70.0 \text{ m}$$

The width of the river is $\boxed{70.0 \text{ m}}$.

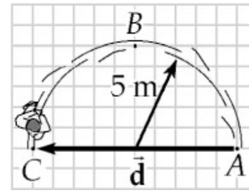
P3.8 We are given $\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}$. When two vectors are added graphically, the second vector is positioned with its tail at the tip of the first vector. The resultant then runs from the tail of the first vector to the tip of the second vector. In this case, vector $\vec{\mathbf{A}}$ will be positioned with its tail at the origin and its tip at the point (0, 29). The resultant is then drawn, starting at the origin (tail of first vector) and going 14 units in the negative y direction to the point (0, −14). The second vector, $\vec{\mathbf{B}}$, must then start from the tip of $\vec{\mathbf{A}}$ at point (0, 29) and end on the tip of $\vec{\mathbf{R}}$ at point (0, −14) as shown in the sketch at the right. From this, it is seen that



ANS. FIG. P3.8

$\boxed{\vec{\mathbf{B}} \text{ is 43 units in the negative } y \text{ direction}}$

P3.9 In solving this problem we must contrast displacement with distance traveled. We draw a diagram of the skater's path in ANS. FIG. P3.9, which is the view from a hovering helicopter so that we can see the circular path as circular in shape. To start with a concrete example, we have chosen to draw motion *ABC* around one half of a circle of radius 5 m.



ANS. FIG. P3.9

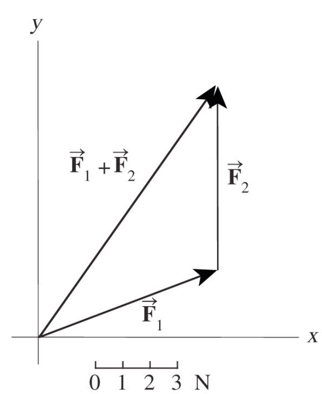
The displacement, shown as $\vec{\mathbf{d}}$ in the diagram, is the straight-line change in position from starting point *A* to finish *C*. In the specific case we have chosen to draw, it lies along a diameter of the circle. Its magnitude is $|\vec{\mathbf{d}}| = |-10.0\hat{\mathbf{i}}| = 10.0$ m.

The distance skated is greater than the straight-line displacement. The distance follows the curved path of the semicircle (*ABC*). Its length is half of the circumference: $s = \dfrac{1}{2}(2\pi r) = 5.00\pi \text{ m} = 15.7 \text{ m}$.

A straight line is the shortest distance between two points. For any nonzero displacement, less or more than across a semicircle, the distance along the path will be greater than the displacement magnitude. Therefore:

$$\boxed{\begin{array}{l}\text{The situation can never be true because the distance is}\\ \text{an arc of a circle between two points, whereas the}\\ \text{magnitude of the displacement vector is a straight-line}\\ \text{cord of the circle between the same points.}\end{array}}$$

P3.10 We find the resultant $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ graphically by placing the tail of $\vec{\mathbf{F}}_2$ at the head of $\vec{\mathbf{F}}_1$. The resultant force vector $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ is of magnitude $\boxed{9.5 \text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x \text{ axis}}$.



ANS. FIG. P3.10

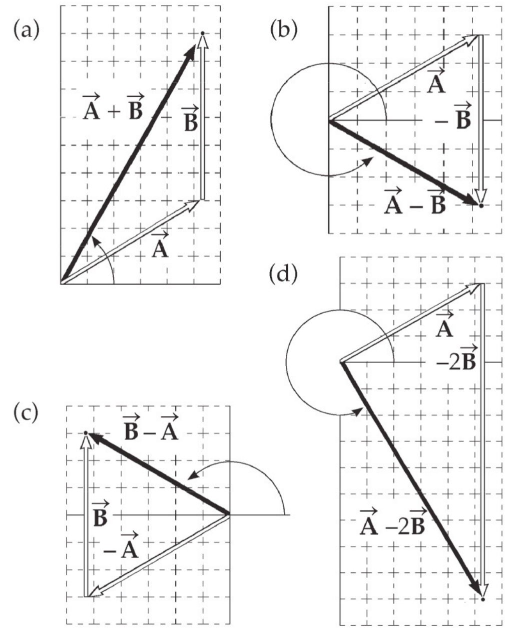
P3.11 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\vec{\mathbf{A}}+\vec{\mathbf{B}}=\boxed{5.2\text{ m at }60^\circ}$

(b) $\vec{\mathbf{A}}-\vec{\mathbf{B}}=\boxed{3.0\text{ m at }330^\circ}$

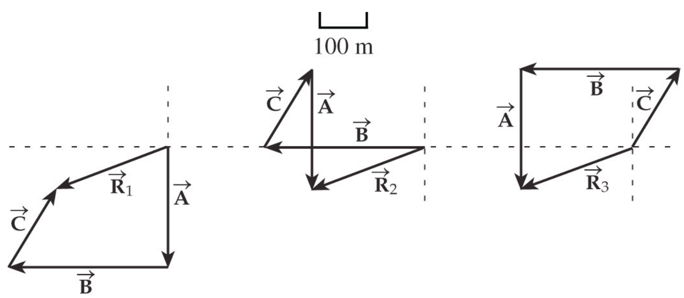
(c) $\vec{\mathbf{B}}-\vec{\mathbf{A}}=\boxed{3.0\text{ m at }150^\circ}$

(d) $\vec{\mathbf{A}}-2\mathbf{B}=\boxed{5.2\text{ m at }300^\circ}$



ANS. FIG. P3.11

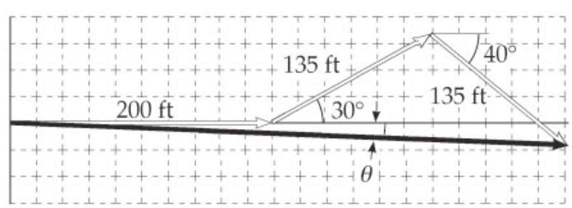
P3.12 (a) The three diagrams are shown in ANS. FIG. P3.12a below.



ANS. FIG. P3.12a

(b) The diagrams in ANS. FIG. P3.12a represent the graphical solutions for the three vector sums: $\vec{\mathbf{R}}_1=\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$, $\vec{\mathbf{R}}_2=\vec{\mathbf{B}}+\vec{\mathbf{C}}+\vec{\mathbf{A}}$, and $\vec{\mathbf{R}}_3=\vec{\mathbf{C}}+\vec{\mathbf{B}}+\vec{\mathbf{A}}$.

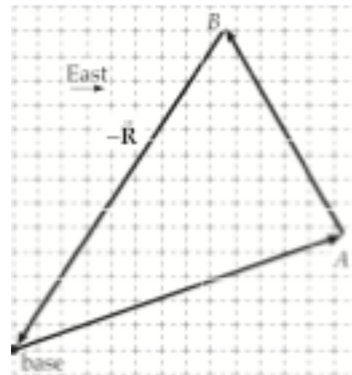
P3.13 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be $\boxed{d=420\text{ ft and }\theta=-3^\circ}$.



(Scale: 1unit = 20 ft)

ANS. FIG. P3.13

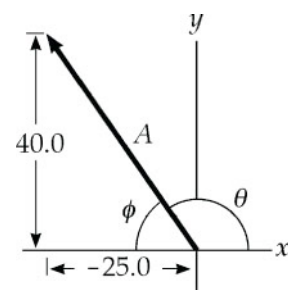
***P3.14** ANS. FIG. P3.14 shows the graphical addition of the vector from the base camp to lake A to the vector connecting lakes A and B, with a scale of 1 unit = 20 km. The distance from lake B to base camp is then the negative of this resultant vector, or $-\vec{\mathbf{R}} = \boxed{\text{310 km at 57° S of W}}$.



