4

Motion in Two Dimensions

CHAPTER OUTLINE

* An asterisk indicates an item new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ4.1 The car's acceleration must have an inward component and a forward component: answer (e). Another argument: Draw a final velocity vector of two units west. Add to it a vector of one unit south. This represents subtracting the initial velocity from the final velocity, on the way to finding the acceleration. The direction of the resultant is that of vector (e).

OQ4.2 (i) The 45° angle means that at point *A* the horizontal and vertical velocity components are equal. The horizontal velocity component is the same at *A*, *B*, and *C*. The vertical velocity component is zero at *B* and negative at *C*. The assembled answer is $a = b = c > d = 0 > e$.

(ii) The *x component* of acceleration is everywhere zero and the *y component* is everywhere $-9.80\ \text{m/s}^2$. Then we have $a = c = 0 > b = d = e$.

OQ4.3 Because gravity pulls downward, the horizontal and vertical motions of a projectile are independent of each other. Both balls have zero initial vertical components of velocity, and both have the same vertical accelerations, $-g$; therefore, both balls will have identical vertical motions: they will reach the ground at the same time. Answer (b).

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OQ4.4 The projectile on the moon is in flight for a time interval six times larger, with the same range of vertical speeds and with the same constant horizontal speed as on Earth. Then its maximum altitude is (d) six times larger.

OQ4.5 The acceleration of a car traveling at constant speed in a circular path is directed toward the center of the circle. Answer (d).

OQ4.6 The acceleration of gravity near the surface of the Moon acts the same way as on Earth, it is constant and it changes only the vertical component of velocity. Answers (b) and (c).

OQ4.7 The projectile on the Moon is in flight for a time interval six times larger, with the same range of vertical speeds and with the same constant horizontal speed as on Earth. Then its range is (d) six times larger.

OQ4.8 Let the positive x direction be that of the girl's motion. The x component of the velocity of the ball relative to the ground is (+5 – 12) m/s = –7 m/s. The x-velocity of the ball relative to the girl is (–7 – 8) m/s = –15 m/s. The relative speed of the ball is +15 m/s, answer (d).

OQ4.9 Both wrench and boat have identical horizontal motions because gravity influences the vertical motion of the wrench only. Assuming neither air resistance nor the wind influences the horizontal motion of the wrench, the wrench will land at the base of the mast. Answer (b).

OQ4.10 While in the air, the baseball is a projectile whose velocity always has a constant horizontal component ($v_x = v_{xi}$) and a vertical component that changes at a constant rate ($\Delta v_y / \Delta t = a_y = -g$). At the highest point on the path, the vertical velocity of the ball is momentarily zero. Thus, at this point, the resultant velocity of the ball is horizontal and its acceleration continues to be directed downward ($a_x = 0$, $a_y = -g$). The only correct choice given for this question is (c).

OQ4.11 The period $T = 2\pi r/v$ changes by a factor of 4/4 = 1. The answer is (a).

OQ4.12 The centripetal acceleration $a = v^2/r$ becomes $(3v)^2/(3r) = 3v^2/r$, so it is 3 times larger. The answer is (b).

OQ4.13 (a) Yes (b) No: The escaping jet exhaust exerts an extra force on the plane. (c) No (d) Yes (e) No: The stone is only a few times more dense than water, so friction is a significant force on the stone. The answer is (a) and (d).

OQ4.14 With radius half as large, speed should be smaller by a factor of $1/\sqrt{2}$, so that $a = v^2/r$ can be the same. The answer is (d).

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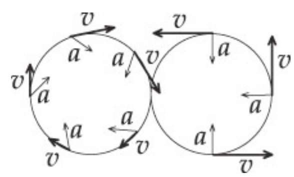
ANSWERS TO CONCEPTUAL QUESTIONS

CQ4.1 A parabola results, because the originally forward velocity component stays constant and the rocket motor gives the spacecraft constant acceleration in a perpendicular direction. These are the same conditions for a projectile, for which the velocity is constant in the horizontal direction and there is a constant acceleration in the perpendicular direction. Therefore, a curve of the same shape is the result.

CQ4.2 The skater starts at the center of the eight, goes clockwise around the left circle and then counterclockwise around the right circle.



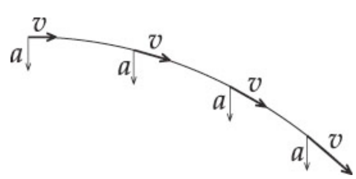
CQ4.3 No, you cannot determine the instantaneous velocity because the points could be separated by a finite displacement, but you can determine the average velocity. Recall the definition of average velocity:

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\Delta\vec{\mathbf{x}}}{\Delta t}$$

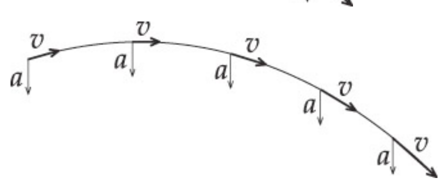
CQ4.4 (a) On a straight and level road that does not curve to left or right. (b) Either in a circle or straight ahead on a level road. The acceleration magnitude can be constant either with a nonzero or with a zero value.

CQ4.5 (a) Yes, the projectile is in free fall. (b) Its vertical component of acceleration is the downward acceleration of gravity. (c) Its horizontal component of acceleration is zero.

CQ4.6 (a)



(b)



CQ4.7 (a) No. Its velocity is constant in magnitude and direction. (b) Yes. The particle is continuously changing the direction of its velocity vector.

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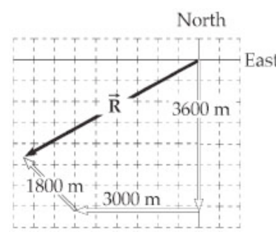
SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 4.1 The Position, Velocity, and Acceleration Vectors

P4.1 We must use the method of vector addition and the definitions of average velocity and of average speed.



ANS. FIG. P4.1

(a) For each segment of the motion we model the car as a particle under constant velocity. Her displacements are

$$\vec{\mathbf{R}} = (20.0\text{ m/s})(180\text{ s})\ \textbf{south}$$
$$+ (25.0\text{ m/s})(120\text{ s})\ \textbf{west}$$
$$+ (30.0\text{ m/s})(60.0\text{ s})\ \textbf{northwest}$$

Choosing $\hat{\mathbf{i}} = \text{east and } \hat{\mathbf{j}} = \text{north},$ we have

$$\vec{\mathbf{R}} = (3.60\text{ km})(-\hat{\mathbf{j}}) + (3.00\text{ km})(-\hat{\mathbf{i}}) + (1.80\text{ km})\cos 45°(-\hat{\mathbf{i}})$$
$$+ (1.80\text{ km})\sin 45°(\hat{\mathbf{j}})$$

$$\vec{\mathbf{R}} = (3.00 + 1.27)\text{ km}(-\hat{\mathbf{i}}) + (1.27 - 3.60)\text{ km}(\hat{\mathbf{j}})$$
$$= (-4.27\hat{\mathbf{i}} - 2.33\hat{\mathbf{j}})\text{ km}$$

The answer can also be written as

$$\vec{\mathbf{R}} = \sqrt{(-4.27\text{ km})^2 + (-2.33\text{ km})^2} = 4.87\text{ km}$$

$$\text{at } \tan^{-1}\left(\frac{2.33}{4.27}\right) = 28.6°$$

or $\boxed{4.87\text{ km at }28.6°\text{ S of W}}$

(b) The total distance or path length traveled is (3.60 + 3.00 + 1.80) km = 8.40 km, so

$$\text{average speed} = \left(\frac{8.40\text{ km}}{6.00\text{ min}}\right)\left(\frac{1.00\text{ min}}{60.0\text{ s}}\right)\left(\frac{1\,000\text{ m}}{\text{km}}\right) = \boxed{23.3\text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87\times 10^3\text{ m}}{360\text{ s}} = \boxed{13.5\text{ m/s along }\vec{\mathbf{R}}}$$

P4.2 The sun projects onto the ground the x component of the hawk's velocity:

$$(5.00\text{ m/s})\cos(-60.0°) = \boxed{2.50\text{ m/s}}$$

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***P4.3** (a) For the average velocity, we have

$$\vec{\mathbf{v}}_{\text{avg}} = \left(\frac{x(4.00) - x(2.00)}{4.00\text{ s} - 2.00\text{ s}}\right)\hat{\mathbf{i}} + \left(\frac{y(4.00) - y(2.00)}{4.00\text{ s} - 2.00\text{ s}}\right)\hat{\mathbf{j}}$$

$$= \left(\frac{5.00\text{ m} - 3.00\text{ m}}{2.00\text{ s}}\right)\hat{\mathbf{i}} + \left(\frac{3.00\text{ m} - 1.50\text{ m}}{2.00\text{ s}}\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_{\text{avg}} = \boxed{\left(1.00\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) For the velocity components, we have

$$v_x = \frac{dx}{dt} = a = 1.00\text{ m/s}$$

and $v_y = \dfrac{dy}{dt} = 2ct = \left(0.250\text{ m/s}^2\right)t$

Therefore,

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (1.00\text{ m/s})\hat{\mathbf{i}} + \left(0.250\text{ m/s}^2\right)t\,\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{v}}(t = 2.00\text{ s}) = (1.00\text{ m/s})\hat{\mathbf{i}} + (0.500\text{ m/s})\hat{\mathbf{j}}}$$

and the speed is

$$\left|\vec{\mathbf{v}}(t = 2.00\text{ s})\right| = \sqrt{(1.00\text{ m/s})^2 + (0.500\text{ m/s})^2} = \boxed{1.12\text{ m/s}}$$

P4.4 (a) From $x = -5.00\sin\omega t$, we determine the components of the velocity by taking the time derivatives of x and y:

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00\sin\omega t) = -5.00\omega\cos\omega t$$

and $v_y = \dfrac{dy}{dt} = \left(\dfrac{d}{dt}\right)(4.00 - 5.00\cos\omega t) = 0 + 5.00\omega\sin\omega t$

At $t = 0$,

$$\vec{\mathbf{v}} = \left(-5.00\omega\cos 0\right)\hat{\mathbf{i}} + \left(5.00\omega\sin 0\right)\hat{\mathbf{j}} = \boxed{-5.00\omega\,\hat{\mathbf{i}}\text{ m/s}}$$

(b) Acceleration is the time derivative of the velocity, so

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-5.00\omega\cos\omega t) = +5.00\omega^2\sin\omega t$$

and $a_y = \dfrac{dv_y}{dt} = \left(\dfrac{d}{dt}\right)(5.00\omega\sin\omega t) = 5.00\omega^2\cos\omega t$

At t = 0,

$$\vec{\mathbf{a}} = \left(5.00\omega^2\sin 0\right)\hat{\mathbf{i}} + \left(5.00\omega^2\cos 0\right)\hat{\mathbf{j}} = \boxed{5.00\omega^2\hat{\mathbf{j}}\text{ m/s}^2}$$

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(c) $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boxed{(4.00\text{ m})\hat{\mathbf{j}} + (5.00\text{ m})(-\sin\omega t\hat{\mathbf{i}} - \cos\omega t\hat{\mathbf{j}})}$

$$\vec{\mathbf{v}} = \boxed{(5.00\text{ m})\omega\left[-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right]}$$

$$\vec{\mathbf{a}} = \boxed{(5.00\text{ m})\omega^2\left[\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]}$$

(d) the object moves in a circle of radius 5.00 m centered at (0, 4.00 m)

P4.5 (a) The x and y equations combine to give us the expression for $\vec{\mathbf{r}}$:

$\vec{\mathbf{r}} = 18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$, where $\vec{\mathbf{r}}$ is in meters and t is in seconds.

(b) We differentiate the expression for $\vec{\mathbf{r}}$ with respect to time:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d}{dt}\left[18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}\right]$$

$$= \frac{d}{dt}(18.0t)\hat{\mathbf{i}} + \frac{d}{dt}\left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$$

$\vec{\mathbf{v}} = 18.0\hat{\mathbf{i}} + \left[4.00 - (9.80)t\right]\hat{\mathbf{j}}$, where $\vec{\mathbf{v}}$ is in meters per second and t is in seconds.

(c) We differentiate the expression for $\vec{\mathbf{v}}$ with respect to time:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{d}{dt}\left\{18.0\hat{\mathbf{i}} + \left[4.00 - (9.80)t\right]\hat{\mathbf{j}}\right\}$$

$$= \frac{d}{dt}(18.0)\hat{\mathbf{i}} + \frac{d}{dt}\left[4.00 - (9.80)t\right]\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{a}} = -9.80\hat{\mathbf{j}}\text{ m/s}^2}$$

(d) By substitution,

$$\vec{\mathbf{r}}(3.00\text{ s}) = \boxed{(54.0\text{ m})\hat{\mathbf{i}} - (32.1\text{ m})\hat{\mathbf{j}}}$$

$$\vec{\mathbf{v}}(3.00\text{ s}) = \boxed{(18.0\text{ m/s})\hat{\mathbf{i}} - (25.4\text{ m/s})\hat{\mathbf{j}}}$$

$$\vec{\mathbf{a}}(3.00\text{ s}) = \boxed{(-9.80\text{ m/s}^2)\hat{\mathbf{j}}}$$

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Section 4.2 Two-Dimensional Motion with Constant Acceleration

P4.6 We use the vector versions of the kinematic equations for motion in two dimensions. We write the initial position, initial velocity, and acceleration of the particle in vector form:

$$\vec{\mathbf{a}} = 3.00\hat{\mathbf{j}}\ \text{m/s}^2;\ \vec{\mathbf{v}}_i = 5.00\hat{\mathbf{i}}\ \text{m/s};\ \vec{\mathbf{r}}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$$

(a) The position of the particle is given by Equation 4.9:

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2 = (5.00\ \text{m/s})t\hat{\mathbf{i}} + \frac{1}{2}(3.00\ \text{m/s}^2)t^2\hat{\mathbf{j}}$$
$$= \boxed{5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}}$$

where r is in m and t in s.

(b) The velocity of the particle is given by Equation 4.8:

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \boxed{5.00\hat{\mathbf{i}} + 3.00t\ \hat{\mathbf{j}}}$$

where v is in m/s and t in s.

(c) To obtain the particle's position at t = 2.00 s, we plug into the equation obtained in part (a):

$$\vec{\mathbf{r}}_f = (5.00\ \text{m/s})(2.00\ \text{s})\hat{\mathbf{i}} + \left(1.50\ \text{m/s}^2\right)(2.00\ \text{s})^2\hat{\mathbf{j}}$$
$$= \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \text{m}$$

so $x_f = \boxed{10.0\ \text{m}}$, $y_f = \boxed{6.00\ \text{m}}$

(d) To obtain the particle's speed at t = 2.00 s, we plug into the equation obtained in part (b):

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = (5.00\ \text{m/s})\hat{\mathbf{i}} + \left(3.00\ \text{m/s}^2\right)(2.00\ \text{s})\hat{\mathbf{j}}$$
$$= \left(5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00\ \text{m/s})^2 + (6.00\ \text{m/s})^2} = \boxed{7.81\ \text{m/s}}$$

P4.7 (a) We differentiate the equation for the vector position of the particle with respect to time to obtain its velocity:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \left(\frac{d}{dt}\right)\left(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}\right) = \boxed{-12.0t\hat{\mathbf{j}}\ \text{m/s}}$$

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(b) Differentiating the expression for velocity with respect to time gives the particle's acceleration:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\hat{\mathbf{j}}\right) = \boxed{-12.0\hat{\mathbf{j}}\ \text{m/s}^2}$$

(c) By substitution, when $t = 1.00$ s,

$$\boxed{\vec{\mathbf{r}} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)\text{m};\ \vec{\mathbf{v}} = -12.0\hat{\mathbf{j}}\ \text{m/s}}$$

***P4.8** (a) For the x component of the motion we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$.

$$0.01\ \text{m} = 0 + \left(1.80\times10^7\ \text{m/s}\right)t + \frac{1}{2}\left(8\times10^{14}\ \text{m/s}^2\right)t^2$$

$$\left(4\times10^{14}\ \text{m/s}^2\right)t^2 + \left(1.80\times10^7\ \text{m/s}\right)t - 10^{-2}\ \text{m} = 0$$

$$t = \left(\frac{1}{2\left(4\times10^{14}\ \text{m/s}^2\right)}\right)\left[-1.80\times10^7\ \text{m/s} \pm\sqrt{\left(1.8\times10^7\ \text{m/s}\right)^2 - 4\left(4\times10^{14}\ \text{m/s}^2\right)\left(-10^{-2}\text{m}\right)}\right]$$

$$= \frac{-1.8\times10^7 \pm 1.84\times10^7\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2}$$

We choose the + sign to represent the physical situation:

$$t = \frac{4.39\times10^5\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2} = 5.49\times10^{-10}\ \text{s}$$

Here

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\ &= 0 + 0 + \frac{1}{2}\left(1.6\times10^{15}\ \text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right)^2 \\ &= 2.41\times10^{-4}\ \text{m} \end{aligned}$$

