7

Energy of a System

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ7.1 Answer (c). Assuming that the cabinet has negligible speed during the operation, all of the work Alex does is used in increasing the gravitational potential energy of the cabinet-Earth system. However, in addition to increasing the gravitational potential energy of the cabinet-Earth system by the same amount as Alex did, John must do work overcoming the friction between the cabinet and ramp. This means that the total work done by John is greater than that done by Alex.

OQ7.2 Answer (d). The work–energy theorem states that $W_{\text{net}} = \Delta K = K_f - K_i$. Thus, if $W_{\text{net}} = 0$, then $K_f - K_i$ or $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, which leads to the conclusion that the speed is unchanged ($v_f = v_i$). The velocity of the particle involves both magnitude (speed) and direction. The work–energy theorem shows that the magnitude or speed is unchanged when $W_{net} = 0$, but makes no statement about the direction of the velocity.

OQ7.3 Answer (a). The work done on the wheelbarrow by the worker is

$$W = (F\cos\theta)\Delta x = (50\text{ N})(5.0\text{ m}) = +250\text{ J}$$

OQ7.4 Answer (c). The system consisting of the cart's fixed, initial kinetic energy is the mechanical energy that can be transformed due to friction from the surface. Therefore, the loss of mechanical energy is $\Delta E_{\text{mech}} = -f_k d = -(6\text{ N})(0.06\text{ m}) = 0.36\text{ J}.$ This product must remain the same in all cases. For the cart rolling through gravel, $-(9\text{ N})(d) = 0.36\text{ J}$ tells us $d = 4$ cm.

OQ7.5 The answer is a > b = e > d > c. Each dot product has magnitude (1)·(1)·cos θ, where θ is the angle between the two factors. Thus for (a) we have cos 0 = 1. For (b) and (e), cos 45° = 0.707. For (c), cos 180° = −1. For (d), cos 90° = 0.

OQ7.6 Answer (c). The net work needed to accelerate the object from $v = 0$ to v is

$$W_1 = KE_{1f} - KE_{1i} = \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}mv^2$$

The work required to accelerate the object from speed v to speed $2v$ is

$$W_2 = KE_{2f} - KE_{2i} = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\left(4v^2 - v^2\right) = 3\left(\frac{1}{2}mv^2\right) = 3W_1$$

OQ7.7 Answer (e). As the block falls freely, only the conservative gravitational force acts on it. Therefore, mechanical energy is conserved, or $KE_f + PE_f = KE_i + PE_i$. Assuming that the block is released from rest ($KE_i = 0$), and taking $y = 0$ at ground level ($PE_f = 0$), we have that

$$KE_f = PE_i \quad \text{or} \quad \frac{1}{2}mv_f^2 = mgy \quad \text{and} \quad y_i = \frac{v_f^2}{2g}$$

Thus, to double the final speed, it is necessary to increase the initial height by a factor of four.

OQ7.8 (i) Answer (b). Tension is perpendicular to the motion. (ii) Answer (c). Air resistance is opposite to the motion.

OQ7.9 Answer (e). Kinetic energy is proportional to mass.

OQ7.10 (i) Answers (c) and (e). The force of block on spring is equal in magnitude and opposite to the force of spring on block.

(ii) Answers (c) and (e). The spring tension exerts equal-magnitude forces toward the center of the spring on objects at both ends.

OQ7.11 Answer (a). Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

OQ7.12 Answer (b). Since the rollers on the ramp used by David were frictionless, he did not do any work overcoming nonconservative forces as he slid the block up the ramp. Neglecting any change in kinetic energy of the block (either because the speed was constant or was essentially zero during the lifting process), the work done by either Mark or David equals the increase in the gravitational potential energy of the block-Earth system as the block is lifted from the ground to the truck bed. Because they lift identical blocks through the same vertical distance, they do equal amounts of work.

OQ7.13 (i) Answer: a = b = c = d. The gravitational acceleration is quite precisely constant at locations separated by much less than the radius of the planet.

(ii) Answer: c = d > a = b. The mass but not the elevation affects the gravitational force.

(iii) Answer: c > b = d > a. Gravitational potential energy of the object-Earth system is proportional to mass times height.

OQ7.14 Answer (d). $4.00\text{ J} = \frac{1}{2}k(0.100\text{ m})^2$. Therefore, $k = 800$ N/m and to stretch the spring to 0.200 m requires extra work

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\text{ J} = 12.0\text{ J}$$

OQ7.15 Answer (a). The system consisting of the cart's fixed, initial kinetic energy is the mechanical energy that can be transformed due to friction from the surface. Therefore, the loss of mechanical energy is $\Delta E_{\text{mech}} = -f_k d = -(6\text{ N})(0.06\text{ m}) = 0.36\text{ J}$. This product must remain the same in all cases. For the cart rolling through gravel, $-(f_k)(0.18\text{ m}) = 0.36\text{ J}$ tells us $f_k = 2$ N.

OQ7.16 Answer (c). The ice cube is in neutral equilibrium. Its zero acceleration is evidence for equilibrium.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ7.1 Yes. The floor of a rising elevator does work on a passenger. A normal force exerted by a stationary solid surface does no work.

CQ7.2 Yes. Object 1 exerts some forward force on object 2 as they move through the same displacement. By Newton's third law, object 2 exerts an equal-size force in the opposite direction on object 1. In $W = F\Delta r\cos\theta$, the factors F and Δr are the same, and θ differs by 180°, so object 2 does −15.0 J of work on object 1. The energy transfer is 15 J from object 1 to object 2, which can be counted as a change in energy of −15 J for object 1 and a change in energy of +15 J for object 2.

CQ7.3 It is sometimes true. If the object is a particle initially at rest, the net work done on the object is equal to its final kinetic energy. If the object is not a particle, the work could go into (or come out of) some other form of energy. If the object is initially moving, its initial kinetic energy must be added to the total work to find the final kinetic energy.

CQ7.4 The scalar product of two vectors is positive if the angle between them is between 0° and 90°, including 0°. The scalar product is negative when $90° < \theta \leq 180°$.

CQ7.5 No. Kinetic energy is always positive. Mass and squared speed are both positive.

CQ7.6 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release. He extends this distance by taking a step forward.

CQ7.7 (a) Positive work is done by the chicken on the dirt.

(b) The person does no work on anything in the environment. Perhaps some extra chemical energy goes through being energy transmitted electrically and is converted into internal energy in his brain; but it would be very hard to quantify "extra."

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

CQ7.8 (a) Not necessarily. It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

CQ7.9 The gravitational energy of the key-Earth system is lowest when the key is on the floor letter-side-down. The average height of particles in the key is lowest in that configuration. As described by $F = -dU/dx$, a force pushes the key downhill in potential energy toward the bottom of a graph of potential energy versus orientation angle. Friction removes mechanical energy from the key-Earth system, tending to leave the key in its minimum-potential energy configuration.

CQ7.10 There is no violation. Choose the book as the system. You did positive work (average force and displacement are in same direction) and the Earth did negative work (average force and displacement are in opposite directions) on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

CQ7.11 $k' = 2k$. Think of the original spring as being composed of two half-springs. The same force F that stretches the whole spring by x stretches each of the half-springs by $x/2$; therefore, the spring constant for each of the half-springs is $k' = [F/(x/2)] = 2(F/x) = 2k$.

CQ7.12 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

CQ7.13 Yes. As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

CQ7.14 Force of tension on a ball moving in a circle on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 7.2 Work Done by a Constant Force

P7.1 (a) The 35-N force applied by the shopper makes a 25° angle with the displacement of the cart (horizontal). The work done on the cart by the shopper is then

$$W_{\text{shopper}} = (F\cos\theta)\Delta x = (35.0\text{ N})(50.0\text{ m})\cos 25.0°$$
$$= \boxed{1.59\times 10^{3}\text{ J}}$$

(b) The force exerted by the shopper is now completely horizontal and will be equal to the friction force, since the cart stays at a constant velocity. In part (a), the shopper's force had a downward vertical component, increasing the normal force on the cart, and thereby the friction force. Because there is no vertical component here, the friction force will be less, and the $\boxed{\text{the force is smaller}}$ than before.

(c) Since the horizontal component of the force is less in part (b), the work performed by the shopper on the cart over the same 50.0-m distance is $\boxed{\text{the same}}$ as in part (b).

P7.2 (a) The work done on the raindrop by the gravitational force is given by

$$W = mgh = \left(3.35\times10^{-5}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(100\text{ m}\right) = \boxed{3.28\times10^{-2}\text{ J}}$$

(b) Since the raindrop is falling at constant velocity, all forces acting on the drop must be in balance, and $R = mg$, so

$$W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$$

P7.3 (a) The work done by a constant force is given by

$$W = Fd\cos\theta$$

where θ is the angle between the force and the displacement of the object. In this case, $F = -mg$ and $\theta = 180°$, giving

$$W = (281.5\text{ kg})(9.80\text{ m/s}^2)[(17.1\text{ cm})(1\text{ m}/10^2\text{ cm})] = \boxed{472\text{ J}}$$

(b) If the object moved upward at constant speed, the net force acting on it was zero. Therefore, the magnitude of the upward force applied by the lifter must have been equal to the weight of the object:

$$F = mg = (281.5\text{ kg})(9.80\text{ m/s}^2) = 2.76\times10^3\text{ N} = \boxed{2.76\text{ kN}}$$

P7.4 Assuming the mass is lifted at constant velocity, the total upward force exerted by the two men equals the weight of the mass: $F_{\text{total}} = mg = (653.2\text{ kg})(9.80\text{ m/s}^2) = 6.40\times10^3\text{ N}$. They exert this upward force through a total upward displacement of 96 inches (4 inches per lift for each of 24 lifts). The total work would then be

$$W_{\text{total}} = (6.40\times10^3\text{ N})[(96\text{ in})(0.025\,4\text{ m}/1\text{ in})] = \boxed{1.56\times10^4\text{ J}}$$

P7.5 We apply the definition of work by a constant force in the first three parts, but then in the fourth part we add up the answers. The total (net) work is the sum of the amounts of work done by the individual forces, and is the work done by the total (net) force. This identification is not represented by an equation in the chapter text, but is something you know by thinking about it, without relying on an equation in a list. The definition of work by a constant force is $W = F\Delta r\cos\theta$.