ANS. FIG. P3.14

Section 3.4 Components of a Vector and Unit Vectors

P3.15 First we should visualize the vector either in our mind or with a sketch, as shown in ANS. FIG. P3.15. The magnitude of the vector can be found by the Pythagorean theorem:



ANS. FIG. P3.15

$$A_x = -25.0$$

$$A_y = 40.0$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2}$$

$$= \boxed{47.2 \text{ units}}$$

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}$$

so

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan^{-1}\left(\frac{40.0}{25.0}\right) = \tan^{-1}(1.60) = 58.0°$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°: $\theta = 180° - 58° = \boxed{122°}$

P3.16 We can calculate the components of the vector A using $(A_x, A_y) = (A\cos\theta, A\sin\theta)$ if the angle θ is measured from the $+x$ axis, which is true here. For $A = 35.0$ units and $\theta = 325°$,

$$\boxed{A_x = 28.7 \text{ units}, A_y = -20.1 \text{ units}}$$

P3.17 (a) $\boxed{\text{Yes}}$.

(b) Let v represent the speed of the camper. The northward component of its velocity is v cos 8.50°. To avoid crowding the minivan we require $v \cos 8.50° \geq 28$ m/s.

$$\boxed{\begin{array}{l}\text{We can satisfy this requirement simply by taking}\\ v \geq (28.0 \text{ m/s})/\cos 8.50° = 28.3 \text{ m/s}.\end{array}}$$

P3.18 The person would have to walk

$$(3.10 \text{ km})\sin 25.0° = \boxed{1.31 \text{ km north}}$$

and $$(3.10 \text{ km})\cos 25.0° = \boxed{2.81 \text{ km east}}$$

P3.19 Do not think of sin θ = opposite/hypotenuse, but jump right to $y = R \sin \theta$. The angle does not need to fit inside a triangle. We find the x and y components of each vector using $x = r \cos \theta$ and $y = r \sin \theta$. In unit vector notation, $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$.

(a) $x = 12.8 \cos 150°$, $y = 12.8 \sin 150°$, and $\boxed{(x, y) = (-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}) \text{ m}}$

(b) $x = 3.30 \cos 60.0°$, $y = 3.30 \sin 60.0°$, and $\boxed{(x, y) = (1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}) \text{ cm}}$

(c) $x = 22.0 \cos 215°$, $y = 22.0 \sin 215°$, and $\boxed{(x, y) = (-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}) \text{ in}}$

P3.20 (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00 \text{ blocks}$$

and the angle the resultant makes with the x axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00 \text{ blocks at } 53.1° \text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.00 \text{ blocks}}$.

P3.21 Let $+x$ be East and $+y$ be North. We can sum the total x and y displacements of the spelunker as

$$\sum x = 250 \text{ m} + (125 \text{ m})\cos 30° = 358 \text{ m}$$
$$\sum y = 75 \text{ m} + (125 \text{ m})\sin 30° - 150 \text{ m} = -12.5 \text{ m}$$

the total displacement is then

$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358 \text{ m})^2 + (-12.5 \text{ m})^2} = 358 \text{ m}$$

at an angle of

$$\theta = \tan^{-1}\left(\frac{\sum y}{\sum x}\right) = \tan^{-1}\left(-\frac{12.5 \text{ m}}{358 \text{ m}}\right) = -2.00°$$

or $\boxed{\vec{\mathbf{d}} = 358 \text{ m at } 2.00° \text{ S of E}}$

P3.22 We use the numbers given in Problem 3.11:

$$\vec{\mathbf{A}} = 3.00 \text{ m}, \ \theta_A = 30.0°$$

$$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60 \text{ m},$$

$$A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50 \text{ m}$$

So $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = (2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}) \text{ m}$

$$\vec{\mathbf{B}} = 3.00 \text{ m}, \theta_B = 90.0°$$

$$B_x = 0, B_y = 3.00 \text{ m} \rightarrow \vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} \text{ m}$$

then $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)\text{m}}$

P3.23 We can get answers in unit-vector form just by doing calculations with each term labeled with an $\hat{\mathbf{i}}$ or a $\hat{\mathbf{j}}$. There are, in a sense, only two vectors to calculate, since parts (c), (d), and (e) just ask about the magnitudes and directions of the answers to (a) and (b). Note that the whole numbers appearing in the problem statement are assumed to have three significant figures.

We use the property of vector addition that states that the components of $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ are computed as $R_x = A_x + B_x$ and $R_y = A_y + B_y$.

(a) $\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) + \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$

(b) $\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) - \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$

(c) $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $\left|\vec{\mathbf{A}} - \vec{\mathbf{B}}\right| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6°}$

P3.24 The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$d_{\text{DC east}} = d_{\text{DA east}} + d_{\text{AC east}}$$
$$= (730\text{ mi})\cos 5.00° - (560\text{ mi})\sin 21.0° = 527\text{ miles}$$

$$d_{\text{DC north}} = d_{\text{DA north}} + d_{\text{AC north}}$$
$$= (730\text{ mi})\sin 5.00° + (560\text{ mi})\cos 21.0° = 586\text{ miles}$$

By the Pythagorean theorem,

$$d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788\text{ mi}$$

Then, $\theta = \tan^{-1}\left(\frac{d_{\text{DC north}}}{d_{\text{DC east}}}\right) = 48.0°$

Thus, Chicago is $\boxed{\text{788 miles at 48.0° northeast of Dallas}}$.

P3.25 We use the unit-vector addition method. It is just as easy to add three displacements as to add two. We take the direction east to be along $+\hat{\mathbf{i}}$. The three displacements can be written as:

$$\vec{\mathbf{d}}_1 = (-3.50\text{ m})\hat{\mathbf{j}}$$
$$\vec{\mathbf{d}}_2 = (8.20\text{ m})\cos 45.0°\hat{\mathbf{i}} + (8.20\text{ m})\sin 45.0°\hat{\mathbf{j}}$$
$$= (5.80\text{ m})\hat{\mathbf{i}} + (5.80\text{ m})\hat{\mathbf{j}}$$
$$\vec{\mathbf{d}}_3 = (-15.0\text{ m})\hat{\mathbf{i}}$$

The resultant is

$$\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 = (-15.0\text{ m} + 5.80\text{ m})\hat{\mathbf{i}} + (5.80\text{ m} - 3.50\text{ m})\hat{\mathbf{j}}$$
$$= (-9.20\text{ m})\hat{\mathbf{i}} + (2.30\text{ m})\hat{\mathbf{j}}$$

(or 9.20 m west and 2.30 m north).

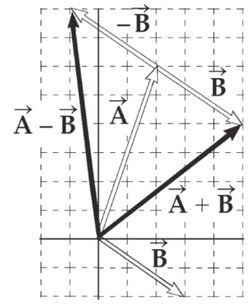
The magnitude of the resultant displacement is

$$|\vec{\mathbf{R}}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20\text{ m})^2 + (2.30\text{ m})^2} = \boxed{9.48\text{ m}}$$

The direction of the resultant vector is given by

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2.30 \text{ m}}{-9.20 \text{ m}}\right) = \boxed{166°}$$

P3.26 (a) See figure to the right.



ANS. FIG. P3.26

(b) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$= \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}$

$= \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$

(c) $\vec{\mathbf{C}} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$

$\vec{\mathbf{D}} = \sqrt{(-1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{-1.00}\right)$

$\vec{\mathbf{D}} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$

P3.27 We first tabulate the three strokes of the novice golfer, with the x direction corresponding to East and the y direction corresponding to North. The sum of the displacement in each of the directions is shown as the last row of the table.

East	North
x (m)	y (m)
0	4.00
1.41	1.41
–0.500	–0.866
+0.914	4.55

The "hole-in-one" single displacement is then

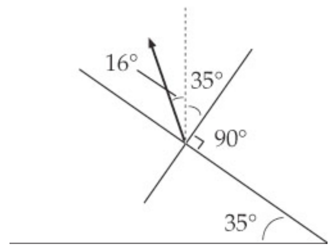
$$\left|\vec{\mathbf{R}}\right| = \sqrt{|x|^2 + |y|^2} = \sqrt{(0.914 \text{ m})^2 + (4.55 \text{ m})^2} = 4.64 \text{ m}$$

The angle of the displacement with the horizontal is

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4.55 \text{ m}}{0.914 \text{ m}}\right) = 78.6°$$

The expert golfer would accomplish the hole in one with the displacement $\boxed{4.64 \text{ m at } 78.6° \text{ N of E}}$.

