6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ6.1 (a) A > C = D > B = E = 0. At constant speed, centripetal acceleration is largest when radius is smallest. A straight path has infinite radius of curvature. (b) Velocity is north at *A*, west at *B*, and south at *C*. (c) Acceleration is west at *A*, nonexistent at *B*, east at *C*, to be radially inward.

OQ6.2 Answer (a). Her speed increases, until she reaches terminal speed.

OQ6.3 (a) Yes. Its path is an arc of a circle; the direction of its velocity is changing. (b) No. Its speed is not changing.

OQ6.4 (a) Yes, point *C*. Total acceleration here is centripetal acceleration, straight up. (b) Yes, point *A*. The speed at *A* is zero where the bob is reversing direction. Total acceleration here is tangential acceleration, to the right and downward perpendicular to the cord. (c) No. (d) Yes, point *B*. Total acceleration here is to the right and either downwards or upwards depending on whether the magnitude of the centripetal acceleration is smaller or larger than the magnitude of the tangential acceleration.

OQ6.5 Answer (b). The magnitude of acceleration decreases as the speed increases because the air resistance force increases, counterbalancing more and more of the gravitational force.

OQ6.6 (a) No. When $v = 0$, $v^2/r = 0$.

(b) Yes. Its speed is changing because it is reversing direction.

OQ6.7 (i) Answer (c). The iPod shifts backward relative to the student's hand. The cord then pulls the iPod upward and forward, to make it gain speed horizontally forward along with the airplane. (ii) Answer (b). The angle stays constant while the plane has constant acceleration. This experiment is described in the book *Science from your Airplane Window* by Elizabeth Wood.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ6.1 (a) Friction, either static or kinetic, exerted by the roadway where it meets the rubber tires accelerates the car forward and then maintains its speed by counterbalancing resistance forces. Most of the time static friction is at work. But even kinetic friction (racers starting) will still move the car forward, although not as efficiently. (b) The air around the propeller pushes forward on its blades. Evidence is that the propeller blade pushes the air toward the back of the plane. (c) The water pushes the blade of the oar toward the bow. Evidence is that the blade of the oar pushes the water toward the stern.

CQ6.2 The drag force is proportional to the speed squared and to the effective area of the falling object. At terminal velocity, the drag and gravity forces are in balance. When the parachute opens, its effective area increases greatly, causing the drag force to increase greatly. Because the drag and gravity forces are no longer in balance, the greater drag force causes the speed to decrease, causing the drag force to decrease until it and the force of gravity are in balance again.

CQ6.3 The speed changes. The tangential force component causes tangential acceleration.

CQ6.4 (a) The object will move in a circle at a constant speed.

(b) The object will move in a straight line at a changing speed.

CQ6.5 The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, "g," is changed inside the elevator. "g" $= g \pm a$

CQ6.6 I would not accept that statement for two reasons. First, to be "beyond the pull of gravity," one would have to be infinitely far away from all other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

CQ6.7 This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.

CQ6.8 (a) The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. (b) When moving with terminal speed, an object is in equilibrium and has zero acceleration.

CQ6.9 Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration to keep blood flowing up to the pilot's brain.

CQ6.10 The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.

CQ6.11 The current consensus is that the laws of physics are probabilistic in nature on the fundamental level. For example, the Uncertainty Principle (to be discussed later) states that the position and velocity (actually, momentum) of any particle cannot both be known exactly, so the resulting predictions cannot be exact. For another example, the moment of the decay of any given radioactive atomic nucleus cannot be predicted, only the average rate of decay of a large number of nuclei can be predicted—in this sense, quantum mechanics implies that the future is indeterminate. How the laws of physics are related to our sense of free will is open to debate.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 6.1 Extending the Particle in Uniform Circular Motion Model

P6.1 We are given $m = 3.00$ kg, $r = 0.800$ m. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{max} = Mg = (25.0 \text{ kg})(9.80 \text{ m/s}^2) = 245 \text{ N}$$

When the 3.00-kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

so $T = \dfrac{mv^2}{r}$

Then

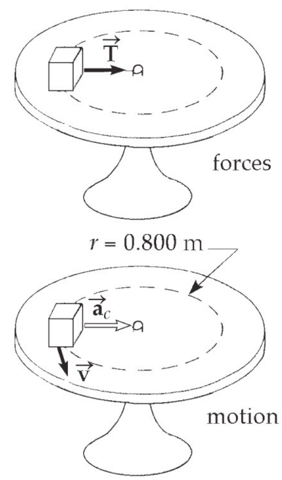
$$v^2 = \frac{rT}{m} = \frac{(0.800 \text{ m})T}{3.00 \text{ kg}} \le \frac{(0.800 \text{ m})T_{max}}{3.00 \text{ kg}}$$
$$= \frac{(0.800 \text{ m})(245 \text{ N})}{3.00 \text{ kg}} = 65.3 \text{ m}^2/\text{s}^2$$

This represents the maximum value of v^2, or

$$0 \le v \le \sqrt{65.3} \text{ m/s}$$

which gives

$$\boxed{0 \le v \le 8.08 \text{ m/s}}$$



ANS. FIG. P6.1

P6.2 (a) The astronaut's orbital speed is found from Newton's second law, with

$$\sum F_y = ma_y: \; mg_{moon} \text{ down} = \frac{mv^2}{r} \text{ down}$$

solving for the velocity gives

$$v = \sqrt{g_{moon} r} = \sqrt{(1.52 \text{ m/s}^2)(1.7 \times 10^6 \text{ m} + 100 \times 10^3 \text{ m})}$$
$$v = \boxed{1.65 \times 10^3 \text{ m/s}}$$

(b) To find the period, we use $v = \dfrac{2\pi r}{T}$ and solve for T:

$$T = \frac{2\pi(1.8 \times 10^6 \text{ m})}{1.65 \times 10^3 \text{ m/s}} = \boxed{6.84 \times 10^3 \text{ s}} = 1.90 \text{ h}$$

P6.3 (a) The force acting on the electron in the Bohr model of the hydrogen atom is directed radially inward and is equal to

$$F = \frac{mv^2}{r} = \frac{(9.11\times10^{-31}\ \text{kg})(2.20\times10^{6}\ \text{m/s})^2}{0.529\times10^{-10}\ \text{m}}$$
$$= \boxed{8.33\times10^{-8}\ \text{N inward}}$$

(b) $a = \dfrac{v^2}{r} = \dfrac{(2.20\times10^{6}\ \text{m/s})^2}{0.529\times10^{-10}\ \text{m}} = \boxed{9.15\times10^{22}\ \text{m/s}^2\ \text{inward}}$

P6.4 In $\sum F = m\dfrac{v^2}{r}$, both m and r are unknown but remain constant. Symbolically, write

$$\sum F_{\text{slow}} = \left(\frac{m}{r}\right)(14.0\ \text{m/s})^2 \text{ and } \sum F_{\text{fast}} = \left(\frac{m}{r}\right)(18.0\ \text{m/s})^2$$

Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\dfrac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130\ \text{N}) = \boxed{215\ \text{N}}$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

P6.5 We neglecting relativistic effects. With 1 u = 1.661 x 10^{-27} kg, and from Newton's second law, we obtain

$$F = ma_c = \frac{mv^2}{r}$$
$$= (2\times1.661\times10^{-27}\ \text{kg})\frac{(2.998\times10^{7}\ \text{m/s})^2}{(0.480\ \text{m})}$$
$$= \boxed{6.22\times10^{-12}\ \text{N}}$$

P6.6 (a) The car's speed around the curve is found from

$$v = \frac{235\ \text{m}}{36.0\ \text{s}} = 6.53\ \text{m/s}$$

This is the answer to part (b) of this problem. We calculate the radius of the curve from $\frac{1}{4}(2\pi r) = 235\ \text{m}$, which gives r = 150 m.

The car's acceleration at point B is then

$$\vec{\mathbf{a}}_r = \left(\frac{v^2}{r}\right) \text{ toward the center}$$

$$= \frac{(6.53 \text{ m/s})^2}{150 \text{ m}} \text{ at } 35.0° \text{ north of west}$$

$$= (0.285 \text{ m/s}^2)\left(\cos 35.0°\left(-\hat{\mathbf{i}}\right) + \sin 35.0°\hat{\mathbf{j}}\right)$$

$$= \boxed{\left(-0.233\hat{\mathbf{i}} + 0.163\hat{\mathbf{j}}\right) \text{ m/s}^2}$$

(b) From part (a), $v = \boxed{6.53 \text{ m/s}}$

(c) We find the average acceleration from

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right)}{\Delta t}$$

$$= \frac{\left(6.53\hat{\mathbf{j}} - 6.53\hat{\mathbf{i}}\right) \text{ m/s}}{36.0 \text{ s}}$$

$$= \boxed{\left(-0.181\hat{\mathbf{i}} + 0.181\hat{\mathbf{j}}\right) \text{ m/s}^2}$$

P6.7 Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the 3.00 m/s^2 centripetal acceleration:

$$a_c = v^2/r \quad \text{so} \quad v = \sqrt{a_c r} = \sqrt{(3.00 \text{ m/s}^2)(60.0 \text{ m})} = 13.4 \text{ m/s}$$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$:

$$T = \frac{2\pi r}{v} = \frac{2\pi(60.0 \text{ m})}{13.4 \text{ m/s}} = 28.1 \text{ s}$$

so the frequency of rotation is

$$f = \frac{1}{T} = \frac{1}{28.1 \text{ s}} = \left(\frac{1}{28.1 \text{ s}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.14 \text{ rev/min}}$$

P6.8 ANS. FIG. P6.8 shows the free-body diagram for this problem.

(a) The forces acting on the pendulum in the vertical direction must be in balance since the acceleration of the bob in this direction is zero. From Newton's second law in the y direction,

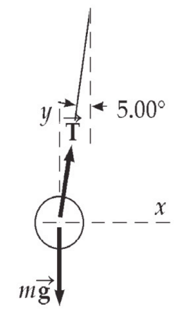
$$\sum F_y = T\cos\theta - mg = 0$$

Solving for the tension T gives

$$T = \frac{mg}{\cos\theta} = \frac{(80.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 5.00°} = 787\text{ N}$$

In vector form,

$$\vec{\mathbf{T}} = T\sin\theta\hat{\mathbf{i}} + T\cos\theta\hat{\mathbf{j}}$$
$$= \boxed{(68.6\text{ N})\hat{\mathbf{i}} + (784\text{ N})\hat{\mathbf{j}}}$$



ANS. FIG. P6.8

(b) From Newton's second law in the x direction,

$$\sum F_x = T\sin\theta = ma_c$$

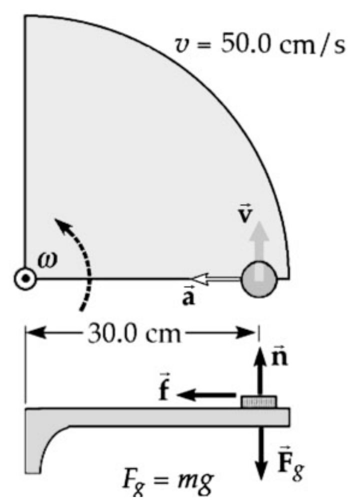
which gives

$$a_c = \frac{T\sin\theta}{m} = \frac{(787\text{ N})\sin 5.00°}{80.0\text{ kg}} = \boxed{0.857\text{ m/s}^2}$$

toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

P6.9 ANS. FIG. P6.9 shows the constant maximum speed of the turntable and the centripetal acceleration of the coin.



ANS. FIG. P6.9

(a) The force of $\boxed{\text{static friction}}$ causes the centripetal acceleration.