(a) The applied force does work given by

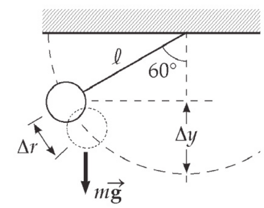
$$W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.6 **METHOD ONE**

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12.0\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then



ANS. FIG. P7.6

$$\cos\theta = \cos(90° + \phi) = -\sin\phi$$

The work done by the gravitational force on Spiderman is

$$W = \int_i^f F\cos\theta\, dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12.0\text{ m})d\phi$$

$$= -mg(12.0\text{ m})\int_0^{60°}\sin\phi\, d\phi$$

$$= (-80.0\text{ kg})(9.80\text{ m/s}^2)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12.0\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times 10^3\text{ J}}$$

METHOD TWO

The force of gravity on Spiderman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y coordinate below the tree limb is –12 m. His final y coordinate is $(-12.0\text{ m})\cos 60.0° = -6.00\text{ m}$. His change in elevation is –6.00 m – (–12.0 m). The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6.00\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}$$

Section 7.3 The Scalar Product of Two Vectors

P7.7 $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)$$
$$+ A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)$$
$$+ A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)$$

And since $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$ and $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$,

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

P7.8 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$$

P7.9 $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \left(3.00\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}\right) - \left(-\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{k}}\right) = 4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}$

$$\vec{\mathbf{C}}\cdot\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right) = \left(2.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}\right) = 0 + (-2.00) + (+18.0)$$
$$= \boxed{16.0}$$

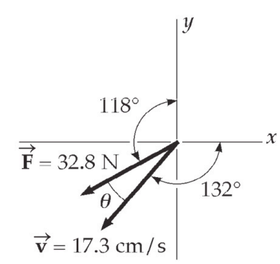
P7.10 We must first find the angle between the two vectors. It is

$$\theta = (360° - 132°) - (118° + 90.0°) = 20.0°$$

Then

$$\vec{\mathbf{F}}\cdot\vec{\mathbf{r}} = Fr\cos\theta = (32.8\text{ N})(0.173\text{ m})\cos 20.0°$$

or $\vec{\mathbf{F}}\cdot\vec{\mathbf{r}} = 5.33\text{ N}\cdot\text{m} = \boxed{5.33\text{ J}}$



ANS. FIG. P7.10

P7.11 (a) We use the mathematical representation of the definition of work.

$$W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = F_x x + F_y y = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m}$$
$$= \boxed{16.0\text{ J}}$$

(b) $\theta = \cos^{-1}\left(\dfrac{\vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}}{F\Delta r}\right)$

$$= \cos^{-1}\frac{16\text{ N}\cdot\text{m}}{\sqrt{(6.00\text{ N})^2 + (-2.00\text{ N})^2}\cdot\sqrt{(3.00\text{ m})^2 + (1.00\text{ m})^2}}$$
$$= \boxed{36.9°}$$

P7.12 (a) $\vec{\mathbf{A}} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$

$$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\frac{12.0 + 8.00}{\sqrt{13.0}\cdot\sqrt{32.0}}\right) = \boxed{11.3°}$$

(b) $\vec{\mathbf{A}} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$$\cos\theta = \left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \frac{-6.00 - 16.0}{\sqrt{20.0}\cdot\sqrt{29.0}} \quad\rightarrow\quad \theta = \boxed{156°}$$

(c) $\vec{\mathbf{A}} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$

$$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\frac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$$

P7.13 Let θ represent the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Turning by 25.0° makes the dot product larger, so the angle between $\vec{\mathbf{C}}$ and $\vec{\mathbf{B}}$ must be smaller. We call it $\theta - 25.0°$. Then we have

$$5A\cos\theta = 30 \quad\text{and}\quad 5A\cos(\theta - 25.0°) = 35$$

Then

$$A\cos\theta = 6 \quad\text{and}\quad A(\cos\theta\cos 25.0° + \sin\theta\sin 25.0°) = 7$$

Dividing,

$$\cos 25.0° + \tan\theta\sin 25.0° = 7/6$$

or $$\tan\theta = (7/6 - \cos 25.0°)/\sin 25.0° = 0.616$$

Which gives $\theta = 31.6°$. Then the direction angle of A is

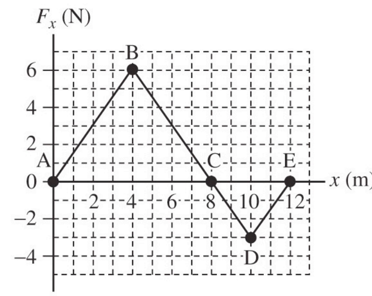
$$60.0° - 31.6° = 28.4°$$

Substituting back,

$$A\cos 31.6° = 6 \quad\text{so}\quad \vec{\mathbf{A}} = \boxed{7.05\text{ m at }28.4°}$$

Section 7.4 Work Done by a Varying Force

P7.14 $W = \int_i^f F dx =$ area under curve from x_i to x_f



ANS. FIG. P7.14

(a) $x_i = 0$ and $x_f = 800$ m

$$W_{0\to 8} = \text{area of triangle ABC}$$

$$= \left(\frac{1}{2}\right)\text{AC} \times \text{height}$$

$$W_{0\to 8} = \left(\frac{1}{2}\right) \times 8.00 \text{ m} \times 6.00 \text{ N}$$

$$= \boxed{24.0 \text{ J}}$$

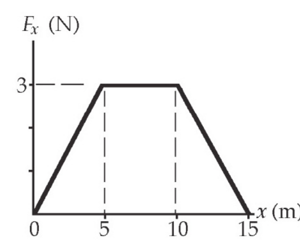
(b) $x_i = 8.00$ m and $x_f = 10.0$ m

$$W_{8\to 10} = \text{area of } \Delta\text{CDE} = \left(\frac{1}{2}\right)\text{CE} \times \text{height},$$

$$W_{8\to 10} = \left(\frac{1}{2}\right) \times (2.00 \text{ m}) \times (-3.00 \text{ N}) = \boxed{-3.00 \text{ J}}$$

(c) $W_{0\to 10} = W_{0\to 8} + W_{0\to 10} = 24.0 + (-3.00) = \boxed{21.0 \text{ J}}$

P7.15 We use the graphical representation of the definition of work. W equals the area under the force-displacement curve. This definition is still written $W = \int F_x \, dx$ but it is computed geometrically by identifying triangles and rectangles on the graph.



ANS. FIG. P7.15

(a) For the region $0 \le x \le 5.00$ m,

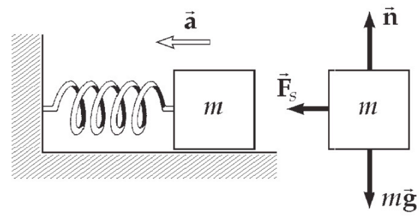
$$W = \frac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$, $W = (3.00 \text{ N})(5.00 \text{ m}) = \boxed{15.0 \text{ J}}$

(c) For the region $10.00 \le x \le 15.0$, $W = \dfrac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$

(d) For the region $0 \le x \le 15.0$, $W = (7.50 + 7.50 + 15.0) \text{ J} = \boxed{30.0 \text{ J}}$

P7.16 $\sum F_x = ma_x$: $kx = ma$



ANS. FIG. P7.16

$$k = \frac{ma}{x}$$

$$= \frac{(4.70\times 10^{-3}\text{ kg})\,(0.800)\,(9.80\text{ m/s}^2)}{0.500\times 10^{-2}\text{m}}$$

$$= \boxed{7.37\text{ N/m}}$$

P7.17 When the load of mass $M = 4.00$ kg is hanging on the spring in equilibrium, the upward force exerted by the spring on the load is equal in magnitude to the downward force that the Earth exerts on the load, given by $w = Mg$. Then we can write Hooke's law as $Mg = +kx$. The spring constant, force constant, stiffness constant, or Hooke's-law constant of the spring is given by

$$k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{2.50\times 10^{-2}\text{ m}} = 1.57\times 10^3\text{ N/m}$$

(a) For the 1.50-kg mass,

$$y = \frac{mg}{k} = \frac{(1.50\text{ kg})(9.80\text{ m/s}^2)}{1.57\times 10^3\text{ N/m}} = 0.009\ 38\text{ m} = \boxed{0.938\text{ cm}}$$

(b) $$\text{Work} = \frac{1}{2}ky^2 = \frac{1}{2}\left(1.57\times 10^3\text{ N/m}\right)\left(4.00\times 10^{-2}\text{ m}\right)^2 = \boxed{1.25\text{ J}}$$

P7.18 In $F = -kx$, F refers to the size of the force that the spring exerts on each end. It pulls down on the doorframe in part (a) in just as real a sense as it pulls on the second person in part (b).