So, $\boxed{\vec{\mathbf{r}}_f = \left(10.0\hat{\mathbf{i}} + 0.241\hat{\mathbf{j}}\right)\ \text{mm}}$

(b)

$$\begin{aligned} \vec{\mathbf{v}}_f &= \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t \\ &= 1.80\times10^7\hat{\mathbf{i}}\ \text{m/s} \\ &\quad + \left(8\times10^{14}\hat{\mathbf{i}}\ \text{m/s}^2 + 1.6\times10^{15}\hat{\mathbf{j}}\ \text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right) \\ &= \left(1.80\times10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(4.39\times10^5\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}} \\ &= \boxed{\left(1.84\times10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}}} \end{aligned}$$

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(c) $\left|\vec{\mathbf{v}}_f\right| = \sqrt{(1.84\times10^7\ \text{m/s})^2 + (8.78\times10^5\ \text{m/s})^2} = \boxed{1.85\times10^7\ \text{m/s}}$

(d) $\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right) = \tan^{-1}\left(\dfrac{8.78\times10^5}{1.84\times10^7}\right) = \boxed{2.73°}$

P4.9 Model the fish as a particle under constant acceleration. We use our old standard equations for constant-acceleration straight-line motion, with x and y subscripts to make them apply to parts of the whole motion. At $t = 0$,

$$\vec{\mathbf{v}}_i = \left(4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right)\ \text{m/s and } \hat{\mathbf{r}}_i = (10.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})\ \text{m}$$

At the first "final" point we consider, 20.0 s later,

$$\vec{\mathbf{v}}_f = (20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})\ \text{m/s}$$

(a) $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{20.0\ \text{m/s} - 4.00\ \text{m/s}}{20.0\ \text{s}} = \boxed{0.800\ \text{m/s}^2}$

$a_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{-5.00\ \text{m/s} - 1.00\ \text{m/s}}{20.0\ \text{s}} = \boxed{-0.300\ \text{m/s}^2}$

(b) $\theta = \tan^{-1}\left(\dfrac{-0.300\ \text{m/s}^2}{0.800\ \text{m/s}^2}\right) = -20.6° = \boxed{339°\ \text{from} + x\ \text{axis}}$

(c) At $t = 25.0$ s the fish's position is specified by its coordinates and the direction of its motion is specified by the direction angle of its velocity:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$= 10.0\ \text{m} + (4.00\ \text{m/s})(25.0\ \text{s}) + \frac{1}{2}(0.800\ \text{m/s}^2)(25.0\ \text{s})^2$$
$$= \boxed{360\ \text{m}}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$= -4.00\ \text{m} + (1.00\ \text{m/s})(25.0\ \text{s}) + \frac{1}{2}(-0.300\ \text{m/s}^2)(25.0\ \text{s})^2$$
$$= \boxed{-72.7\ \text{m}}$$

$$v_{xf} = v_{xi} + a_x t = 4.00\ \text{m/s} + \left(0.800\ \text{m/s}^2\right)(25.0\ \text{s}) = 24\ \text{m/s}$$
$$v_{yf} = v_{yi} + a_y t = 1.00\ \text{m/s} - \left(0.300\ \text{m/s}^2\right)(25.0\ \text{s}) = -6.50\ \text{m/s}$$
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50\ \text{m/s}}{24.0\ \text{m/s}}\right) = \boxed{-15.2°}$$

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P4.10 The directions of the position, velocity, and acceleration vectors are given with respect to the x axis, and we know that the components of a vector with magnitude A and direction θ are given by $A_x = A\cos\theta$ and $A_y = A\sin\theta$; thus we have

$$\vec{\mathbf{r}}_i = 29.0\cos 95.0°\,\hat{\mathbf{i}} + 29.0\sin 95.0°\,\hat{\mathbf{j}} = -2.53\,\hat{\mathbf{i}} + 28.9\,\hat{\mathbf{j}}$$
$$\vec{\mathbf{v}}_i = 4.50\cos 40.0°\,\hat{\mathbf{i}} + 4.50\sin 40.0°\,\hat{\mathbf{j}} = 3.45\,\hat{\mathbf{i}} + 2.89\,\hat{\mathbf{j}}$$
$$\vec{\mathbf{a}} = 1.90\cos 200°\,\hat{\mathbf{i}} + 1.90\sin 200°\,\hat{\mathbf{j}} = -1.79\,\hat{\mathbf{i}} + -0.650\,\hat{\mathbf{j}}$$

where $\vec{\mathbf{r}}$ is in m, $\vec{\mathbf{v}}$ in m/s, $\vec{\mathbf{a}}$ in m/s^2, and t in s.

(a) From $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$,

$$\boxed{\vec{\mathbf{v}}_f = (3.45 - 1.79t)\hat{\mathbf{i}} + (2.89 - 0.650t)\hat{\mathbf{j}}}$$

where $\vec{\mathbf{v}}$ in m/s and t in s.

(b) The car's position vector is given by

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$
$$= (-2.53 + 3.45t + \frac{1}{2}(-1.79)t^2)\hat{\mathbf{i}} + (28.9 + 2.89t + \frac{1}{2}(-0.650)t^2)\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{r}}_f = (-2.53 + 3.45t - 0.893t^2)\hat{\mathbf{i}} + (28.9 + 2.89t - 0.325t^2)\hat{\mathbf{j}}}$$

where $\vec{\mathbf{r}}$ is in m and t in s.

Section 4.3 Projectile Motion

P4.11 At the maximum height $v_y = 0$, and the time to reach this height is found from

$$v_{yf} = v_{yi} + a_y t \text{ as } t = \frac{v_{yf} - v_{yi}}{a_y} = \frac{0 - v_{yi}}{-g} = \frac{v_{yi}}{g}.$$

The vertical displacement that has occurred during this time is

$$(\Delta y)_{\max} = v_{y,\text{avg}}t = \left(\frac{v_{yf} + v_{yi}}{2}\right)t = \left(\frac{0 + v_{yi}}{2}\right)\left(\frac{v_{yi}}{g}\right) = \frac{v_{yi}^2}{2g}$$

Thus, if $(\Delta y)_{\max} = 12\text{ ft}\left(\frac{1\text{ m}}{3.281\text{ ft}}\right) = 3.66\text{ m}$, then

$$v_{yi} = \sqrt{2g(\Delta y)_{\max}} = \sqrt{2(9.80\text{ m/s}^2)(3.66\text{ m})} = 8.47\text{ m/s}$$

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and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_i = \frac{v_{yi}}{\sin\theta} = \frac{8.47 \text{ m/s}}{\sin 45°} = \boxed{12.0 \text{ m/s}}$$

***P4.12** From Equation 4.13 with R = 15.0 m, v_i = 3.00 m/s, and $\theta_{\max}$ = 45.0°:

$$g_{planet} = \frac{v_i^2 \sin 2\theta}{R} = \frac{v_i^2 \sin 90°}{R} = \frac{9.00 \text{ m}^2/\text{s}^2}{15.0 \text{ m}} = \boxed{0.600 \text{ m/s}^2}$$

P4.13 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If t is the time at which it hits the ground, then since there is no horizontal acceleration,

$$x_f = v_{xi}t \rightarrow t = x_f/v_{xi} \rightarrow t = (1.40 \text{ m}/v_{xi})$$

At time t, it has fallen a distance of 1.22 m with a downward acceleration of 9.80 m/s^2. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$0 = 1.22 \text{ m} - (4.90 \text{ m/s}^2)(1.40 \text{ m}/v_{xi})^2$$

Thus,

$$v_{xi} = \sqrt{\frac{(4.90 \text{ m/s}^2)(1.40 \text{ m})^2}{1.22 \text{ m}}} = \boxed{2.81 \text{ m/s}}$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t \rightarrow v_{yf} = v_{yi} + (-g)(1.40 \text{ m}/v_{xi})$$
$$v_{yf} = 0 + (-9.80 \text{ m/s}^2)(1.40 \text{ m}/2.81 \text{ m/s}) = -4.89 \text{ m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.89 \text{ m/s}}{2.81 \text{ m/s}}\right) = -60.2°$$

The mug's velocity is 60.2° below the horizontal when it strikes the ground.

P4.14 The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time t are

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow x_f = 0 + v_{xi}t \rightarrow x_f = v_{xi}t$$

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and

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2 \rightarrow y_f = -0 + 0 - \frac{1}{2}gt^2 \rightarrow y_f = -\frac{1}{2}gt^2$$

(a) When the mug reaches the floor, $y_f = h$ and $x_f = d$, so

$$-h = -\frac{1}{2}gt^2 \rightarrow h = \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$$

is the time of impact, and

$$x_f = v_{xi}t \rightarrow d = v_{xi}t \rightarrow v_{xi} = \frac{d}{t}$$

$$\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$$

(b) Just before impact, the x component of velocity is still

$$v_{xf} = v_{xi}$$

while the y component is

$$v_{yf} = v_{yi} + at \rightarrow v_{yf} = 0 - gt = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\left(\frac{-\sqrt{2gh}}{d\sqrt{\frac{g}{2h}}}\right)$$

$$\theta = \tan^{-1}\left(\frac{-2h}{d}\right) = -\tan^{-1}\left(\frac{2h}{d}\right)$$

because the x component of velocity is positive (forward) and the y component is negative (downward).

The direction of the mug's velocity is $\tan^{-1}(2h/d)$ below the horizontal.

P4.15 We ignore the trivial case where the angle of projection equals zero degrees.

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}; \quad R = \frac{v_i^2(\sin 2\theta_i)}{g}; \quad 3h = R$$

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so $$\frac{3v_i^2 \sin^2\theta_i}{2g} = \frac{v_i^2(\sin 2\theta_i)}{g}$$

or $$\frac{2}{3} = \frac{\sin^2\theta_i}{\sin 2\theta_i} = \frac{\tan\theta_i}{2}$$

thus, $$\theta_i = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{53.1°}$$

P4.16 The horizontal range of the projectile is found from $x = v_{xi}t = v_i \cos\theta_i t$. Plugging in numbers,

$$x = (300 \text{ m/s})(\cos 55.0°)(42.0 \text{ s})$$

$$x = \boxed{7.23 \times 10^3 \text{ m}}$$

The vertical position of the projectile is found from

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

Plugging in numbers,

$$y = (300 \text{ m/s})(\sin 55.0°)(42.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(42.0 \text{ s})^2$$

$$= \boxed{1.68 \times 10^3 \text{ m}}$$

P4.17 (a) The vertical component of the salmon's velocity as it leaves the water is

$$v_{yi} = +v_i \sin\theta = +(6.26 \text{ m/s}) \sin 45.0° \simeq +4.43 \text{ m/s}$$

When the salmon returns to water level at the end of the leap, the vertical component of velocity will be

$$v_{yf} = -v_{yi} \simeq -4.43 \text{ m/s}$$

If the salmon jumps out of the water at $t = 0$, the time interval required for it to return to the water is

$$\Delta t_1 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-4.43 \text{ m/s} - 4.43 \text{ m/s}}{-9.80 \text{ m/s}^2} \simeq 0.903 \text{ s}$$

The horizontal distance traveled during the leap is

$$L = v_{xi}\Delta t_i = (v_i \cos\theta)\Delta t_i$$
$$= (6.26 \text{ m/s})\cos 45.0°(0.903 \text{ s}) = 4.00 \text{ m}$$

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To travel this same distance underwater, at speed v = 3.58 m/s, requires a time interval of

$$\Delta t_2 = \frac{L}{v} = \frac{4.00\text{ m}}{3.58\text{ m/s}} \simeq 1.12\text{ s}$$

The average horizontal speed for the full porpoising maneuver is then

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{2L}{\Delta t_1 + \Delta t_2} = \frac{2(4.00\text{ m})}{0.903\text{ s} + 1.12\text{ s}} = \boxed{3.96\text{ m/s}}$$

(b) From (a), the total time interval for the porpoising maneuver is

$$\Delta t = 0.903\text{ s} + 1.12\text{ s} = 2.02\text{ s}$$

Without porpoising, the time interval to travel distance $2L$ is

$$\Delta t_2 = \frac{2L}{v} = \frac{8.00\text{ m}}{3.58\text{ m/s}} \simeq 2.23\text{ s}$$

The percentage difference is

$$\frac{\Delta t_1 - \Delta t_2}{\Delta t_2} \times 100\% = -9.6\%$$

$\boxed{\text{Porpoising reduces the time interval by 9.6\%.}}$

P4.18 (a) We ignore the trivial case where the angle of projection equals zero degrees. Because the projectile motion takes place over level ground, we can use Equations 4.12 and 4.13:

$$R = h \rightarrow \frac{v_i^2 \sin 2\theta_i}{g} = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

Expanding,

$$2\sin 2\theta_i = \sin^2\theta_i$$
$$4\sin\theta_i \cos\theta_i = \sin^2\theta_i$$
$$\tan\theta_i = 4$$
$$\theta_i = \tan^{-1}(4) = \boxed{76.0^\circ}$$

(b) The maximum range is attained for $\theta_i = 45°$:

$$R = \frac{v_i^2 \sin[2(76.0^\circ)]}{g} \text{ and } R_{\max} = \frac{v_i^2 \sin[2(45.0^\circ)]}{g} = \frac{v_i^2}{g}$$

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then

$$R_{\max} = \frac{v_i^2 \sin[2(76.0°)]}{g \sin[2(76.0°)]} = \frac{R}{\sin[2(76.0°)]}$$

$$R_{\max} = \boxed{2.13R}$$

(c) Since g divides out, the answer is the same on every planet.

***P4.19** Consider the motion from original zero height to maximum height *h:*

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right) \text{ gives } 0 = v_{yi}^2 - 2g(h-0)$$

or $$v_{yi} = \sqrt{2gh}$$

Now consider the motion from the original point to half the maximum height:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right) \text{ gives } v_{yh}^2 = 2gh + 2(-g)\left(\frac{1}{2}h - 0\right)$$

so $$v_{yh} = \sqrt{gh}$$

At maximum height, the speed is $v_x = \frac{1}{2}\sqrt{v_x^2 + v_{yh}^2} = \frac{1}{2}\sqrt{v_x^2 + gh}$

Solving,

$$v_x = \sqrt{\frac{gh}{3}}$$

Now the projection angle is

$$\theta_i = \tan^{-1}\frac{v_{yi}}{v_x} = \tan^{-1}\frac{\sqrt{2gh}}{\sqrt{gh/3}} = \tan^{-1}\sqrt{6} = \boxed{67.8°}$$

P4.20 (a) $x_f = v_{xi}t = (8.00 \text{ m/s})\cos 20.0° \,(3.00 \text{ s}) = \boxed{22.6 \text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}gt^2$$

$$y_f = (8.00 \text{ m/s})\sin 20.0°(3.00 \text{ s}) + \frac{1}{2}(9.80 \text{ m/s}^2)(3.00 \text{ s})^2$$

$$= \boxed{52.3 \text{ m}}$$

(c) $$10.0 \text{ m} = (8.00 \text{ m/s})(\sin 20.0°)t + \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

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Suppressing units,

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18 \text{ s}}$$

P4.21 The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos\theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \dfrac{d}{v_i \cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \ \sin\theta_i\left(\frac{d}{v_i \cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos\theta_i}\right)^2$$

Therefore, the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d\tan\theta_i - \frac{gd^2}{2v_i^2\cos^2\theta_i}}$$

P4.22 (a) The time of flight of a water drop is given by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = y_1 - \frac{1}{2}gt^2$$

For $t_1 > 0$, the root is $t_1 = \sqrt{\dfrac{2y_1}{g}} = \sqrt{\dfrac{2(2.35 \text{ m})}{9.8 \text{ m/s}^2}} = 0.693$ s.

The horizontal range of a water drop is

$$\begin{aligned} x_{f1} &= x_i + v_{xi}t + \frac{1}{2}a_x t^2 \\ &= 0 + 1.70 \text{ m/s } (0.693 \text{ s}) + 0 = 1.18 \text{ m} \end{aligned}$$

This is about the width of a town sidewalk, so $\boxed{\text{there is}}$ space for a walkway behind the waterfall. Unless the lip of the channel is well designed, water may drip on the visitors. A tall or inattentive person may get his or her head wet.

(b) Now the flight time t_2 is given by

$$0 = y_2 + 0 - \frac{1}{2}gt_2^2, \text{ where } y_2 = \frac{y_1}{12}:$$

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2y_1}{g(12)}} = \frac{1}{\sqrt{12}} \times \sqrt{\frac{2y_1}{g}} = \frac{t_1}{\sqrt{12}}$$

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From the same equation as in part (a) for horizontal range, $x_2 = v_2 t_2$, where $x_2 = x_1/12$:

$$x_2 = v_2 t_2 \to \frac{x_1}{12} = v_2 \frac{t_1}{\sqrt{12}}$$

$$v_2 = \frac{x_1}{t_1\sqrt{12}} = \frac{v_1}{\sqrt{12}} = \frac{1.70 \text{ m/s}}{\sqrt{12}} = \boxed{0.491 \text{ m/s}}$$

The rule that the scale factor for speed is the square root of the scale factor for distance is Froude's law, published in 1870.