P3.28 We take the x axis along the slope downhill. (Students, get used to this choice!) The y axis is perpendicular to the slope, at 35.0° to the vertical. Then the displacement of the snow makes an angle of 90.0° + 35.0° + 16.0° = 141° with the x axis.



ANS. FIG. P3.28

(a) $\boxed{\begin{array}{l}\text{Its component parallel to the surface is}\\ (1.50 \text{ m}) \cos 141° = -1.17 \text{ m},\\ \text{or } 1.17 \text{ m toward the top of the hill.}\end{array}}$

(b) $\boxed{\begin{array}{l}\text{Its component perpendicular to the surface is}\\ (1.50 \text{ m}) \sin 141° = 0.944 \text{ m, or } 0.944 \text{ m away}\\ \text{from the snow.}\end{array}}$

P3.29 (a) The single force is obtained by summing the two forces:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

$$\vec{\mathbf{F}} = 120 \cos(60.0°)\hat{\mathbf{i}} + 120 \sin(60.0°)\hat{\mathbf{j}}$$
$$- 80.0 \cos(75.0°)\hat{\mathbf{i}} + 80.0 \sin(75.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right) \text{ N}$$

We can also express this force in terms of its magnitude and direction:

$$\left|\vec{\mathbf{F}}\right| = \sqrt{39.3^2 + 181^2} \text{ N} = \boxed{185 \text{ N}}$$

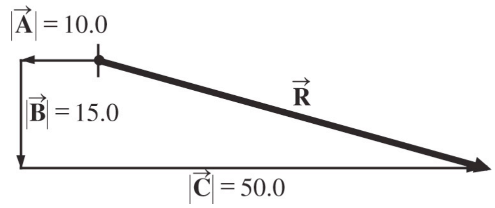
$$\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$$

(b) A force equal and opposite the resultant force from part (a) is required for the total force to equal zero:

$$\vec{\mathbf{F}}_3 = -\vec{\mathbf{F}} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right) \text{ N}}$$

P3.30 ANS. FIG. P3.30 is a graphical depiction of the three displacements the football undergoes, with $\vec{\mathbf{A}}$ corresponding to the 10.0-yard backward run, $\vec{\mathbf{B}}$ corresponding to the 15.0-yard sideways run, and $\vec{\mathbf{C}}$ corresponding to the 50.0-yard downfield pass. The resultant vector is then

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = -10.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}} + 50.0\hat{\mathbf{i}}$$
$$= 40.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}$$
$$\left|\vec{\mathbf{R}}\right| = \left[(40.0)^2 + (-15.0)^2\right]^{1/2} = \boxed{42.7 \text{ yards}}$$



ANS. FIG. P3.30

P3.31 (a) We add the components of the three vectors:

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 6\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$$
$$\left|\vec{\mathbf{D}}\right| = \sqrt{6^2 + 2^2} = \boxed{6.32 \text{ m at } \theta = 342°}$$

(b) Again, using the components of the three vectors,

$$\vec{\mathbf{E}} = -\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}} = -2\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$$
$$\left|\vec{\mathbf{E}}\right| = \sqrt{2^2 + 12^2} = \boxed{12.2 \text{ m at } \theta = 99.5°}$$

P3.32 We are given $\vec{\mathbf{A}} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$, and $\vec{\mathbf{B}} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$, and $\vec{\mathbf{A}} - \vec{\mathbf{B}} + 3\vec{\mathbf{C}} = 0$. Solving for $\vec{\mathbf{C}}$ gives

$$3\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$
$$\vec{\mathbf{C}} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}} \text{ or } \quad C_x = \boxed{7.30 \text{ cm}}; C_y = \boxed{-7.20 \text{ cm}}$$

P3.33 Hold your fingertip at the center of the front edge of your study desk, defined as point *O*. Move your finger 8 cm to the right, then 12 cm vertically up, and then 4 cm horizontally away from you. Its location relative to the starting point represents position vector $\vec{\mathbf{A}}$. Move three-fourths of the way straight back toward *O*. Now your fingertip is at the location of $\vec{\mathbf{B}}$. Now move your finger 50 cm straight through *O*, through your left thigh, and down toward the floor. Its position vector now is $\vec{\mathbf{C}}$.

We use unit-vector notation throughout. There is no adding to do here, but just multiplication of a vector by two different scalars.

(a) $\vec{\mathbf{A}} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\vec{\mathbf{B}} = \dfrac{\vec{\mathbf{A}}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\vec{\mathbf{C}} = -3\vec{\mathbf{A}} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.34 We are given $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$. The magnitude of the vector is therefore

$$\left|\vec{\mathbf{B}}\right| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

And the angle of the vector with the three coordinate axes is

$$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°} \text{ is the angle with the } x \text{ axis}$$

$$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°} \text{ is the angle with the } y \text{ axis}$$

$$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°} \text{ is the angle with the } z \text{ axis}$$

P3.35 The component description of $\vec{\mathbf{A}}$ is just restated to constitute the answer to part (a): $A_x = -3.00$, $A_y = 2.00$.

(a) $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \boxed{-3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}}$

(b) $\left|\vec{\mathbf{A}}\right| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{2.00}{-3.00}\right) = -33.7°$$

θ is in the second quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$.

(c) $R_x = 0$, $R_y = -4.00$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, thus $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00$, $B_y = R_y - A_y = -4.00 - 2.00 = -6.00$.

Therefore, $\vec{\mathbf{B}} = \boxed{3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}$.

P3.36 We carry out the prescribed mathematical operations using unit vectors.

(a) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\text{ m}}$

$$\left|\vec{\mathbf{C}}\right| = \sqrt{(5.00\text{ m})^2 + (1.00\text{ m})^2 + (3.00\text{ m})^2} = \boxed{5.92\text{ m}}$$

(b) $\vec{\mathbf{D}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right)\text{m}}$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{(4.00\text{ m})^2 + (11.0\text{ m})^2 + (15.0\text{ m})^2} = \boxed{19.0\text{ m}}$$

P3.37 (a) Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0$$

Substituting $a = 1.33b - 4.33$ into the second equation, we find

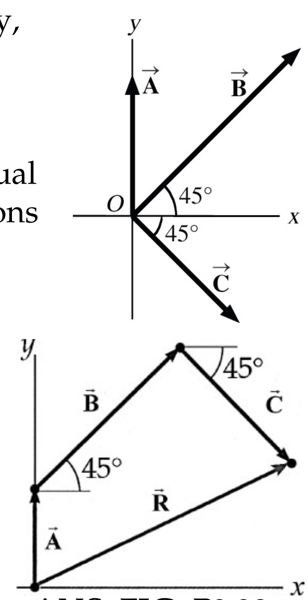
$$-8(1.33b - 4.33) + 3b + 19 = 0 \rightarrow 7.67b = 53.67 \rightarrow b = 7.00$$

and so $a = 1.33(7.00) - 4.33 = 5.00$.

Thus $\boxed{a = 5.00,\ b = 7.00}$. Therefore, $5.00\vec{\mathbf{A}} + 7.00\vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$.

(b) In order for vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation, as each component gives us one equation.

P3.38 The given diagram shows the vectors individually, but not their addition. The second diagram represents a map view of the motion of the ball. According to the definition of a displacement, we ignore any departure from straightness of the actual path of the ball. We model each of the three motions as straight. The simplified problem is solved by straightforward application of the component method of vector addition. It works for adding two, three, or any number of vectors.