(a) Consider the upward force exerted by the bottom end of the spring, which undergoes a downward displacement that we count as negative:

$$k = -F/x = -(7.50\text{ kg})(9.80\text{ m/s}^2)/(-0.415\text{ m} + 0.350\text{ m})$$

$$= -73.5\text{ N}/(-0.065\text{ m}) = \boxed{1.13\text{ kN/m}}$$

(b) Consider the end of the spring on the right, which exerts a force to the left:

$$x = -F/k = -(-190\text{ N})/(1130\text{ N/m}) = 0.168\text{ m}$$

The length of the spring is then

$$0.350\text{ m} + 0.168\text{ m} = \boxed{0.518\text{ m} = 51.8\text{ cm}}$$

P7.19 (a) Spring constant is given by $F = kx$:

$$k = \frac{F}{x} = \frac{230\text{ N}}{0.400\text{ m}} = \boxed{575\text{ N/m}}$$

(b) $$\text{Work } = F_{\text{avg}}x = \left(\frac{230\text{ N} - 0}{2}\right)(0.400\text{ m}) = \boxed{46.0\text{ J}}$$

P7.20 The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= (1.5\text{ kg})(9.8\text{ m/s}^2)\left(\frac{1}{1\,200\text{ N/m}} + \frac{1}{1\,800\text{ N/m}}\right)$$

$$= \boxed{2.04 \times 10^{-2}\text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg(1/k_1 + 1/k_2)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

$$= \left(\frac{1}{1\,200\text{ N/m}} + \frac{1}{1\,800\text{ N/m}}\right)^{-1} = \boxed{720\text{ N/m}}$$

P7.21 (a) The force mg is the tension in each of the springs. The bottom of the upper (first) spring moves down by distance $x_1 = |F|/k_1 = mg/k_1$. The top of the second spring moves down by this distance, and the second spring also stretches by $x_2 = mg/k_2$. The bottom of the lower spring then moves down by distance

$$x_{\text{total}} = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = \boxed{mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

(b) From the last equation we have

$$mg = \frac{x_1 + x_2}{\dfrac{1}{k_1} + \dfrac{1}{k_2}}$$

This is of the form

$$|F| = \left(\frac{1}{1/k_1 + 1/k_2}\right)(x_1 + x_2)$$

The downward displacement is opposite in direction to the upward force the springs exert on the load, so we may write $F = -k_{eff}\, x_{\text{total}}$, with the effective spring constant for the pair of springs given by

$$k_{eff} = \boxed{\frac{1}{1/k_1 + 1/k_2}}$$

P7.22 $$[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$$

P7.23 (a) If the weight of the first tray stretches all four springs by a distance equal to the thickness of the tray, then the proportionality expressed by Hooke's law guarantees that each additional tray will have the same effect, so that the top surface of the top tray can always have the same elevation above the floor if springs with the right spring constant are used.

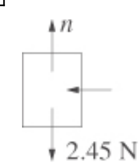
(b) The weight of a tray is (0.580 kg)(9.8 m/s^2) = 5.68 N. The force $\frac{1}{4}(5.68\ \text{N}) = 1.42\ \text{N}$ should stretch one spring by 0.450 cm, so its spring constant is

$$k = \frac{|F_s|}{x} = \frac{1.42\ \text{N}}{0.004\,5\ \text{m}} = \boxed{316\ \text{N/m}}$$

(c) We did not need to know the length or width of the tray.

P7.24 The spring exerts on each block an outward force of magnitude

$$|F_s| = kx = (3.85\ \text{N/m})(0.08\ \text{m}) = 0.308\ \text{N}$$



ANS. FIG. P7.24

Take the $+x$ direction to the right. For the light block on the left, the vertical forces are given by

$$F_g = mg = (0.250\ \text{kg})(9.80\ \text{m/s}^2) = 2.45\ \text{N}$$

and $\sum F_y = 0$

so $n - 2.45\ \text{N} = 0 \quad \rightarrow \quad n = 2.45\ \text{N}$

Similarly, for the heavier block,

$$n = F_g = (0.500\ \text{kg})(9.80\ \text{m/s}^2) = 4.90\ \text{N}$$

(a) For the block on the left,

$$\sum F_x = ma_x: \quad -0.308\text{ N} = (0.250\text{ kg})a$$

$$a = \boxed{-1.23\text{ m/s}^2}$$

For the heavier block,

$$+0.308\text{ N} = (0.500\text{ kg})a$$

$$a = \boxed{0.616\text{ m/s}^2}$$

(b) For the block on the left, $f_k = \mu_k n = 0.100(2.45\text{ N}) = 0.245\text{ N}$.

$$\sum F_x = ma_x$$

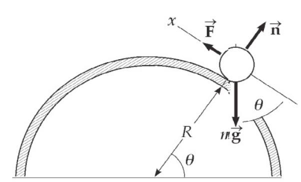
$$-0.308\text{ N} + 0.245\text{ N} = (0.250\text{ kg})a$$

$$a = \boxed{-0.252\text{ m/s}^2 \text{ if the force of static friction is not too large}}.$$

For the block on the right, $f_k = \mu_k n = 0.490$ N. The maximum force of static friction would be larger, so no motion would begin and the acceleration is $\boxed{\text{zero}}$.

(c) Left block: $f_k = 0.462(2.45\text{ N}) = 1.13$ N. The maximum static friction force would be larger, so the spring force would produce no motion of this block or of the right-hand block, which could feel even more friction force. For both, $a = \boxed{0}$.

P7.25 (a) The radius to the object makes angle θ with the horizontal. Taking the x axis in the direction of motion tangent to the cylinder, the object's weight makes an angle θ with the $-x$ axis. Then,



ANS. FIG. P7.25

$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

(b) $W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

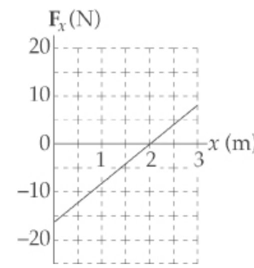
$$W = \int_0^{\pi/2} mg\cos\theta R\,d\theta = mgR\sin\theta\Big|_0^{\pi/2} = mgR(1-0) = \boxed{mgR}$$

P7.26 The force is given by $F_x = (8x - 16)$ N.

(a) See ANS. FIG. P7.26 to the right.

(b) $$W_{\text{net}} = \frac{-(2.00\text{ m})(16.0\text{ N})}{2} + \frac{(1.00\text{ m})(8.00\text{ N})}{2}$$

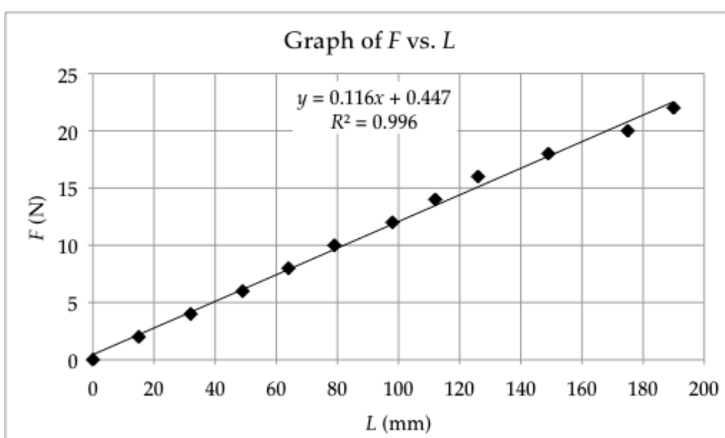
$$= \boxed{-12.0\text{ J}}$$



ANS. FIG. P7.26

P7.27 (a)

F (N)	*L* (mm)	*F* (N)	*L* (mm)
0.00	0.00	12.0	98.0
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190



ANS FIG. P7.27(a)

(b) By least-squares fitting, its slope is 0.116 N/mm = $\boxed{116\text{ N/m}}$.

(c) To draw the straight line we use all the points listed and also the origin. If the coils of the spring touched each other, a bend or nonlinearity could show up at the bottom end of the graph. If the spring were stretched "too far," a nonlinearity could show up at the top end. But there is no visible evidence for a bend in the graph near either end.