P4.23 (a) From the particle under constant velocity model in the x direction, find the time at which the ball arrives at the goal:

$$x_f = x_i + v_i t \to t = \frac{x_f - x_i}{v_{xi}} = \frac{36.0 \text{ m} - 0}{(20 \text{ m/s}) \cos 53.0°} = 2.99 \text{ s}$$

From the particle under constant acceleration model in the y direction, find the height of the ball at this time:

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$y_f = 0 + (20.0 \text{ m/s}) \sin 53.0°(2.99 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(2.99 \text{ s})^2$$

$$y_f = 3.94 \text{ m}$$

Therefore, the ball clears the crossbar by

$$3.94 \text{ m} - 3.05 \text{ m} = \boxed{0.89 \text{ m}}$$

(b) Use the particle under constant acceleration model to find the time at which the ball is at its highest point in its trajectory:

$$v_{yf} = v_{yi} - gt \to t = \frac{v_{yf} - v_{yi}}{g} = \frac{(20.0 \text{ m/s})\sin 53.0° - 0}{9.80 \text{ m/s}^2} = 1.63 \text{ s}$$

Because this is earlier than the time at which the ball reaches the goal, the ball clears the goal on its way down.

P4.24 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2\left(-9.80 \text{ m/s}^2\right)(1.85 - 1.02) \text{ m}$. From this, $v_{yi} = 4.03 \text{ m/s}$. [Note that this is the answer to part (c) of this problem.]

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For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80\text{ m/s}^2)(0.900 - 1.85)$ m. Thus, the vertical velocity just before he lands is $v_{yf} = -4.32$ m/s.

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32\text{ m/s} = 4.03\text{ m/s} + (-9.80\text{ m/s}^2)t$$

or $t = \boxed{0.852\text{ s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80\text{ m} = v_{xi}(0.852\text{ s})$$

which yields $v_{xi} = \boxed{3.29\text{ m/s}}$.

(c) $v_{yi} = \boxed{4.03\text{ m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\dfrac{v_{yi}}{v_{xi}} = \tan^{-1}\left(\dfrac{4.03\text{ m/s}}{3.29\text{ m/s}}\right) = \boxed{50.8°}$

(e) Similarly for the deer, the upward part of the flight gives

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = v_{yi}^2 + 2\left(-9.80\text{ m/s}^2\right)(2.50 - 1.20)\text{ m}$$

so $v_{yi} = 5.04$ m/s

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields

$$v_{yf}^2 = 0 + 2\left(-9.80\text{ m/s}^2\right)(0.700 - 2.50)\text{ m and } v_{yf} = -5.94\text{ m/s}.$$

The hang time is then found as $v_{yf} = v_{yi} + a_y t$:

$$-5.94\text{ m/s} = 5.04\text{ m/s} + (-9.80\text{ m/s}^2)t \text{ and}$$

$$\boxed{t = 1.12\text{ s}}$$

P4.25 (a) For the horizontal motion, we have $x_f = d = 24$ m:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$24\text{ m} = 0 + v_i(\cos 53°)(2.2\text{ s}) + 0$$

$$v_i = \boxed{18.1\text{ m/s}}$$

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(b) As it passes over the wall, the ball is above the street by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_f = 0 + (18.1\ \text{m/s})(\sin\ 53°)(2.2\ \text{s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(2.2\ \text{s})^2 = 8.13\ \text{m}$$

So it clears the parapet by 8.13 m – 7 m = $\boxed{1.13\ \text{m}}$.

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation:

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

or $$6\ \text{m} = (\tan 53°)x_f - \left(\frac{9.8\,\text{m/s}^2}{2(18.1\ \text{m/s})^2\cos^2 53°}\right)x_f^2$$

Solving,

$$(0.041\ 2\ \text{m}^{-1})x_f^2 - 1.33x_f + 6\ \text{m} = 0$$

and, suppressing units,

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.041\ 2)(6)}}{2(0.041\ 2)}$$

This yields two results:

x_f = 26.8 m or 5.44 m

The ball passes twice through the level of the roof.

It hits the roof at distance from the wall

26.8 m – 24 m = $\boxed{2.79\ \text{m}}$

P4.26 We match the given equations:

$$x_f = 0 + (11.2\ \text{m/s})(\cos\ 18.5°)t$$

$$0.360\ \text{m} = 0.840\ \text{m} + (11.2\ \text{m/s})(\sin\ 18.5°)t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

to the equations for the coordinates of the final position of a projectile:

$$x_f = x_i + v_{xi}t$$

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

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For the equations to represent the same functions of time, all coefficients must agree: $x_i = 0$, $y_i = 0.840$ m, $v_{xi} = (11.2\text{ m/s})\cos 18.5°$, $v_{yi} = (11.2\text{ m/s})\sin 18.5°$, and $g = 9.80\text{ m/s}^2$.

(a) Then the original position of the athlete's center of mass is the point with coordinates $(x_i,\ y_i) = \boxed{(0,\ 0.840\text{ m})}$. That is, his original position has position vector $\vec{\mathbf{r}} = 0\hat{\mathbf{i}} + 0.840\hat{\mathbf{j}}$ m.

(b) His original velocity is $\vec{\mathbf{v}}_i = (11.2\text{ m/s})(\cos 18.5°)\hat{\mathbf{i}} + (11.2\text{ m/s})(\sin 18.5°)\hat{\mathbf{j}} = \boxed{11.2\text{ m/s at }18.5°}$ above the x axis.

(c) From $(4.90\text{ m/s}^2)t^2 - (3.55\text{ m/s})t - 0.48\text{ m} = 0$, we find the time of flight, which must be positive. Suppressing units,

$$t = \frac{-(-3.55) + \sqrt{(-3.55)^2 - 4(4.90)(-0.48)}}{2(4.90)} = 0.841\text{ s}$$

Then $x_f = (11.2\text{ m/s})\cos 18.5°(0.841\text{ s}) = \boxed{8.94\text{ m}}$.

P4.27 Model the rock as a projectile, moving with constant horizontal velocity, zero initial vertical velocity, and with constant vertical acceleration. Note that the sound waves from the splash travel in a straight line to the soccer player's ears. The time of flight of the rock follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-40.0\text{ m} = 0 + 0 + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)t^2$$

$$t = 2.86\text{ s}$$

The extra time 3.00 s – 2.86 s = 0.140 s is the time required for the sound she hears to travel straight back to the player. It covers distance

$$(343\text{ m/s})\ 0.143\text{ s} = 49.0\text{ m} = \sqrt{x^2 + (40.0\text{ m})^2}$$

where x represents the horizontal distance the rock travels. Solving for x gives $x = 28.3$ m. Since the rock moves with constant speed in the x direction and travels horizontally during the 2.86 s that it is in flight,

$$x = 28.3\text{ m} = v_{xi}t + 0t^2$$

$$\therefore v_{xi} = \frac{28.3\text{ m}}{2.86\text{ s}} = \boxed{9.91\text{ m/s}}$$

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P4.28 The initial velocity components of the projectile are

$$x_i = 0 \quad \text{and} \quad y_i = h$$

$$v_{xi} = v_i \cos\theta \quad \text{and} \quad v_{yi} = v_i \sin\theta$$

while the constant acceleration components are

$$a_x = 0 \quad \text{and} \quad a_y = -g$$

The coordinates of the projectile are

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = (v_i \cos\theta)t \quad \text{and}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = h + (v_i \sin\theta)t - \frac{1}{2}gt^2$$

and the components of velocity are

$$v_{xf} = v_{xi} + a_x t = v_i \cos\theta \quad \text{and}$$

$$v_{yf} = v_{yi} + a_y t = v_i \sin\theta - gt$$

(a) We know that when the projectile reaches its maximum height, $v_{yf} = 0$:

$$v_{yf} = v_i \sin\theta - gt = 0 \rightarrow \boxed{t = \frac{v_i \sin\theta}{g}}$$

(b) At the maximum height, $y = h_{\text{max}}$:

$$h_{\text{max}} = h + (v_i \sin\theta)t - \frac{1}{2}gt^2$$

$$h_{\text{max}} = h + v_i \sin\theta \frac{v_i \sin\theta}{g} - \frac{1}{2}g\left(\frac{v_i \sin\theta}{g}\right)^2$$

$$\boxed{h_{\text{max}} = h + \frac{(v_i \sin\theta)^2}{2g}}$$

P4.29 (a) Initial coordinates: $\boxed{x_i = 0.00 \text{ m}, y_i = 0.00 \text{ m}}$

(b) Components of initial velocity: $\boxed{v_{xi} = 18.0 \text{ m/s}, \ v_{yi} = 0}$

(c) $\boxed{\text{Free fall motion, with constant downward acceleration } g = 9.80 \text{ m/s}^2.}$

(d) $\boxed{\text{Constant velocity motion in the horizontal direction.}}$ There is no horizontal acceleration from gravity.

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(e) $v_{xf} = v_{xi} + a_x t \quad \rightarrow \quad \boxed{v_{xf} = v_{xi}}$

$v_{yf} = v_{yi} + a_y t \quad \rightarrow \quad \boxed{v_{yf} = -gt}$

(f) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad \rightarrow \quad \boxed{x_f = v_{xi}t}$

$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \quad \rightarrow \quad \boxed{y_f = -\frac{1}{2}gt^2}$

(g) We find the time of impact:

$$y_f = -\frac{1}{2}gt^2$$

$$-h = -\frac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(50.0\text{ m})}{9.80\text{ m/s}^2}} = \boxed{3.19\text{ s}}$$

(h) At impact, $v_{xf} = v_{xi} = 18.0$ m/s, and the vertical component is

$$v_{yf} = -gt$$

$$= -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh} = -\sqrt{2(9.80\text{ m/s}^2)(50.0\text{ m})} = -31.3\text{ m/s}$$

Thus,

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(18.0\text{ m/s})^2 + (-31.3\text{ m/s})^2} = \boxed{36.1\text{ m/s}}$$

and

$$\theta_f = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-31.3}{18.0}\right) = \boxed{-60.1°}$$

which in this case means the velocity points into the fourth quadrant because its y component is negative.

P4.30 (a) When a projectile is launched with speed v_i at angle θ_i above the horizontal, the initial velocity components are $v_{xi} = v_i \cos\theta_i$ and $v_{yi} = v_i \sin\theta_i$. Neglecting air resistance, the vertical velocity when the projectile returns to the level from which it was launched (in this case, the ground) will be $v_y = -v_{yi}$. From this information, the total time of flight is found from $v_{yf} = v_{yi} + a_y t$ to be

$$t_{\text{total}} = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-v_{yi} - v_{yi}}{-g} = \frac{2v_{yi}}{g} \quad \text{or} \quad t_{\text{total}} = \frac{2v_i \sin\theta_i}{g}$$

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Since the horizontal velocity of a projectile with no air resistance is constant, the horizontal distance it will travel in this time (i.e., its range) is given by

$$R = v_{xi}t_{\text{total}} = (v_i\cos\theta_i)\left(\frac{2v_i\sin\theta_i}{g}\right) = \frac{v_i^2}{g}(2\sin\theta_i\cos\theta_i)$$
$$= \frac{v_i^2\sin(2\theta_i)}{g}$$

Thus, if the projectile is to have a range of $R = 81.1$ m when launched at an angle of $\theta_i = 45.0°$, the required initial speed is

$$v_i = \sqrt{\frac{Rg}{\sin(2\theta_i)}} = \sqrt{\frac{(81.1\text{ m})(9.80\text{ m/s}^2)}{\sin(90.0°)}} = \boxed{28.2\text{ m/s}}$$

(b) With $v_i = 28.2$ m/s and $\theta_i = 45.0°$ the total time of flight (as found above) will be

$$t_{\text{total}} = \frac{2v_i\sin\theta_i}{g} = \frac{2(28.2\text{ m/s})\sin(45.0°)}{9.80\text{ m/s}^2} = \boxed{4.07\text{ s}}$$

(c) Note that at $\theta_i = 45.0°$, and that sin $(2\theta_i)$ will decrease as θ_i is increased above this optimum launch angle. Thus, if the range is to be kept constant while the launch angle is increased above 45.0°, we see from $v_i = \sqrt{Rg/\sin(2\theta_i)}$ that

$\boxed{\text{the required initial velocity will increase}}$.

Observe that for $\theta_i < 90°$, the function $\sin\theta_i$ increases as θ_i is increased. Thus, increasing the launch angle above 45.0° while keeping the range constant means that both v_i and $\sin\theta_i$ will increase. Considering the expression for t_{total} given above, we see that $\boxed{\text{the total time of flight will increase}}$.

P4.31 We first consider the vertical motion of the stone as it falls toward the water. The initial y velocity component of the stone is

$$v_{yi} = v_i\sin\theta = -(4.00\text{ m/s})\sin 60.0° = -3.46\text{ m/s}$$

and its y coordinate is

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2 = h + (v_i\sin\theta)t - \frac{1}{2}gt^2$$
$$y_f = 2.50 - 3.46t - 4.90t^2$$

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where y is in m and t in s. We have taken the water's surface to be at $y = 0$. At the water,

$$4.90t^2 + 3.46t - 2.50 = 0$$

Solving for the positive root of the equation, we get

$$t = \frac{-3.46 + \sqrt{(3.46)^2 - 4(4.90)(-2.50)}}{2(4.90)}$$
$$t = \frac{-3.46 + 7.81}{9.80}$$
$$t = 0.443 \text{ s}$$

The y component of velocity of the stone when it reaches the water at this time t is

$$v_{yf} = v_{yi} + a_y t = -3.46 - gt = -7.81 \text{ m/s}$$

After the stone enters to water, its speed, and therefore the magnitude of each velocity component, is reduced by one-half. Thus, the y component of the velocity of the stone in the water is

$$v_{yi} = (-7.81 \text{ m/s})/2 = -3.91 \text{ m/s},$$

and this component remains constant until the stone reaches the bottom. As the stone moves through the water, its y coordinate is

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_f = -3.91t$$

The stone reaches the bottom of the pool when $y_f = -3.00$ m:

$$y_f = -3.91t = -3.00 \rightarrow t = 0.767 \text{ s}$$

The total time interval the stone takes to reach the bottom of the pool is

$$\Delta t = 0.443 \text{ s} + 0.767 \text{ s} = \boxed{1.21 \text{ s}}$$

***P4.32** (a) The time for the ball to reach the fence is

$$t = \frac{\Delta x}{v_{xi}} = \frac{130 \text{ m}}{v_i \cos 35.0°} = \frac{159 \text{ m}}{v_i}$$

At this time, the ball must be $\Delta y = 21.0 \text{ m} - 1.00 \text{ m} = 20.0 \text{ m}$ above its launch position, so

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$

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gives

$$20.0 \text{ m} = (v_i \sin 35.0°)\left(\frac{159 \text{ m}}{v_i}\right) - \frac{1}{2}(9.80 \text{ m/s}^2)\left(\frac{159 \text{ m}}{v_i}\right)^2$$

or

$$(159 \text{ m})\sin 35.0° - 20.0 \text{ m} = \frac{(4.90 \text{ m/s}^2)(159 \text{ m})^2}{v_i^2}$$

from which we obtain

$$v_i = \sqrt{\frac{(4.90 \text{ m/s}^2)(159 \text{ m})^2}{(159 \text{ m})\sin 35.0° - 20.0 \text{ m}}} = \boxed{41.7 \text{ m/s}}$$

(b) From our equation for the time of flight above,

$$t = \frac{159 \text{ m}}{v_i} = \frac{159 \text{ m}}{41.7 \text{ m/s}} = \boxed{3.81 \text{ s}}$$

(c) When the ball reaches the wall (at t = 3.81 s),

$$v_x = v_i \cos 35.0° = (41.7 \text{ m/s})\cos 35.0° = \boxed{34.1 \text{ m/s}}$$

$$\begin{aligned} v_y &= v_i \sin 35.0° + a_y t \\ &= (41.7 \text{ m/s})\sin 35.0° - (9.80 \text{ m/s}^2)(3.81 \text{ s}) \\ &= \boxed{-13.4 \text{ m/s}} \end{aligned}$$

and $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(34.1 \text{ m/s})^2 + (-13.4 \text{ m/s})^2} = \boxed{36.7 \text{ m/s}}$

Section 4.4 Analysis Model: Particle in Uniform Circular Motion

P4.33 Model the discus as a particle in uniform circular motion. We evaluate its centripetal acceleration from the standard equation proved in the text.

$$a_c = \frac{v^2}{r} = \frac{(20.0 \text{ m/s})^2}{1.06 \text{ m}} = \boxed{377 \text{ m/s}^2}$$

The mass is unnecessary information.