(b) From ANS. FIG. P6.9,

$$ma\hat{\mathbf{i}} = f\,\hat{\mathbf{i}} + n\hat{\mathbf{j}} + mg(-\hat{\mathbf{j}})$$

$$\sum F_y = 0 = n - mg$$

thus, $n = mg$ and

$$\sum F_r = m\frac{v^2}{r} = f = \mu n = \mu mg$$

Then,

$$\mu = \frac{v^2}{rg} = \frac{(50.0\text{ cm/s})^2}{(30.0\text{ cm})(980\text{ cm/s}^2)} = \boxed{0.085\ 0}$$

P6.10 We solve for the tensions in the two strings:

$$F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$$

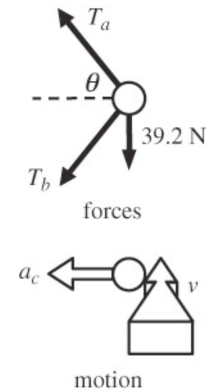
The angle θ is given by

$$\theta = \sin^{-1}\left(\frac{1.50\text{ m}}{2.00\text{ m}}\right) = 48.6°$$

The radius of the circle is then

$$r = (2.00\text{ m})\cos 48.6° = 1.32\text{ m}$$

Applying Newton's second law,



ANS. FIG. P6.10

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$T_a\cos 48.6° + T_b\cos 48.6° = \frac{(4.00\text{ kg})(3.00\text{ m/s})^2}{1.32\text{ m}}$$

$$T_a + T_b = \frac{27.27\text{ N}}{\cos 48.6°} = 41.2\text{ N} \quad \textbf{[1]}$$

$$\sum F_y = ma_y: \quad T_a\sin 48.6° - T_b\sin 48.6° - 39.2\text{ N} = 0$$

$$T_a - T_b = \frac{39.2\text{ N}}{\sin 48.6°} = 52.3\text{ N} \quad \textbf{[2]}$$

To solve simultaneously, we add the equations in T_a and T_b:

$$(T_a + T_b) + (T_a - T_b) = 41.2\text{ N} + 52.3\text{ N}$$

$$T_a = \frac{93.8\text{ N}}{2} = 46.9\text{ N}$$

This means that $T_b = 41.2\text{ N} - T_a = -5.7\text{ N}$, which we may interpret as meaning the lower string pushes rather than pulls!

The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.

To answer the **What if?**, we go back to equation [2] above and substitute mg for the weight of the object. Then,

$$\sum F_y = ma_y: \quad T_a\sin 48.6° - T_b\sin 48.6° - mg = 0$$

$$T_a - T_b = \frac{(4.00\text{ kg})g}{\sin 48.6°} = 5.33g$$

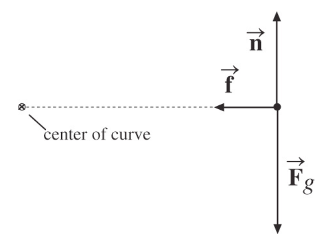
We then add this equation to equation [2] to obtain

$$(T_a + T_b) + (T_a - T_b) = 41.2\text{ N} + 5.33g$$

or $$T_a = 20.6\text{ N} + 2.67g \text{ and } T_b = 41.2\text{ N} - T_a = 41.2\text{ N} - 2.67g$$

For this situation to be possible, T_b must be > 0, or $g < 7.72\text{ m/s}^2$. This is certainly the case on the surface of the Moon and on Mars.

P6.11 Call the mass of the egg crate m. The forces on it are its weight $F_g = mg$ vertically down, the normal force n of the truck bed vertically up, and static friction f_s directed to oppose relative sliding motion of the crate on the truck bed. The friction force is directed radially inward. It is the only horizontal force on the crate, so it must provide the centripetal acceleration. When the truck has maximum speed, friction f_s will have its maximum value with $f_s = \mu_s n$.



ANS. FIG. P6.11

Newton's second law in component form becomes

$$\sum F_y = ma_y \quad \text{giving} \quad n - mg = 0 \quad \text{or} \quad n = mg$$

$$\sum F_x = ma_x \quad \text{giving} \quad f_s = ma_r$$

From these three equations,

$$\mu_s n \le \frac{mv^2}{r} \quad \text{and} \quad \mu_s mg \le \frac{mv^2}{r}$$

The mass divides out. The maximum speed is then

$$v \le \sqrt{\mu_s rg} = \sqrt{0.600(35.0\text{ m})(9.80\text{ m/s}^2)} \rightarrow v \le \boxed{14.3\text{ m/s}}$$

Section 6.2 Nonuniform Circular Motion

P6.12 (a) The external forces acting on the water are

the gravitational force

and the contact force exerted on the water by the pail.

(b) The contact force exerted by the pail is the most important in causing the water to move in a circle. If the gravitational force acted alone, the water would follow the parabolic path of a projectile.

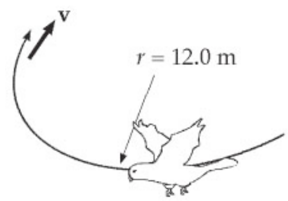
(c) When the pail is inverted at the top of the circular path, it cannot hold the water up to prevent it from falling out. If the water is not to spill, the pail must be moving fast enough that the required centripetal force is at least as large as the gravitational force. That is, we must have

$$m\frac{v^2}{r} \geq mg \quad \text{or} \quad v \geq \sqrt{rg} = \sqrt{(1.00\,\text{m})(9.80\,\text{m/s}^2)} = \boxed{3.13\ \text{m/s}}$$

(d) If the pail were to suddenly disappear when it is at the top of the circle and moving at 3.13 m/s, the water would follow the parabolic path of a projectile launched with initial velocity components of $v_{xi} = 3.13$ m/s, $v_{yi} = 0$.

P6.13 (a) The hawk's centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(4.00\ \text{m/s})^2}{12.0\ \text{m}} = \boxed{1.33\ \text{m/s}^2}$$



ANS. FIG. P6.13

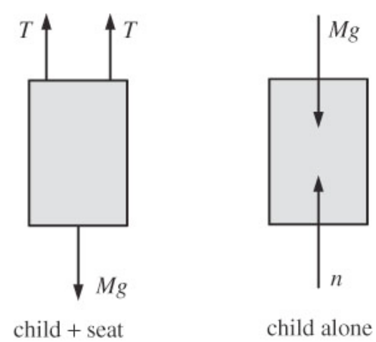
(b) The magnitude of the acceleration vector is

$$a = \sqrt{a_c^2 + a_t^2}$$
$$= \sqrt{(1.33\ \text{m/s}^2)^2 + (1.20\ \text{m/s}^2)^2} = \boxed{1.79\ \text{m/s}^2}$$

at an angle

$$\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \tan^{-1}\left(\frac{1.33\ \text{m/s}^2}{1.20\ \text{m/s}^2}\right) = \boxed{48.0°\ \text{inward}}$$

6.14 We first draw a force diagram that shows the forces acting on the child-seat system and apply Newton's second law to solve the problem. The child's path is an arc of a circle, since the top ends of the chains are fixed. Then at the lowest point the child's motion is changing in direction: He moves with centripetal acceleration even as his speed is not changing and his tangential acceleration is zero.



ANS. FIG. P6.14

(a) ANS. FIG. P6.14 shows that the only forces acting on the system of child + seat are the tensions in the two chains and the weight of the boy:

$$\sum F = F_{\text{net}} = 2T - mg = ma = \frac{mv^2}{r}$$

with

$$F_{\text{net}} = 2T - mg = 2(350 \text{ N}) - (40.0 \text{ kg})(9.80 \text{ m/s}^2) = 308 \text{ N}$$

solving for v gives

$$v = \sqrt{\frac{F_{\text{net}} r}{m}} = \sqrt{\frac{(308 \text{ N})(3.00 \text{ m})}{40.0 \text{ kg}}} = \boxed{4.81 \text{ m/s}}$$

(b) The normal force from the seat on the child accelerates the child in the same way that the total tension in the chain accelerates the child-seat system. Therefore, $n = 2T = \boxed{700 \text{ N}}$.

P6.15 See the forces acting on seat (child) in ANS. FIG. P6.14.

(a) $\sum F = 2T - Mg = \frac{Mv^2}{R}$

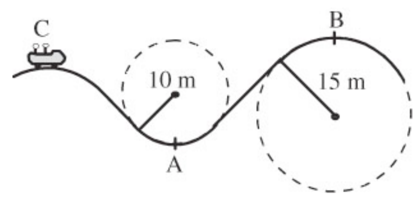
$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

$$\boxed{v = \sqrt{(2T - Mg)\left(\frac{R}{M}\right)}}$$

(b) $n - Mg = F = \frac{Mv^2}{R}$

$$\boxed{n = Mg + \frac{Mv^2}{R}}$$

P6.16 (a) We apply Newton's second law at point A, with $v = 20.0$ m/s, n = force of track on roller coaster, and $R = 10.0$ m:



ANS. FIG. P6.16

$$\sum F = \frac{Mv^2}{R} = n - Mg$$

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500 \text{ kg})(9.80 \text{ m/s}^2) + \frac{(500 \text{ kg})(20.0 \text{ m/s}^2)}{10.0 \text{ m}}$$

$$n = 4\,900 \text{ N} + 20\,000 \text{ N} = \boxed{2.49 \times 10^4 \text{ N}}$$

(b) At point B, the centripetal acceleration is now downward, and Newton's second law now gives

$$\sum F = n - Mg = -\frac{Mv^2}{R}$$

The maximum speed at B corresponds to the case where the rollercoaster begins to fly off the track, or when $n = 0$. Then,

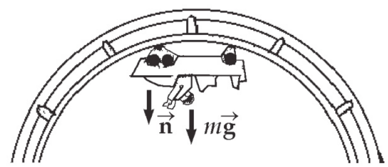
$$-Mg = -\frac{Mv_{\text{max}}^2}{R}$$

which gives

$$v_{\text{max}} = \sqrt{Rg} = \sqrt{(15.0\text{ m})(9.80\text{ m/s}^2)} = \boxed{12.1\text{ m/s}}$$

P6.17 (a) $a_c = \dfrac{v^2}{r}$

$$r = \frac{v^2}{a_c} = \frac{(13.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{8.62\text{ m}}$$



ANS. FIG. P6.17

(b) Let n be the force exerted by the rail.

Newton's second law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg,\text{ downward}}$$

(c) $a_c = \dfrac{v^2}{r}$, or $a_c = \dfrac{(13.0\text{ m/s})^2}{20.0\text{ m}} = \boxed{8.45\text{ m/s}^2}$

(d) If the force exerted by the rail is n_1,

then $n_1 + Mg = \dfrac{Mv^2}{r} = Ma_c$

$n_1 = M(a_c - g)$ which is < 0 since $a_c = 8.45\text{ m/s}^2$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars.

In a teardrop-shaped loop, the radius of curvature r decreases, causing the centripetal acceleration to increase. The speed would decrease as the car rises (because of gravity), but the overall effect is that the required centripetal force increases, meaning the normal force increases--there is less danger if not wearing a seatbelt.

P6.18 (a) Consider radial forces on the object, taking inward as positive.

$$\sum F_r = ma_r: \quad T - mg\cos\theta = \frac{mv^2}{r}$$

Solving for the tension gives

$$T = mg\cos\theta + \frac{mv^2}{r}$$
$$= (0.500 \text{ kg})(9.80 \text{ m/s}^2)\cos 20.0°$$
$$+ (0.500 \text{ kg})(8.00 \text{ m/s})^2/2.00 \text{ m}$$
$$= 4.60 \text{ N} + 16.0 \text{ N} = \boxed{20.6 \text{ N}}$$

(b) We already found the radial component of acceleration,

$$a_r = \frac{v^2}{r} = \frac{(8.00 \text{ m/s})^2}{2.00 \text{ m}} = \boxed{32.0 \text{ m/s}^2 \text{ inward}}$$

Consider the tangential forces on the object:

$$\sum F_t = ma_t: \quad mg\sin\theta = ma_t$$

Solving for the tangential component of acceleration gives

$$a_t = g\sin\theta = \left(9.80 \text{ m/s}^2\right)\sin 20.0°$$
$$= \boxed{3.35 \text{ m/s}^2 \text{ downward tangent to the circle}}$$

(c) The magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(32.0 \text{ m/s}^2\right)^2 + \left(3.35 \text{ m/s}^2\right)^2} = 32.2 \text{ m/s}^2$$

at an angle of

$$\tan^{-1}\left(\frac{3.35 \text{ m/s}^2}{32.0 \text{ m/s}^2}\right) = 5.98°$$

Thus, the acceleration is

$$\boxed{32.2 \text{ m/s}^2 \text{ inward and below the cord at } 5.98°}$$

(d) No change.