(d) In the equation $F = kx$, the spring constant k is the slope of the F-versus-x graph.

$$\boxed{k = 116\ \text{N/m}}$$

(e) $F = kx = (116\ \text{N/m})(0.105\ \text{m}) = \boxed{12.2\ \text{N}}$

P7.28 (a) We find the work done by the gas on the bullet by integrating the function given:

$$W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

$$W = \int_0^{0.600\ \text{m}} \left(15\,000\ \text{N} + 10\,000x\ \text{N/m} - 25\,000x^2\ \text{N/m}^2\right) dx \cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3}\bigg|_0^{0.600\ \text{m}}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

(b) Similarly,

$$W = (15.0\ \text{kN})(1.00\ \text{m}) + \frac{(10.0\ \text{kN/m})(1.00\ \text{m})^2}{2} - \frac{\left(25.0\ \text{kN/m}^2\right)(1.00\ \text{m})^3}{3}$$

$$W = 11.67\ \text{kJ} = \boxed{11.7\ \text{kJ}}$$

(c) $$\frac{11.7\ \text{kJ} - 9.00\ \text{kJ}}{9.00\ \text{kJ}} \times 100\% = 29.6\%$$

$$\boxed{\text{The work is greater by 29.6\%.}}$$

P7.29 $$W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_0^{5\ \text{m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right) \text{N} \cdot dx\hat{\mathbf{i}}$$

$$\int_0^{5\ \text{m}} (4\ \text{N/m})x\,dx + 0 = (4\ \text{N/m})\frac{x^2}{2}\bigg|_0^{5\ \text{m}} = \boxed{50.0\ \text{J}}$$

P7.30 We read the coordinates of the two specified points from the graph as

$$a = (5\text{ cm}, -2\text{ N})\text{ and }b = (25\text{ cm}, 8\text{ N})$$

We can then write u as a function of v by first finding the slope of the curve:

$$\text{slope} = \frac{u_b - u_a}{v_b - v_a} = \frac{8\text{ N} - (-2\text{ N})}{25\text{ cm} - 5\text{ cm}} = 0.5\text{ N/cm}$$

The y intercept of the curve can be found from $u = mv + b$, where $m = 0.5$ N/cm is the slope of the curve, and b is the y intercept. Plugging in point a, we obtain

$$u = mv + b$$
$$-2\text{ N} = (0.5\text{ N/cm})(5\text{ cm}) + b$$
$$b = -4.5\text{ N}$$

Then,

$$u = mv + b = (0.5\text{ N/cm})v - 4.5\text{ N}$$

(a) Integrating the function above, suppressing units, gives

$$\int_a^b u\,dv = \int_5^{25}(0.5v - 4.5)\,dv = \left[0.5v^2/2 - 4.5v\right]_5^{25}$$
$$= 0.25(625 - 25) - 4.5(25 - 5)$$
$$= 150 - 90 = 60\text{ N}\cdot\text{cm} = \boxed{0.600\text{ J}}$$

(b) Reversing the limits of integration just gives us the negative of the quantity:

$$\int_b^a u\,dv = \boxed{-0.600\text{ J}}$$

(c) This is an entirely different integral. It is larger because all of the area to be counted up is positive (to the right of $v = 0$) instead of partly negative (below $u = 0$).

$$\int_a^b v\,du = \int_{-2}^{8}(2u + 9)\,du = \left[2u^2/2 + 9u\right]_{-2}^{8}$$
$$= 64 - (-2)^2 + 9(8 + 2)$$
$$= 60 + 90 = 150\text{ N}\cdot\text{cm} = \boxed{1.50\text{ J}}$$

Section 7.5 Kinetic Energy and the Work-Kinetic Energy Theorem

P7.31 $\vec{\mathbf{v}}_i = \left(6.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)$ m/s^2

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{37.0}$ m/s

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00\text{ kg})\left(37.0\text{ m}^2/\text{s}^2\right) = \boxed{55.5\text{ J}}$$

(b) $\vec{\mathbf{v}}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$$v_f^2 = \vec{\mathbf{v}}_f \cdot \vec{\mathbf{v}}_f = 64.0 + 16.0 = 80.0\text{ m}^2/\text{s}^2$$

$$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 55.5 = \boxed{64.5\text{ J}}$$

P7.32 (a) Since the applied force is horizontal, it is in the direction of the displacement, giving $\theta = 0°$. The work done by this force is then

$$W_{F_0} = \left(F_0\cos\theta\right)\Delta x = F_0(\cos 0)\Delta x = F_0\Delta x$$

and

$$F_0 = \frac{W_{F_0}}{\Delta x} = \frac{350\text{ J}}{12.0\text{ m}} = \boxed{29.2\text{ N}}$$

(b) If the applied force is greater than 29.2 N, the crate would accelerate in the direction of the force, so its [speed would increase] with time.

(c) If the applied force is less than 29.2 N, the [crate would slow down and come to rest.]

P7.33 (a) $K_A = \frac{1}{2}(0.600\text{ kg})(2.00\text{ m/s})^2 = \boxed{1.20\text{ J}}$

(b) $\frac{1}{2}mv_B^2 = K_B$: $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50\text{ J})}{0.600\text{ kg}}} = \boxed{5.00\text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50\text{ J} - 1.20\text{ J} = \boxed{6.30\text{ J}}$

P7.34 (a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50\text{ J})}{4.00\text{ kg}}} = \boxed{1.94\text{ m/s}}$$

(b) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5\text{ J})}{4.00\text{ kg}}} = \boxed{3.35\text{ m/s}}$$

(c) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0\text{ J})}{4.00\text{ kg}}} = \boxed{3.87\text{ m/s}}$$

P7.35 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ the distance it moves the piling.

$$\sum W = \Delta K: \quad W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

so $$(mg)(h+d)\cos 0° + (\bar{F})(d)\cos 180° = 0 - 0$$

Thus,

$$\bar{F} = \frac{(mg)(h+d)}{d} = \frac{(2\,100\text{ kg})(9.80\text{ m/s}^2)(5.12\text{ m})}{0.120\text{ m}}$$

$$= \boxed{8.78\times 10^5\text{ N}}$$

The force on the pile driver is $\boxed{\text{upward.}}$

P7.36 (a) $v_f = 0.096(3.00\times 10^8\text{ m/s}) = 2.88\times 10^7\text{ m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(9.11\times 10^{-31}\text{ kg})(2.88\times 10^7\text{ m/s})^2 = \boxed{3.78\times 10^{-16}\text{ J}}$$

(b) $K_i + W = K_f : 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\text{ m})\cos 0° = 3.78\times 10^{-16}\text{ J}$$

$$F = \boxed{1.35\times 10^{-14}\text{ N}}$$

(c) $\sum F = ma$: $\quad a = \dfrac{\sum F}{m} = \dfrac{1.35\times10^{-14}\text{ N}}{9.11\times10^{-31}\text{ kg}} = \boxed{1.48\times10^{+16}\text{ m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t$: $\quad 2.88\times10^7\text{ m/s} = 0 + \left(1.48\times10^{16}\text{ m/s}^2\right)t$

$t = \boxed{1.94\times10^{-9}\text{ s}}$

P7.37 (a) $K_i + \sum W = K_f = \dfrac{1}{2}mv_f^2$

$0 + \sum W = \dfrac{1}{2}(15.0\times10^{-3}\text{ kg})(780\text{ m/s})^2 = \boxed{4.56\text{ kJ}}$

(b) As shown in part (a), the net work performed on the bullet is $\boxed{4.56\text{ kJ.}}$

(c) $F = \dfrac{W}{\Delta r\cos\theta} = \dfrac{4.56\times10^3\text{ J}}{(0.720\text{ m})\cos 0°} = \boxed{6.34\text{ kN}}$

(d) $a = \dfrac{v_f^2 - v_i^2}{2x_f} = \dfrac{(780\text{ m/s})^2 - 0}{2(0.720\text{ m})} = \boxed{422\text{ km/s}^2}$

(e) $\sum F = ma = (15\times10^{-3}\text{ kg})(422\times10^3\text{ m/s}^2) = \boxed{6.34\text{ kN}}$

(f) $\boxed{\text{The forces are the same. The two theories agree.}}$

P7.38 (a) As the bullet moves the hero's hand, work is done on the bullet to decrease its kinetic energy. The average force is opposite to the displacement of the bullet:

$$W_{\text{net}} = F_{\text{avg}}\Delta x\cos\theta = -F_{\text{avg}}\Delta x = \Delta K$$

$$F_{\text{avg}} = \frac{\Delta K}{-\Delta x} = \frac{0 - \frac{1}{2}\left(7.80\times10^{-3}\text{ kg}\right)\left(575\text{ m/s}\right)^2}{-0.055\ 0\text{ m}}$$

$$\boxed{F_{\text{avg}} = 2.34 \times 10^4\text{ N, opposite to the direction of motion}}$$

(b) If the average force is constant, the bullet will have a constant acceleration and its average velocity while stopping is $\bar{v} = (v_f + v_i)/2$. The time required to stop is then

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{2(\Delta x)}{v_f + v_i} = \frac{2(5.50\times10^{-2}\text{m})}{0 + 575\text{ m/s}} = \boxed{1.91\times10^{-4}\text{s}}$$

P7.39 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x{}^2 + v_y{}^2)$

$$= \frac{1}{2}(5.75\text{ kg})[(5.00\text{ m/s})^2 + (-3.00\text{ m/s})^2] = \boxed{97.8\text{ J}}$$

(b) We know $F_x = ma_x$ and $F_y = ma_y$. At $t = 0$, $x_i = y_i = 0$, and $v_{xi} = 5.00$ m/s, $v_{yi} = -3.00$ m/s; at $t = 2.00$ s, $x_f = 8.50$ m, $y_f = 5.00$ m.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$a_x = \frac{2(x_f - x_i - v_{xi}t)}{t^2} = \frac{2[8.50\text{ m} - 0 - (5.00\text{ m/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= -0.75\text{ m/s}^2$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$a_y = \frac{2(y_f - y_i - v_{yi}t)}{t^2} = \frac{2[5.00\text{ m} - 0 - (-3.00\text{ m/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= 5.50\text{ m/s}^2$$

$$F_x = ma_x = (5.75\text{ kg})(-0.75\text{ m/s}^2) = -4.31\text{ N}$$

$$F_y = ma_y = (5.75\text{ kg})(5.50\text{ m/s}^2) = 31.6\text{ N}$$

$$\boxed{\vec{\mathbf{F}} = \left(-4.31\hat{\mathbf{i}} + 31.6\hat{\mathbf{j}}\right)\text{ N}}$$