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P4.34 Centripetal acceleration is given by $a = \frac{v^2}{R}$. To find the velocity of a point at the equator, we note that this point travels through $2\pi R_E$ (where $R_E = 6.37 \times 10^6$ m is Earth's radius) in 24.0 hours. Then,

$$v = \frac{2\pi R_E}{T} = \frac{2\pi(6.37 \times 10^6 \text{ m})}{(24 \text{ h})(3\,600 \text{ s/h})} = 463 \text{ m/s}$$

and,

$$a = \frac{v^2}{R} = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}}$$

$$= \boxed{0.033\,7 \text{ m/s}^2 \text{ directed toward the center of Earth}}$$

***P4.35** Centripetal acceleration is given by $a_c = \frac{v^2}{r}$. Let *f* represent the rotation rate. Each revolution carries each bit of metal through distance $2\pi r$, so $v = 2\pi r f$ and

$$a_c = \frac{v^2}{r} = 4\pi^2 r f^2 = 100g$$

A smaller radius implies smaller acceleration. To meet the criterion for each bit of metal we consider the minimum radius:

$$f = \left(\frac{100g}{4\pi^2 r}\right)^{1/2} = \left(\frac{100 \cdot 9.8 \text{ m/s}^2}{4\pi^2 (0.021 \text{ m})}\right)^{1/2}$$

$$= 34.4 \ \frac{1}{\text{s}}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.06 \times 10^3 \text{ rev/min}}$$

***P4.36** The radius of the tire is $r = 0.500$ m. The speed of the stone on its outer edge is

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500 \text{ m})}{(60.0 \text{ s}/200 \text{ rev})} = \boxed{10.5 \text{ m/s}}$$

and its acceleration is

$$a = \frac{v^2}{R} = \frac{(10.5 \text{ m/s})^2}{0.500 \text{ m}} = \boxed{219 \text{ m/s}^2 \text{ inward}}$$

P4.37 Centripetal acceleration is $a_c = \frac{v^2}{r} \rightarrow v = \sqrt{a_c r}$, where $a_c = 20.0g$, and speed v is in meters per second if r is in meters.

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We can convert the speed into a rotation rate, in rev/min, by using the relations 1 revolution = $2\pi r$, and 1 min = 60 s:

$$v = \sqrt{a_c r} = \sqrt{a_c r}\left(\frac{1\ \text{rev}}{2\pi r}\right) = \frac{1\ \text{rev}}{2\pi}\sqrt{\frac{a_c}{r}}$$

$$= \frac{1\ \text{rev}}{2\pi}\sqrt{\frac{20.0\left(9.80\ \text{m/s}^2\right)}{29.0\ \text{ft}}\left(\frac{3.281\ \text{ft}}{1\ \text{m}}\right)}\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

$$= \boxed{45.0\ \text{rev/min}}$$

P4.38 (a) Using the definition of speed and noting that the ball travels in a circular path,

$$v = \frac{d}{\Delta t} = \frac{2\pi R}{T}$$

where R is the radius of the circle and T is the period, that is, the time interval required for the ball to go around once. For the periods given in the problem,

$$8.00\ \text{rev/s} \rightarrow T = \frac{1}{8.00\ \text{rev/s}} = 0.125\ \text{s}$$

$$6.00\ \text{rev/s} \rightarrow T = \frac{1}{6.00\ \text{rev/s}} = 0.167\ \text{s}$$

Therefore, the speeds in the two cases are:

$$8.00\ \text{rev/s} \rightarrow v = \frac{2\pi(0.600\ \text{m})}{0.125\ \text{s}} = 30.2\,\text{m/s}$$

$$6.00\ \text{rev/s} \rightarrow v = \frac{2\pi(0.900\ \text{m})}{0.167\ \text{s}} = 33.9\,\text{m/s}$$

Therefore, $\boxed{6.00\ \text{rev/s}}$ gives the greater speed of the ball.

(b) Acceleration $= \dfrac{v^2}{r} = \dfrac{(9.60\pi\ \text{m/s})^2}{0.600\ \text{m}} = \boxed{1.52 \times 10^3\ \text{m/s}^2}$.

(c) At 6.00 rev/s, acceleration $= \dfrac{(10.8\pi\ \text{m/s})^2}{0.900\ \text{m}} = \boxed{1.28 \times 10^3\ \text{m/s}^2}$. So 8 rev/s gives the higher acceleration.

***P4.39** The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration: $a_c = g$. So

$$\frac{v^2}{r} = g$$

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Solving for the velocity,

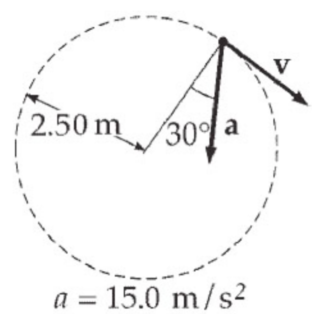
$$v=\sqrt{rg}=\sqrt{(6{,}400+600)(10^3\ \text{m})(8.21\ \text{m/s}^2)}$$
$$=\boxed{7.58\times10^3\ \text{m/s}}$$
$$v=\frac{2\pi r}{T}$$

and

$$T=\frac{2\pi r}{v}=\frac{2\pi(7{,}000\times10^3\ \text{m})}{7.58\times10^3\ \text{m/s}}=\boxed{5.80\times10^3\ \text{s}}$$
$$T=5.80\times10^3\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right)=96.7\ \text{min}$$

Section 4.5 Tangential and Radial Acceleration

P4.40 From the given magnitude and direction of the acceleration we can find both the centripetal and the tangential components. From the centripetal acceleration and radius we can find the speed in part (b). r = 2.50 m, a = 15.0 m/s^2.



ANS. FIG. P4.40

(a) The acceleration has an inward radial component:

$$a_c=a\cos 30.0°=(15.0\ \text{m/s}^2)(\cos 30°)$$
$$=\boxed{13.0\ \text{m/s}^2}$$

(b) The speed at the instant shown can be found by using

$$a_c=\frac{v^2}{r}$$
$$v^2=ra_c=2.50\ \text{m}(13.0\ \text{m/s})$$
$$=32.5\ \text{m}^2/\text{s}^2$$
$$v=\sqrt{32.5}\ \text{m/s}=\boxed{5.70\ \text{m/s}}$$

(c) $a^2=a_t^2+a_r^2$

so $a_t=\sqrt{a^2-a_r^2}=\sqrt{(15.0\ \text{m/s}^2)^2-(13.0\ \text{m/s}^2)^2}=\boxed{7.50\ \text{m/s}^2}$

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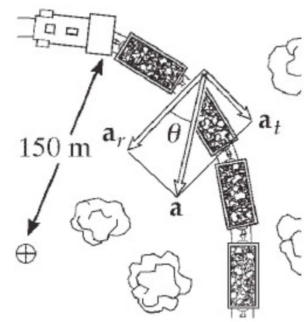
P4.41 Since the train is changing both its speed and direction, the acceleration vector will be the vector sum of the tangential and radial acceleration components. The tangential acceleration can be found from the changing speed and elapsed time, while the radial acceleration can be found from the radius of curvature and the train's speed.



ANS. FIG. P4.41

First, let's convert the speed units from km/h to m/s:

$$v_i = 90.0 \text{ km/h} = (90.0 \text{ km/h})(10^3 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 25.0 \text{ m/s}$$

$$v_f = 50.0 \text{ km/h} = (50.0 \text{ km/h})(10^3 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 13.9 \text{ m/s}$$

The tangential acceleration and radial acceleration are, respectively,

$$a_t = \frac{\Delta v}{\Delta t} = \frac{13.9 \text{ m/s} - 25.0 \text{ m/s}}{15.0 \text{ s}} = -0.741 \text{ m/s}^2 \quad \text{(backward)}$$

and $$a_r = \frac{v^2}{r} = \frac{(13.9 \text{ m/s})^2}{150 \text{ m}} = 1.29 \text{ m/s} \quad \text{(inward)}$$

so $$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(1.29 \text{ m/s}^2\right)^2 + \left(-0.741 \text{ m/s}^2\right)^2} = 1.48 \text{ m/s}^2$$

at an angle of

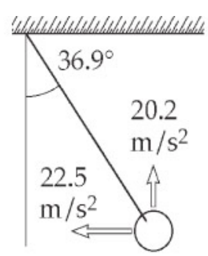
$$\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741 \text{ m/s}^2}{1.29 \text{ m/s}^2}\right) = 29.9°$$

therefore, $\vec{\mathbf{a}} = \boxed{1.48 \text{ m/s}^2 \text{ inward and } 29.9° \text{ backward}}$

P4.42 (a) See ANS. FIG. P4.42.

(b) The components of the 20.2 m/s^2 and the 22.5 m/s^2 accelerations along the rope together constitute the centripetal acceleration:

$$a_c = \left(22.5 \text{ m/s}^2\right) \cos(90.0° - 36.9°) + \left(20.2 \text{ m/s}^2\right) \cos 36.9° = \boxed{29.7 \text{ m/s}^2}$$



ANS. FIG. P4.42

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67 \text{ m/s}$ tangent to the circle.

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P4.43 The particle's centripetal acceleration is $v^2/r = (3\text{ m/s})^2/2\text{ m} = 4.50\text{ m/s}^2$. The total acceleration magnitude can be larger than or equal to this, but not smaller.

(a) Yes. The particle can be either speeding up or slowing down, with a tangential component of acceleration of magnitude $\sqrt{6^2 - 4.5^2} = 3.97\text{ m/s}$.

(b) No. The magnitude of the acceleration cannot be less than $v^2/r = 4.5\text{ m/s}^2$.

Section 4.6 Relative Velocity and Relative Acceleration

***P4.44** The westward speed of the airplane is the horizontal component of its velocity vector, and the northward speed of the wind is the vertical component of its velocity vector, which has magnitude and direction given by

$$v = \sqrt{(150\text{ km/h})^2 + (30.0\text{ km/h})^2} = \boxed{153\text{ km/h}}$$

$$\theta = \tan^{-1}\left(\frac{30.0\text{ km/h}}{150\text{ km/h}}\right) = \boxed{11.3° \text{ north of west}}$$

P4.45 The airplane (AP) travels through the air (W) that can move relative to the ground (G). The airplane is to make a displacement of 750 km north. Treat north as positive y and west as positive x.

(a) The wind (W) is blowing at 35.0 km/h, south. The northern component of the airplane's velocity relative to the ground is

$$(v_{\text{AP,G}})_y = (v_{\text{AP,W}})_y + (v_{\text{W,G}})_y = 630\text{ km/h} - 35.0\text{ km/h} = 595\text{ km/h}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\Delta y = \left(v_{\text{AP,G}}\right)_y \Delta t \rightarrow \quad \Delta t = \frac{\Delta y}{\left(v_{\text{AP,G}}\right)_y} = \frac{750\text{ km}}{595\text{ km/h}} = 1.26\text{ h}}$$

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(b) The wind (W) is blowing at 35.0 km/h, north. The northern component of the airplane's velocity relative to the ground is

$$(v_{\text{AP,G}})_y = (v_{\text{AP,W}})_y + (v_{\text{W,G}})_y = 630\ \text{km/h} + 35.0\ \text{km/h}$$
$$= 665\ \text{km/h}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\Delta t = \frac{\Delta y}{\left(v_{\text{AP,G}}\right)_y} = \frac{750\ \text{km}}{665\ \text{km/h}} = 1.13\ \text{h}}$$

(c) Now, the wind (W) is blowing at 35.0 km/h, east. The airplane must travel directly north to reach its destination, so it must head somewhat west and north so that the east component of the wind's velocity is cancelled by the airplane's west component of velocity. If the airplane heads at an angle θ measured west of north, then

$$(v_{\text{AP,G}})_x = (v_{\text{AP,W}})_x + (v_{\text{W,G}})_x$$
$$= (630\ \text{km/h})\sin\theta + (-35.0\ \text{km/h}) = 0$$

$$\sin\theta = 35.0/630 \rightarrow \theta = 3.18°$$

The northern component of the airplane's velocity relative to the ground is

$$(v_{\text{AP,G}})_y = (v_{\text{AP,W}})_y + (v_{\text{W,G}})_y = (630\ \text{km/h})\cos 3.18° + 0$$
$$= 629\ \text{km/h}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\Delta t = \frac{\Delta y}{\left(v_{\text{AP,G}}\right)_y} = \frac{750\ \text{km}}{629\ \text{km/h}} = 1.19\ \text{h}}$$

P4.46 Consider the direction the first beltway (B1) moves to be the positive direction. The first beltway moves relative to the ground (G) with velocity $v_{\text{B1,G}} = v_1$.

(a) The woman's velocity relative to the ground is $v_{\text{WG}} = v_{\text{W,B1}} + v_{\text{B1,G}} = v_1 + 0 = v_1$. The time interval required for the woman to travel distance L relative to the ground is

$$\boxed{\Delta t_{\text{woman}} = \frac{L}{v_1}}$$

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(b) The man's (M) velocity relative to the ground is $v_{MG} = v_{M,B1} + v_{B1,G} = v_2 + v_1$. The time interval required for the man to travel distance L relative to the ground is

$$\boxed{\Delta t_{\text{man}} = \frac{L}{v_1 + v_2}}$$

(c) The second beltway (B2) moves in the negative direction; its velocity is $v_{B2,G} = -v_1$, and the child (C) rides on the second beltway; his velocity relative to the ground is

$$v_{CG} = v_{C,B2} + v_{B2,G} = 0 - v_1 = -v_1$$

The man's velocity relative to the child is

$$v_{MC} = v_{M,B1} + v_{B1,G} + v_{G,B2} + v_{B2,C}$$
$$v_{MC} = v_{M,B1} + v_{B1,G} - v_{B2,G} - v_{C,B2}$$
$$v_{MC} = v_2 + v_1 - (-v_1) + 0 = v_1 + 2\,v_2$$

so, the time interval required for the man to travel distance L relative to the child is

$$\boxed{\Delta t_{\text{man}} = \frac{L}{v_1 + 2v_2}}$$

P4.47 Both police car (P) and motorist (M) move relative to the ground (G). Treating west as the positive direction, the components of their velocities (in km/h) are:

$$v_{PG} = 95.0 \text{ km/h (west)} \qquad v_{PG} = 80 \text{ km/h (west)}$$

(a) $v_{MP} = v_{MG} + v_{GP} = v_{MG} - v_{PG} = 80.0 \text{ km/h} - 95.0 \text{ km/h} = -15.0$
$= \boxed{15.0 \text{ km/h, east}}$

(b) $v_{PM} = -v_{MP} + \boxed{15.0 \text{ km/h, west}}$

(c) Relative to the motorist, the police car approaches at 15.0 km/h:

$$d = v\Delta t$$

$$\rightarrow \Delta t = \frac{d}{v} = \frac{0.250 \text{ km}}{15.0 \text{ km/h}} = \left(1.67 \times 10^{-2} \text{ h}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{60.0 \text{ s}}$$

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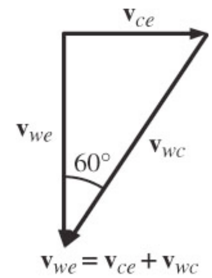
We define the following velocity vectors:

$\vec{\mathbf{v}}_{ce}$ = the velocity of the car relative to the Earth

$\vec{\mathbf{v}}_{wc}$ = the velocity of the water relative to the car

$\vec{\mathbf{v}}_{we}$ = the velocity of the water relative to the Earth



ANS. FIG. P4.48

These velocities are related as shown in ANS. FIG. P4.48

(a) Since $\vec{\mathbf{v}}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0$ km/h or

$$\vec{\mathbf{v}}_{wc} = \boxed{57.7 \text{ km/h at } 60.0° \text{ west of vertical}}$$

(b) Since $\vec{\mathbf{v}}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7 \text{ km/h})\cos 60.0°$$
$$= \boxed{28.9 \text{ km/h downward}}$$

P4.49 (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a = \sqrt{(2.50 \text{ m/s})^2 + (9.80 \text{ m/s})^2} = \boxed{10.1 \text{ m/s}^2}$$

$$\tan\theta = \frac{2.50 \text{ m/s}^2}{9.80 \text{ m/s}^2} = 0.255$$

$$\theta = \boxed{14.3° \text{ to the south from the vertical}}$$

To this observer, the bolt moves as if it were in a gravitational field of 9.80 m/s^2 down + 2.50 m/s^2 south.

(b) $a = \boxed{9.80 \text{ m/s}^2 \text{ vertically downward}}$

(c) If it is at rest relative to the ceiling at release, the bolt moves on a straight line download and southward at 14.3 degrees from the vertical.

(d) The bolt moves on a parabola with a vertical axis.

P4.50 The total time interval in the river is the longer time spent swimming upstream (against the current) plus the shorter time swimming downstream (with the current). For each part, we will use the basic equation $t = d/v$, where v is the speed of the student relative to the shore.

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(a) Total time = time upstream plus time downstream:

$$t_{up} = \frac{1\,000 \text{ m}}{1.20 \text{ m/s} - 0.500 \text{ m/s}} = 1.43 \times 10^3 \text{ s}$$

$$t_{down} = \frac{1\,000 \text{ m}}{1.20 \text{ m/s} + 0.500 \text{ m/s}} = 588 \text{ s}$$

Therefore, $t_{total} = 1.43 \times 10^3 \text{ s} + 588 \text{ s} = \boxed{2.02 \times 10^3 \text{ s}}$.