(e) If the object is swinging down it is gaining speed, and if the object is swinging up it is losing speed, but the forces are the same; therefore, its acceleration is regardless of the direction of swing.

P6.19 Let the tension at the lowest point be T. From Newton's second law, $\sum F = ma$ and

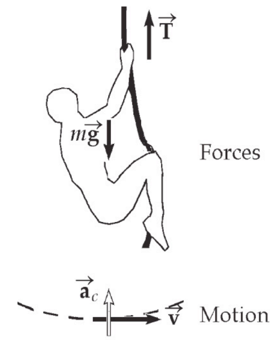
$$T - mg = ma_c = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right)$$

$$T = (85.0\ \text{kg})\left[9.80\ \text{m/s}^2 + \frac{(8.00\ \text{m/s})^2}{10.0\ \text{m}}\right]$$

$$= 1.38\ \text{kN} > 1\,000\ \text{N}$$

$$\boxed{\text{He doesn't make it across the river because the vine breaks.}}$$



ANS. FIG. P6.19

Section 6.3 Motion in Accelerated Frames

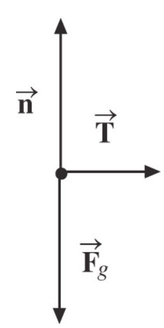
P6.20 (a) From $\sum F_x = Ma$, we obtain

$$a = \frac{T}{M} = \frac{18.0\ \text{N}}{5.00\ \text{kg}} = \boxed{3.60\ \text{m/s}^2} \text{ to the right}$$

(b) If v = const, $a = 0$, so $\boxed{T = 0}$. (This is also an equilibrium situation.)

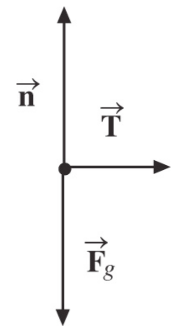
(c) $\boxed{\text{Someone in the car (noninertial observer) claims that the forces on the mass along } x \text{ are } T \text{ and a fictitious force } (-Ma).}$

(d) $\boxed{\text{Someone at rest outside the car (inertial observer) claims that } T \text{ is the only force on } M \text{ in the } x \text{ direction.}}$



ANS. FIG. P6.20

P6.21 The only forces acting on the suspended object are the force of gravity $m\vec{\mathbf{g}}$ and the force of tension T forward and upward at angle θ with the vertical, as shown in the free-body diagram in ANS. FIG. P6.21. Applying Newton's second law in the x and y directions,



ANS. FIG. P6.21

$$\sum F_x = T\sin\theta = ma \qquad [1]$$

$$\sum F_y = T\cos\theta - mg = 0$$

or $$T\cos\theta = mg \qquad [2]$$

(a) Dividing equation [1] by [2] gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.306$$

Solving for θ, $\theta = \boxed{17.0°}$

(b) From equation [1],

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\ \text{kg})(3.00\ \text{m/s}^2)}{\sin(17.0°)} = \boxed{5.12\ \text{N}}$$

P6.22 In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r} = \frac{(5.70\ \text{m/s})^2}{2.40\ \text{m}} = 13.5\ \text{m/s}^2$$

In her own noninertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2} = \sqrt{(9.80\ \text{m/s}^2)^2 + (13.5\ \text{m/s}^2)^2} = 16.7\ \text{m/s}^2$$

This is larger than g by a factor of $\dfrac{16.7\ \text{m/s}}{9.80\ \text{m/s}} = 1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F = 1.71(55.0\ \text{N}) = \boxed{93.8\ \text{N}}$$

P6.23 The scale reads the upward normal force exerted by the floor on the passenger. The maximum force occurs during upward acceleration (when starting an upward trip or ending a downward trip). The minimum normal force occurs with downward acceleration. For each respective situation,

$$\sum F_y = ma_y \quad \text{becomes for starting} \quad +591\text{ N} - mg = +ma$$

$$\text{and for stopping} \quad +391\text{ N} - mg = -ma$$

where a represents the magnitude of the acceleration.

(a) These two simultaneous equations can be added to eliminate a and solve for mg:

$$+591\text{ N} - mg + 391\text{ N} - mg = 0$$

or $$982\text{ N} - 2mg = 0$$

$$F_g = mg = \frac{982\text{ N}}{2} = \boxed{491\text{ N}}$$

(b) From the definition of weight, $m = \dfrac{F_g}{g} = \dfrac{491\text{ N}}{9.80\text{ m/s}^2} = \boxed{50.1\text{ kg}}$

(c) Substituting back gives $+591\text{ N} - 491\text{ N} = (50.1\text{ kg})a$, or

$$a = \frac{100\text{ N}}{50.1\text{ kg}} = \boxed{2.00\text{ m/s}^2}$$

P6.24 Consider forces on the backpack as it slides in the Earth frame of reference.

$$\sum F_y = ma_y: \quad +n - mg = ma,\ n = m(g+a),\ f_k = \mu_k m(g+a)$$

$$\sum F_x = ma_x: \quad -\mu_k m(g+a) = ma_x$$

The motion across the floor is described by

$$L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k(g+a)t^2$$

We solve for μ_k:

$$vt - L = \frac{1}{2}\mu_k(g+a)t^2$$

$$\boxed{\mu_k = \frac{2(vt-L)}{(g+a)t^2}}$$

P6.25 The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.120\text{ m})}{7.25\text{ s}} = 0.104\text{ m/s}$$

The top layer of water feels a downward force of gravity mg and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m\left(0.104 \text{ m/s}\right)^2}{0.12 \text{ m}} = m\,9.01\times10^{-2} \text{ m/s}^2$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\left(\frac{0.090\ 1 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) = \boxed{0.527°}$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

Section 6.4 Motion in the Presence of Velocity-Dependent Resistive Forces

P6.26 (a) $\rho = \dfrac{m}{V}$, $A = 0.020\ 1 \text{ m}^2$, $R = \dfrac{1}{2}\rho_{\text{air}} A D v_T^2 = mg$

$$m = \rho_{\text{bead}} V = 0.830 \text{ g/cm}^3\left[\frac{4}{3}\pi\left(8.00 \text{ cm}\right)^3\right] = 1.78 \text{ kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2\left(1.78 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)}{0.500\left(1.20 \text{ kg/m}^3\right)\left(0.020\ 1 \text{ m}^2\right)}} = \boxed{53.8 \text{ m/s}}$$

(b) From $v_f^2 = v_i^2 + 2gh = 0 + 2gh$, we solve for h:

$$h = \frac{v_f^2}{2g} = \frac{\left(53.8 \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{148 \text{ m}}$$

P6.27 With 100 km/h = 27.8 m/s, the resistive force is

$$R = \frac{1}{2}D\rho A v^2 = \frac{1}{2}(0.250)\left(1.20 \text{ kg/m}^3\right)\left(2.20 \text{ m}^2\right)\left(27.8 \text{ m/s}\right)^2$$
$$= 255 \text{ N}$$

$$a = -\frac{R}{m} = -\frac{255 \text{ N}}{1\ 200 \text{ kg}} = \boxed{-0.212 \text{ m/s}^2}$$

P6.28 Given m = 80.0 kg, v_T = 50.0 m/s, we write

$$mg = \frac{D\rho A v_T^2}{2}$$

which gives

$$\frac{D\rho A}{2} = \frac{mg}{v_T^2} = 0.314 \text{ kg/m}$$

(a) At v = 30.0 m/s,

$$a = g - \frac{D\rho A v^2/2}{m} = 9.80 \text{ m/s}^2 - \frac{(0.314 \text{ kg/m})(30.0 \text{ m/s})^2}{80.0 \text{ kg}}$$

$$= \boxed{6.27 \text{ m/s}^2 \text{ downward}}$$

(b) At v = 50.0 m/s, terminal velocity has been reached.

$$\sum F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{784 \text{ N directed up}}$$

(c) At v = 30.0 m/s,

$$\frac{D\rho A v^2}{2} = (0.314 \text{ kg/m})(30.0 \text{ m/s})^2 = \boxed{283 \text{ N upward}}$$

P6.29 Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward):

$$F = mg + bv$$

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi(8.92 \times 10^3 \text{ kg/m}^3)(2.00 \times 10^{-2} \text{ m})^3$$

$$= 0.299 \text{ kg}$$

The applied force is then

$$F = mg + bv = (0.299 \text{ kg})(9.80 \text{ m/s}^2) + (0.950 \text{ kg/s})(9.00 \times 10^{-2} \text{ m/s})$$

$$= \boxed{3.01 \text{ N}}$$

P6.30 (a) The acceleration of the Styrofoam is given by

$$a = g - Bv$$

When $v = v_T$, $a = 0$ and $g = Bv_T \rightarrow B = \dfrac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus,

$$v_T = \frac{h}{\Delta t} = \frac{1.50\text{ m}}{5.00\text{ s}} = 0.300\text{ m/s}$$

Then

$$\boxed{B = \frac{g}{v_T} = \frac{9.80\text{ m/s}^2}{0.300\text{ m/s}} = 32.7\text{ s}^{-1}}$$

(b) At $t = 0$, $v = 0$, and $a = g = \boxed{9.80\text{ m/s}^2\text{ down}}$

(c) When $v = 0.150$ m/s,

$$\begin{aligned} a &= g - Bv \\ &= 9.80\text{ m/s}^2 - (32.7\text{ s}^{-1})(0.150\text{ m/s}) \\ &= \boxed{4.90\text{ m/s}^2\text{ down}} \end{aligned}$$

P6.31 We have a particle under a net force in the special case of a resistive force proportional to speed, and also under the influence of the gravitational force.