(c) We can obtain the particle's speed at $t = 2.00$ s from the particle under constant acceleration model, or from the nonisolated system model. From the former,

$$v_{xf} = v_{xi} + a_x t = (5.00\text{ m/s}) + \left(-0.75\text{ m/s}^2\right)(2.00\text{ s}) = 3.50\text{ m/s}$$

$$v_{xf} = v_{yi} + a_y t = (-3.00\text{ m/s}) + \left(5.50\text{ m/s}^2\right)(2.00\text{ s}) = 8.00\text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(3.50\text{ m/s})^2 + (8.00\text{ m/s})^2} = \boxed{8.73\text{ m/s}}$$

From the nonisolated system model,

$$\sum W = \Delta K: \quad W_{\text{ext}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The work done by the force is given by

$$W_{\text{ext}} = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = F_x\Delta r_x + F_y\Delta r_y$$

$$= (-4.31\text{ N})(8.50\text{ m}) + (31.6\text{ N})(5.00\text{ m}) = 121\text{ J}$$

then,

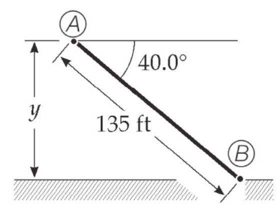
$$\frac{1}{2}mv_f^2 = W_{ext} + \frac{1}{2}mv_i^2 = 121\text{ J} + 97.8\text{ J} = 219\text{ J}$$

which gives

$$v_f = \sqrt{\frac{2(219\text{ J})}{5.75\text{ kg}}} = \boxed{8.73\text{ m/s}}$$

Section 7.6 Potential Energy of a System

P7.40 (a) With our choice for the zero level for potential energy of the car-Earth system when the car is at point Ⓑ,



ANS. FIG. P7.40

$$\boxed{U_B = 0}$$

When the car is at point Ⓐ, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With 135 ft = 41.1 m, this height is found as:

$$y = (41.1\text{ m})\sin 40.0° = 26.4\text{ m}$$

Thus,

$$U_A = (1\,000\text{ kg})(9.80\text{ m/s}^2)(26.4\text{ m}) = \boxed{2.59\times10^5\text{ J}}$$

The change in potential energy of the car-Earth system as the car moves from Ⓐ to Ⓑ is

$$U_B - U_A = 0 - 2.59\times10^5\text{ J} = \boxed{-2.59\times10^5\text{ J}}$$

(b) With our choice of the zero configuration for the potential energy of the car-Earth system when the car is at point Ⓐ, we have $\boxed{U_A = 0}$. The potential energy of the system when the car is at point Ⓑ is given by $U_B = mgy$, where y is the vertical distance of point Ⓑ below point Ⓐ. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000 \text{ kg})(9.80 \text{ m/s}^2)(-26.5 \text{ m}) = \boxed{-2.59 \times 10^5 \text{ J}}$$

The change in potential energy when the car moves from Ⓐ to Ⓑ is

$$U_B - U_A = -2.59 \times 10^5 \text{ J} - 0 = \boxed{-2.59 \times 10^5 \text{ J}}$$

P7.41 Use $U = mgy$, where y is measured relative to a reference level. Here, we measure y to be relative to the top edge of the well, where we take $y = 0$.

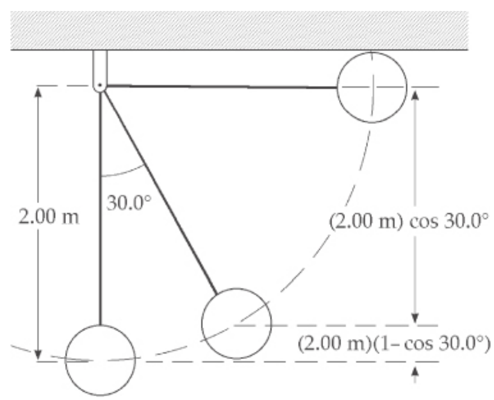
(a) $y = 1.3$ m: $U = mgy = (0.20 \text{ kg})(9.80 \text{ m/s}^2)(1.3 \text{ m}) = \boxed{+2.5 \text{ J}}$

(b) $y = -5.0$ m: $U = mgy = (0.20 \text{ kg})(9.80 \text{ m/s}^2)(-5.0 \text{ m}) = \boxed{-9.8 \text{ J}}$

(c) $\Delta U = U_f - U_i = (-9.8 \text{ J}) - (2.5 \text{ J}) = -12.3 = \boxed{-12 \text{ J}}$

P7.42 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the swing is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy$$
$$= (400 \text{ N})(2.00 \text{ m})$$
$$= \boxed{800 \text{ J}}$$



ANS. FIG. P7.42

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of (2.00 m) (1 – cos 30.0°) above the lowest point of the arc. Thus,

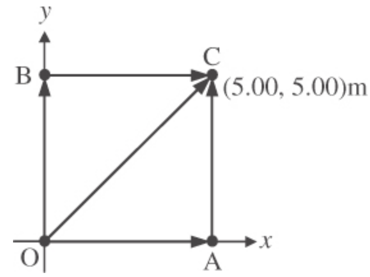
$$U_g = mgy = (400 \text{ N})(2.00 \text{ m})(1 - \cos 30.0°) = \boxed{107 \text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

Section 7.7 Conservative and Nonconservative Forces

P7.43 The gravitational force is downward:

$$F_g = mg = (4.00 \text{ kg})(9.80 \text{ m/s}^2) = 39.2 \text{ N}$$



ANS. FIG. P7.43

(a) Work along OAC = work along OA + work along AC

$$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$$
$$= (39.2 \text{ N})(5.00 \text{ m})(0) + (39.2 \text{ N})(5.00 \text{ m})(-1)$$
$$= \boxed{-196 \text{ J}}$$

(b) W along OBC = W along OB + W along BC

$$= (39.2 \text{ N})(5.00 \text{ m})\cos 180° + (39.2 \text{ N})(5.00 \text{ m})\cos 90.0°$$
$$= \boxed{-196 \text{ J}}$$

(c) Work along OC = F_g(OC) cos 135°

$$= (39.2 \text{ N})\left(5.00 \times \sqrt{2} \text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196 \text{ J}}$$

(d) The results should all be the same, since the gravitational force is conservative.

P7.44 (a) $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, and if the force is constant, this can be written as

$W = \vec{\mathbf{F}} \cdot \int d\vec{\mathbf{r}} = \boxed{\vec{\mathbf{F}} \cdot \left(\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i\right)}$, which depends only on the end points, and not on the path.

(b) $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int \left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right) \cdot \left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right)$

$$= (3.00 \text{ N}) \int_0^{5.00 \text{ m}} dx + (4.00 \text{ N}) \int_0^{5.00 \text{ m}} dy$$

$$W = (3.00 \text{ N})x\Big|_0^{5.00 \text{ m}} + (4.00 \text{ N})y\Big|_0^{5.00 \text{ m}} = 15.0 \text{ J} + 20.0 \text{ J} = \boxed{35.0 \text{ J}}$$

The same calculation applies for all paths.

P7.45 In the following integrals, remember that

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}=\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}=1 \quad \text{and} \quad \hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=0$$

(a) The work done on the particle in its first section of motion is

$$W_{OA} = \int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} 2y\,dx$$

and since along this path, $y = 0$, that means $W_{OA} = 0$.

In the next part of its path,

$$W_{AC} = \int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} x^2\,dy$$

For $x = 5.00$ m, $W_{AC} = 125$ J

and $W_{OAC} = 0 + 125 = \boxed{125\text{ J}}$.

(b) Following the same steps,

$$W_{OB} = \int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} x^2\,dy$$

Since along this path, $x = 0$, that means $W_{OB} = 0$.

$$W_{BC} = \int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int_0^{5.00\text{ m}} 2y\,dx$$

Since $y = 5.00$ m, $W_{BC} = 50.0$ J.

$$W_{OAC} = 0 + 125 = \boxed{125\text{ J}}$$

(c) $W_{OC} = \int\left(dx\hat{\mathbf{i}}+dy\hat{\mathbf{j}}\right)\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right) = \int\left(2y\,dx + x^2\,dy\right)$

Since $x = y$ along OC, $W_{OC} = \int_0^{5.00\text{ m}}\left(2x+x^2\right)dx = \boxed{66.7\text{ J}}$

(d) $\boxed{F \text{ is nonconservative.}}$

(e) $\boxed{\text{The work done on the particle depends on the path followed by the particle.}}$

P7.46 Along each step of motion, to overcome friction you must push with a force of 3.00 N in the direction of travel along the path, so in the expression for work, $\cos\theta = \cos 0° = 1$.

(a) $W = (3.00\text{ N})(5.00\text{ m})(1) + (3.00\text{ N})(5.00\text{ m})(1) = \boxed{30.0\text{ J}}$

(b) The distance CO is $(5.00^2 + 5.00^2)^{1/2}$ m = 7.07 m

$$W = (3.00\text{ N})(5.00\text{ m})(1) + (3.00\text{ N})(5.00\text{ m})(1) + (3.00\text{ N})(7.07\text{ m})(1) = \boxed{51.2\text{ J}}$$

(c) $W = (3.00\text{ N})(7.07\text{ m})(1) + (3.00\text{ N})(7.07\text{ m})(1) = \boxed{42.4\text{ J}}$

(d) Friction is a nonconservative force.