(b) Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}$.

(c) Swimming with the current does not compensate for the time lost swimming against the current.

P4.51 The student must swim faster than the current to travel upstream.

(a) The speed of the student relative to shore is $v_{up} = c - v$ while swimming upstream (against the current), and $v_{down} = c + v$ while swimming downstream (with the current).

Note, The student must swim faster than the current to travel upstream. The time interval required to travel distance d upstream is then

$$\Delta t_{up} = \frac{d}{v_{up}} = \frac{d}{c - v}$$

and the time interval required to swim the same distance d downstream is

$$\Delta t_{down} = \frac{d}{v_{down}} = \frac{d}{c + v}$$

The time interval for the round trip is therefore

$$\Delta t = \Delta t_{up} + \Delta t_{down} = \frac{d}{c - v} + \frac{d}{c + v} = d\frac{(c + v) + (c - v)}{(c - v)(c + v)}$$

$$\boxed{\Delta t = \frac{2dc}{c^2 - v^2}}$$

(b) In still water, $v = 0$, so $v_{up} = v_{down} = c$; the equation for the time interval for the complete trip reduces to

$$\boxed{\Delta t = \frac{2d}{c}}$$

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(c) The equation for the time interval for the complete trip can be written as

$$\Delta t = \frac{2dc}{c^2 - v^2} = \frac{2d}{c\left(1 - \frac{v^2}{c^2}\right)}$$

Because the denominator is always smaller than c, $\boxed{\text{swimming with and against the current is always longer than in still water}}$.

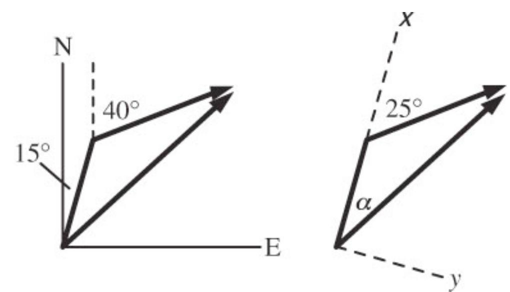
P4.52 Choose the x axis along the 20-km distance. The y components of the displacements of the ship and the speedboat must agree:

$$(26 \text{ km/h})t\sin(40.0° - 15.0°) = (50 \text{ km/h})t\sin\alpha$$

$$\alpha = \sin^{-1}\left(\frac{11.0 \text{ km/h}}{50 \text{ km/h}}\right) = 12.7°$$

The speedboat should head

$$15.0° + 12.7° = \boxed{27.7° \text{ E of N}}$$



ANS. FIG. P4.52

P4.53 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

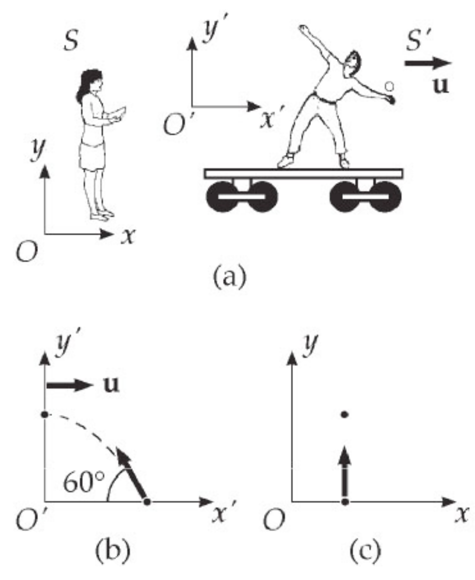
$$\frac{v'_y}{v'_x} = \tan 60.0° = \sqrt{3}$$

Let u represent the speed of S' relative to S. Then because there is no x motion in S, we can write $v_x = v'_x + u = 0$ so that $v'_x = -u = -10.0$ m/s. Hence the ball is thrown backwards in S'. Then,

$$v'_y = v'_y = \sqrt{3}\left|v'_x\right| = 10.0\sqrt{3} \text{ m/s}$$

Using $v_y^2 = 2gh$ we find

$$h = \frac{(10.0\sqrt{3} \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{15.3 \text{ m}}$$



ANS. FIG. P4.53

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The motion of the ball as seen by the student in S' is shown in ANS. FIG. P4.53(b). The view of the professor in S is shown in ANS. FIG. P4.53(c).

P4.54 (a) For the boy to catch the can at the same location on the truck bed, he must throw it $\boxed{\text{straight up, at } 0° \text{ to the vertical}}$.

(b) We find the time of flight of the can by considering its horizontal motion:

$$16.0 \text{ m} = (9.50 \text{ m/s})t + 0 \rightarrow t = 1.68 \text{ s}$$

For the free fall of the can, $y_f = y_i + v_{yi}t - \frac{1}{2}a_y t^2$:

$$0 = 0 + v_{yi}(1.68 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.68 \text{ s})^2$$

which gives $v_{yi} = \boxed{8.25 \text{ m/s}}$.

(c) The boy sees the can always over his head, traveling in $\boxed{\text{a straight up and down line}}$.

(d) The ground observer sees the can move as a projectile traveling in $\boxed{\text{a symmetric parabola opening downward}}$.

(e) Its initial velocity is

$$\sqrt{(9.50 \text{ m/s})^2 + (8.25 \text{ m/s})^2} = \boxed{12.6 \text{ m/s north}}$$

at an angle of

$$\tan^{-1}\left(\frac{8.25 \text{ m/s}}{9.50 \text{ m/s}}\right) = \boxed{41.0° \text{ above the horizontal}}$$

Additional Problems

***P4.55** After the string breaks the ball is a projectile, and reaches the ground at time t:

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2$$

$$-1.20 \text{ m} = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

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so t = 0.495 s. Its constant horizontal speed is

$$v_x = \frac{x}{t} = \frac{2.00 \text{ m}}{0.495 \text{ s}} = 4.04 \text{ m/s}$$

so before the string breaks

$$a_c = \frac{v_x^2}{r} = \frac{(4.04 \text{ m/s})^2}{0.300 \text{ m}} = \boxed{54.4 \text{ m/s}^2}$$

***P4.56** The maximum height of the ball is given by Equation 4.12:

$$h = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

Equation 4.13 then gives the horizontal range of the ball:

$$R = \frac{v_i^2 \sin 2\theta_i}{g} = \frac{2v_i^2 \sin \theta_i \cos \theta_i}{g}$$

If $h = \dfrac{R}{6}$, Equation 4.12 yields

$$v_i \sin \theta_i = \sqrt{\frac{gR}{3}} \qquad \textbf{[1]}$$

Substituting equation [1] above into Equation 4.13 gives

$$R = \frac{2\left(\sqrt{gR/3}\right) v_i \cos \theta_i}{g}$$

which reduces to

$$v_i \cos \theta_i = \frac{1}{2}\sqrt{3gR} \qquad \textbf{[2]}$$

(a) From $v_{yf} = v_{yi} + a_y t$, the time to reach the peak of the path $\left(\text{where } v_{yf} = 0\right)$ is found to be

$$t_{peak} = v_i \sin \theta_i / g$$

Using equation [1], this gives

$$t_{peak} = \sqrt{\frac{R}{3g}}$$

The total time of the ball's flight is then

$$\boxed{t_{flight} = 2t_{peak} = 2\sqrt{\frac{R}{3g}}}$$

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(b) At the path's peak, the ball moves horizontally with speed

$$v_{peak} = v_{xi} = v_i \cos\theta_i$$

Using equation [2], this becomes

$$v_{peak} = \boxed{\frac{1}{2}\sqrt{3gR}}$$

(c) The initial vertical component of velocity is $v_{yi} = v_i \sin\theta_i$. From equation [1],

$$v_{yi} = \boxed{\sqrt{\frac{gR}{3}}}$$

(d) Squaring equations [1] and [2] and adding the results,

$$v_i^2\left(\sin^2\theta_i + \cos^2\theta_i\right) = \frac{gR}{3} + \frac{3gR}{4} = \frac{13gR}{12}$$

Thus, the initial speed is

$$v_i = \boxed{\sqrt{\frac{13gR}{12}}}$$

(e) Dividing equation [1] by [2] yields

$$\tan\theta_i = \frac{v_i \sin\theta_i}{v_i \cos\theta_i} = \left[\frac{\left(\sqrt{gR/3}\right)}{\left(\frac{1}{2}\sqrt{3gR}\right)}\right] = \frac{2}{3}$$

Therefore,

$$\theta_i = \tan^{-1}\left(\frac{2}{3}\right) = \boxed{33.7^\circ}$$

(f) For a given initial speed, the projection angle yielding maximum peak height is $\theta_i = 90.0^\circ$. With the speed found in (d), Equation 4.12 then yields

$$h_{\max} = \frac{(13gR/12)\sin^2 90.0^\circ}{2g} = \boxed{\frac{13}{24}R}$$

(g) For a given initial speed, the projection angle yielding maximum range is $\theta_i = 45.0^\circ$. With the speed found in (d), Equation 4.13 then gives

$$R_{\max} = \frac{(13gR/12)\sin 90.0^\circ}{g} = \boxed{\frac{13}{12}R}$$

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P4.57 We choose positive y to be in the downward direction. The ball when released has velocity components $v_{xi} = v$ and $v_{yi} = 0$, where v is the speed of the man. We can find the length of the time interval the ball takes to fall the distance h using

$$\Delta y = \frac{1}{2}a_y(\Delta t)^2 = \frac{1}{2}g(\Delta t)^2 \to \Delta t = \sqrt{\frac{2h}{g}}$$

The horizontal displacement of the ball during this time interval is

$$\Delta x = v_{xi}\ \Delta t = v\sqrt{\frac{2h}{g}} = 7.00h$$

Solve for the speed:

$$v = \sqrt{\frac{49.0gh}{2}} = \sqrt{\frac{49.0(9.80\text{ m/s}^2)h}{2}} = 15.5\sqrt{h}$$

where h is in m and v in m/s.

If we express the height as a function of speed, we have

$$h = (4.16\times 10^{-2})v^2$$

where h is in m and v is in m/s.

For a normally proportioned adult, h is about 0.50 m, which would mean that $v = 15.5\sqrt{0.50} = 11$ m/s, which is about 39 km/h; no normal adult could walk "briskly" at that speed. If the speed were a realistic typical speed of 4 km/h, from our equation for h, we find that the height would be about 4 cm, much too low for a normal adult.

P4.58 (a) From $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$, we have

$$\int_i^f d\vec{\mathbf{v}} = \int_i^f \vec{\mathbf{a}}\,dt = \Delta\vec{\mathbf{v}}$$

Then

$$\vec{\mathbf{v}} - 5\hat{\mathbf{i}}\text{ m/s} = \int_0^t 6\,t^{1/2}dt\,\hat{\mathbf{j}} = 6\left.\frac{t^{3/2}}{3/2}\right|_0^t\hat{\mathbf{j}} = 4\,t^{3/2}\,\hat{\mathbf{j}}\text{ m/s}$$

so $\vec{\mathbf{v}} = \boxed{\left(5\,\hat{\mathbf{i}} + 4\,t^{3/2}\,\hat{\mathbf{j}}\right)\text{ m/s}}$.

(b) From $\vec{\mathbf{v}} = d\vec{\mathbf{r}}/dt$, we have

$$\int_i^f d\vec{\mathbf{r}} = \int_i^f \vec{\mathbf{v}}\,dt = \Delta\vec{\mathbf{r}}$$

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Then

$$\vec{\mathbf{r}} - 0 = \int_0^t \left(5\,\hat{\mathbf{i}} + 4\,t^{3/2}\,\hat{\mathbf{j}}\right)dt = \left(5t\,\hat{\mathbf{i}} + 4\frac{t^{5/2}}{5/2}\hat{\mathbf{j}}\right)\Bigg|_0^t$$

$$= \boxed{\left(5t\,\hat{\mathbf{i}} + 1.6\,t^{5/2}\,\hat{\mathbf{j}}\right)\text{ m}}$$

P4.59 (a) The speed at the top is

$$v_x = v_i \cos\theta_i = (143\text{ m/s})\cos 45° = \boxed{101\text{ m/s}}$$

(b) In free fall the plane reaches altitude given by

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = \left(143\text{ m/s }\sin 45°\right)^2 + 2\left(-9.80\text{ m/s}^2\right)\left(y_f - 31\ 000\text{ ft}\right)$$

$$y_f = 31\ 000\text{ ft } + 522\text{ m}\left(\frac{3.28\text{ ft}}{1\text{ m}}\right) = \boxed{3.27 \times 10^4\text{ ft}}$$

(c) For the whole free-fall motion $v_{yf} = v_{yi} + a_y t$:

$$-101\text{ m/s} = +101\text{ m/s} - \left(9.80\text{ m/s}^2\right)t$$

$$t = \boxed{20.6\text{ s}}$$

P4.60 (a) The acceleration is that of gravity: $\boxed{9.80\text{ m/s}^2\text{, downward.}}$

(b) The horizontal component of the initial velocity is $v_{xi} = v_i \cos 40.0°$ $= 0.766v_i$, and the time required for the ball to move 10.0 m horizontally is

$$t = \frac{\Delta x}{v_{xi}} = \frac{10.0\text{ m}}{0.766\ v_i} = \frac{13.1\text{ m}}{v_i}$$

At this time, the vertical displacement of the ball must be

$$\Delta y = y_f - y_i = 3.05\text{ m} - 2.00\text{ m} = 1.05\text{ m}$$

Thus, $\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$ becomes

$$1.05\text{ m} = \left(\cancel{v_i}\ \sin 40.0°\right)\frac{13.1\text{ m}}{\cancel{v_i}} + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\frac{(13.1\text{ m})^2}{v_i^2}$$

or $\quad 1.05\text{ m} = 8.39\text{ m} - \dfrac{835\text{ m}^3/\text{s}^2}{v_i^2}$

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which yields

$$v_i = \sqrt{\frac{835\ \text{m}^3/\text{s}^2}{8.39\ \text{m} - 1.05\ \text{m}}} = \boxed{10.7\ \text{m/s}}$$

P4.61 Both Lisa and Jill start from rest. Their accelerations are

$$\vec{\mathbf{a}}_\text{L} = (3.00\,\hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}})\ \text{m/s}^2$$

$$\vec{\mathbf{a}}_\text{J} = (1.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})\ \text{m/s}^2$$

Integrating these, and knowing that they start from rest, we find their velocities:

$$\vec{\mathbf{v}}_\text{L} = (3.00t\,\hat{\mathbf{i}} - 2.00t\,\hat{\mathbf{j}})\ \text{m/s}$$

$$\vec{\mathbf{v}}_\text{J} = (1.00t\,\hat{\mathbf{i}} + 3.00t\,\hat{\mathbf{j}})\ \text{m/s}$$

Integrating again, and knowing that they start from the origin, we find their positions:

$$\vec{\mathbf{r}}_\text{L} = (1.50t^2\,\hat{\mathbf{i}} - 1.00t^2\,\hat{\mathbf{j}})\ \text{m}$$

$$\vec{\mathbf{r}}_\text{J} = (0.50\,t^2\,\hat{\mathbf{i}} + 1.50t^2\,\hat{\mathbf{j}})\ \text{m}$$

All of the above are with respect to the ground (G).