(a) The speed v varies with time according to Equation 6.6,

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

where $v_T = mg/b$ is the terminal speed. Hence,

$$b = \frac{mg}{v_T} = \frac{(3.00 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{2.00 \times 10^{-2}\text{ m/s}} = \boxed{1.47\text{ N}\cdot\text{s/m}}$$

(b) To find the time interval for v to reach $0.632v_T$, we substitute $v = 0.632v_T$ into Equation 6.6, giving

$$0.632v_T = v_T(1 - e^{-bt/m}) \qquad \text{or} \qquad 0.368 = e^{-(1.47t/0.003\,00)}$$

Solve for t by taking the natural logarithm of each side of the equation:

$$\ln(0.368) = -\frac{1.47\,t}{3.00 \times 10^{-3}} \qquad \text{or} \qquad -1 = -\frac{1.47\,t}{3.00 \times 10^{-3}}$$

or $$t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04\times 10^{-3}\ \text{s}}$$

(c) At terminal speed, $R = v_T b = mg$. Therefore,

$$R = v_T b = mg = \left(3.00\times 10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = \boxed{2.94\times 10^{-2}\ \text{N}}$$

P6.32 We write

$$-kmv^2 = -\frac{1}{2}D\rho A v^2$$

so

$$k = \frac{D\rho A}{2m} = \frac{0.305\left(1.20\ \text{kg/m}^3\right)\left(4.2\times 10^{-3}\ \text{m}^2\right)}{2(0.145\ \text{kg})} = 5.3\times 10^{-3}/\text{m}$$

solving for the velocity as the ball crosses home plate gives

$$v = v_i e^{-kx} = (40.2\ \text{m/s})e^{-\left(5.3\times 10^{-3}/\text{m}\right)(18.3\ \text{m})} = \boxed{36.5\ \text{m/s}}$$

P6.33 We start with Newton's second law,

$$\sum F = ma$$

substituting,

$$-kmv^2 = m\frac{dv}{dt}$$

$$-kdt = \frac{dv}{v^2}$$

$$-k\int_0^t dt = \int_{v_i}^{v} v^{-2}\,dv$$

integrating both sides gives

$$-k(t-0) = \left.\frac{v^{-1}}{-1}\right|_{v_i}^{v} = -\frac{1}{v} + \frac{1}{v_i}$$

$$\frac{1}{v} = \frac{1}{v_i} + kt = \frac{1 + v_i kt}{v_i}$$

$$\boxed{v = \frac{v_i}{1 + v_i kt}}$$

P6.34 (a) Since the window is vertical, the normal force is horizontal and is given by n = 4.00 N. To find the vertical component of the force, we note that the force of kinetic friction is given by

$$f_k = \mu_k n = 0.900(4.00\text{ N}) = 3.60\text{ N upward}$$

to oppose downward motion. Newton's second law then becomes

$$\sum F_y = ma_y: \quad +3.6\text{ N} - (0.16\text{ kg})(9.8\text{ m/s}^2) + P_y = 0$$

$$P_y = -2.03\text{ N} = \boxed{2.03\text{ N down}}$$

(b) Now, with the increased downward force, Newton's second law gives

$$\sum F_y = ma_y:$$

$$+3.60\text{ N} - (0.160\text{ kg})(9.80\text{ m/s}^2) - 1.25(2.03\text{ N}) = 0.160\text{ kg } a_y$$

then

$$a_y = -0.508\text{ N}/0.16\text{ kg} = -3.18\text{ m/s}^2 = \boxed{3.18\text{ m/s}^2\text{ down}}$$

(c) At terminal velocity,

$$\sum F_y = ma_y: \quad +(20.0\text{ N}\cdot\text{s/m})v_T - (0.160\text{ kg})(9.80\text{ m/s}^2) - 1.25(2.03\text{ N}) = 0$$

Solving for the terminal velocity gives

$$v_T = 4.11\text{ N}/(20\text{ N}\cdot\text{s/m}) = \boxed{0.205\text{ m/s down}}$$

P6.35 (a) We must fit the equation $v = v_i e^{-ct}$ to the two data points:

At t = 0, v = 10.0 m/s, so $v = v_i e^{-ct}$ becomes

$$10.0\text{ m/s} = v_i e^0 = (v_i)(1)$$

which gives v_i = 10.0 m/s

At t = 20.0 s, v = 5.00 m/s so the equation becomes

$$5.00\text{ m/s} = (10.0\text{ m/s})e^{-c(20.0\text{ s})}$$

giving $\quad 0.500 = e^{-c(20.0\text{ s})}$

or $\quad -20.0c = \ln\left(\frac{1}{2}\right) \rightarrow c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47\times 10^{-2}\text{ s}^{-1}}$

(b) At t = 40.0 s

$$v = (10.0\text{ m/s})e^{-40.0c} = (10.0\text{ m/s})(0.250) = \boxed{2.50\text{ m/s}}$$

(c) The acceleration is the rate of change of the velocity:

$$a = \frac{dv}{dt} = \frac{d}{dt}v_i e^{-ct} = v_i\left(e^{-ct}\right)(-c) = -c\left(v_i e^{-ct}\right)$$
$$= \boxed{-cv}$$

Thus, the acceleration is a negative constant times the speed.

P6.36 In $R = \frac{1}{2}D\rho A v^2$, we estimate that the coefficient of drag for an open palm is $D = 1.00$, the density of air is $\rho = 1.20\ \text{kg/m}^3$, the area of an open palm is $A = (0.100\ \text{m})(0.160\ \text{m}) = 1.60 \times 10^{-2}\ \text{m}^2$, and $v = 29.0$ m/s (65 miles per hour). The resistance force is then

$$R = \frac{1}{2}(1.00)\left(1.20\ \text{kg/m}^3\right)\left(1.60\times 10^{-2}\ \text{m}^2\right)(29.0\ \text{m/s})^2 = 8.07\ \text{N}$$

or $R \sim \boxed{10^1\ \text{N}}$

Additional Problems

P6.37 Because the car travels at a constant speed, it has no tangential acceleration, but it does have centripetal acceleration because it travels along a circular arc. The direction of the centripetal acceleration is toward the center of curvature, and the direction of velocity is tangent to the curve.

Point A

direction of velocity: $\boxed{\text{East}}$

direction of the centripetal acceleration: $\boxed{\text{South}}$

Point B

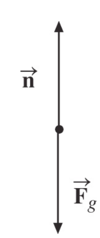
direction of velocity: $\boxed{\text{South}}$

direction of the centripetal acceleration: $\boxed{\text{West}}$

P6.38 The free-body diagram of the passenger is shown in ANS. FIG. P6.38. From Newton's second law,

$$\sum F_y = ma_y$$

$$n - mg = \frac{mv^2}{r}$$



ANS. FIG. P6.38

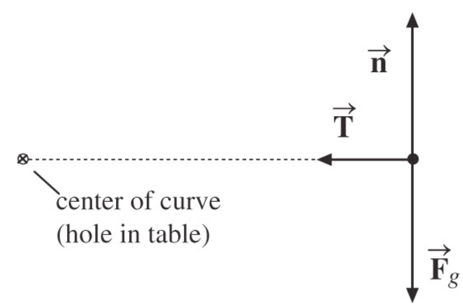
which gives

$$n = mg + \frac{mv^2}{r}$$

$$= (50\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(50.0\ \text{kg})(19\ \text{m/s})^2}{25\ \text{m}}$$

$$= \boxed{1.2 \times 10^3\ \text{N}}$$

P6.39 The free-body diagram of the rock is shown in ANS. FIG. P6.39. Take the x direction inward toward the center of the circle. The mass of the rock does not change. We know when $r_1 = 2.50$ m, $v_1 = 20.4$ m/s, and $T_1 = 50.0$ N. To find T_2 when $r_2 = 1.00$ m, and $v_2 = 51.0$ m/s, we use Newton's second law in the horizontal direction:



ANS. FIG. P6.39

$$\sum F_x = ma_x$$

In both cases,

$$T_1 = \frac{mv_1^2}{r_1} \quad \text{and} \quad T_2 = \frac{mv_2^2}{r_2}$$

Taking the ratio of the two tensions gives

$$\frac{T_2}{T_1} = \frac{v_2^2}{v_1^2}\frac{r_1}{r_2} = \left(\frac{51.0\ \text{m/s}}{20.4\ \text{m/s}}\right)^2 \left(\frac{2.50\ \text{m}}{1.00\ \text{m}}\right) = 15.6$$

then

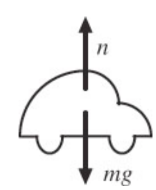
$$T_2 = 15.6T_1 = 15.6(50.0\ \text{N}) = \boxed{781\ \text{N}}$$

We assume the tension in the string is not altered by friction from the hole in the table.

P6.40 (a) We first convert the speed of the car to SI units:

$$v = (30\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right)$$

$$= 8.33\ \text{m/s}$$



ANS. FIG. P6.40

Newton's second law in the vertical direction then gives

$$\sum F_y = ma_y: \qquad +n - mg = -\frac{mv^2}{r}$$

Solving for the normal force,

$$n = m\left(g - \frac{v^2}{r}\right)$$

$$= (1\,800 \text{ kg})\left[9.80 \text{ m/s}^2 - \frac{(8.33 \text{ m/s})^2}{20.4 \text{ m}}\right]$$

$$= \boxed{1.15 \times 10^4 \text{ N up}}$$

(b) At the maximum speed, the weight of the car is just enough to provide the centripetal force, so $n = 0$. Then $mg = \frac{mv^2}{r}$ and

$$v = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(20.4 \text{ m})} = \boxed{14.1 \text{ m/s}} = 50.9 \text{ km/h}$$

P6.41 (a) The free-body diagram in ANS. FIG. P6.40 shows the forces on the car in the vertical direction. Newton's second law then gives

$$\sum F_y = ma_y = \frac{mv^2}{R}$$

$$mg - n = \frac{mv^2}{R} \qquad n = \boxed{mg - \frac{mv^2}{R}}$$

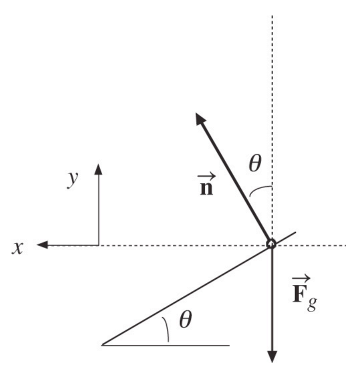
(b) When $n = 0$, $\qquad mg = \frac{mv^2}{R}$

Then, $v = \boxed{\sqrt{gR}}$

A more gently curved bump, with larger radius, allows the car to have a higher speed without leaving the road. This speed is proportional to the square root of the radius.

P6.42 The free-body diagram for the object is shown in ANS. FIG. P6.42. The object travels in a circle of radius $r = L\cos\theta$ about the vertical rod.

Taking inward toward the center of the circle as the positive x direction, we have



ANS. FIG. P6.42

$$\sum F_x = ma_x: \quad n\sin\theta = \frac{mv^2}{r}$$

$$\sum F_y = ma_y:$$

$$n\cos\theta - mg = 0 \rightarrow n\cos\theta = mg$$

Dividing, we find

$$\frac{n\sin\theta}{n\cos\theta}=\frac{mv^2/r}{gr} \quad\rightarrow\quad \tan\theta=\frac{v^2}{gr}$$

Solving for v gives

$$v^2 = gr\tan\theta$$

$$v^2 = g(L\cos\theta)\tan\theta$$

$$\boxed{v=(gL\sin\theta)^{1/2}}$$

P6.43 Let v_i represent the speed of the object at time 0. We have

$$\int_{v_i}^{v}\frac{dv}{v}=-\frac{b}{m}\int_i^t dt \qquad \ln v\Big|_{v_i}^{v}=-\frac{b}{m}t\Big|_i^t$$

$$\ln v-\ln v_i=-\frac{b}{m}(t-0) \qquad \ln\left(v/v_i\right)=-\frac{bt}{m}$$

$$v/v_i=e^{-bt/m} \qquad \boxed{v=v_ie^{-bt/m}}$$

From its original value, the speed decreases rapidly at first and then more and more slowly, asymptotically approaching zero.

In this model the object keeps losing speed forever. It travels a finite distance in stopping.

The distance it travels is given by

$$\int_0^r dr=v_i\int_0^t e^{-bt/m}dt$$

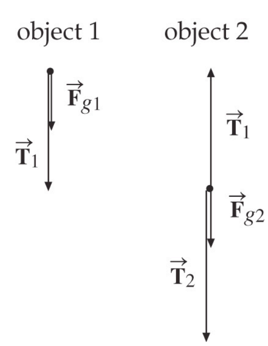
$$r=-\frac{m}{b}v_i\int_0^t e^{-bt/m}\left(-\frac{b}{m}dt\right)=-\frac{m}{b}v_i\,e^{-bt/m}\Big|_0^t$$

$$=-\frac{m}{b}v_i\left(e^{-bt/m}-1\right)=\frac{mv_i}{b}\left(1-e^{-bt/m}\right)$$

As t goes to infinity, the distance approaches $\frac{mv_i}{b}(1-0)=mv_i/b$.

P6.44 The radius of the path of object 1 is twice that of object 2. Because the strings are always "collinear," both objects take the same time interval to travel around their respective circles; therefore, the speed of object 1 is twice that of object 2.

The free-body diagrams are shown in ANS. FIG. P6.44. We are given $m_1 = 4.00$ kg, $m_2 = 3.00$ kg, $v = 4.00$ m/s, and $\ell = 0.500$ m.