ANS. FIG. P3.38

(a) We find the two components of each of the three vectors

$$A_x = (20.0\text{ units})\cos 90° = 0$$

and $A_y = (20.0\text{ units})\sin 90° = 20.0\text{ units}$

$$B_x = (40.0\text{ units})\cos 45° = 28.3\text{ units}$$

and $$B_y = (40.0\text{ units})\sin 45° = 28.3\text{ units}$$

$$C_x = (30.0\text{ units})\cos 315° = 21.2\text{ units}$$

and $$C_y = (30.0\text{ units})\sin 315° = -21.2\text{ units}$$

Now adding,

$$R_x = A_x + B_x + C_x = (0 + 28.3 + 21.2)\text{ units} = 49.5\text{ units}$$

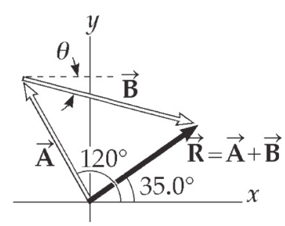
and $$R_y = A_y + B_y + C_y = (20 + 28.3 - 21.2)\text{ units} = 27.1\text{ units}$$

so $$\vec{\mathbf{R}} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$$

(b) $$\left|\vec{\mathbf{R}}\right| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7°}$$

P3.39 We will use the component method for a precise answer. We already know the total displacement, so the algebra of solving a vector equation will guide us to do a subtraction.



ANS. FIG. P3.39

We have $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$:

$$A_x = 150\cos 120° = -75.0\text{ cm}$$

$$A_y = 150\sin 120° = 130\text{ cm}$$

$$R_x = 140\cos 35.0° = 115\text{ cm}$$

$$R_y = 140\sin 35.0° = 80.3\text{ cm}$$

Therefore,

$$\vec{\mathbf{B}} = [115 - (-75)\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = \left(190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}\right)\text{ cm}$$

$$\left|\vec{\mathbf{B}}\right| = \sqrt{190^2 + 49.7^2} = \boxed{196\text{ cm}}$$

$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}$$

P3.40 First, we sum the components of the two vectors for the male:

$$d_{3mx} = d_{1mx} + d_{2mx} = 0 + (100\text{ cm})\cos 23.0° = 92.1\text{ cm}$$
$$d_{3my} = d_{1my} + d_{2my} = 104\text{ cm} + (100\text{ cm})\sin 23.0° = 143.1\text{ cm}$$

$$\boxed{\begin{array}{l}\text{magnitude: } d_{3m} = \sqrt{(92.1\text{ cm})^2 + (143.1\text{ cm})^2} = 170.1\text{ cm} \\ \text{direction: } \tan^{-1}(143.1/92.1) = 57.2° \text{ above } +x \text{ axis (first quadrant)}\end{array}}$$

followed by the components of the two vectors for the female:

$$d_{3fx} = d_{1fx} + d_{2fx} = 0 + (86.0\text{ cm})\cos 28.0° = 75.9\text{ cm}$$
$$d_{3fy} = d_{1fy} + d_{2fy} = 84.0\text{ cm} + (86.0\text{ cm})\sin 28.0° = 124.4\text{ cm}$$

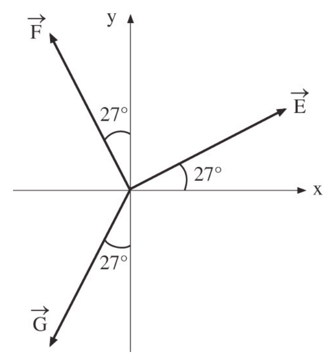
$$\boxed{\begin{array}{l}\text{magnitude: } d_{3f} = \sqrt{(75.9\text{ cm})^2 + (124.4\text{ cm})^2} = 145.7\text{ cm} \\ \text{direction: } \tan^{-1}(124.4/75.9) = 58.6° \text{ above } +x \text{ axis (first quadrant)}\end{array}}$$

P3.41 (a) $\vec{\mathbf{E}} = (17.0\text{ cm})\cos(27.0°)\hat{\mathbf{i}} + (17.0\text{ cm})\sin(27.0°)\hat{\mathbf{j}}$

$\vec{\mathbf{E}} = \boxed{(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}})\text{ cm}}$

(b) $\vec{\mathbf{F}} = (17.0\text{ cm})\cos(117.0°)\hat{\mathbf{i}} + (17.0\text{ cm})\sin(117.0°)\hat{\mathbf{j}}$

$\vec{\mathbf{F}} = \boxed{\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)\text{ cm}}$



ANS. FIG. P3.41

Note that we did not need to explicitly identify the angle with the positive x axis, but by doing so, we don't have to keep track of minus signs for the components.

(c) $\vec{\mathbf{G}} = [(-17.0\text{ cm})\cos(243.0°)]\hat{\mathbf{i}} + [(-17.0\text{ cm})\sin(243.0°)]\hat{\mathbf{j}}$

$\boxed{\vec{\mathbf{G}} = \left(-7.72\hat{\mathbf{i}} - 15.1\hat{\mathbf{j}}\right)\text{ cm}}$

P3.42 The position vector from radar station to ship is

$$\vec{\mathbf{S}} = \left(17.3\sin 136°\hat{\mathbf{i}} + 17.3\cos 136°\hat{\mathbf{j}}\right)\text{ km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right)\text{ km}$$

From station to plane, the position vector is

$$\vec{\mathbf{P}} = \left(19.6\sin 153°\hat{\mathbf{i}} + 19.6\cos 153°\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km}$$

or

$$\vec{\mathbf{P}} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km}$$

(a) To fly to the ship, the plane must undergo displacement

$$\vec{\mathbf{D}} = \vec{\mathbf{S}} - \vec{\mathbf{P}} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right) \text{ km}}$$

(b) The distance the plane must travel is

$$D = \left|\vec{\mathbf{D}}\right| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2} \text{ km} = \boxed{6.31 \text{ km}}$$

P3.43 The hurricane's first displacement is

$$(41.0 \text{ km/h})(3.00 \text{ h}) \text{ at } 60.0° \text{ N of W}$$

and its second displacement is

$$(25.0 \text{ km/h})(1.50 \text{ h}) \text{ due North}$$

With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\begin{aligned} &\left[(41.0 \text{ km/h})\cos 60.0°\right](3.00 \text{ h})\left(-\hat{\mathbf{i}}\right) \\ &\qquad + \left[(41.0 \text{ km/h})\sin 60.0°\right](3.00 \text{ h})\hat{\mathbf{j}} \\ &\qquad + (25.0 \text{ km/h})(1.50 \text{ h})\hat{\mathbf{j}} \\ &= 61.5 \text{ km}\left(-\hat{\mathbf{i}}\right) + 144 \text{ km } \hat{\mathbf{j}} \end{aligned}$$

with magnitude $\sqrt{(61.5 \text{ km})^2 + (144 \text{ km})^2} = \boxed{157 \text{ km}}$.

P3.44 Note that each shopper must make a choice whether to turn 90° to the left or right, each time he or she makes a turn. One set of such choices, following the rules in the problem, results in the shopper heading in the positive y direction and then again in the positive x direction.