Section 7.8 Relationship Between Conservative Forces and Potential Energy

P7.47 We use the relation of force to potential energy as the force is the negative derivative of the potential energy with respect to distance:

$$U(r) = \frac{A}{r}$$

$$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$$

If A is positive, the positive value of radial force indicates a force of repulsion.

P7.48 We need to be very careful in identifying internal and external work on the book-Earth system. The first 20.0 J, done by the librarian on the system, is external work, so the system now contains an additional 20.0 J compared to the initial configuration. When the book falls and the system returns to the initial configuration, the 20.0 J of work done by the gravitational force from the Earth is *internal* work. This work only transforms the gravitational potential energy of the system to kinetic energy. It does *not* add more energy to the system. Therefore, the book hits the ground with 20.0 J of kinetic energy. The book-Earth system now has zero gravitational potential energy, for a total energy of 20.0 J, which is the energy put into the system by the librarian.

P7.49

$$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial(3x^3y - 7x)}{\partial x} = -(9x^2y - 7) = 7 - 9x^2y$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial(3x^3y - 7x)}{\partial y} = -(3x^3 - 0) = -3x^3$$

Thus, the force acting at the point (x, y) is

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7 - 9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$$

P7.50 (a) We use Equation 7.27 relating the potential energy of the system to the conservative force acting on the particle, with $U_i = 0$:

$$U = U_f - U_i = U_f - 0$$

$$= -\int_0^x \left(-Ax + Bx^2\right)dx = A\frac{x^2}{2} - B\frac{x^3}{3}\bigg|_0^x = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$$

(b) From (a), $U(2.00\text{ m}) = 2A - 2.67B$, and $U(3.00\text{ m}) = 4.5A - 9B$.

$$\boxed{\Delta U = (4.5A - 9B) - (2A - 2.67B) = 2.5A - 6.33B}$$

(c) If we consider the particle alone as a system, the change in its kinetic energy is the work done by the force on the particle: $W = \Delta K$. For the entire system of which this particle is a member, this work is internal work and equal to the negative of the change in potential energy of the system:

$$\boxed{\Delta K = -\Delta U = -2.5A + 6.33B}$$

P7.51 (a) For a particle moving along the x axis, the definition of work by a variable force is

$$W_F = \int_{x_i}^{x_f} F_x dx$$

Here $F_x = (2x + 4)$ N, $x_i = 1.00$ m, and $x_f = 5.00$ m.

So

$$W_F = \int_{1.00\text{ m}}^{5.00\text{ m}} (2x + 4)dx \text{ N}\cdot\text{m} = x^2 + 4x\Big]_{1.00\text{ m}}^{5.00\text{ m}} \text{ N}\cdot\text{m}$$

$$= \left(5^2 + 20 - 1 - 4\right)\text{ J} = \boxed{40.0\text{ J}}$$

(b) The change in potential energy of the system is the negative of the internal work done by the conservative force on the particle: $\Delta U = -W_{\text{int}} = \boxed{-40.0\text{ J}}$

(c) From $\Delta K = K_f - \dfrac{mv_1^2}{2}$, we obtain

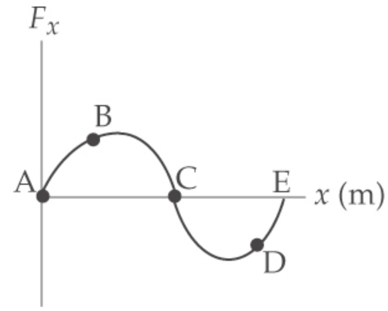
$$K_f = \Delta K + \frac{mv_1^2}{2} = 40.0\text{ J} + \frac{(5.00\text{ kg})(3.00\text{ m/s})^2}{2} = \boxed{62.5\text{ J}}$$

Section 7.9 Energy Diagrams and Equilibrium of a System

P7.52 (a) F_x is zero at points A, C, and E; F_x is positive at point B and negative at point D.

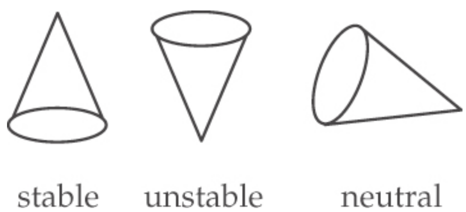
(b) A and E are unstable, and C is stable.

(c) ANS. FIG. P7.52 shows the curve for F_x vs. x for this problem.



ANS. FIG. P7.52

P7.53 The figure below shows the three equilibrium configurations for a right circular cone.



ANS. FIG. P7.53

Additional Problems

P7.54 (a) $\vec{\mathbf{F}} = -\dfrac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}}$

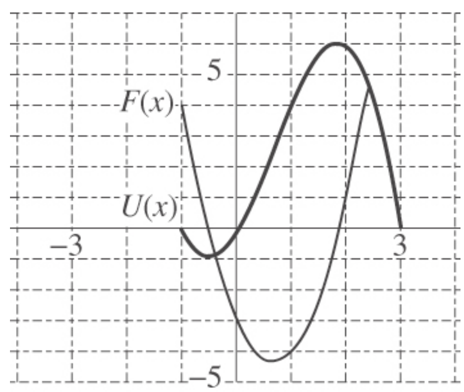
$$= \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$$

(b) $F = 0$ when

$x = \boxed{1.87 \text{ and } -0.535}$.

(c) The stable point is at $x = -0.535$, point of minimum $U(x)$.

The unstable point is at $x = 1.87$, maximum in $U(x)$.



ANS. FIG. P7.54

P7.55 Initially, the ball's velocity is

$$\vec{\mathbf{v}} = (40.0 \text{ m/s}) \cos 30.0°\hat{\mathbf{i}} + (40.0 \text{ m/s}) \sin 30.0°\hat{\mathbf{j}}$$

At its apex, the ball's velocity is

$$\vec{\mathbf{v}} = (40.0 \text{ m/s}) \cos 30.0°\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6 \text{ m/s})\hat{\mathbf{i}}$$

The ball's kinetic energy of the ball at this point is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$$

P7.56 We evaluate $\int_{12.8}^{23.7} \frac{375dx}{x^3 + 3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3 + 3.75(12.8)} + \frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \cdots \frac{375(0.100)}{(23.6)^3 + 3.75(23.6)} = 0.806$$

and

$$\frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \frac{375(0.100)}{(13.0)^3 + 3.75(13.0)} + \cdots \frac{375(0.100)}{(23.7)^3 + 3.75(23.7)} = 0.791$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799 \text{ N}\cdot\text{m}}$.

P7.57 (a) The equivalent spring constant for the stel balls is

$$k = \frac{|F|}{|x|} = \frac{16\ 000 \text{ N}}{0.000\ 2 \text{ m}} = \boxed{8 \times 10^7 \text{ N/m}}$$

(b) $\boxed{\text{A time interval}}$. If the interaction occupied no time, the force exerted by each ball on the other could be infinite, and that cannot happen.

(c) We assume that steel has the density of its main constituent, iron, shown in Table 14.1. Then its mass is

$$\rho V = \rho\left(\frac{4}{3}\right)\pi r^3 = \left(\frac{4\pi}{3}\right)(7\ 860 \text{ kg/m}^3)(0.025\ 4 \text{ m/2})^3$$
$$= 0.067\ 4 \text{ kg}$$

its kinetic energy is then

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.067\ 4\ \text{kg})(5\ \text{m/s})^2 = \boxed{0.8\ \text{J}}$$

(d) Imagine one ball running into an infinitely hard wall and bouncing off elastically. The original kinetic energy becomes elastic potential energy

$$0.843\ \text{J} = (1/2)(8 \times 10^7\ \text{N/m})\, x^2 \qquad x = 0.145\ \text{mm} \approx \boxed{0.15\,\text{mm}}$$

(e) The ball does not really stop with constant acceleration, but imagine it moving 0.145 mm forward with average speed (5 m/s + 0)/2 = 2.5 m/s. The time interval over which it stops is then

$$0.145\ \text{mm}/(2.5\ \text{m/s}) = 6 \times 10^{-5}\ \text{s} \approx \boxed{10^{-4}\ \text{s}}$$

P7.58 The work done by the applied force is

$$W = \int_i^f F_{\text{applied}}dx = \int_0^{x_{\max}} -\left[-\left(k_1x + k_2x^2\right)\right]dx$$

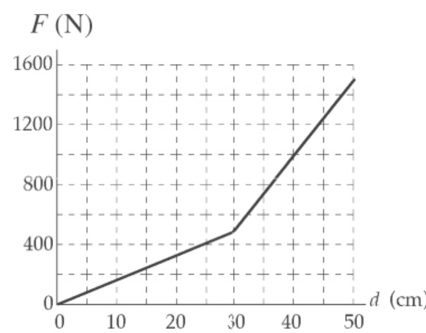
$$= \int_0^{x_{\max}} k_1x\,dx + \int_0^{x_{\max}} k_2x^2dx = k_1\left.\frac{x^2}{2}\right|_0^{x_{\max}} + k_2\left.\frac{x^3}{3}\right|_0^{x_{\max}}$$

$$= \boxed{k_1\frac{x_{\max}^2}{2} + k_2\frac{x_{\max}^3}{3}}$$

P7.59 Compare an initial picture of the rolling car with a final picture with both springs compressed. From conservation of energy, we have

$$K_i + \sum W = K_f$$

Work by both springs changes the car's kinetic energy.