ANS. FIG. P6.44

Taking down as the positive direction, we have

$$\text{Object 1: } T_1 + m_1 g = \frac{m_1 v_1^2}{r_1}, \quad \text{where } v_1 = 2v,\ r_1 = 2\ell.$$

$$\text{Object 2: } T_2 - T_1 + m_2 g = \frac{m_2 v_2^2}{r_2}, \quad \text{where } v_2 = v,\ r_2 = 2\ell.$$

(a) From above:

$$T_1 = \frac{m_1 v_1^2}{r_1} - m_1 g = m_1\left(\frac{v_1^{\ 2}}{r_1} - g\right)$$

$$T_1 = (4.00\text{ kg})\left[\frac{[2(4.00\text{ m/s})]^2}{2(0.500\text{ m})} - 9.80\text{ m/s}^2\right]$$

$$T_1 = 216.8\text{ N} = \boxed{217\text{ N}}$$

(b) From above:

$$T_2 = T_1 + \frac{m_2 v_2^2}{r_2} - m_2 g$$

$$T_2 = T_1 + m_2\left(\frac{v_2^{\ 2}}{r_2} - g\right)$$

$$T_2 = T_1 + (3.00\text{ kg})\left[\frac{(4.00\text{ m/s})^2}{0.500\text{ m}} - 9.80\text{ m/s}^2\right]$$

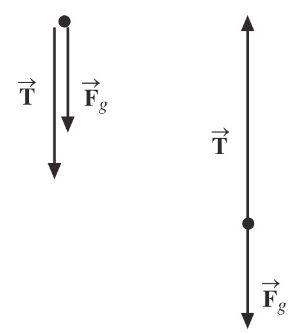
$$T_2 = 216.8\text{ N} + 66.6\text{ N} = 283.4\text{ N} = \boxed{283\text{ N}}$$

(c) $\boxed{\text{From above, } T_2 > T_1 \text{ always, so string 2 will break first.}}$

P6.45 (a) At each point on the vertical circular path, two forces are acting on the ball (see ANS. FIG. P6.45):

(1) The downward gravitational force with constant magnitude $F_g = mg$

(2) The tension force in the string, always directed toward the center of the path



ANS. FIG. P6.45

(b) ANS. FIG. P6.45 shows the forces acting on the ball when it is at the highest point on the path (left-hand diagram) and when it is at the bottom of the circular path (right-hand diagram). Note that the gravitational force has the same magnitude and direction at each point on the circular path. The tension force varies in magnitude at different points and is always directed toward the center of the path.

(c) At the top of the circle, $F_c = mv^2/r = T + F_g$, or

$$T = \frac{mv^2}{r} - F_g = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

$$= (0.275\ \text{kg})\left[\frac{(5.20\ \text{m/s})^2}{0.850\ \text{m}} - 9.80\ \text{m/s}^2\right] = \boxed{6.05\ \text{N}}$$

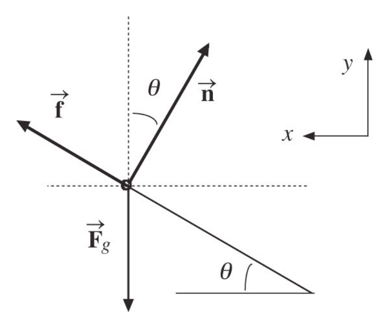
(d) At the bottom of the circle, $F_c = mv^2/r = T - F_g = T - mg$, and solving for the speed gives

$$v^2 = \frac{r}{m}(T - mg) = r\left(\frac{T}{m} - g\right) \quad \text{and} \quad v = \sqrt{r\left(\frac{T}{m} - g\right)}$$

If the string is at the breaking point at the bottom of the circle, then $T = 22.5$ N, and the speed of the object at this point must be

$$v = \sqrt{(0.850\ \text{m})\left(\frac{22.5\ \text{N}}{0.275\ \text{kg}} - 9.80\ \text{m/s}^2\right)} = \boxed{7.82\ \text{m/s}}$$

P6.46 The free-body diagram is shown on the right, where it is assumed that friction points up the incline, otherwise, the child would slide down the incline. The net force is directed left toward the center of the circular path in which the child travels. The radius of this path is $R = d\cos\theta$.



ANS. FIG. P6.46

Three forces act on the child, a normal force, static friction, and gravity. The relations of their force components are:

$$\sum F_x: \quad f_s \cos\theta - n \sin\theta = mv^2/R \qquad \textbf{[1]}$$

$$\sum F_y: \quad f_s \sin\theta + n\cos\theta - mg = 0 \rightarrow$$

$$f_s \sin\theta + n\cos\theta = mg \qquad \textbf{[2]}$$

Solve for the static friction and normal force.

To solve for static friction, multiply equation [1] by $\cos\theta$ and equation [2] by $\sin\theta$ and add:

$$\cos\theta\left[f_s\cos\theta - n\sin\theta\right] + \sin\theta\left[f_s\sin\theta - n\cos\theta\right] = \cos\theta\left(\frac{mv^2}{R}\right) + \sin\theta(mg)$$

$$f_s = mg\sin\theta + \left(\frac{mv^2}{R}\right)\cos\theta$$

To solve for the normal force, multiply equation [1] by $-\sin\theta$ and equation [2] by $\cos\theta$ and add:

$$-\sin\theta\left[f_s\cos\theta - n\sin\theta\right] + \cos\theta\left[f_s\sin\theta - n\cos\theta\right] = -\sin\theta\left(\frac{mv^2}{R}\right) + \cos\theta(mg)$$

$$n = mg\cos\theta - \left(\frac{mv^2}{R}\right)\sin\theta$$

In the above, we have used $\sin^2\theta + \cos^2\theta = 1$.

If the above equations are to be consistent, static friction and the normal force must satisfy the condition $f_s \le \mu_s n$; this means

$$(mg)\sin\theta + (mv^2/R)\cos\theta \le \mu_s[(mg)\cos\theta - (mv^2/R)\sin\theta] \rightarrow$$
$$v^2(\cos\theta + \mu_s\sin\theta) \le g\,R(\mu_s\cos\theta - \sin\theta)$$

Using this result, and that $R = d\cos\theta$, we have the requirement that

$$v \le \sqrt{\frac{gd\cos\theta(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}}$$

If this condition cannot be met, if v is too large, the physical situation cannot exist.

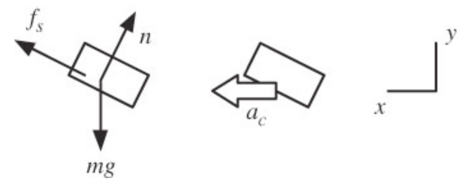
The values given in the problem are d = 5.32 m, μ_s = 0.700, θ = 20.0°, and v = 3.75 m/s. Check whether the given value of v satisfies the above condition:

$$\sqrt{\frac{(9.80\text{ m/s}^2)(5.32\text{ m})\cos 20.0°\left[(0.700)\cos 20.0° - \sin 20.0°\right]}{(\cos 20.0° + 0.700\sin 20.0°)}}$$
$$= 3.62\text{ m/s}$$

The situation is impossible because the speed of the child given in the problem is too large: static friction could not keep the child in place on the incline.

P6.47 (a) The speed of the bag is

$$\frac{2\pi(7.46\ \text{m})}{38\ \text{s}} = 1.23\ \text{m/s}$$



ANS. FIG. P6.47

The total force on it must add to

$$ma_c = \frac{mv^2}{r}$$

$$= \frac{(30\ \text{kg})(1.23\ \text{m/s})^2}{7.46\ \text{m}} = 6.12\ \text{N}$$

Newton's second law gives

$$\sum F_x = ma_x: \quad f_s\cos 20.0° - n\sin 20.0° = 6.12\ \text{N}$$

$$\sum F_y = ma_y: \quad f_s\sin 20.0° + n\cos 20.0° - (30.0\ \text{kg})(9.80\ \text{m/s}^2) = 0$$

Solving for the normal force gives

$$n = \frac{f_s\cos 20.0° - 6.12\ \text{N}}{\sin 20.0°}$$

Substituting,

$$f_s\sin 20.0° + f_s\frac{\cos^2 20.0°}{\sin 20.0°} - (6.12\ \text{N})\frac{\cos 20.0°}{\sin 20.0°} = 294\ \text{N}$$

$$f_s(2.92) = 294\ \text{N} + 16.8\ \text{N}$$

$$f_s = \boxed{106\ \text{N}}$$

(b) The speed of the bag is now

$$v = \frac{2\pi(7.94\ \text{m})}{34\ \text{s}} = 1.47\ \text{m/s}$$

which corresponds to a total force of

$$ma_c = \frac{mv^2}{r}$$

$$= \frac{(30\ \text{kg})(1.47\ \text{m/s})^2}{7.94\ \text{m}} = 8.13\ \text{N}$$

Newton's second law then gives

$$f_s\cos 20 - n\sin 20 = 8.13\ \text{N}$$

$$f_s\sin 20 + n\cos 20 = 294\ \text{N}$$

Solving for n,

$$n = \frac{f_s\cos 20.0° - 8.13\ \text{N}}{\sin 20.0°}$$

Substituting,

$$f_s \sin 20.0° + f_s \frac{\cos^2 20.0°}{\sin 20.0°} - (8.13\ \text{N}) \frac{\cos 20.0°}{\sin 20.0°} = 294\ \text{N}$$

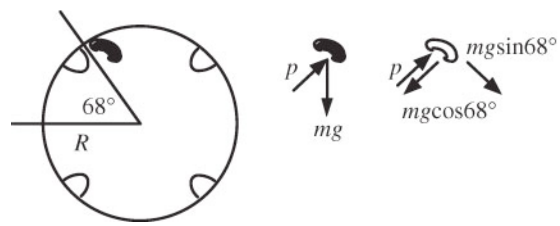
$$f_s (2.92) = 294\ \text{N} + 22.4\ \text{N}$$

$$f_s = 108\ \text{N}$$

$$n = \frac{(108\ \text{N})\cos 20.0° - 8.13\ \text{N}}{\sin 20.0°} = 273\ \text{N}$$

$$\mu_s = \frac{f_s}{n} = \frac{108\ \text{N}}{273\ \text{N}} = \boxed{0.396}$$

P6.48 When the cloth is at a lower angle θ, the radial component of $\sum F = ma$ reads



ANS. FIG. P6.48

$$n + mg \sin\theta = \frac{mv^2}{r}$$

At $\theta = 68.0°$, the normal force drops to zero and $g \sin 68° = \frac{v^2}{r}$:

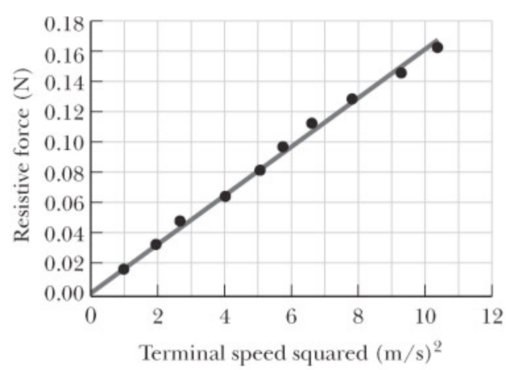
$$v = \sqrt{rg \sin 68°} = \sqrt{(0.33\ \text{m})(9.8\ \text{m/s}^2)\sin 68°} = 1.73\ \text{m/s}$$

The rate of revolution is

$$\text{angular speed} = (1.73\ \text{m/s})\left(\frac{1\ \text{rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi(0.33\ \text{m})}\right)$$

$$= \boxed{0.835\ \text{rev/s}} = 50.1\ \text{rev/min}$$

P6.49 The graph in Figure 6.16b is shown in ANS. FIG. P6.49.