Find the magnitude of the sum of the displacements:

$$\vec{\mathbf{d}} = (8.00 \text{ m})\hat{\mathbf{i}} + (3.00 \text{ m})\hat{\mathbf{j}} + (4.00 \text{ m})\hat{\mathbf{i}} = (12.00 \text{ m})\hat{\mathbf{i}} + (3.00 \text{ m})\hat{\mathbf{j}}$$

$$\text{magnitude: d} = \sqrt{(12.00 \text{ m})^2 + (3.00 \text{ m})^2} = 12.4 \text{ m}$$

$$\boxed{\text{Impossible because 12.4 m is greater than 5.00 m.}}$$

P3.45 The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$, where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 8\,040$ m/30 s = 268 m/s. The position vector as a function of time is

$$\vec{\mathbf{P}} = (268 \text{ m/s})t\hat{\mathbf{i}} + (7.60 \times 10^3 \text{ m})\hat{\mathbf{j}}$$

At $t = 45.0$ s, $\vec{\mathbf{P}} = \left[1.21\times10^4\,\hat{\mathbf{i}} + 7.60\times10^3\,\hat{\mathbf{j}}\right]$ m. The magnitude is

$$\vec{\mathbf{P}} = \sqrt{\left(1.21\times10^4\right)^2 + \left(7.60\times10^3\right)^2}\ \text{m} = \boxed{1.43\times10^4\ \text{m}}$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{7.60\times10^3}{1.21\times10^4}\right) = \boxed{32.2^\circ\ \text{above the horizontal}}$$

P3.46 The displacement from the start to the finish is

$$16\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) = (11\hat{\mathbf{i}} + 9\hat{\mathbf{j}})$$

The displacement from the starting point to A is $f\left(11\hat{\mathbf{i}} + 9\hat{\mathbf{j}}\right)$ meters.

(a) The position vector of point A is

$$5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + f\left(11\hat{\mathbf{i}} + 9\hat{\mathbf{j}}\right) = \boxed{\left[(5+11f)\hat{\mathbf{i}} + (3+9f)\hat{\mathbf{j}}\right]\ \text{m}}$$

(b) For $f = 0$ we have the position vector $\boxed{(5+0)\hat{\mathbf{i}} + (3+0)\hat{\mathbf{j}}\ \text{meters.}}$

(c) This is reasonable because it is the location of the starting point, $5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ meters.

(d) For $f = 1 = 100\%$, we have position vector
$(5+11)\hat{\mathbf{i}} + (3+9)\hat{\mathbf{j}}$ meters $= \boxed{16\hat{\mathbf{i}} + 12\hat{\mathbf{j}}\ \text{meters}}$.

(e) This is reasonable because we have completed the trip, and this is the position vector of the endpoint.

P3.47 Let the positive x direction be eastward, the positive y direction be vertically upward, and the positive z direction be southward. The total displacement is then

$$\begin{aligned}\vec{\mathbf{d}} &= \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right)\ \text{cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\ \text{cm}\\ &= \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\ \text{cm}\end{aligned}$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}\ \text{cm} = \boxed{10.4\ \text{cm}}$.

(b) Its angle with the y axis follows from
$\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5^\circ}$.

Additional Problems

P3.48 The Pythagorean theorem and the definition of the tangent will be the starting points for our calculation.

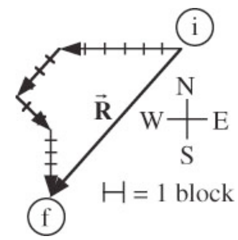
(a) Take the wall as the xy plane so that the coordinates are $x = 2.00$ m and $y = 1.00$ m; and the fly is located at point P. The distance between two points in the xy plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

so here $d = \sqrt{(2.00\text{ m} - 0)^2 + (1.00\text{ m} - 0)^2} = \boxed{2.24\text{ m}}$

(b) $\theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{1.00\text{ m}}{2.00\text{ m}}\right) = 26.6°$, so $\vec{\mathbf{r}} = \boxed{2.24\text{ m, }26.6°}$

P3.49 We note that $-\hat{\mathbf{i}}$ = west and $-\hat{\mathbf{j}}$ = south. The given mathematical representation of the trip can be written as 6.30 b west + 4.00 b at 40° south of west +3.00 b at 50° south of east +5.00 b south.



ANS. FIG. P3.49

(a) The figure on the right shows a map of the successive displacements that the bus undergoes.

(b) The total odometer distance is the sum of the magnitudes of the four displacements:

$$6.30\text{ b} + 4.00\text{ b} + 3.00\text{ b} + 5.00\text{ b} = \boxed{18.3\text{ b}}$$

(c)
$$\begin{aligned}
\vec{\mathbf{R}} &= (-6.30 - 3.06 + 1.93)\text{ b}\hat{\mathbf{i}} + (-2.57 - 2.30 - 5.00)\text{ b}\hat{\mathbf{j}} \\
&= -7.44\text{ b}\hat{\mathbf{i}} - 9.87\text{ b}\hat{\mathbf{j}} \\
&= \sqrt{(7.44\text{ b})^2 + (9.87\text{ b})^2}\ \text{at}\ \tan^{-1}\left(\frac{9.87}{7.44}\right)\ \text{south of west} \\
&= 12.4\text{ b at }53.0°\text{ south of west} \\
&= \boxed{12.4\text{ b at }233°\text{ counterclockwise from east}}
\end{aligned}$$

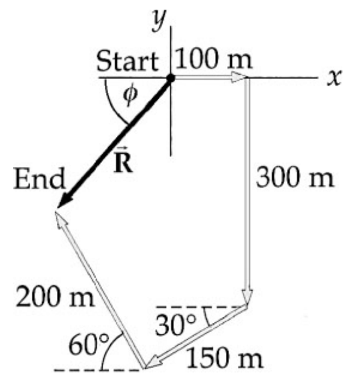
P3.50 To find the new speed and direction of the aircraft, we add the vector components of the wind to the vector velocity of the aircraft:

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (300 + 100\cos 30.0°)\hat{\mathbf{i}} + (100\sin 30.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}} = \left(387\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)\text{ mi/h}$$

$$|\vec{\mathbf{v}}| = \boxed{390\text{ mi/h at }7.37°\text{ N of E}}$$

P3.51 On our version of the diagram we have drawn in the resultant from the tail of the first arrow to the head of the last arrow. The resultant displacement $\vec{\mathbf{R}}$ is equal to the sum of the four individual displacements, $\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4$. We translate from the pictorial representation to a mathematical representation by writing the individual displacements in unit-vector notation:



ANS. FIG. P3.51

$$\vec{\mathbf{d}}_1 = 100\hat{\mathbf{i}} \text{ m}$$
$$\vec{\mathbf{d}}_2 = -300\hat{\mathbf{j}} \text{ m}$$
$$\vec{\mathbf{d}}_3 = (-150 \cos 30°)\hat{\mathbf{i}} \text{ m} + (-150 \sin 30°)\hat{\mathbf{j}} \text{ m} = -130\hat{\mathbf{i}} \text{ m} - 75\hat{\mathbf{j}} \text{ m}$$
$$\vec{\mathbf{d}}_4 = (-200 \cos 60°)\hat{\mathbf{i}} \text{ m} + (200 \sin 60°)\hat{\mathbf{j}} \text{ m} = -100\hat{\mathbf{i}} \text{ m} + 173\hat{\mathbf{j}} \text{ m}$$

Summing the components together, we find

$$R_x = d_{1x} + d_{2x} + d_{3x} + d_{4x} = (100 + 0 - 130 - 100) \text{ m} = -130 \text{ m}$$
$$R_y = d_{1y} + d_{2y} + d_{3y} + d_{4y} = (0 - 300 - 75 + 173) \text{ m} = -202 \text{ m}$$

so altogether

$$\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right)\text{m}}$$

Its magnitude is

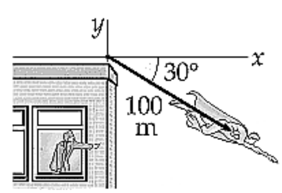
$$\left|\vec{\mathbf{R}}\right| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240 \text{ m}}$$

We calculate the angle $\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-202}{-130}\right) = 57.2°$.