(a) In general, Lisa's velocity with respect to Jill is

$$\vec{\mathbf{v}}_\text{LJ} = \vec{\mathbf{v}}_\text{LG} + \vec{\mathbf{v}}_\text{GJ} = \vec{\mathbf{v}}_\text{LG} - \vec{\mathbf{v}}_\text{JG}$$
$$\vec{\mathbf{v}}_\text{LJ} = \vec{\mathbf{v}}_\text{L} - \vec{\mathbf{v}}_\text{J} = (3.00t\,\hat{\mathbf{i}} - 2.00t\,\hat{\mathbf{j}}) - (1.00t\,\hat{\mathbf{i}} + 3.00t\,\hat{\mathbf{j}})$$
$$\vec{\mathbf{v}}_\text{LJ} = (2.00t\,\hat{\mathbf{i}} - 5.00t\,\hat{\mathbf{j}})$$

When $t = 5.00\ \text{s}, \vec{\mathbf{v}}_\text{LJ} = (10.0\,\hat{\mathbf{i}} - 25.0\,\hat{\mathbf{j}})\ \text{m/s},$ so the speed (magnitude) is

$$\boxed{v = \sqrt{(10.0)^2 + (25.0)^2} = 26.9\ \text{m/s}}$$

(b) In general, Lisa's position with respect to Jill is

$$\vec{\mathbf{r}}_\text{LJ} = \vec{\mathbf{r}}_\text{L} - \vec{\mathbf{r}}_\text{J} = (1.50t^2\,\hat{\mathbf{i}} - 1.00t^2\,\hat{\mathbf{j}}) - (0.50t^2\,\hat{\mathbf{i}} + 1.50t^2\,\hat{\mathbf{j}})$$
$$\vec{\mathbf{r}}_\text{LJ} = (1.00t^2\,\hat{\mathbf{i}} - 2.50t^2\,\hat{\mathbf{j}})$$

When $t = 5.00\ \text{s}, \vec{\mathbf{r}}_\text{LJ} = (25.0\,\hat{\mathbf{i}} - 62.5\,\hat{\mathbf{j}})\ \text{m},$ and their distance apart is

$$\boxed{d = \sqrt{(25.0\ \text{m})^2 + (62.5\ \text{m})^2} = 67.3\ \text{m}}$$

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(c) In general, Lisa's acceleration with respect to Jill is

$$\vec{\mathbf{a}}_{LJ} = \vec{\mathbf{a}}_L - \vec{\mathbf{a}}_J = (3.00\,\hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}}) - (1.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})$$

$$\vec{\mathbf{a}}_{LJ} = \boxed{(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})\ \text{m/s}^2}$$

P4.62 (a) The stone's initial velocity components (at $t = 0$) are v_{xi} and $v_{yi} = 0$, and the stone falls through a vertical displacement $\Delta y = -h$. We find the time t when the stone strikes the ground using

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2 \rightarrow -h = 0 - \frac{1}{2}gt^2 \rightarrow \boxed{t = \sqrt{\frac{2h}{g}}}$$

(b) To find the stone's initial horizontal component of velocity, we know at the above time t, the stone's horizontal displacement is $\Delta x = d$:

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow d = v_{xi}t \rightarrow v_{ox} = \frac{d}{t} \rightarrow \boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$$

(c) The vertical component of velocity at time t is

$$v_{yf} = v_{yi} + a_y t = 0 - gt \rightarrow v_{yf} = -g\sqrt{\frac{2h}{g}} \rightarrow v_{yf} = -\sqrt{2gh}$$

and the horizontal component does not change; therefore, the speed of the stone as it reaches the ocean is

$$\boxed{v_f = \sqrt{v_{xf}{}^2 + v_{yf}{}^2} = \sqrt{\left(\frac{d^2 g}{2h}\right) + (2gh)}}$$

(d) From above,

$$\theta_f = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-\sqrt{2gh}}{d\sqrt{\frac{g}{2h}}}\right)$$

$$\theta_f = -\tan^{-1}\left(\frac{2h}{d}\right)$$

which means the velocity points below the horizontal by angle

$$\boxed{\theta_f = \tan^{-1}\left(\frac{2h}{d}\right)}$$

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P4.63 We use a fixed coordinate system that, viewed from above, has its positive x axis passing through point A when the flea jumps, and its positive y axis 90° counterclockwise from its x axis. Its positive z axis is upward. The turntable rotates clockwise. At $t = 0$, the flea jumps straight up relative to the turntable, but the turntable is spinning, so the flea has both horizontal and vertical components of velocity relative to the fixed coordinate axes. Because the turntable is spinning clockwise, the horizontal velocity of the flea is in the negative y directon:

$$v_y = \left(-33.3\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi(10.0\text{ cm})}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = -34.9\text{ cm/s}$$

The vertical motion of the flea is independent of its horizontal motion. The time interval the flea takes to rise to a height h of 5.00 cm is the same time interval the flea takes to drop back to the turntable. We find the interval to drop using

$$z_f = z_i + v_{zi}t + \frac{1}{2}a_z t^2 \rightarrow 0 = h - \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$$

where h is in m and t in s. Substituting, we find

$$t = \sqrt{\frac{2(0.050\ 0\text{ m})}{9.80\text{ m/s}^2}} = 0.101\text{ s}$$

The total time interval for the flea to leave the surface of the turntable and return is twice this: $\Delta t = 0.202$ s.

(a) Find the clockwise angle the turntable rotates through in the time interval Δt:

$$\begin{aligned}\Delta\theta &= \left(\frac{33.3\text{ rev}}{\text{min}}\right)(0.202\text{ s}) \\ &= \left[\left(\frac{33.3\text{ rev}}{\text{min}}\right)\left(\frac{360°}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right](0.202\text{ s}) \\ &= 40.4°\end{aligned}$$

Point A lies 10.0 cm from the origin. When the flea jumps, the line passing from the origin to point A coincides with the positive x axis, but when the flea lands, the line makes an angle of –40.4° with the positive x axis:

$$\vec{\mathbf{r}}_A = [10.0\cos(-40.4°)]\hat{\mathbf{i}} + [10.0\sin(-40.4°)]\hat{\mathbf{j}}$$

$$\vec{\mathbf{r}}_A = \boxed{(7.61\hat{\mathbf{i}} - 6.48\hat{\mathbf{j}})\text{cm}}$$

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(b) During this time interval, the flea goes through a horizontal y displacement

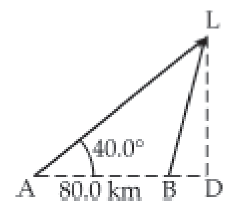
$$\Delta y = v_y \Delta t = (-34.9 \text{ cm/s})(0.202 \text{ s}) = -7.05 \text{ cm}.$$

The flea has no motion parallel to the x axis; therefore, the position of point B where the flea lands is

$$\vec{\mathbf{r}}_B = \boxed{(10.0\hat{\mathbf{i}} - 7.05\hat{\mathbf{j}}) \text{ cm}}$$

***P4.64** ANS. FIG. P4.64 shows the triangles ALB and ALD. To find the length $\overline{AL}$, we write

$$\overline{AL} = v_1 t = (90.0 \text{ km/h})(2.50 \text{ h}) = 225 \text{ km}$$



ANS. FIG. P4.64

To find the distance travelled by the second couple, we need to determine the length $\overline{BD}$:

$$\overline{BD} = \overline{AD} - \overline{AB}$$
$$= \overline{AL}\cos 40.0° - 80.0 \text{ km} = 92.4 \text{ km}$$

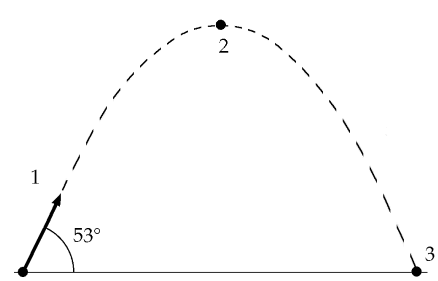
Then, from the triangle BLD in ANS. FIG. P4.64,

$$\overline{BL} = \sqrt{(\overline{BD})^2 + (\overline{DL})^2}$$
$$= \sqrt{(92.4 \text{ km})^2 + (\overline{AL}\sin 40.0°)^2} = 172 \text{ km}$$

Note that the law of cosines can also be used for the triangle ABL to solve for the length BD. Since Car 2 travels this distance in 2.50 h, its constant speed is

$$v_2 = \frac{172 \text{ km}}{2.5 \text{ h}} = \boxed{68.8 \text{ km/h}}$$

***P4.65** Consider the rocket's trajectory in 3 parts as shown in the diagram on the right. Our initial conditions give:



ANS. FIG. P4.65

$$a_y = (30.0 \text{ m/s}^2)\sin 53.0° = 24.0 \text{ m/s}^2$$

$$a_x = (30.0 \text{ m/s}^2)\cos 53.0° = 18.1 \text{ m/s}^2$$

$$v_{yi} = (100 \text{ m/s})\sin 53.0° = 79.9 \text{ m/s}$$

$$v_{xi} = (100 \text{ m/s})\cos 53.0° = 60.2 \text{ m/s}$$

The distances traveled during each phase of the motion are given in Table P4.65 below.

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<u>Path Part #1:</u>

$$\begin{aligned} v_{yf} &= v_{yi} + a_y t \\ &= 79.9 \text{ m/s} + \left(24.0 \text{ m/s}^2\right)(3.00 \text{ s}) \\ &= 152 \text{ m/s} \end{aligned}$$

$$\begin{aligned} v_{xf} &= v_{xi} + a_x t \\ &= 60.2 \text{ m/s} + \left(18.1 \text{ m/s}^2\right)(3.00 \text{ s}) \\ &= 114 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \Delta y &= v_{yi} t + \frac{1}{2} a_y t^2 \\ &= (79.9 \text{ m/s})(3.00 \text{ s}) + \frac{1}{2}\left(24.0 \text{ m/s}^2\right)(3.00 \text{ s})^2 \\ &= 347 \text{ m} \end{aligned}$$

$$\begin{aligned} \Delta x &= v_{xi} t + \frac{1}{2} a_x t^2 \\ &= (60.2 \text{ m/s})(3.00 \text{ s}) + \frac{1}{2}\left(18.1 \text{ m/s}^2\right)(3.00 \text{ s})^2 \\ &= 262 \text{ m} \end{aligned}$$

<u>Path Part #2</u>:

Now $a_x = 0$, $a_y = -9.80 \text{ m/s}^2$, $v_{xf} = v_{xi} = 114 \text{ m/s}$, $v_{yi} = 152 \text{ m/s}$, and $v_{yf} = 0$, so

$$\begin{aligned} v_{yf} &= v_{yi} + a_y t \\ 0 &= 152 \text{ m/s} - \left(9.80 \text{ m/s}^2\right) t \end{aligned}$$

which gives $t = 15.5$ s

$$\Delta x = v_{xf} t = (114 \text{ m/s})(15.5 \text{ s}) = 1.77 \times 10^3 \text{ m}$$

$$\Delta y = (152 \text{ m/s})(15.5 \text{ s}) - \frac{1}{2}\left(9.80 \text{ m/s}^2\right)(15.5 \text{ s})^2 = 1.17 \times 10^3 \text{ m}$$

<u>Path Path #3</u>:

With $v_{yi} = 0$, $a_x = 0$, $a_y = -9.80 \text{ m/s}^2$, and $v_{xf} = v_{xi} = 114 \text{ m/s}$, then

$$\begin{aligned} \left(v_{yf}\right)^2 - \left(v_{yi}\right)^2 &= 2a\Delta y \\ \left(v_{yf}\right)^2 - 0 &= 2\left(-9.80 \text{ m/s}^2\right)\left(-1.52 \times 10^3 \text{ m}\right) \end{aligned}$$

which gives $v_{yf} = -173 \text{ m/s}$

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We find the time from $v_{yf} = v_{yi} - gt$, which gives

$-173\text{ m/s} - 0 = -\left(9.80\text{ m/s}^2\right)t$, or $t = 17.6$ s

$\Delta x = v_{xf}t = 114(17.6) = 2.02 \times 10^3\text{ m}$

(a) $\Delta y(\max) = \boxed{1.52 \times 10^3\text{ m}}$

(b) $t(\text{net}) = 3.00\text{ s} + 15.5\text{ s} + 17.6\text{ s} = \boxed{36.1\text{ s}}$

(c) $\Delta x(\text{net}) = 262\text{ m} + 1.77 \times 10^3\text{ m} + 2.02 \times 10^3\text{ m}$

$\Delta x(\text{net}) = \boxed{4.05 \times 10^3\text{ m}}$

	Path Part		
	#1	#2	#3
a_y	24.0	–9.80	–9.80
a_x	18.1	0.0	0.0
v_{yf}	152	0.0	–173
v_{xf}	114	114	114
v_{yi}	79.9	152	0.0
v_{xi}	60.2	114	114
Δy	347	1.17×10^3	-1.52×10^3
Δx	262	1.77×10^3	2.02×10^3
t	3.00	15.5	17.6

Table P4.65

***P4.66** Take the origin at the mouth of the cannon. We have $x_f = v_{xi}t$, which gives

$$2\,000\text{ m} = (1\,000\text{ m/s})\cos\theta_i t$$

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Therefore,

$$t = \frac{2.00\text{ s}}{\cos\theta_i}$$

From $y_f = v_{yi}t + \frac{1}{2}a_y t^2$:

$$800\text{ m} = (1\,000\text{ m/s})\sin\theta_i t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$800\text{ m} = (1\,000\text{ m/s})\sin\theta_i\left(\frac{2.00\text{ s}}{\cos\theta_i}\right) - \frac{1}{2}(9.80\text{ m/s}^2)\left(\frac{2.00\text{ s}}{\cos\theta_i}\right)^2$$

$$800\text{ m}(\cos^2\theta_i) = 2\,000\text{ m}(\sin\theta_i\cos\theta_i) - 19.6\text{ m}$$

$$19.6\text{ m} + 800\text{ m}(\cos^2\theta_i) = 2\,000\text{ m}\sqrt{1-\cos^2\theta_i}(\cos\theta_i)$$

$$384 + (31\,360)\cos^2\theta_i + (640\,000)\cos^4\theta_i$$
$$= (4\,000\,000)\cos^2\theta_i - (4\,000\,000)\cos^4\theta_i$$

$$4\,640\,000\cos^4\theta_i - 3\,968\,640\cos^2\theta_i + 384 = 0$$

$$\cos^2\theta_i = \frac{3\,968\,640 \pm \sqrt{(3\,968\,640)^2 - 4(4\,640\,000)(384)}}{9\,280\,000}$$

$$\cos\theta_i = 0.925 \text{ or } \cos\theta_i = 0.009\,84$$

$$\theta_i = \boxed{22.4° \text{ or } 89.4°} \qquad \text{(Both solutions are valid.)}$$

P4.67 Given the initial velocity, we can calculate the height change of the ball as it moves 130 m horizontally. So this is what we do, expecting the answer to be inconsistent with grazing the top of the bleachers. We assume the ball field is horizontal. We think of the ball as a particle in free fall (moving with constant acceleration) between the point just after it leaves the bat until it crosses above the cheap seats.

The initial components of velocity are

$$v_{xi} = v_i\cos\theta = 41.7\cos 35.0° = 34.2\text{ m/s}$$
$$v_{yi} = v_i\sin\theta = 41.7\sin 35.0° = 23.9\text{ m/s}$$

We find the time when the ball has traveled through a horizontal displacement of 130 m:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow x_f = x_i + v_{xi}t \rightarrow t = (x_f - x_i)/v_{xi}$$

$$t = \frac{130\text{ m} - 0}{34.2\text{ m/s}} = 3.80\text{ s}$$

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Now we find the vertical position of the ball at this time:

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2 = 0 + v_{yi}t - \frac{1}{2}t^2$$
$$y_f = (23.9\ \text{m/s})(3.80\ \text{s}) - (4.90\ \text{m/s}^2)(3.80\ \text{s})^2 = 20.1\ \text{m}$$

The ball would not be high enough to have cleared the 24.0-m-high bleachers.

P4.68 At any time t, the two drops have identical y coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2|x(t)| = 2(v_{xi}t) = 2(v_i \cos\theta_i)t = \boxed{2v_i t \cos\theta_i}$$

P4.69 (a) The Moon's gravitational acceleration is the probe's centripetal acceleration: (For the Moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$
$$\frac{1}{6}(9.80\ \text{m/s}^2) = \frac{v^2}{1.74 \times 10^6\ \text{m}}$$
$$v = \sqrt{2.84 \times 10^6\ \text{m}^2/\text{s}^2} = \boxed{1.69\ \text{km/s}}$$

(b) The time interval can be found from

$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74 \times 10^6\ \text{m})}{1.69 \times 10^3\ \text{m/s}} = 6.47 \times 10^3\ \text{s} = \boxed{1.80\ \text{h}}$$

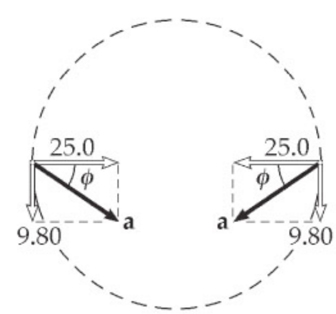
P4.70 (a) The length of the cord is given as $r = 1.00$ m. At the positions with $\theta = 90.0°$ and $270°$,

$$a_c = \frac{v^2}{r} = \frac{(5.00\ \text{m/s})^2}{1.00\ \text{m}} = \boxed{25.0\ \text{m/s}^2}$$

(b) The tangential acceleration is only the acceleration due to gravity,

$$a_t = g = \boxed{9.80\ \text{m/s}^2}$$

(c) See ANS. FIG. P4.70.



ANS. FIG. P4.70

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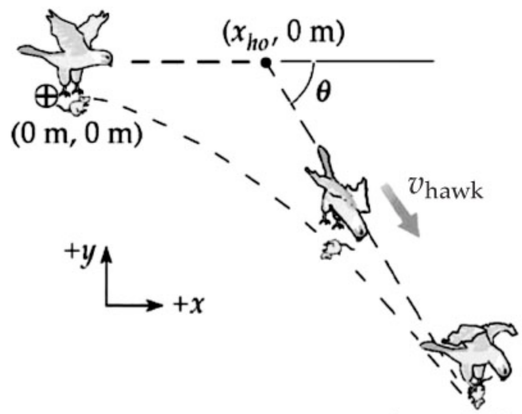
(d) The magnitude and direction of the total acceleration at these positions is given by

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(25.0\ \text{m/s}^2\right)^2 + \left(9.80\ \text{m/s}^2\right)^2} = \boxed{26.8\ \text{m/s}^2}$$

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{9.80\ \text{m/s}^2}{25.0\ \text{m/s}^2}\right) = \boxed{21.4°}$$

P4.71 We know the distance that the mouse and hawk move down, but to find the diving speed of the hawk, we must know the time interval of descent, so we will solve part (c) first. If the hawk and mouse both maintain their original horizontal velocity of 10 m/s (as the mouse should without air resistance), then the hawk only needs to think about diving straight down, but to a ground-based observer, the path will appear to be a straight line angled less than 90° below horizontal.