ANS. FIG. P6.49

(a) The graph line is straight, so we may use any two points on it to find the slope. It is convenient to take the origin as one point, and we read (9.9 m^2/s^2, 0.16 N) as the coordinates of another point. Then the slope is

$$\text{slope} = \frac{0.160\ \text{N} - 0}{9.9\ \text{m}^2/\text{s}^2} = \boxed{0.016\ 2\ \text{kg/m}}$$

(b) In $R = \frac{1}{2}D\rho A v^2$, we identify the vertical-axis variable as R and the horizontal-axis variable as v^2. Then the slope is

$$\text{slope} = \frac{R}{v^2} = \frac{\frac{1}{2}D\rho A v^2}{v^2} = \boxed{\frac{1}{2}D\rho A}$$

(c) We follow the directions in the problem statement:

$$\frac{1}{2}D\rho A = 0.0162\ \text{kg/m}$$

$$D = \frac{2(0.0162\ \text{kg/m})}{(1.20\ \text{kg/m}^3)\pi(0.105\ \text{m})^2} = \boxed{0.778}$$

(d) From the table, the eighth point is at force

$$mg = 8(1.64\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2) = 0.129\ \text{N}$$

and horizontal coordinate $(2.80\ \text{m/s})^2$. The vertical coordinate of the line is here

$$(0.016\,2\ \text{kg/m})(2.80\ \text{m/s})^2 = 0.127\ \text{N}$$

The scatter percentage is

$$\frac{0.129\ \text{N} - 0.127\ \text{N}}{0.127\ \text{N}} = \boxed{1.5\%}$$

(e) The interpretation of the graph can be stated thus:

For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as 2%. This proportionality agrees with that described by the theoretical equation $R = \frac{1}{2}D\rho A v^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

P6.50 (a) The forces acting on the ice cube are the Earth's gravitational force, straight down, and the basin's normal force, upward and inward at 35.0° with the vertical. We choose the x and y axes to be horizontal and vertical, so that the acceleration is purely in the x direction. Then

$$\sum F_x = ma_x: \quad n \sin 35° = mv^2/R$$

$$\sum F_y = ma_y: \quad n \cos 35° - mg = 0$$

Dividing eliminates the normal force:

$$n \sin 35.0°/n \cos\ 35.0° = mv^2/Rmg$$

$$\tan 35.0° = v^2/Rg$$

$$\boxed{v = \sqrt{Rg \tan 35.0°} = \sqrt{\left(6.86\ \text{m/s}^2\right)R}}$$

(b) $\boxed{\text{The mass is unnecessary.}}$

(c) The answer to (a) indicates that the speed is proportional to the square root of the radius, so $\boxed{\text{increasing the radius will make the}}$ $\boxed{\text{required speed increase.}}$

(d) The period of revolution is given by

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{Rg \tan 35.0°}} = \left(2.40\ \text{s}/\sqrt{\text{m}}\right)\sqrt{R}$$

$\boxed{\text{When the radius increases, the period increases.}}$

(e) $\boxed{\text{On a larger circle, the ice cube's speed is proportional to } \sqrt{R} \text{ but the distance it travels is proportional to } R\text{, so the time interval required is proportional to } R/\sqrt{R} = \sqrt{R}.}$

P6.51 Take the positive x axis up the hill. Newton's second law in the x direction then gives

$$\sum F_x = ma_x: \quad +T \sin\theta - mg \sin\phi = ma$$

from which we obtain

$$a = \frac{T}{m}\sin\theta - g\sin\phi \qquad \textbf{[1]}$$

In the y direction,

$$\sum F_y = ma_y: \quad +T \cos\theta - mg \cos\phi = 0$$

Solving for the tension gives

$$T = \frac{mg\cos\phi}{\cos\theta} \qquad \textbf{[2]}$$

Substituting for T from [2] into [1] gives

$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$

$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

P6.52 (a) We first convert miles per hour to feet per second:

$$v = (300\ \text{mi/h})\left(\frac{88.0\ \text{ft/s}}{60.0\ \text{mi/h}}\right) = 440\ \text{ft/s} \text{ at the top of the loop}$$

and $v = 450\ \text{mi/h} = 660\ \text{ft/s}$ at the bottom of the loop.

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160\ \text{lb} + \left(\frac{160\ \text{lb}}{32.0\ \text{ft/s}^2}\right)\frac{(660\ \text{ft/s})^2}{1\,200\ \text{ft}} = \boxed{1\,975\ \text{lb}}$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = 160\ \text{lb} - \left(\frac{160\ \text{lb}}{32.0\ \text{ft/s}^2}\right)\frac{(440\ \text{ft/s})^2}{1\,200\ \text{ft}} = \boxed{-647\ \text{lb}}$$

Since the plane is upside down, the seat exerts this downward force as a normal force.

(c) When $F_g' = 0$, then $mg = \frac{mv^2}{R}$. If we vary the aircraft's R and v such that this equation is satisfied, then the pilot feels weightless.

P6.53 (a) The only horizontal force on the car is the force of friction, with a maximum value determined by the surface roughness (described by the coefficient of static friction) and the normal force (here equal to the gravitational force on the car).

(b) From Newton's second law in one dimension,

$$\sum F_x = ma_x: \quad -f = ma \quad \rightarrow \quad a = -\frac{f}{m} = \left(v^2 - v_0^2\right)/2\left(x - x_0\right)$$

solving for the stopping distance gives

$$x - x_0 = \frac{m\left(v^2 - v_0^2\right)}{2f} = \frac{(1\ 200\ \text{kg})\left[0^2 - (20.0\ \text{m/s})^2\right]}{2(-7\ 000\ \text{N})} = \boxed{34.3\ \text{m}}$$

(c) Newton's second law now gives

$$f = \frac{mv^2}{r}$$

or $$r = \frac{mv^2}{f} = \frac{(1\,200\ \text{kg})(20.0\ \text{m/s})^2}{7\ 000\ \text{N}} = \boxed{68.6\ \text{m}}$$

A top view shows that you can avoid running into the wall by turning through a quarter-circle, if you start at least this far away from the wall.

(d) Braking is better. You should not turn the wheel. If you used any of the available friction force to change the direction of the car, it would be unavailable to slow the car, and the stopping distance would be longer.

(e) The conclusion is true in general. The radius of the curve you can barely make is twice your minimum stopping distance.

P6.54 (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or $T = \boxed{m_2 g}$.

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = \boxed{m_2 g}$.

(c) From $F_c = \dfrac{m_1 v^2}{R}$,

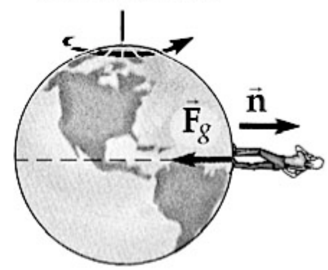
we have $v = \sqrt{\dfrac{RF_c}{m_1}} = \boxed{\sqrt{\left(\dfrac{m_2}{m_1}\right) gR}}$

(d) $\boxed{\text{The puck will spiral inward, gaining speed as it does so.}}$ It gains speed because the extra-large string tension pulls at an angle of less than 90° to the direction of the inward-spiraling velocity, producing forward tangential acceleration as well as inward radial acceleration of the puck.

(e) $\boxed{\text{The puck will spiral outward, slowing down as it does so.}}$

P6.55 (a) The gravitational force exerted by the planet on the person is

$$mg = (75.0\text{ kg})(9.80\text{ m/s}^2)$$
$$= \boxed{735\text{ N}}\text{ down}$$



ANS. FIG. P6.55

Let n represent the force exerted on the person by a scale, which is an upward force whose size is her "apparent weight." The true weight is mg down. For the person at the equator, summing up forces on the object in the direction towards the Earth's center gives $\sum F = ma$:

$$mg - n = ma_c$$

where $a_c = v^2/R_E = 0.033\,7\text{ m/s}^2$

is the centripetal acceleration directed toward the center of the Earth.

Thus, we can solve part (c) before part (b) by noting that

$$n = m(g - a_c) < mg$$

(c) or $mg = n + ma_c > n$.

$\boxed{\text{The gravitational force is greater. The normal force is smaller, just as one experiences at the top of a moving ferris wheel.}}$

(b) If $m = 75.0$ kg and $g = 9.80\text{ m/s}^2$, at the equator we have

$$n = m(g - a_c) = (75.0\text{ kg})(9.800\text{ m/s}^2 - 0.033\,7\text{ m/s}^2) = \boxed{732\text{ N}}$$

P6.56 (a) $v = v_i + kx$ implies the acceleration is

$$\boxed{a = \frac{dv}{dt} = 0 + k\frac{dx}{dt} = +kv}$$

(b) The total force is

$$\sum F = ma = m(+kv)$$

As a vector, the force is parallel or antiparallel to the velocity:

$$\boxed{\sum \vec{\mathbf{F}} = km\vec{\mathbf{v}}}$$

(c) For k positive, some feedback mechanism could be used to impose such a force on an object for a while. The object's speed rises exponentially.

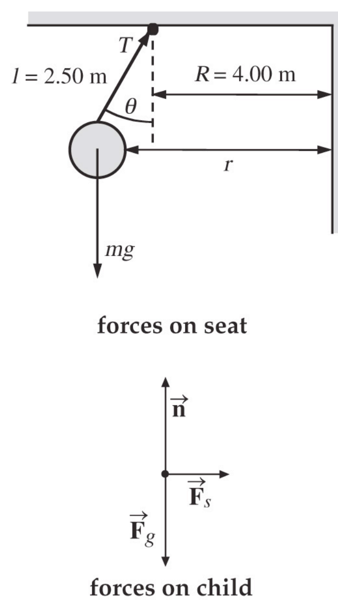
(d) For k negative, think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed.

P6.57 (a) As shown in the free-body diagram on the right, the mass at the end of the chain is in vertical equilibrium. Thus,

$$T\cos\theta = mg \qquad \textbf{[1]}$$

Horizontally, the mass is accelerating toward the center of a circle of radius r:

$$T\sin\theta = ma_r = \frac{mv^2}{r} \qquad \textbf{[2]}$$



ANS. FIG. P6.57

Here, r is the sum of the radius of the circular platform $R = D/2 = 4.00$ m and $2.50\sin\theta$:

$$r = (2.50\sin\theta + 4.00)\text{ m}$$
$$r = (2.50\sin 28.0° + 4.00)\text{ m}$$
$$= 5.17\text{ m}$$

We solve for the tension T from [1]:

$$T\cos\theta = mg \rightarrow T = \frac{mg}{\cos\theta}$$

and substitute into [2] to obtain

$$\tan\theta = \frac{a_r}{g} = \frac{v^2}{gr}$$

$$v^2 = gr\tan\theta = (9.80\text{ m/s}^2)(5.17\text{ m})(\tan 28.0°)$$

$$v = \boxed{5.19\text{ m/s}}$$

(b) The free-body diagram for the child is shown in ANS. FIG. P6.57.

(c) $T = \dfrac{mg}{\cos\theta} = \dfrac{(40.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 28.0°} = \boxed{444\text{ N}}$

P6.58 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \dfrac{2v}{g}$, where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \dfrac{2\pi R}{t}$, so $t = \dfrac{2v}{g} = \dfrac{2\pi R}{v}$.

Thus, $v^2 = \pi R g$ and $v = \boxed{\sqrt{\pi R g}}$.