The resultant points into the third quadrant instead of the first quadrant. The angle counterclockwise from the $+x$ axis is

$$\theta = 180 + \phi = \boxed{237°}$$

***P3.52** The superhero follows a straight-line path at 30.0° below the horizontal. If his displacement is 100 m, then the coordinates of the superhero are:



ANS. FIG. P3.52

$$x = (100 \text{ m})\cos(-30.0°) = \boxed{86.6 \text{ m}}$$
$$y = (100 \text{ m})\sin(-30.0°) = \boxed{-50.0 \text{ m}}$$

P3.53 (a) Take the x axis along the tail section of the snake. The displacement from tail to head is

$$\begin{aligned}(240\text{ m})\hat{\mathbf{i}}+\left[(420-240)\text{ m}\right]\cos(180°-105°)\hat{\mathbf{i}}\\ -(180\text{ m})\sin 75°\hat{\mathbf{j}}=287\text{ m}\hat{\mathbf{i}}-174\text{ m}\hat{\mathbf{j}}\end{aligned}$$

Its magnitude is $\sqrt{(287)^2+(174)^2}\text{ m}=335\text{ m}.$

From $v=\dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: }\Delta t=\frac{\text{distance}}{v}=\frac{335\text{ m}(\text{h})(1\text{ km})(3600\text{ s})}{(12\text{ km})(1000\text{ m})(1\text{ h})}=101\text{ s}$$

$$\text{Olaf: }\Delta t=\frac{420\text{ m}\cdot\text{s}}{3.33\text{ m}}=126\text{ s}$$

Inge wins by $126-101=\boxed{25.4\text{ s}}$.

(b) Olaf must run the race in the same time:

$$v=\frac{d}{\Delta t}=\frac{420\text{ m}}{101\text{ s}}\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{\text{km}}{10^3\text{ m}}\right)=\boxed{15.0\text{ km/h}}$$

P3.54 The position vector from the ground under the controller of the first airplane is

$$\begin{aligned}\vec{\mathbf{r}}_1&=(19.2\text{ km})(\cos 25°)\hat{\mathbf{i}}+(19.2\text{ km})(\sin 25°)\hat{\mathbf{j}}+(0.8\text{ km})\hat{\mathbf{k}}\\&=\left(17.4\hat{\mathbf{i}}+8.11\hat{\mathbf{j}}+0.8\hat{\mathbf{k}}\right)\text{ km}\end{aligned}$$

The second is at

$$\begin{aligned}\vec{\mathbf{r}}_2&=(17.6\text{ km})(\cos 20°)\hat{\mathbf{i}}+(17.6\text{ km})(\sin 20°)\hat{\mathbf{j}}+(1.1\text{ km})\hat{\mathbf{k}}\\&=\left(16.5\hat{\mathbf{i}}+6.02\hat{\mathbf{j}}+1.1\hat{\mathbf{k}}\right)\text{ km}\end{aligned}$$

Now the displacement from the first plane to the second is

$$\vec{\mathbf{r}}_2-\vec{\mathbf{r}}_1=\left(-0.863\hat{\mathbf{i}}-2.09\hat{\mathbf{j}}+0.3\hat{\mathbf{k}}\right)\text{ km}$$

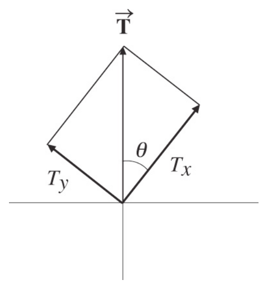
with magnitude

$$\sqrt{(0.863)^2+(2.09)^2+(0.3)^2}\text{ km}=\boxed{2.29\text{ km}}$$

P3.55 (a) The tensions T_x and T_y act as an equivalent tension T (see ANS. FIG. P3.55) which supports the downward weight; thus, the combination is equivalent to 0.150 N, upward. We know that T_x = 0.127 N, and the tensions are perpendicular to each other, so their combined magnitude is

$$T = \sqrt{T_x^2 + T_y^2} = 0.150\text{ N} \to T_y^2 = (0.150\text{ N})^2 - T_x^2$$

$$T_y^2 = (0.150\text{ N})^2 - (0.127\text{ N})^2 \to T_y = 0.078\text{ N}$$



ANS. FIG. P3.55

(b) From the figure, $\theta = \tan^{-1}\left(T_y/T_x\right) = 32.1°$. The angle the x axis makes with the horizontal axis is $90° - \theta = \boxed{57.9°}$.

(c) From the figure, the angle the y axis makes with the horizontal axis is $\theta = \boxed{32.1°}$.

P3.56 (a) Consider the rectangle in the figure to have height H and width W. The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are related by $\vec{\mathbf{A}} + \overrightarrow{\mathbf{ab}} + \overrightarrow{\mathbf{bc}} = \vec{\mathbf{B}}$, where

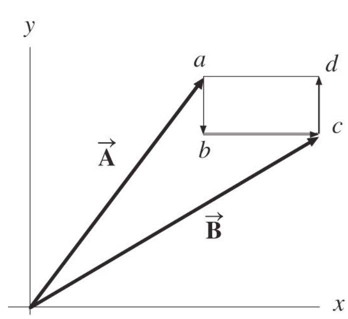
$$\vec{\mathbf{A}} = (10.0\text{ m})(\cos 50.0°)\hat{\mathbf{i}} + (10.0\text{ m})(\sin 50.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}} = \left(6.42\hat{\mathbf{i}} + 7.66\hat{\mathbf{j}}\right)\text{ m}$$

$$\vec{\mathbf{B}} = (12.0\text{ m})(\cos 30.0°)\hat{\mathbf{i}} + (12.0\text{ m})(\sin 30.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} = \left(10.4\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\text{ m}$$

$$\overrightarrow{\mathbf{ab}} = -\text{H}\hat{\mathbf{j}} \text{ and } \overrightarrow{\mathbf{bc}} = \text{W}\hat{\mathbf{i}}$$



ANS. FIG. P3.56

Therefore,

$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = \overrightarrow{\mathbf{ab}} + \overrightarrow{\mathbf{bc}}$$

$$\left(3.96\hat{\mathbf{i}} - 1.66\hat{\mathbf{j}}\right) \text{ m} = \text{W}\hat{\mathbf{i}} - \text{H}\hat{\mathbf{j}} \rightarrow \text{W} = 3.96 \text{ m and H} = 1.66 \text{ m}$$

$$\boxed{\text{The perimeter measures } 2(\text{H} + \text{W}) = 11.24 \text{ m.}}$$

(b) The vector from the origin to the upper-right corner of the rectangle (point *d*) is

$$\vec{\mathbf{B}} + \text{H}\hat{\mathbf{j}} = 10.4 \text{ m}\hat{\mathbf{i}} + (6.00 \text{ m} + 1.66 \text{ m})\hat{\mathbf{j}} = 10.4 \text{ m}\hat{\mathbf{i}} + 7.66 \text{ m}\hat{\mathbf{j}}$$

$$\boxed{\begin{array}{l}\text{magnitude: } \sqrt{(10.4 \text{ m})^2 + (7.66 \text{ m})^2} = 12.9 \text{ m} \\ \text{direction: } \tan^{-1}(7.66/10.4) = 36.4° \text{ above } +x \text{ axis (first quadrant)}\end{array}}$$

P3.57 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

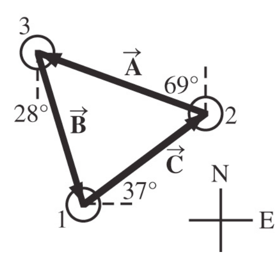
(b) $\left|\vec{\mathbf{R}}\right| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c) $$\cos\theta_x = \frac{R_x}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{57.7° \text{ from } +x}$$

$$\cos\theta_y = \frac{R_y}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_y = \cos^{-1}\left(\frac{R_y}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{74.5° \text{ from } +y}$$

$$\cos\theta_z = \frac{R_z}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_z = \cos^{-1}\left(\frac{R_z}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{36.7° \text{ from } +z}$$

P3.58 Let *A* represent the distance from island 2 to island 3. The displacement is $\vec{\mathbf{A}} = A$ at 159°. Represent the displacement from 3 to 1 as $\vec{\mathbf{B}} = B$ at 298°. We have 4.76 km at $37° + \vec{\mathbf{A}} + \vec{\mathbf{B}} = 0$.