ANS. FIG. P4.71

We begin with the simple calculation of the free-fall time interval for the mouse.

(c) The mouse falls a total vertical distance y = 200 m – 3.00 m = 197 m. The time interval of fall is found from (with $v_{yi} = 0$)

$$y = v_{yi}t - \frac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2(197\ \text{m})}{9.80\ \text{m/s}^2}} = \boxed{6.34\ \text{s}}$$

(a) To find the diving speed of the hawk, we must first calculate the total distance covered from the vertical and horizontal components. We already know the vertical distance y; we just need the horizontal distance during the same time interval (minus the 2.00-s late start).

$$x = v_{xi}(t - 2.00\ \text{s}) = (10.0\ \text{m/s})(6.34\ \text{s} - 2.00\ \text{s}) = 43.4\ \text{m}$$

The total distance is

$$d = \sqrt{x^2 + y^2} = \sqrt{(43.4\ \text{m})^2 + (197\ \text{m})^2} = 202\ \text{m}$$

So the hawk's diving speed is

$$v = \frac{\Delta d}{\Delta t} = \frac{\sqrt{(197\ \text{m})^2 + (43.4\ \text{m})^2}}{4.34\ \text{s}} = \boxed{46.5\ \text{m/s}}$$

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(b) at an angle below the horizontal of

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{197\text{ m}}{43.4\text{ m}}\right) = \boxed{77.6°}$$

P4.72 (a) We find the x coordinate from $x = 12t$. We find the y coordinate from $49t - 4.9t^2$. Then we find the projectile's distance from the origin as $(x^2 + y^2)^{1/2}$, with these results:

***t* (s)**	0	1	2	3	4	5	6	7	8	9	10
***r* (m)**	0	45.7	82.0	109	127	136	138	133	124	117	120

(b) From the table, it looks like the magnitude of r is largest at a bit less than 6 s.

The vector $\vec{\mathbf{v}}$ tells how $\vec{\mathbf{r}}$ is changing. If $\vec{\mathbf{v}}$ at a particular point has a component along $\vec{\mathbf{r}}$, then $\vec{\mathbf{r}}$ will be increasing in magnitude (if $\vec{\mathbf{v}}$ is at an angle less than 90° from $\vec{\mathbf{r}}$) or decreasing (if the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ is more than 90°). To be at a maximum, the distance from the origin must be momentarily staying constant, and the only way this can happen is for the angle between velocity and displacement to be a right angle. Then $\vec{\mathbf{r}}$ will be changing in direction at that point, but not in magnitude.

(c) When $t = 5.70$ s, $r = \boxed{138\text{ m.}}$

(d) We can require $dr^2/dt = 0 = (d/dt)[(12t)^2 + (49t - 4.9t^2)^2]$, which results in the solution.

P4.73 (a) The time of flight must be positive. It is determined by

$$y_f = y_i + v_{yi}t + (1/2)a_y t^2 \rightarrow \quad 0 = 1.20 + v_i \sin 35.0° t - 4.90t^2$$

From the quadratic formula, and suppressing units, we find

$$t = \frac{0.574v_i + \sqrt{0.329v_i^2 + 23.52}}{9.80}$$

Then the range follows from $x = v_{xi}t + 0 = v_0 t$ as

$$\boxed{x(v_i) = v_i\sqrt{0.164\,3 + 0.002\,299v_i^2} + 0.047\,94v_i^2}$$

where x is in meters and v_i is in meters per second.

(b) Substituting $v_i = 0.100$ gives $x(v_i = 0.100) = \boxed{0.041\,0\text{ m}}$

(c) Substituting $v_i = 100$ gives $x(v_i = 100) = \boxed{961\text{ m}}$

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(d) When v_i is small, v_i^2 becomes negligible. The expression $x(v_i)$ simplifies to $v_i\sqrt{0.164\,3+0}+0=\boxed{0.405\ v_i}$. Note that this gives nearly the answer to part (b).

(e) When v_i is large, v_i is negligible in comparison to v_i^2. Then $x(v_i)$ simplifies to

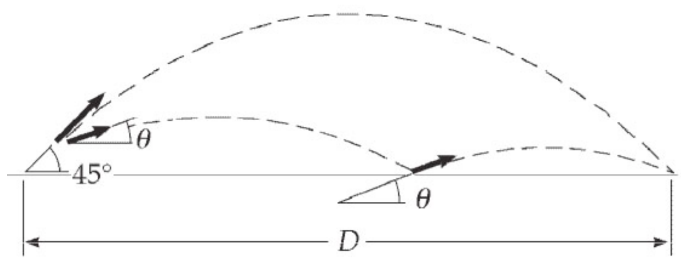
$$x(v_i)\cong v_i\sqrt{0+0.002\,299\,v_i^2}+0.047\,94\,v_i^2=\boxed{0.0959\ v_i^2}$$

This nearly gives the answer to part (c).

(f) The graph of x versus v_i starts from the origin as a straight line with slope 0.405 s. Then it curves upward above this tangent line, getting closer and closer to the parabola $x=(0.095\,9\ \mathrm{s}^2/\mathrm{m})v_i^2$.

P4.74 The special conditions allowing use of the horizontal range equation applies. For the ball thrown at 45°,

$$D=R_{45}=\frac{v_i^2\sin 90°}{g}$$



ANS. FIG. P4.74

For the bouncing ball,

$$D=R_1+R_2=\frac{v_i^2\sin 2\theta}{g}+\frac{(v_i/2)^2\sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g}=\frac{v_i^2\sin 2\theta}{g}+\frac{v_i^2\sin 2\theta}{4g}$$

$$\sin 2\theta=\frac{4}{5}$$

$$\boxed{\theta=26.6°}$$

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(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$
$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing. So for the ball thrown at 45.0°:

$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

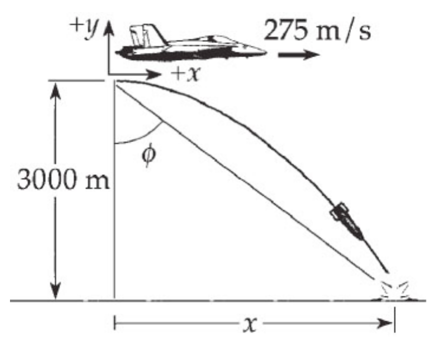
For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2(v_i/2)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{3v_i \sin 26.6°/g}{2v_i \sin 45.0°/g} = \frac{1.34}{1.41} = \boxed{0.949}$$

P4.75 We model the bomb as a particle with constant acceleration, equal to the downward free-fall acceleration, from the moment after release until the moment before impact. After we find its range it will be a right-triangle problem to find the bombsight angle.



ANS. FIG. P4.75

(a) We take the origin at the point under the plane at bomb release. In its horizontal flight, the bomb has $v_{yi} = 0$ and $v_{xi} = 275$ m/s. We represent the height of the plane as y.

Then, $\Delta y = -\frac{1}{2}gt^2$; $\Delta x = v_i t$

Combining the equations to eliminate t gives:

$$\Delta y = -\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2$$

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From this, $(\Delta x)^2 = \left(\dfrac{-2\Delta y}{g}\right)v_i^2$. Thus

$$\Delta x = v_i\sqrt{\frac{-2\Delta y}{g}} = (275\text{ m/s})\sqrt{\frac{-2(-3\,000\text{ m})}{9.80\text{ m/s}^2}}$$
$$= 6.80\times 10^3\text{ m} = \boxed{6.80\text{ km}}$$

(b) The plane has the same velocity as the bomb in the x direction. Therefore, the plane will be $\boxed{\text{3 000 m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi = \dfrac{\Delta x}{\Delta y}$;

therefore, $\phi = \tan^{-1} = \left(\dfrac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\dfrac{6\,800\text{ m}}{3\,000\text{ m}}\right) = \boxed{66.2°}$.

P4.76 Equation of bank: $y^2 = 16x$ **[1]**

Equations of motion: $x = v_i t$ **[2]**

$y = -\dfrac{1}{2}gt^2$ **[3]**

Substitute for t from [2] into [3]: $y = -\dfrac{1}{2}g\left(\dfrac{x^2}{v_i^2}\right)$. Equate y from the bank equation to y from the equations of motion:

$$16x = \left[-\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2x^4}{4v_i^4} - 16x = x\left(\frac{g^2x^3}{4v_i^4} - 16\right) = 0$$

From this, $x = 0$ or $x^3 = \dfrac{64v_i^4}{g^2}$ and $x = 4\left(\dfrac{10^4}{9.80^2}\right)^{1/3}\text{ m} = \boxed{18.8\text{ m}}$.

Also, $y = -\dfrac{1}{2}g\left(\dfrac{x^2}{v_i^2}\right) = -\dfrac{1}{2}\dfrac{(9.80\text{ m/s}^2)(18.8\text{ m})^2}{(10.0\text{ m/s})^2} = \boxed{-17.3\text{ m}}$

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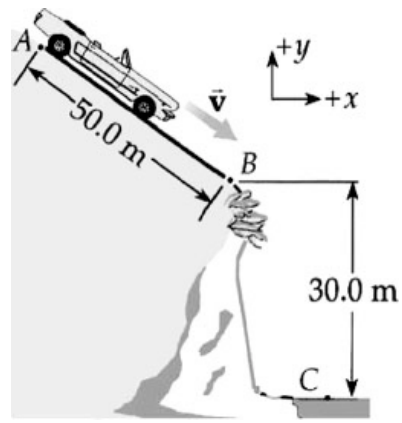
P4.77 The car has one acceleration while it is on the slope and a different acceleration when it is falling, so we must take the motion apart into two different sections. Our standard equations only describe a chunk of motion during which acceleration stays constant. We imagine the acceleration to change instantaneously at the brink of the cliff, but the velocity and the position must be the same just before point *B* and just after point *B*.



ANS. FIG. P4.77

(a) From point *A* to point *B* (along the incline), the car can be modeled as a particle under constant acceleration in one dimension, starting from rest ($v_i = 0$). Therefore, taking Δx to be the position along the incline,

$$v_f^2 - v_i^2 = 2a\Delta x$$

$$v_f^2 - 0 = 2(4.00 \text{ m/s}^2)(50.0 \text{ m})$$

$$v_f = \boxed{20.0 \text{ m/s}}$$

(b) We can find the elapsed time interval from

$$v_f = v_i + at$$

$$20.0 \text{ m/s} = 0 + \left(4.00 \text{ m/s}^2\right)t$$

$$t = \boxed{5.00 \text{ s}}$$

(c) Initial free-fall conditions give us $v_{xi} = 20.0 \cos 37.0° = 16.0$ m/s and $v_{yi} = -20.0 \sin 37.0° = -12.0$ m/s. Since $a_x = 0$, $v_{xf} = v_{xi}$ and

$$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}^2}$$

$$= -\sqrt{2\left(-9.80 \text{ m/s}^2\right)(-30.0 \text{ m}) + (-12.0 \text{ m/s})^2}$$

$$= -27.1 \text{ m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(16.0 \text{ m/s})^2 + (-27.1 \text{ m/s})^2}$$

$$= \boxed{31.5 \text{ m/s at } 59.4° \text{ below the horizontal}}$$

(d) From point *B* to *C*, the time is

$$t_1 = 5 \text{ s}; \; t_2 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-27.1 \text{ m/s} + 12.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 1.53 \text{ s}$$

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The total elapsed time interval is

$$t = t_1 + t_2 = \boxed{6.53 \text{ s}}$$

(e) The horizontal distance covered is

$$\Delta x = v_{xi} t_2 = (16.0 \text{ m/s})(1.53 \text{ s}) = \boxed{24.5 \text{ m}}$$

P4.78 (a) Coyote: $\Delta x = \frac{1}{2} a t^2 \rightarrow 70.0 \text{ m} = \frac{1}{2}(15.0 \text{ m/s}^2)\, t^2$

Roadrunner: $\Delta x = v_{xi} t \rightarrow 70.0 \text{ m} = v_{xi} t$

Solving the above, we get

$$v_{xi} = \boxed{22.9 \text{ m/s}} \text{ and } t = \boxed{3.06 \text{ s}}$$

(b) At the edge of the cliff, $v_{xi} = at = (15.0 \text{ m/s}^2)(3.06 \text{ s}) = 45.8 \text{ m/s}$

Substituting $\Delta y = -100 \text{ m}$ into $\Delta y = \frac{1}{2} a_y t^2$, we find

$$-100 \text{ m} = \frac{1}{2}\left(-9.80 \text{ m/s}^2\right) t^2$$

$$t = 4.52 \text{ s}$$

$$\Delta x = v_i t + \frac{1}{2} a_x t^2$$

$$= (45.8 \text{ m/s})(4.52 \text{ s}) + \frac{1}{2}\left(15.0 \text{ m/s}^2\right)(4.52 \text{ s})^2$$

Solving, $\Delta x = \boxed{360 \text{ m}}$.

(c) For the Coyote's motion through the air,

$$v_{xf} = v_{xi} + a_x t = 45.8 \text{ m/s} + \left(15 \text{ m/s}^2\right)(4.52 \text{ s}) = \boxed{114 \text{ m/s}}$$

$$v_{yf} = v_{yi} + a_y t = 0 - \left(9.80 \text{ m/s}^2\right)(4.52 \text{ s}) = \boxed{-44.3 \text{ m/s}}$$

P4.79 (a) Reference frame: Earth

The ice chest floats downstream 2 km in time interval Δt, so

$$2 \text{ km} = v_{ow} \Delta t \rightarrow \Delta t = 2 \text{ km}/v_{ow}$$

The upstream motion of the boat is described by

$$d = (v - v_{ow})(15 \text{ min})$$

and the downstream motion is described by

$$d + 2 \text{ km} = (v - v_{ow})(\Delta t - 15 \text{ min})$$

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We substitute the above expressions for Δt and d:

$$(v - v_{ow})(15\text{ min}) + 2\text{ km} = (v + v_{ow})\left(\frac{2\text{ km}}{v_{ow}} - 15\text{ min}\right)$$

$$v(15\text{ min}) - v_{ow}(15\text{ min}) + 2\text{ km}$$
$$= \frac{v}{v_{ow}}(2\text{ km}) + 2\text{ km} - v(15\text{ min}) - v_{ow}(15\text{ min})$$

$$v(30\text{ min}) = \frac{v}{v_{ow}}(2\text{ km})$$

$$v_{ow} = \boxed{4.00\text{ km/h}}$$

(b) Reference frame: water

After the boat travels so that it and its starting point are 2 km apart, the chest enters the water, where, in the frame of the water, it is motionless. The boat then travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point where the chest is at rest in the water. Thus, the boat travels for a total time interval of 30 min. During this same time interval, the starting point approaches the chest at speed v_{ow}, traveling 2 km. Thus,

$$v_{ow} = \frac{\Delta x}{\Delta t_{\text{total}}} = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}$$

P4.80 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6\text{ m})}{0.1\text{ s}} \approx 9\text{ m/s}$$

and its centripetal acceleration is $\frac{v^2}{r} \approx \frac{(9\text{ m/s})^2}{0.6\text{ m}} \boxed{\sim 10^2\text{ m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

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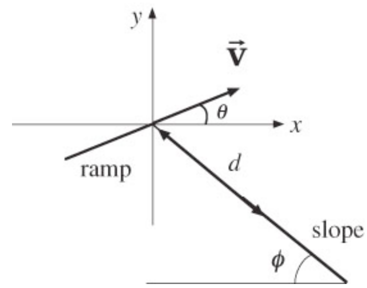
Challenge Problems

P4.81 ANS. FIG. P4.81 indicates that a line extending along the slope will past through the end of the ramp, so we may take the position of the skier as she leaves the ramp to be the origin of our coordinate system.



ANS. FIG. P4.81

(a) Measured from the end of the ramp, the skier lands a distance d down the slope at time t:

$$\Delta x = v_{xi}t$$
$$\rightarrow d\cos 50.0° = (10.0\text{ m/s})(\cos 15.0°)t$$

and

$$\Delta y = v_{yi}t + \frac{1}{2}gt^2 \rightarrow$$
$$-d\sin 50.0° = (10.0\text{ m/s})(\sin 15.0°)t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

Solving, $d = \boxed{43.2\text{ m}}$ and $t = 2.88$ s.