(b) The putty is dislodged when F, the force holding it to the wheel, is

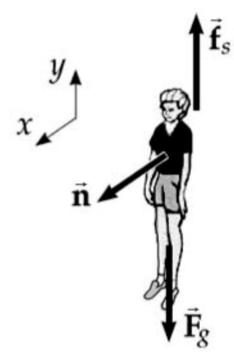
$$F = \frac{mv^2}{R} = \boxed{m\pi g}$$

P6.59 (a) The wall's normal force pushes inward:

$$\sum F_{\text{inward}} = ma_{\text{inward}}$$

becomes

$$n = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 Rm}{T^2}$$



ANS. FIG. P6.59

The friction and weight balance:

$$\sum F_{\text{upward}} = ma_{\text{upward}}$$

becomes

$$+f - mg = 0$$

so with the person just ready to start sliding down,

$$f_s = \mu_s n = mg$$

Substituting,

$$\mu_s n = \mu_s \frac{4\pi^2 Rm}{T^2} = mg$$

Solving,

$$T^2 = \frac{4\pi^2 R\mu_s}{g}$$

gives

$$\boxed{T = \sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

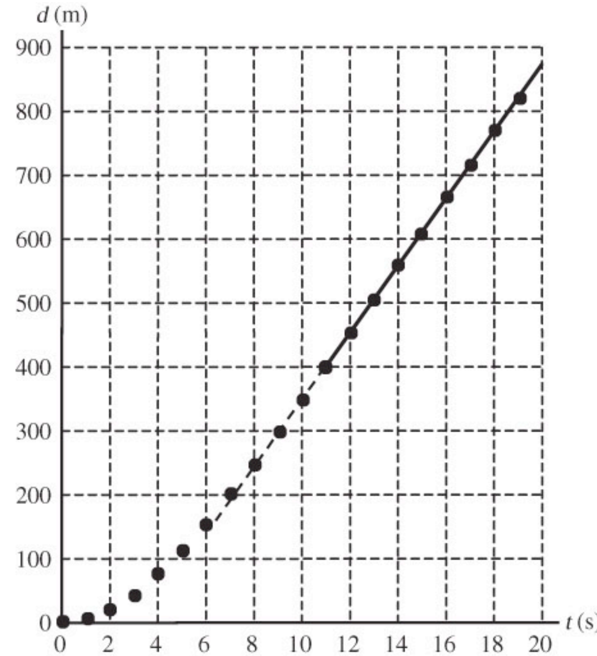
(b) The gravitational and friction forces remain constant. (Static friction adjusts to support the weight.) The normal force increases. The person remains in motion with the wall.

(c) The gravitational force remains constant. The normal and friction forces decrease. The person slides relative to the wall and downward into the pit.

P6.60 (a)

t(s)	d (m)	t(s)	d (m)
1.00	4.88	11.0	399
2.00	18.9	12.0	452
3.00	42.1	13.0	505
4.00	43.8	14.0	558
5.00	112	15.0	611
6.00	154	16.0	664
7.00	199	17.0	717
8.00	246	18.0	770
9.00	296	19.0	823
10.0	347	20.0	876

(b)



(c) A straight line fits the points from t = 11.0 s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876 \text{ m} - 399 \text{ m}}{20.0 \text{ s} - 11.0 \text{ s}} = \boxed{53.0 \text{ m/s}}$$

P6.61 (a) If the car is about to slip *down* the incline, f is directed up the incline.

$$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$$

where $f = \mu_s n$. Substituting,

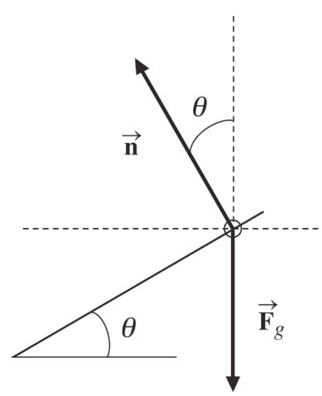
$$n = \frac{mg}{\cos\theta(1 + \mu_s \tan\theta)}$$

and $f = \dfrac{\mu_s mg}{\cos\theta(1 + \mu_s \tan\theta)}$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{\min}^2}{R}$

yields

$$\boxed{v_{\min} = \sqrt{\frac{Rg(\tan\theta - \mu_s)}{1 + \mu_s \tan\theta}}}$$



ANS. FIG. P6.61

When the car is about to slip *up* the incline, f is directed down the incline.

Then,

$$\sum F_y = n\cos\theta - f\sin\theta - mg = 0, \quad \text{with } f = \mu_s n$$

This yields

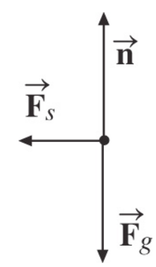
$$n = \frac{mg}{\cos\theta(1-\mu_s\tan\theta)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta(1-\mu_s\tan\theta)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\frac{v_{\max}^2}{R}$, which gives

$$\boxed{v_{\max} = \sqrt{\frac{Rg(\tan\theta+\mu_s)}{1-\mu_s\tan\theta}}}$$

(b) If $v_{\min} = \sqrt{\frac{Rg(\tan\theta-\mu_s)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

P6.62 There are three forces on the child, a vertical normal force, a horizontal force (combination of friction and a horizontal force from a seat belt), and gravity.



ANS. FIG. P6.62

$$\sum F_x: \quad F_s = mv^2/R$$

$$\sum F_y: \quad n - mg = 0 \rightarrow n = mg$$

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{(mv^2/R)^2 + (mg)^2}$$

with a direction of

$$\theta = \tan^{-1}\left[\frac{mg}{mv^2/R}\right] = \tan^{-1}\left[\frac{gR}{v^2}\right] \text{ above the horizontal}$$

For m = 40.0 kg and R = 10.0 m:

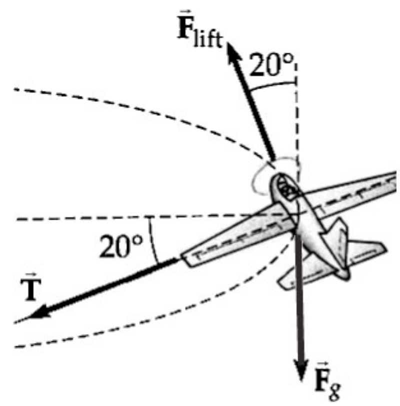
$$F_{\text{net}} = \sqrt{\left[\frac{(40.0\text{ kg})(3.00\text{ m/s})^2}{10.0\text{ m}}\right]^2 + \left[(40.0\text{ kg})(9.80\text{ m/s}^2)\right]^2}$$

$$\boxed{F_{\text{net}} = 394\text{ N}}$$

direction: $\theta = \tan^{-1}\left[\frac{(9.80\text{ m/s}^2)(10.0\text{ m})}{(3.00\text{ m/s})^2}\right] \rightarrow \boxed{\theta = 84.7°}$

P6.63 The plane's acceleration is toward the center of the circle of motion, so it is horizontal. The radius of the circle of motion is (60.0 m) cos 20.0° = 56.4 m and the acceleration is

$$a_c = \frac{v^2}{r} = \frac{(35 \text{ m/s})^2}{56.4 \text{ m}}$$
$$= 21.7 \text{ m/s}^2$$



ANS. FIG. P6.63

We can also calculate the weight of the airplane:

$$F_g = mg$$
$$= (0.750 \text{ kg})(9.80 \text{ m/s}^2)$$
$$= 7.35 \text{ N}$$

We define our axes for convenience. In this case, two of the forces—one of them our force of interest—are directed along the 20.0° line. We define the x axis to be directed in the $+\vec{T}$ direction, and the y axis to be directed in the direction of lift. With these definitions, the x component of the centripetal acceleration is

$$a_{cx} = a_c \cos 20.0°$$

and $\sum F_x = ma_x$ yields $T + F_g \sin 20.0° = ma_{cx}$

Solving for T,

$$T = ma_{cx} - F_g \sin 20.0°$$

Substituting,

$$T = (0.750 \text{ kg})(21.7 \text{ m/s}^2) \cos 20.0° - (7.35 \text{ N}) \sin 20.0°$$

Computing,

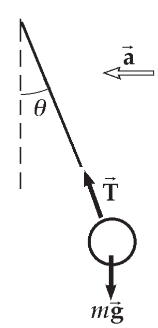
$$T = 15.3 \text{ N} - 2.51 \text{ N} = \boxed{12.8 \text{ N}}$$

***P6.64** (a) While the car negotiates the curve, the accelerometer is at the angle θ.

Horizontally: $T \sin\theta = \dfrac{mv^2}{r}$

Vertically: $T \cos\theta = mg$

where r is the radius of the curve, and v is the speed of the car.



ANS. FIG. P6.64

By division, $\tan\theta = \dfrac{v^2}{rg}$

Then

$$a_c = \frac{v^2}{r} = g\tan\theta:$$

$$a_c = \left(9.80\ \text{m/s}^2\right)\tan 15.0°$$

$$a_c = \boxed{2.63\ \text{m/s}^2}$$

(b) $r = \dfrac{v^2}{a_c}$ gives $r = \dfrac{(23.0\ \text{m/s})^2}{2.63\ \text{m/s}^2} = \boxed{201\ \text{m}}$

(c) $v^2 = rg\tan\theta = (201\ \text{m})\left(9.80\ \text{m/s}^2\right)\tan 9.00°$

$$v = \boxed{17.7\ \text{m/s}}$$

Challenge Problems

P6.65 We find the terminal speed from

$$v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right] \qquad \textbf{[1]}$$

where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$: $v \to v_T = \dfrac{mg}{b}$

At $t = 5.54$ s: $0.500v_T = v_T\left[1 - \exp\left(\dfrac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

Solving,

$$\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500$$

$$\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693$$

$$b = \frac{(9.00\ \text{kg})(0.693)}{5.54\ \text{s}} = 1.13\ \text{kg/s}$$

(a) From $v_T = \frac{mg}{b}$, we have

$$v_T = \frac{(9.00\ \text{kg})(9.80\ \text{m/s}^2)}{1.13\ \text{kg/s}} = \boxed{78.3\ \text{m/s}}$$

(b) We substitute $0.750v_T$ on the left-hand side of equation [1]:

$$0.750v_T = v_T\left[1 - \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right)\right]$$

and solve for t:

$$\exp\left(\frac{-1.13t}{9.00\ \text{s}}\right) = 0.250$$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\ \text{s} = \boxed{11.1\ \text{s}}$$

(c) We differentiate equation [1] with respect to time,

$$\frac{dx}{dt} = \left(\frac{mg}{b}\right)\left[1 - \exp\left(-\frac{bt}{m}\right)\right]$$

then, integrate both sides

$$\int_{x_0}^{x} dx = \int_0^t \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right]dt$$

$$x - x_0 = \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Bigg|_0^t$$

$$= \frac{mgt}{b} + \left(\frac{m^2 g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right) - 1\right]$$

At $t = 5.54$ s,

$$x = (9.00\ \text{kg})(9.80\ \text{m/s}^2)\left(\frac{5.54\ \text{s}}{1.13\ \text{kg/s}}\right)$$

$$+\left(\frac{(9.00\ \text{kg})^2(9.80\ \text{m/s}^2)}{(1.13\ \text{kg/s})^2}\right)\left[\exp(-0.693) - 1\right]$$

$$x = 434\ \text{m} + 626\ \text{m}(-0.500) = \boxed{121\ \text{m}}$$

P6.66 (a) From Problem 6.33,

$$v = \frac{dx}{dt} = \frac{v_i}{1 + v_i kt}$$

$$\int_0^x dx = \int_0^t v_i \frac{dt}{1 + v_i kt} = \frac{1}{k}\int_0^t \frac{v_i k dt}{1 + v_i kt}$$

$$x\Big|_0^x = \frac{1}{k}\ln\left(1 + v_i kt\right)\Big|_0^t$$

$$x - 0 = \frac{1}{k}\left[\ln\left(1 + v_i kt\right) - \ln 1\right]$$

$$\boxed{x = \frac{1}{k}\ln\left(1 + v_i kt\right)}$$

(b) We have $\ln\left(1 + v_i kt\right) = kx$

$$1 + v_i kt = e^{kx} \quad \text{so} \quad v = \frac{v_i}{1 + v_i kt} = \frac{v_i}{e^{kx}} = \boxed{v_i e^{-kx} = v}$$

P6.67 Let the x axis point eastward, the y axis upward, and the z axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{\left(9.80 \text{ m/s}^2\right)\left(285 \text{ m}\right)}{\sin 96.0^\circ}} = 53.0 \text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = \left(53.0 \text{ m/s}\right)\left(\sin 48.0^\circ\right)t - \left(4.90 \text{ m/s}^2\right)t^2$$

giving $t = \boxed{8.04 \text{ s}}$.