ANS. FIG. P3.58

For the *x* components:

$$(4.76 \text{ km})\cos 37° + A\cos 159° + B\cos 298° = 0$$

$$3.80 \text{ km} - 0.934A + 0.470B = 0$$

$$B = -8.10 \text{ km} + 1.99A$$

For the *y* components:

$$(4.76 \text{ km})\sin 37° + A\sin 159° + B\sin 298° = 0$$

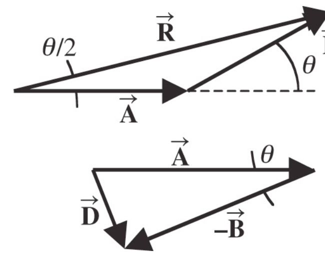
$$2.86 \text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating B by substitution:

$$2.86\text{ km}+0.358A-0.883(-8.10\text{ km}+1.99A)=0$$
$$2.86\text{ km}+0.358A+7.15\text{ km}-1.76A=0$$
$$10.0\text{ km}=1.40A$$
$$A=\boxed{7.17\text{ km}}$$

(b) $B=-8.10\text{ km}+1.99(7.17\text{ km})=\boxed{6.15\text{ km}}$

P3.59 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180°-\theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$. The magnitude of $\vec{\mathbf{R}}$ is then $R=2A\cos\left(\dfrac{\theta}{2}\right)$. This can be seen from applying the law of cosines to the isosceles triangle and using the fact that $B=A$.



ANS. FIG. P3.59

Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}}=\vec{\mathbf{A}}-\vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$1-\cos\theta=2\sin^2\left(\frac{\theta}{2}\right)$$

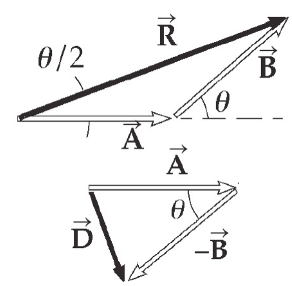
gives the magnitude of $\vec{\mathbf{D}}$ as $D=2A\sin\left(\dfrac{\theta}{2}\right)$.

The problem requires that $R=100D$.

Thus, $2A\cos\left(\dfrac{\theta}{2}\right)=200A\sin\left(\dfrac{\theta}{2}\right)$. This gives

$$\tan\left(\frac{\theta}{2}\right)=0.010\text{ and }\boxed{\theta=1.15°}$$

P3.60 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}}=\vec{\mathbf{A}}+\vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180°-\theta$, $\dfrac{\theta}{2}$, and $\dfrac{\theta}{2}$. The magnitude of $\vec{\mathbf{R}}$ is then $R=2A\cos\left(\dfrac{\theta}{2}\right)$. This can be seen by applying the



ANS. FIG. P3.60

law of cosines to the isosceles triangle and using the fact that $B = A$. Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = nD$ or

$$\cos\left(\frac{\theta}{2}\right) = n\sin\left(\frac{\theta}{2}\right) \text{ giving } \boxed{\theta = 2\tan^{-1}\left(\frac{1}{n}\right)}.$$

The larger R is to be compared to D, the smaller the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ becomes.

P3.61 (a) We write $\vec{\mathbf{B}}$ in terms of the sine and cosine of the angle θ, and add the two vectors:

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(-60 \text{ cm}\hat{\mathbf{j}}\right) + (80 \text{ cm} \cos\theta)\hat{\mathbf{i}} + (80 \text{ cm} \sin\theta)\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (80 \text{ cm} \cos\theta)\hat{\mathbf{i}} + (80 \text{ cm} \sin\theta - 60 \text{ cm})\hat{\mathbf{j}}$$

Dropping units (cm), the magnitude is

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[(80 \cos\theta)^2 + (80 \sin\theta - 60)^2\right]^{1/2}$$

$$= \left[(80)^2\left(\cos^2\theta + \sin^2\theta\right) - 2(80)(60)\sin\theta + (60)^2\right]^{1/2}$$

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[(80)^2 + (60)^2 - 2(80)(60)\sin\theta\right]^{1/2}$$

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{\left[10{,}000 - (9600)\sin\theta\right]^{1/2} \text{ cm}}$$

(b) For $\theta = 270°$, $\sin\theta = -1$, and $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{140 \text{ cm}}$.

(c) For $\theta = 90°$, $\sin\theta = 1$, and $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{20.0 \text{ cm}}$.

(d) They do make sense. The maximum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in the same direction, and it is 60 cm + 80 cm. The minimum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in opposite directions, and it is 80 cm − 60 cm.

P3.62 We perform the integration:

$$\Delta\vec{\mathbf{r}} = \int_0^{0.380\text{ s}} \vec{\mathbf{v}}\,dt = \int_0^{0.380\text{ s}} \left(1.2\hat{\mathbf{i}}\text{ m/s} - 9.8t\hat{\mathbf{j}}\text{ m/s}^2\right)dt$$

$$= 1.2t\;\hat{\mathbf{i}}\text{ m/s}\Big|_0^{0.380\text{ s}} - \left(9.8\hat{\mathbf{j}}\text{ m/s}^2\right)\frac{t^2}{2}\Big|_0^{0.380\text{ s}}$$

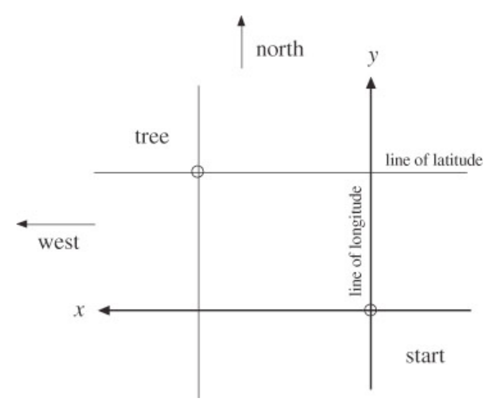
$$= \left(1.2\hat{\mathbf{i}}\text{ m/s}\right)(0.38\text{ s} - 0) - \left(9.8\hat{\mathbf{j}}\text{ m/s}^2\right)\left(\frac{(0.38\text{ s})^2 - 0}{2}\right)$$

$$= \boxed{0.456\hat{\mathbf{i}}\text{ m} - 0.708\hat{\mathbf{j}}\text{ m}}$$

P3.63 (a) $\dfrac{d\vec{\mathbf{r}}}{dt} = \dfrac{d\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{k}}\right)}{dt} = -2\hat{\mathbf{k}} = \boxed{-(2.00\text{ m/s})\hat{\mathbf{k}}}$

(b) The position vector at $t = 0$ is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. At $t = 1$ s, the position is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, and so on. The object is moving straight downward at 2 m/s, so $\dfrac{d\vec{\mathbf{r}}}{dt}$ represents $\boxed{\text{its velocity vector}}$.

P3.64 (a) The very small differences between the angles suggests we may consider this region of Earth to be small enough so that we may consider it to be flat (a plane); therefore, we may consider the lines of latitude and longitude to be parallel and perpendicular, so that we can use them as an *xy* coordinate system. Values of latitude, θ, increase as we travel north, so differences between latitudes can give the *y* coordinate. Values of longitude, ϕ, increase as we travel west, so differences between longitudes can give the *x* coordinate. Therefore, our coordinate system will have $+y$ to the north and $+x$ to the west.



ANS. FIG. P3.64

Since we are near the equator, each line of latitude and longitude may be considered to form a circle with a radius equal to the radius of Earth, $R = 6.36 \times 10^6$ m. Recall the length s of an arc of a circle of radius R that subtends an angle (in radians) $\Delta\theta$ (or $\Delta\phi$) is given by $s = R\Delta\theta$ (or $s = R\Delta\phi$). We can use this equation to find the components of the displacement from the starting point to the tree—these are parallel to the x and y coordinates axes. Therefore, we can regard the origin to be the starting point and the displacements as the x and y coordinates of the tree.