ANS. FIG. P7.59

$$K_i + \frac{1}{2}k_1\left(x_{1i}^2 - x_{1f}^2\right) + \frac{1}{2}k_2\left(x_{2i}^2 - x_{2f}^2\right) = K_f$$

Substituting,

$$\frac{1}{2}mv_i^2 + 0 - \frac{1}{2}(1\,600\ \text{N/m})(0.500\ \text{m})^2 + 0 - \frac{1}{2}(3\,400\ \text{N/m})(0.200\ \text{m})^2 = 0$$

Which gives

$$\frac{1}{2}(6\,000\text{ kg})v_i^2 - 200\text{ J} - 68.0\text{ J} = 0$$

Solving for v_i,

$$v_i = \sqrt{\frac{2(268\text{ J})}{6\,000\text{ kg}}} = \boxed{0.299\text{ m/s}}$$

P7.60 Apply the work-energy theorem to the ball. The spring is initially compressed by $x_{sp,i} = d = 5.00$ cm. After the ball is released from rest, the spring pushes the ball up the incline the distance d, doing positive work on the ball, and gravity does negative work on the ball as it travels up the incline a distance Δx from its starting point. Solve for Δx.

$$K_i + W_s + W_g = K_f$$

$$\frac{1}{2}mv_i^2 + \left(\frac{1}{2}kx_{sp,i}^2 - \frac{1}{2}kx_{sp,f}^2\right) - mg\Delta x\sin\theta = \frac{1}{2}mv_f^2$$

$$0 + \left(\frac{1}{2}kd^2 - 0\right) - mg\Delta x\sin 10.0° = 0$$

$$\Delta x = \frac{kd^2}{2mg\sin 10.0°} = \frac{(1.20\text{ N/cm})(5.00\text{ cm})(0.0500\text{ m})}{2(0.100\text{ kg})\left(9.80\text{ m/s}^2\right)\sin 10.0°}$$
$$= 0.881\text{ m}$$

Thus, the ball travels up the incline a distance of 0.881 m after it is released.

Applying the work-kinetic energy theorem to the ball, one finds that it momentarily comes to rest at a distance up the incline of only 0.881 m. This distance is much smaller than the height of a professional basketball player, so the ball will not reach the upper end of the incline to be put into play in the machine. The ball will simply stop momentarily and roll back to the spring; not an exciting entertainment for any casino visitor!

P7.61 (a) $\vec{\mathbf{F}}_1 = (25.0\text{ N})\left(\cos 35.0°\hat{\mathbf{i}} + \sin 35.0°\hat{\mathbf{j}}\right) = \boxed{\left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)\text{ N}}$

$\vec{\mathbf{F}}_2 = (42.0\text{ N})\left(\cos 150°\hat{\mathbf{i}} + \sin 150°\hat{\mathbf{j}}\right) = \boxed{\left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)\text{ N}}$

(b) $\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \boxed{\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)\text{ N}}$

(c) $\vec{\mathbf{a}} = \dfrac{\sum \vec{\mathbf{F}}}{m} = \boxed{\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right) \text{ m/s}^2}$

(d) $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right) \text{ m/s} + \left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00 \text{ s})$

$$\vec{\mathbf{v}}_f = \boxed{\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right) \text{ m/s}}$$

(e) $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \dfrac{1}{2}\vec{\mathbf{a}}t^2$

$$\vec{\mathbf{r}}_f = 0 + \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)(\text{m/s})(3.00 \text{ s}) + \frac{1}{2}\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00 \text{ s})^2$$

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f = \boxed{\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right) \text{ m}}$$

(f) $K_f = \dfrac{1}{2}mv_f^2 = \dfrac{1}{2}(5.00 \text{ kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48 \text{ kJ}}$

(g) $K_f = \dfrac{1}{2}mv_i^2 + \sum \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$

$$K_f = \frac{1}{2}(5.00 \text{ kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9 \text{ N})(-2.30 \text{ m}) + (35.3 \text{ N})(39.3 \text{ m})\right]$$

$$K_f = 55.6 \text{ J} + 1\,426 \text{ J} = \boxed{1.48 \text{ kJ}}$$

(h) The work-kinetic energy theorem is consistent with Newton's second law, used in deriving it.

P7.62 (a) We write

$$F = ax^b$$

$$1\,000 \text{ N} = a(0.129 \text{ m})^b$$

$$5\,000 \text{ N} = a(0.315 \text{ m})^b$$

Dividing the two equations gives

$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$

$$\ln 5 = b \ln 2.44$$

$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80}$$

$$a = \frac{1\,000\text{ N}}{(0.129\text{ m})^{1.80}} = \boxed{4.01 \times 10^4\text{ N/m}^{1.8}}$$

(b) $$W = \int_i^f F_{\text{applied}}\,dx = \int_0^x ax^b\,dx = \left.\frac{ax^{b+1}}{b+1}\right|_0^x = \frac{ax^{b+1}}{b+1} - 0 = \frac{ax^{b+1}}{b+1}$$

$$W = \frac{(4.01 \times 10^4\text{ N/m}^{1.8})x^{2.8}}{2.80}$$

For $x = 0.250$ m,

$$W = \frac{(4.01 \times 10^4\text{ N/m}^{1.8})(0.250\text{ m})^{2.8}}{2.80}$$
$$= \frac{(4.01 \times 10^4\text{ N/m}^{1.8})(0.250)^{2.8}(\text{m}^{2.8})}{2.80}$$

$$W = \frac{(4.01 \times 10^4\text{ N}\cdot\text{m})(0.250)^{2.8}}{2.80} = \boxed{295\text{ J}}$$

P7.63 The component of the weight force parallel to the incline, $mg\sin\theta$, accelerates the block down the incline through a distance d until it encounters the spring, after which the spring force, pushing up the incline, opposes the weight force and slows the block through a distance x until the block eventually is brought to a momentary stop. The weight force does positive work on the block as it slides down the incline through total distance $(d + x)$, and the spring force does negative work on the block as it slides through distance x. The normal force does no work. Applying the work-energy theorem,

$$K_i + W_g + W_s = K_f$$

$$\frac{1}{2}mv_i^2 + mg\sin\theta(d+x) + \left(\frac{1}{2}kx_{\text{sp},i}^2 - \frac{1}{2}kx_{\text{sp},f}^2\right) = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv^2 + mg\sin\theta(d+x) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

Dividing by m, we have

$$\frac{1}{2}v^2 + g\sin\theta(d+x) - \frac{k}{2m}x^2 = 0 \quad \rightarrow$$

$$\frac{k}{2m}x^2 - (g\sin\theta)x - \left[\frac{v^2}{2} + (g\sin\theta)d\right] = 0$$

Solving for x, we have

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 - 4\left(\frac{k}{2m}\right)\left[-\left(\frac{v^2}{2} + (g\sin\theta)d\right)\right]}}{2\left(\frac{k}{2m}\right)}$$

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

Because distance x must be positive,

$$x = \frac{g\sin\theta + \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

For v = 0.750 m/s, k = 500 N/m, m = 2.50 kg, θ = 20.0°, and g = 9.80 m/s^2, we have $g\sin\theta$ = (9.80 m/s^2)sin20.0° = 3.35 m/s^2 and k/m = (500 N/m)/(2.50 kg) = 200 N/m · kg. Suppressing units, we have

$$x = \frac{3.35 + \sqrt{(3.35)^2 + (200)\left[(0.750)^2 + 2(3.35)(0.300)\right]}}{200}$$

$$= \boxed{0.131\text{ m}}$$

P7.64 The component of the weight force parallel to the incline, $mg\sin\theta$, accelerates the block down the incline through a distance d until it encounters the spring, after which the spring force, pushing up the incline, opposes the weight force and slows the block through a distance x until the block eventually is brought to a momentary stop. The weight force does positive work on the block as it slides down the incline through total distance ($d + x$), and the spring force does negative work on the block as it slides through distance x. The normal force does no work.

Applying the work-energy theorem,

$$K_i + W_g + W_s = K_f$$

$$\frac{1}{2}mv_i^2 + mg\sin\theta(d + x) + \left(\frac{1}{2}kx_{\text{sp},i}^2 - \frac{1}{2}kx_{\text{sp},f}^2\right) = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv^2 + mg\sin\theta(d + x) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

Dividing by m, we have

$$\frac{1}{2}v^2 + g\sin\theta(d+x) - \frac{k}{2m}x^2 = 0 \quad \rightarrow$$

$$\frac{k}{2m}x^2 - (g\sin\theta)x - \left[\frac{v^2}{2} + (g\sin\theta)d\right] = 0$$

Solving for x, we have

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 - 4\left(\frac{k}{2m}\right)\left[-\left(\frac{v^2}{2} + (g\sin\theta)d\right)\right]}}{2\left(\frac{k}{2m}\right)}$$

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

Because distance x must be positive,

$$\boxed{x = \frac{g\sin\theta + \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}}$$

P7.65 (a) The potential energy of the system at point x is given by 5 plus the negative of the work the force does as a particle feeling the force is carried from $x = 0$ to location x.

$$dU = -F dx$$

$$\int_5^U dU = -\int_0^x 8e^{-2x}\, dx$$

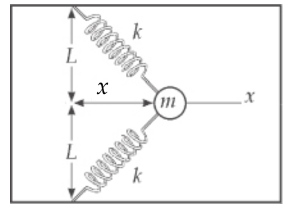
$$U - 5 = -\left(\frac{8}{[-2]}\right)\int_0^x e^{-2x}(-2\, dx)$$

$$U = 5 - \left(\frac{8}{[-2]}\right)e^{-2x}\Big|_0^x = 5 + 4e^{-2x} - 4\cdot 1 = \boxed{1 + 4e^{-2x}}$$

(b) The force must be conservative because the work the force does on the object on which it acts depends only on the original and final positions of the object, not on the path between them. There is a uniquely defined potential energy for the associated force.