(b) Since $a_x = 0$,

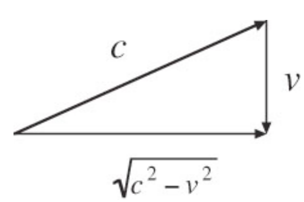
$$v_{xf} = v_{xi} = (10.0\text{ m/s})\cos 15.0° = \boxed{9.66\text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = (10.0\text{ m/s})\sin 15.0° - (9.80\text{ m/s}^2)(2.88\text{ s})$$
$$= \boxed{-25.6\text{ m/s}}$$

(c) Air resistance would ordinarily decrease the values of the range and landing speed. As an airfoil, she can deflect air downward so that the air deflects her upward. This means she can get some lift and increase her distance.

P4.82 (a) For Chris, his speed downstream is $c + v$, while his speed upstream is $c - v$.

Therefore, the total time for Chris is

$$\boxed{\Delta t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2\ L/c}{1 - v^2/c^2}}$$



ANS. FIG. P4.82

(b) Sarah must swim somewhat upstream to counteract the effect from the current. As is shown in the diagram, the magnitude of her cross-stream velocity is $\sqrt{c^2 - v^2}$.

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Thus, the total time for Sarah is

$$\boxed{\Delta t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2\ L/c}{\sqrt{1 - v^2/c^2}}}$$

(c) Since the term $\left(1 - v^2/c^2\right) < 1, \Delta t_1 > \Delta t_2$, so Sarah, who swims cross-stream, returns first.

***P4.83** Let the river flow in the x direction.

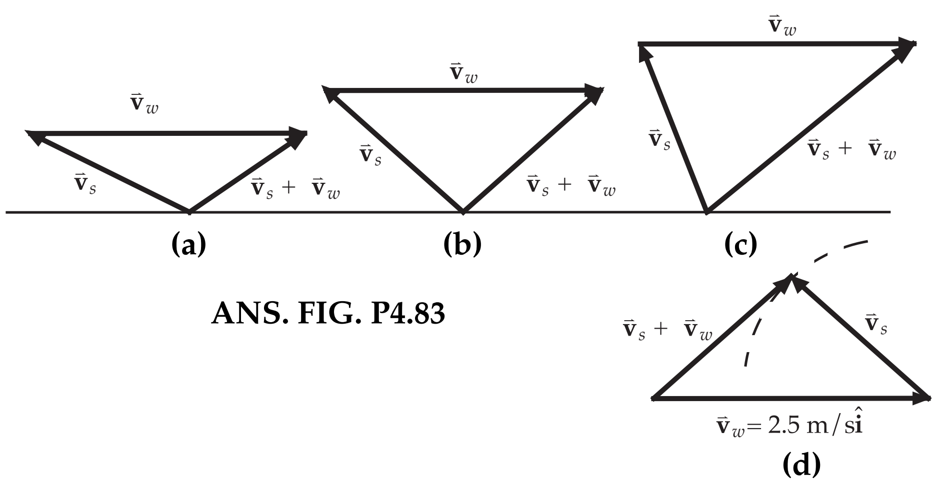
(a) To minimize time, swim perpendicular to the banks in the y direction. You are in the water for time t in $\Delta y = v_y t$,

$$t = \frac{80\text{ m}}{1.5\text{ m/s}} = 53.3\text{ s}$$

(b) The water carries you downstream by

$$\Delta x = v_x t = (2.50\text{ m/s})53.3\text{ s} = \boxed{133\text{ m}}$$

(c) To minimize downstream drift, you should swim so that your resultant velocity $\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$ is perpendicular to your swimming velocity $\vec{\mathbf{v}}_s$ relative to the water. This is shown graphically in the upper row of ANS. FIG. P4.83. Unlike the situations shown in ANS. FIG. P4.83(a) and ANS. FIG. P4.83(b), this condition (shown in ANS. FIG. P4.83(b)) maximizes the angle between the resultant velocity and the shore. The angle between $\vec{\mathbf{v}}_s$ and the shore is given by $\cos\theta = \frac{1.5\text{ m/s}}{2.5\text{ m/s}}$, $\boxed{\theta = 53.1°}$.



ANS. FIG. P4.83

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(d) See ANS. FIG. P4.83(d). Now,
$v_y = v_s \sin\theta = (1.5 \text{ m/s})\sin 53.1° = 1.20 \text{ m/s}$

$$t = \frac{\Delta y}{v_y} = \frac{80 \text{ m}}{1.2 \text{ m/s}} = 66.7 \text{ s}$$

$$\Delta x = v_x t = [2.5 \text{ m/s} - (1.5 \text{ m/s})\cos 53.1°](66.7 \text{ s}) = \boxed{107 \text{ m}}$$

P4.84 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - \frac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2 + x^2 = R^2$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock's surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$:

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock:
$1 > \frac{gR}{v_i^2}$, so

$$\boxed{v_i > \sqrt{gR}}$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$

or $x = R\sqrt{2}$. The distance from the rock's base is

$$x - R = \boxed{\left(\sqrt{2} - 1\right)R}$$

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P4.85 When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25 \text{ km})^2 - (2.15 \text{ km})^2} = 2.437 \text{ km}$$

The vertical displacement of the bomb is

$$y_f = x_f \tan\theta_i - \frac{g x_f^2}{2v_i^2 \cos^2\theta_i}$$

Substituting,

$$-2\,150 \text{ m} = (2\,437 \text{ m})\tan\theta_i - \frac{(9.8 \text{ m/s}^2)(2\,437 \text{ m})^2}{2(280 \text{ m/s})^2 \cos^2\theta_i}$$

or

$$-2\,150 \text{ m} = (2\,437 \text{ m})\tan\theta_i - (371.19 \text{ m})(1 + \tan^2\theta_i)$$

$$\therefore \tan^2\theta_i - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945$$

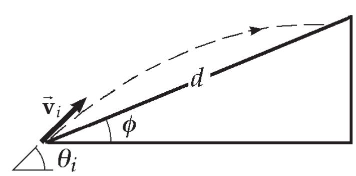
We select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan\theta_i = -0.662, \quad \boxed{\theta_i = -33.5°}$$

P4.86 (a) The horizontal distance traveled by the projectile is given by

$$x_f = v_{xi}t = (v_i \cos\theta_i)t$$

$$\rightarrow t = \frac{x_f}{v_i \cos\theta_i}$$



ANS. FIG. P4.86

We substitute this into the equation for the displacement in y:

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = (\tan\theta_i)(x_f) - \frac{g}{2v_i^2\cos^2\theta_i}x_f^2$$

Now setting $x_f = d\cos\phi$ and $y_f = d\sin\phi$, we have

$$d\sin\phi = (\tan\theta_i)(d\cos\phi) - \frac{g}{2v_i^2\cos^2\theta_i}(d\cos\phi)^2$$

Solving for d yields

$$d = \frac{2v_i^2\cos\theta_i[\sin\theta_i\cos\phi - \sin\phi\cos\theta_i]}{g\cos^2\phi}$$

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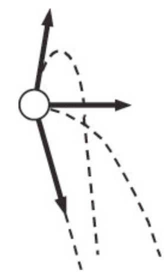
or $$d = \boxed{\frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}}$$

(b) Setting $\frac{d}{d\theta_i}(d) = 0$ leads to

$$\boxed{\theta_i = 45° + \frac{\phi}{2}} \text{ and } \boxed{d_{\max} = \frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$$

P4.87 For the smallest impact angle

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)$$



ANS. FIG. P4.87

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}$$

P4.88 We follow the steps outlined in Example 4.5, eliminating $t = \frac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i \sin\theta\, d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi$$

Clearing the fractions gives

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\frac{d}{d\theta}(d) = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi$$
$$-g\left[\frac{d}{d\theta}(d)\right]\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi$$

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We use the trigonometric identities from Appendix B4:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \text{ and } \sin 2\theta = 2\sin\theta\cos\theta$$

to find

$$\cos\phi\cos 2\theta = \sin 2\theta \sin\phi$$

Next, $\dfrac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \dfrac{1}{\tan 2\theta}$ give $\cot 2\theta = \tan\phi$ so

$$\phi = 90° - 2\theta \text{ and } \theta = 45° - \frac{\phi}{2}$$

P4.89 Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are > 45°, $x = 2\,500$ m, $y = 1\,800$ m, and $v_i = 250$ m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus,

$$t = \frac{x_f}{v_i\cos\theta}$$

Substitute into the expression for y_f:

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$, so $y_f = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f$$

Substitute values, use the quadratic formula, and find

$\tan\theta = 3.905$ or 1.197, which gives $\theta_H = 75.6°$ and $\theta_L = 50.1°$.

$$\text{Range}\left(\text{at } \theta_H\right) = \frac{v_i^2\sin 2\theta_H}{g} = 3.07\times 10^3 \text{ m from enemy ship}$$

$$3.07\times 10^3 \text{ m} - 2\,500 \text{ m} - 300 \text{ m} = 270 \text{ m from shore}$$

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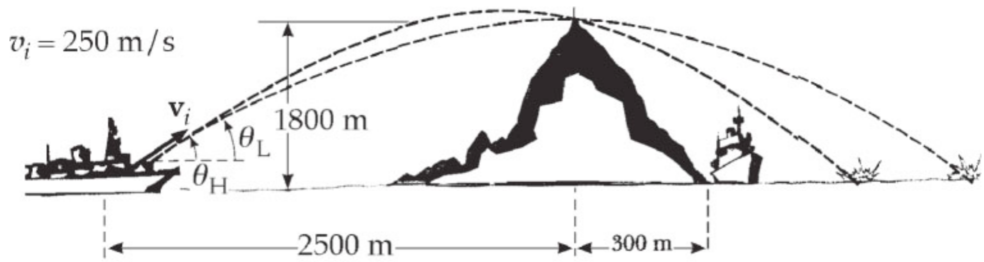
$$\text{Range}\left(\text{at } \theta_L\right) = \frac{v_i^2 \sin 2\theta_L}{g} = 6.28 \times 10^3 \text{ m from enemy ship}$$

$$6.28 \times 10^3 \text{ m} - 2\,500 \text{ m} - 300 \text{ m} = 3.48 \times 10^3 \text{ m from shore}$$

Therefore, the safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48 \times 10^3 \text{ m}}$ from the shore.



ANS. FIG. P4.89

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ANSWERS TO EVEN-NUMBERED PROBLEMS

P4.2 2.50 m/s

P4.4 (a) $-5.00\omega\ \hat{\mathbf{i}}$ m/s; (b) $-5.00\omega^2\ \hat{\mathbf{j}}$ m/s;
(c) $(4.00\text{ m})\hat{\mathbf{j}} + (5.00\text{ m})\left(-\sin\omega t\hat{\mathbf{i}} - \cos\omega t\hat{\mathbf{j}}\right)$,
$(5.00\text{ m})\omega\left[-\cos\omega\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}}\right]$, $(5.00\text{ m})\omega^2\left[\sin\omega t\hat{\mathbf{i}} + \cos\omega\hat{\mathbf{j}}\right]$; (d) a circle of radius 5.00 m centered at (0, 4.00 m)

P4.6 (a) $5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}$; (b) $5.00\hat{\mathbf{i}} + 3.00t\ \hat{\mathbf{j}}$; (c) 10.0 m, 6.00 m; (d) 7.81 m/s

P4.8 (a) $\left(10.0\ \hat{\mathbf{i}} + 0.241\ \hat{\mathbf{j}}\right)$ mm; (b) $\left(1.84\times10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}}$;
(c) 1.85×10^7 m/s; (d) 2.73°

P4.10 (a) $\vec{\mathbf{v}}_f = (3.45 - 1.79t)\hat{\mathbf{i}} + (2.89 - 0.650t)\hat{\mathbf{j}}$;
(b) $\vec{\mathbf{r}}_f = (-25.3 + 3.45t - 0.893t^2)\hat{\mathbf{i}} + \left(28.9 + 2.89t - 0.325t^2\right)\hat{\mathbf{j}}$

P4.12 0.600 m/s^2

P4.14 (a) $v_{xi} = d\sqrt{\dfrac{g}{2h}}$, (b) The direction of the mug's velocity is $\tan^{-1}(2h/d)$ below the horizontal.

P4.16 $x = 7.23 \times 10^3$ m, $y = 1.68 \times 10^3$ m

P4.18 (a) 76.0°, (b) $R_{\max} = 2.13R$, (c) the same on every planet

P4.20 (a) 22.6 m; (b) 52.3 m; (c) 1.18 s

P4.22 (a) there is; (b) 0.491 m/s

P4.24 (a) 0.852 s; (b) 3.29 m/s; (c) 4.03 m/s; (d) 50.8°; (e) t = 1.12 s

P4.26 (a) (0, 0.840 m); (b) 11.2 m/s at 18.5°; (c) 8.94 m

P4.28 (a) $t = v_i \sin\theta/g$; (b) $h_{\max} = h + \dfrac{(v_i \sin\theta)^2}{2g}$

P4.30 (a) 28.2 m/s; (b) 4.07 s; (c) the required initial velocity will increase, the total time of flight will increase

P4.32 (a) 41.7 m/s; (b) 3.81 s; (c) $v_x = 34.1$ m/s, $v_y = -13.4$ m/s, $v = 36.7$ m/s

P4.24 0.033 7 m/s^2 directed toward the center of Earth

P4.36 10.5 m/s, 219 m/s^2 inward

P4.38 (a) 6.00 rev/s; (b) 1.52×10^3 m/s^2; (c) 1.28×10^3 m/s^2

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P4.40 (a) 13.0 m/s^2; (b) 5.70 m/s; (c) 7.50 m/s^2

P4.42 (a) See ANS. FIG. P4.42; (b) 29.7 m/s^2; (c) 6.67 m/s tangent to the circle

P4.44 153 km/h at 11.3° north of west

P4.46 (a) $\Delta t_{\text{woman}} = \dfrac{L}{v_1}$; (b) $\Delta t_{\text{man}} = \dfrac{L}{v_1 + v_2}$; (c) $\Delta t_{\text{man}} = \dfrac{L}{v_1 + 2v_2}$

P4.48 (a) 57.7 km/h at 60.0° west of vertical; (b) 28.9 km/h downward

P4.50 (a) 2.02×10^3 s; (b) 1.67×10^3 s; (c) Swimming with the current does not compensate for the time lost swimming against the current.

P4.52 27.7° E of N

P4.54 (a) straight up, at 0° to the vertical; (b) 8.25 m/s; (c) a straight up and down line; (d) a symmetric parabola opening downward; (e) 12.6 m/s north at $\tan^{-1}(8.25/9.5) = 41.0°$ above the horizontal

P4.56 (a) $2\sqrt{\dfrac{R}{3g}}$; (b) $\dfrac{1}{2}\sqrt{3gR}$; (c) $\sqrt{\dfrac{gR}{3}}$; (d) $\sqrt{\dfrac{13gR}{12}}$; (e) 33.7°; (f) $\dfrac{13}{24}R$; (g) $\dfrac{13}{12}R$

P4.58 (a) $5\hat{\mathbf{i}} + 4t^{3/2}\hat{\mathbf{j}}$; (b) $5t\hat{\mathbf{i}} + 1.6t^{5/2}\hat{\mathbf{j}}$

P4.60 (a) 9.80 m/s^2, downward; (b) 10.7 m/s

P4.62 (a) $t = \sqrt{\dfrac{2h}{g}}$; (b) $v_{xi} = d\sqrt{\dfrac{g}{2h}}$; (c) $v_f = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{\left(\dfrac{d^2 g}{2h}\right) + (2gh)}$; (d) $\theta_f = \tan^{-1}\left(\dfrac{2h}{d}\right)$

P4.64 68.8 km/h

P4.66 22.4° or 89.4°

P4.68 $2v_i t \cos\theta_i$

P4.70 (a) 25.0 m/s^2; (b) 9.80 m/s^2; (c) See ANS. FIG. P4.70; (d) 26.8 m/s^2, 21.4°

P4.72 (a) See table in P4.72(a); (b) From the table, it looks like the magnitude of r is largest at a bit less than 6 s; (c) 138 m; (d) We can require $dr^2/dt = 0 = (d/dt)[(12t)^2 + (49t - 4.9t^2)^2]$, which results in the solution.

P4.74 (a) $\theta = 26.6°$; (b) 0.949

P4.76 18.8 m, −17.3 m

P4.78 (a) 22.9 m/s and 3.06 s; (b) 360 m; (c) 114 m/s, −44.3 m/s

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P4.80 $\sim 10^2\ \text{m/s}^2$

P4.82 (a) $\Delta t_1 = \dfrac{L}{c+v} + \dfrac{L}{c-v} = \dfrac{2L/c}{1-v^2/c^2}$; (b) $\Delta t_2 = \dfrac{2L}{\sqrt{c^2-v^2}} = \dfrac{2L/c}{\sqrt{1-v^2/c^2}}$;
(c) Sarah, who swims cross-stream, returns first

P4.84 (a) $v_i > \sqrt{gR}$; (b) $x - R = \left(\sqrt{2}-1\right)R$

P4.86 (a) See P4.86a for derivation; (b) $d_{\text{max}} = 45° + \dfrac{\phi}{2}$, $\theta_i = \dfrac{v_i^2(1-\sin\phi)}{g\cos^2\phi}$

P4.88 See P4.88 for complete derivation.

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