The angular difference $\Delta\phi$ for longitude values is (west being positive)

$$\begin{aligned}\Delta\phi &= [75.64426° - 75.64238°] \\ &= (0.00188°)(\pi \text{ rad}/180°) \\ &= 3.28 \times 10^{-5} \text{ rad}\end{aligned}$$

corresponding to the x coordinate (displacement west)

$$x = R\Delta\phi = (6.36 \times 10^6 \text{ m})(3.28 \times 10^{-5} \text{ rad}) = 209 \text{ m}$$

The angular difference $\Delta\theta$ for latitude values is (north being positive)

$$\begin{aligned}\Delta\theta &= [0.00162° - (-0.00243°)] \\ &= (0.00405°)(\pi \text{ rad}/180°) \\ &= 7.07 \times 10^{-5} \text{ rad}\end{aligned}$$

corresponding to the y coordinate (displacement north)

$$y = R\Delta\theta = (6.36 \times 10^6 \text{ m})(7.07 \times 10^{-5} \text{ rad}) = 450 \text{ m}$$

The distance to the tree is

$$d = \sqrt{x^2 + y^2} = \sqrt{(209 \text{ m})^2 + (450 \text{ m})^2} = \boxed{496 \text{ m}}$$

The direction to the tree is

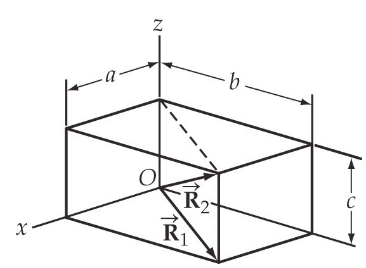
$$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{450 \text{ m}}{209 \text{ m}}\right) = 65.1° = \boxed{65.1° \text{ N of W}}$$

(b) Refer to the arguments above. They are justified because the distances involved are small relative to the radius of Earth.

P3.65 (a) From the picture, $\boxed{\vec{\mathbf{R}}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}.}$

(b) $\boxed{R_1 = \sqrt{a^2 + b^2}}$

(c) $\boxed{\vec{\mathbf{R}}_2 = \vec{\mathbf{R}}_1 + c\hat{\mathbf{k}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}}$



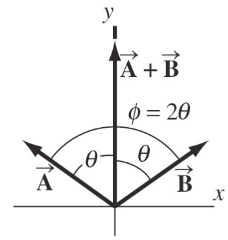
ANS. FIG. P3.65

P3.66 Since

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = 6.00\hat{\mathbf{j}},$$

we have

$$(A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$$



ANS. FIG. P3.66

giving $A_x + B_x = 0 \rightarrow A_x = -B_x.$

Because the vectors have the same magnitude and x components of equal magnitude but of opposite sign, the vectors are reflections of each other in the y axis, as shown in the diagram. Therefore, the two vectors have the same y components:

$$A_y = B_y = (1/2)(6.00) = 3.00$$

Defining θ as the angle between either $\vec{\mathbf{A}}$ or $\vec{\mathbf{B}}$ and the y axis, it is seen that

$$\cos\theta = \frac{A_y}{A} = \frac{B_y}{B} = \frac{3.00}{5.00} = 0.600 \rightarrow \theta = 53.1°$$

The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is then $\boxed{\phi = 2\theta = 106°}$.

Challenge Problem

P3.67 (a) You start at point A: $\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_A = \left(30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}}\right)$ m.

The displacement to B is

$$\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}$$

You cover half of this, $\left(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$, to move to

$$\vec{\mathbf{r}}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}$$

Now the displacement from your current position to C is

$$\vec{\mathbf{r}}_C - \vec{\mathbf{r}}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}$$

You cover one-third, moving to

$$\vec{\mathbf{r}}_3 = \vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}\left(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}$$

The displacement from where you are to D is

$$\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}$$

You traverse one-quarter of it, moving to

$$\vec{\mathbf{r}}_4 = \vec{\mathbf{r}}_3 + \frac{1}{4}\left(\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right)$$
$$= 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}$$

The displacement from your new location to E is

$$\vec{\mathbf{r}}_E - \vec{\mathbf{r}}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\vec{\mathbf{r}}_4 + \Delta\vec{\mathbf{r}}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}$$

The treasure is at $\boxed{(10.0 \text{ m}, 16.0 \text{ m})}$.

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\vec{\mathbf{r}}_A + \frac{1}{2}\left(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right) = \left(\frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B}{2}\right)$$

then to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)}{2} + \frac{\vec{\mathbf{r}}_C - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)/2}{3} = \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C}{3}$$

then to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)}{3} + \frac{\vec{\mathbf{r}}_D - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)/3}{4} = \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D}{4}$$

and last to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)}{4} + \frac{\vec{\mathbf{r}}_E - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)/4}{5}$$
$$= \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D + \vec{\mathbf{r}}_E}{5}$$

This center of mass of the tree distribution is the same location whatever order we take the trees in.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P3.2 (a) 2.31; (b) 1.15

P3.4 (a) (2.17, 1.25) m, (–1.90, 3.29) m; (b) 4.55m

P3.6 (a) r, 180° – θ; (b) 180° + θ; (c) $-\theta$

P3.8 $\vec{\mathbf{B}}$ is 43 units in the negative y direction

P3.10 9.5 N, 57° above the x axis

P3.12 (a) See ANS. FIG. P3.12; (b) The sum of a set of vectors is not affected by the order in which the vectors are added.

P3.14 310 km at 57° S of W

P3.16 $A_x = 28.7$ units, $A_y = -20.1$ units

P3.18 1.31 km north and 2.81 km east

P3.20 (a) 5.00 blocks at 53.1° N of E; (b) 13.00 blocks

P3.22 $\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)$ m

P3.24 788 miles at 48.0° northeast of Dallas

P3.26 (a) See ANS. FIG. P3.24; (b) $5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}, -1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}$; (c) 6.40 at 38.7°, 8.06 at 97.2°

P3.28 (a) Its component parallel to the surface is (1.50 m) cos 141° = –1.17 m, or 1.17 m toward the top of the hill; (b) Its component perpendicular to the surface is (1.50 m) sin 141° = 0.944 m, or 0.944 m away from the snow.

P3.30 42.7 yards

P3.32 $C_x = 7.30$ cm; $C_y = -7.20$ cm

P3.34 59.2° with the x axis, 39.8° with the y axis, 67.4° with the z axis

P3.36 (a) $5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}$, 5.92 m; (b) $(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}})$ m, 19.0) m

P3.38 (a) $49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}$; (b) 56.4, 28.7°

P3.40 magnitude: 170.1 cm, direction: 57.2° above +x axis (first quadrant); magnitude: 145.7 cm, direction: 58.6° above +x axis (first quadrant)

P3.42 (a) $\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)$km; (b) 6.31 km

P3.44 Impossible because 12.4 m is greater than 5.00 m

P3.46 (a) $(5 = 11f)\hat{\mathbf{i}} + (3 + 9f)\hat{\mathbf{j}}$ meters; (b) $(5 + 0)\hat{\mathbf{i}} + (3 + 0)\hat{\mathbf{j}}$ meters; (c) This is reasonable because it is the location of the starting point, $5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ meters. (d) $16\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$ meters; (e) This is reasonable because we have completed the trip, and this is the position vector of the endpoint.

P3.48 2.24 m, 26.6°

P3.50 390 mi/h at 7.37° N of E

P3.52 86.6 m, –50.0 m

P3.54 2.29 km

P3.56 (a) The perimeter measures $2(H + W) = 11.24$ m; (b) magnitude: 12.9 m, direction: 36.4° above $+x$ axis (first quadrant)

P3.58 (a) 7.17 km; (b) 6.15 km

P3.60 $\theta = 2\tan^{-1}\left(\frac{1}{n}\right)$

P3.62 $0.456\hat{\mathbf{i}}\text{ m} - 0.708\hat{\mathbf{j}}\text{ m}$

P3.64 (a) 496 m, 65.1° N of W; (b) The arguments are justified because the distances involved are small relative to the radius of the Earth.

P3.66 $\phi = 2\theta = 106°$