Challenge Problems

P7.66 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components (with $\cos\theta=\dfrac{x}{\sqrt{x^2+L^2}}$) add to



ANS. FIG. P7.66

$$\vec{\mathbf{F}}=-2k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}}\hat{\mathbf{i}}$$

$$=\boxed{-2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)\hat{\mathbf{i}}}$$

(b) Choose $U=0$ at $x=0$. Then at any point the potential energy of the system is

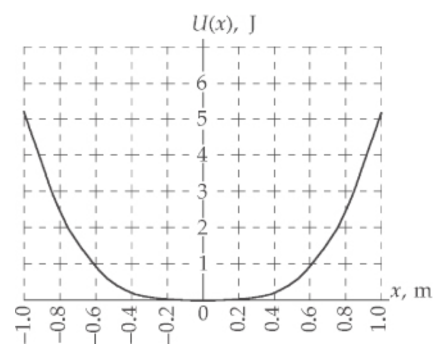
$$U(x)=-\int_0^x F_x\,dx=-\int_0^x\left(-2kx+\frac{2kLx}{\sqrt{x^2+L^2}}\right)dx$$

$$=2k\int_0^x x\,dx-2kL\int_0^x\frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x)=\boxed{kx^2+2kL\left(L-\sqrt{x^2+L^2}\right)}$$

(c) $U(x)=(40.0\text{ N/m})x^2+(96.0\text{ N})\left(1.20\text{ m}-\sqrt{x^2+1.44\text{ m}^2}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x=0$) is $\boxed{x=0}$.



ANS FIG. P7.66(c)

(d) If we consider the particle alone as a system, the change in its kinetic energy is the work done by the force of the springs on the particle: $W = \Delta K$. For the entire system of particle and springs, this work is internal work and equal to the negative of the change in potential energy of the system: $\Delta K = -\Delta U$. From part (c), we evaluate U for $x = 0.500$ m:

$$U = (40.0 \text{ N/m})(0.500 \text{ m})^2 + (96.0 \text{ N})\left(1.20 \text{ m} - \sqrt{(0.500 \text{ m})^2 + 1.44 \text{ m}^2}\right) = 0.400 \text{ J}$$

Now find the speed of the particle:

$$\tfrac{1}{2}mv^2 = -\Delta U$$

$$v = \sqrt{\frac{-2\Delta U}{m}} = \sqrt{\frac{-2}{1.18 \text{ kg}}(0 - 0.400 \text{ J})} = \boxed{0.823 \text{ m/s}}$$

P7.67 (a) We assume the spring lies in the horizontal plane of the motion, then the radius of the puck's motion is $r = L_0 + x$, where $L_0 = 0.155$ m is the unstretched length. The spring force causes the puck's centripetal acceleration:

$$F = mv^2/r \quad \rightarrow \quad kx = m(2\pi r/T)^2/r \quad \rightarrow \quad kT^2x = 4\pi^2 mr$$

Substituting $r = (L_0 + x)$, we have

$$kT^2x = 4\pi^2 m(L_0 + x)$$

$$kx = \frac{(4\pi^2 mL_0)}{T^2} + \frac{x(4\pi^2 m)}{T^2}$$

$$x\left(k - \frac{4\pi^2 m}{T^2}\right) = \frac{4\pi^2 mL_0}{T^2}$$

$$x = \frac{4\pi^2 mL_0/T^2}{k - 4\pi^2 mL_0/T^2}$$

For $k = 4.30$ N/m, $L_0 = 0.155$ m, and $T = 1.30$ s, we have

$$x = \frac{4\pi^2 m(0.155 \text{ m})/(1.30 \text{ s})^2}{4.30 \text{ N/m} - 4\pi^2 m/(1.30 \text{ s})^2}$$

$$= \frac{(3.62 \text{ m/s}^2)m}{4.30 \text{ kg/s}^2 - (23.36/\text{s}^2)m}$$

$$= \frac{(3.62 \text{ m})m}{[4.30 \text{ kg} - (23.36)m]}\frac{1/\text{s}^2}{1/\text{s}^2}$$

$$\boxed{x = \frac{(3.62\text{ m})m}{4.30\text{ kg} - (23.4)m}}$$

(b) For $m = 0.070$ kg,

$$x = \frac{(3.62\text{ m})[0.070\text{ kg}]}{4.30\text{ kg} - 23.36(0.070\text{ kg})} = \boxed{0.095\ 1\text{ m}}$$

(c) We double the puck mass and find

$$x = \frac{(3.6208\text{ m})[0.140\text{ kg}]}{4.30\text{ kg} - 23.360(0.140\text{ kg})} = \boxed{0.492\text{ m}}$$

more than twice as big!

(d) For $m = 0.180$ kg,

$$x = \frac{(3.62\text{ m})[0.180\text{ kg}]}{4.30\text{ kg} - 23.36(0.180\text{ kg})} = \frac{0.652}{0.0952}\text{ m} = \boxed{6.85\text{ m}}$$

We have to get a bigger table!

(e) When the denominator of the fraction goes to zero, the extension becomes infinite. This happens for 4.3 kg – 23.4 $m = 0$; that is for $m = 0.184$ kg. For any larger mass, the spring cannot constrain the motion. The situation is impossible.

(f) The extension is directly proportional to m when m is only a few grams. Then it grows faster and faster, diverging to infinity for $m = 0.184$ kg.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P7.2 (a) 3.28×10^{-2} J; (b) -3.28×10^{-2} J

P7.4 1.56×10^{4} J

P7.6 method one: -4.70×10^{3} J; method two: -4.70 kJ

P7.8 28.9

P7.10 5.33 J

P7.12 (a) 11.3°; (b) 156°; (c) 82.3°

P7.14 (a) 24.0 J; (b) −3.00 J; (c) 21.0 J

P7.16 7.37 N/m

P7.18 (a) 1.13 kN/m; (b) 0.518 m = 51.8 cm

P7.20 (a) 2.04×10^{-2} m; (b) 720 N/m

P7.22 kg/s^2

P7.24 (a) -1.23 m/s^2, 0.616 m/s^2; (b) -0.252 m/s^2 if the force of static friction is not too large, zero; (c) 0

P7.26 (a) See ANS FIG P7.26; (b) −12.0 J

P7.28 (a) 9.00 kJ; (b) 11.7 kJ; (c) The work is greater by 29.6%

P7.30 (a) 0.600 J; (b) −0.600 J; (c) 1.50 J

P7.32 (a) 29.2 N; (b) speed would increase; (c) crate would slow down and come to rest.

P7.34 (a) 1.94 m/s; (b) 3.35 m/s; (c) 3.87 m/s

P7.36 (a) 3.78×10^{-16} J; (b) 1.35×10^{-14} N; (c) $1.48 \times 10^{+16}$ m/s^2; (d) 1.94×10^{-9} s

P7.38 (a) $F_{\text{avg}} = 2.34 \times 10^{4}$ N, opposite to the direction of motion; (b) 1.91×10^{-4} s

P7.40 (a) $U_B = 0$, 2.59×10^{5} J; (b) $U_A = 0$, -2.59×10^{5} J, -2.59×10^{5} J

P7.42 (a) 800 J; (b) 107 J; (c) $U_g = 0$

P7.44 (a) $\vec{\mathbf{F}} \cdot (\vec{\mathbf{r}}_{\text{fi}} - \vec{\mathbf{r}}_i)$, which depends only on end points, and not on the path; (b) 35.0 J

P7.46 (a) 30.0 J; (b) 51.2 J; (c) 42.4 J; (d) Friction is a nonconservative force

P7.48 The book hits the ground with 20.0 J of kinetic energy. The book-Earth now has zero gravitational potential energy, for a total energy of 20.0 J, which is the energy put into the system by the librarian.

P7.50 (a) $\frac{Ax^2}{2} - \frac{Bx^3}{3}$; (b) $\Delta U = (4.5A - 9B) - (2A - 2.67B) = 2.5A - 6.33B$;

(c) $\Delta K = -\Delta U = -2.5A + 6.33B$

P7.52 (a) F_x is zero at points A, C, and E; F_x is positive at point B and negative at point D; (b) A and E are unstable, and C is stable; (c) See ANS FIG P7.52

P7.54 (a) $(3x^2 - 4x - 3)\hat{\mathbf{i}}$; (b) 1.87 and −0.535; (c) See ANS. FIG. P7.54

P7.56 0.799 N · m

P7.58 $k_1 \frac{x_{\text{max}}^2}{2} + k_2 \frac{x_{\text{max}}^3}{3}$

P7.60 The ball will simply stop momentarily and roll back to the spring.

P7.62 (a) $b = 1.80$, $a = 4.01 \times 10^4\ \text{N/m}^{1.8}$; (b) 295 J

P7.64 $$x = \frac{g\sin\theta\sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

P7.66 (a) $-2kx\left(1 - \frac{L}{\sqrt{x^2 + L^2}}\right)\hat{\mathbf{i}}$; (b) $kx^2 + 2kL\left(L - \sqrt{x^2 + L^2}\right)$; (c) See ANS. FIG. P7.66(c), $x = 0$; (d) $v = 0.823$ m/s