(b) $$v_{xi} = \frac{2\pi R_e \cos\phi_i}{86\,400 \text{ s}} = \frac{2\pi\left(6.37 \times 10^6 \text{ m}\right)\cos 35.0^\circ}{86\,400 \text{ s}} = \boxed{379 \text{ m/s}}$$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)\left(360^\circ\right) = \left(\frac{285 \text{ m}}{2\pi\left(6.37 \times 10^6 \text{ m}\right)}\right)\left(360^\circ\right)$$

$$= 2.56 \times 10^{-3} \text{ degrees}$$

The final latitude is then

$$\phi_f = \phi_i - \Delta\phi = 35.0° - 0.002\,56° = 34.997\,4°$$

The cup is moving eastward at a speed

$$v_{xf} = \frac{2\pi R_e \cos\phi_f}{86\,400 \text{ s}}$$

which is larger than the eastward velocity of the tee by

$$\Delta v_x = v_{xf} - v_{xi} = \left(\frac{2\pi R_e}{86\,400 \text{ s}}\right)\left[\cos\phi_f - \cos\phi_i\right]$$

$$= \left(\frac{2\pi R_e}{86\,400 \text{ s}}\right)\left[\cos(\phi - \Delta\phi) - \cos\phi_i\right]$$

$$= \left(\frac{2\pi R_e}{86\,400 \text{ s}}\right)\left[\cos\phi_i \cos\Delta\phi + \sin\phi_i \sin\Delta\phi - \cos\phi_i\right]$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and

$$\Delta v_x \approx \left(\frac{2\pi R_e}{86\,400 \text{ s}}\right)\sin\phi_i \sin\Delta\phi$$

$$\Delta v_x \approx \left[\frac{2\pi\left(6.37\times10^6 \text{ m}\right)}{86\,400 \text{ s}}\right]\sin 35.0° \sin 0.002\,56°$$

$$= \boxed{1.19\times10^{-2} \text{ m/s}}$$

(d) $\Delta x = (\Delta v_x)t = \left(1.19\times10^{-2} \text{ m/s}\right)(8.04 \text{ s}) = 0.095\,5 \text{ m} = \boxed{9.55 \text{ cm}}$

P6.68 (a) We let R represent the radius of the hoop and T represent the period of its rotation. The bead moves in a circle with radius $r = R\sin\theta$ at a speed of

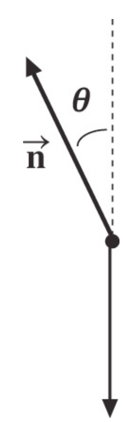
$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$

The normal force has an inward radial component of $n\sin\theta$ and an upward component of $n\cos\theta$.

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta}$$



ANS. FIG. P6.68

Then $\sum F_x = n\sin\theta = m\dfrac{v^2}{r}$ becomes

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m}{R\sin\theta}\left(\frac{2\pi R\sin\theta}{T}\right)^2$$

which reduces to $\dfrac{g\sin\theta}{\cos\theta} = \dfrac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\sin\theta = 0 \quad\Rightarrow\quad \theta = 0°$ **[1]**

and $\cos\theta = \dfrac{gT^2}{4\pi^2 R}$ **[2]**

If R = 15.0 cm and T = 0.450 s, the second solution yields

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.450\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 0.335 \quad \text{or} \quad \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions: $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution [2] above becomes

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.850\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 1.20\text{, which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$.

(c) There is only one solution for (b) because the period is too large.

(d) The equation that the angle must satisfy has two solutions whenever $4\pi^2 R > gT^2$ but only the solution 0° otherwise. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position. Zero is always a solution for the angle.

(e) From the derivation of the solution in (a), there are never more than two solutions.

P6.69 At terminal velocity, the accelerating force of gravity is balanced by friction drag:

$$mg = arv + br^2v^2$$

(a) With $r = 10.0\ \mu\text{m},\ mg = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$

For water, $m = \rho V = 1\,000\ \text{kg/m}^3\left[\frac{4}{3}\pi\left(10^{-5}\ \text{m}\right)^3\right]$

$$mg = 4.11\times10^{-11} = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$$

Assuming v is small, ignore the second term on the right hand side: $\boxed{v = 0.013\ 2\ \text{m/s}}$

(b) With $r = 100\ \mu\text{m},\ mg = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$mg = 4.11\times10^{-8} = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$$

Taking the positive root,

$$v = \frac{-3.10 + \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03\ \text{m/s}}$$

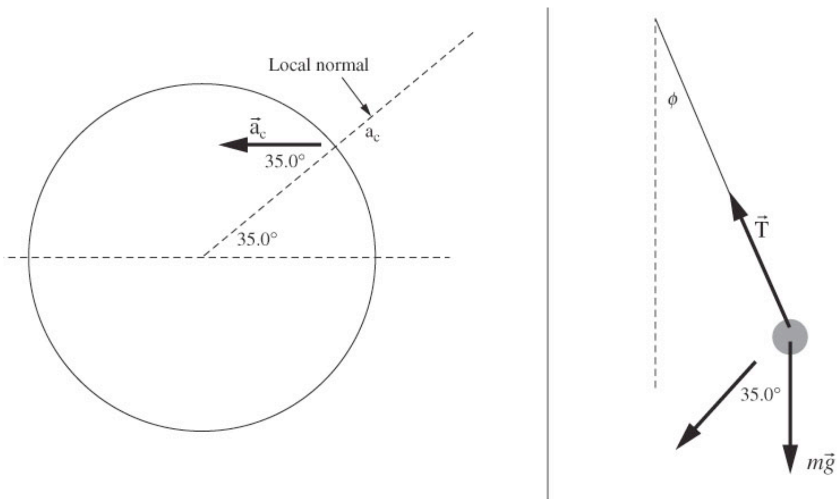
(c) With $r = 1.00\ \text{mm},\ mg = \left(3.10\times10^{-7}\right)v + \left(0.870\times10^{-6}\right)v^2$

Assuming $v > 1$ m/s, and ignoring the first term:

$$4.11\times10^{-5} = \left(0.870\times10^{-6}\right)v^2 \qquad v = \boxed{6.87\ \text{m/s}}$$

P6.70 At a latitude of 35°, the centripetal acceleration of a plumb bob is directed at 35° to the local normal, as can be seen from the following diagram below at left.

Therefore, if we look at a diagram of the forces on the plumb bob and its acceleration with the local normal in a vertical orientation, we see the second diagram in ANS. FIG. P6.70:



ANS. FIG. P6.70

We first find the centripetal acceleration of the plumb bob. The first figure shows that the radius of the circular path of the plumb bob is $R \cos 35.0°$, where R is the radius of the Earth. The acceleration is

$$a_c = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 R\cos 35.0°}{T^2}$$
$$= \frac{4\pi^2\left(6.37 \times 10^6 \text{ m}\right)\cos 35.0°}{(86\ 400 \text{ s})^2} = 0.027\ 6 \text{ m/s}^2$$

Apply the particle under a net force model to the plumb bob in both x and y directions in the second diagram:

$$x\text{:}\ \ T\sin\phi = ma_c \sin 35.0°$$
$$y\text{:}\ \ mg - T\cos\phi = ma_c \cos 35.0°$$

Divide the equations:

$$\tan\phi = \frac{a_c \sin 35.0°}{g - a_c \cos 35.0°}$$

$$\tan\phi = \frac{\left(0.027\ 6 \text{ m/s}^2\right)\sin 35.0°}{9.80 \text{ m/s}^2 - \left(0.027\ 6 \text{ m/s}^2\right)\cos 35.0°} = 1.62 \times 10^{-3}$$

$$\phi = \tan^{-1}\left(1.62 \times 10^{-3}\right) = \boxed{0.092\ 8°}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P6.2 (a) 1.65×10^3 m/s; (b) 6.84×10^3 s

P6.4 215 N, horizontally inward

P6.6 (a) $\left(-0.233\hat{\mathbf{i}}+0.163\hat{\mathbf{j}}\right)$ m/s^2; (b) 6.53 m/s, $\left(-0.181\hat{\mathbf{i}}+0.181\hat{\mathbf{j}}\right)$ m/s^2

P6.8 (a) $(68.6\text{ N})\hat{\mathbf{i}}+(784\text{ N})\hat{\mathbf{j}}$; (b) $a = 0.857$ m/s^2

P6.10 The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.

P6.12 (a) the gravitational force and the contact force exerted on the water by the pail; (b) contact force exerted by the pail; (c) 3.13 m/s; (d) the water would follow the parabolic path of a projectile

P6.14 (a) 4.81 m/s; (b) 700 N

P6.16 (a) 2.49×10^4 N; (b) 12.1 m/s

P6.18 (a) 20.6 N; (b) 32.0 m/s^2 inward, 3.35 m/s^2 downward tangent to the circle; (c) 32.2 m/s^2 inward and below the cord at 5.98°; (d) no change; (e) acceleration is regardless of the direction of swing

P6.20 (a) 3.60 m/s^2; (b) $T = 0$; (c) noninertial observer in the car claims that the forces on the mass along x are T and a fictitious force ($-Ma$); (d) inertial observer outside the car claims that T is the only force on M in the x direction

P6.22 93.8 N

P6.24 $\dfrac{2(vt-L)}{(g+a)t^2}$

P6.26 (a) 53.8 m/s; (b) 148 m

P6.28 (a) 6.27 m/s^2 downward; (b) 784 N directed up; (c) 283 N upward

P6.30 (a) 32.7 s^{-1}; (b) 9.80 m/s^2 down; (c) 4.90 m/s^2 down

P6.32 36.5 m/s

P6.34 (a) 2.03 N down; (b) 3.18 m/s^2 down; (c) 0.205 m/s down

P6.36 10^1 N

P6.38 1.2×10^3 N

P6.40 (a) 1.15×10^4 N up; (b) 14.1 m/s

P6.42 See Problem 6.42 for full derivation.

P6.44 (a) 217 N; (b) 283 N; (c) $T_2 > T_1$ always, so string 2 will break first

P6.46 The situation is impossible because the speed of the child given in the problem is too large: static friction could not keep the child in place on the incline

P6.48 0.835 rev/s

P6.50 (a) $v = \sqrt{Rg \tan 35.0°} = \sqrt{(6.86\ \text{m/s}^2)R}$; (b) the mass is unnecessary; (c) increasing the radius will make the required speed increase; (d) when the radius increases, the period increases; (e) the time interval required is proportional to $R/\sqrt{R} = \sqrt{R}$

P6.52 (a) 1 975 lb; (b) −647 lb; (c) When $F'_g = 0$, then $mg = \dfrac{mv^2}{R}$.

P6.54 (a) m_2g; (b) m_2g; (c) $\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}$; (d) The puck will spiral inward, gaining speed as it does so; (e) The puck will spiral outward, slowing down as it does so

P6.56 (a) $a = +kv$; (b) $\sum \vec{\mathbf{F}} = km\vec{\mathbf{v}}$; (c) some feedback mechanism could be used to impose such a force on an object; (d) think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed

P6.58 (a) $\sqrt{\pi Rg}$; (b) $m\pi g$

P6.60 (a) See table in P6.60 (a); (b) See graph in P6.60 (b); (c) 53.0 m/s

P6.62 84.7°

P6.64 (a) 2.63 m/s²; (b) 201 m; (c) 17.7 m/s

P6.66 (a) $x = \dfrac{1}{k}\ln(1 + v_i kt)$; (b) $v = v_i e^{-kx}$

P6.68 (a) θ = 70.4° and θ = 0°; (b) θ = 0°; (c) the period is too large; (d) Zero is always a solution for the angle; (e) there are never more than two solutions

P6.70 0.092 8°