9

Linear Momentum and Collisions

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ9.1 Think about how much the vector momentum of the Frisbee changes in a horizontal plane. This will be the same in magnitude as your momentum change. Since you start from rest, this quantity directly controls your final speed. Thus (b) is largest and (c) is smallest. In between them, (e) is larger than (a) and (a) is larger than (c). Also (a) is equal to (d), because the ice can exert a normal force to prevent you from recoiling straight down when you throw the Frisbee up. The assembled answer is b > e > a = d > c.

OQ9.2 (a) No: mechanical energy turns into internal energy in the coupling process.

(b) No: the Earth feeds momentum into the boxcar during the downhill rolling process.

(c) Yes: total energy is constant as it turns from gravitational into kinetic.

(d) Yes: If the boxcar starts moving north, the Earth, very slowly, starts moving south.

(e) No: internal energy appears.

(f) Yes: Only forces internal to the two-car system act.

OQ9.3 (i) Answer (c). During the short time the collision lasts, the total system momentum is constant. Whatever momentum one loses the other gains.

(ii) Answer (a). The problem implies that the tractor's momentum is negligible compared to the car's momentum before the collision. It also implies that the car carries most of the kinetic energy of the system. The collision slows down the car and speeds up the tractor, so that they have the same final speed. The faster-moving car loses more energy than the slower tractor gains because a lot of the car's original kinetic energy is converted into internal energy.

OQ9.4 Answer (a). We have $m_1 = 2$ kg, $v_{1i} = 4$ m/s; $m_2 = 1$ kg, and $v_{1i} = 0$. We find the velocity of the 1-kg mass using the equation derived in Section 9.4 for an elastic collision:

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{4\text{ kg}}{3\text{ kg}}\right)(4\text{ m/s}) + \left(\frac{1\text{ kg}}{3\text{ kg}}\right)(0) = 5.33\text{ m/s}$$

OQ9.5 Answer (c). We choose the original direction of motion of the cart as the positive direction. Then, $v_i = 6$ m/s and $v_f = -2$ m/s. The change in the momentum of the cart is

$$\Delta p = mv_f - mv_i = m(v_f - v_i) = (5\text{ kg})(-2\text{ m/s} - 6\text{ m/s})$$
$$= -40\text{ kg}\cdot\text{m/s}.$$

OQ9.6 Answer (c). The impulse given to the ball is $I = F_{\text{avg}}\Delta t = mv_f - mv_i$. Choosing the direction of the final velocity of the ball as the positive direction, this gives

$$F_{\text{avg}} = \frac{m(v_f - v_i)}{\Delta t} = \frac{(57.0 \times 10^{-3}\text{ kg})[25.0\text{ m/s} - (-21.0\text{ m/s})]}{0.060\text{ s}}$$
$$= 43.7\text{ kg}\cdot\text{m/s}^2 = 43.7\text{ N}$$

OQ9.7 Answer (a). The magnitude of momentum is proportional to speed and the kinetic energy is proportional to speed squared. The speed of the rocket becomes 4 times larger, so the kinetic energy becomes 16 times larger.

OQ9.8 Answer (d). The magnitude of momentum is proportional to speed and the kinetic energy is proportional to speed squared. The speed of the rocket becomes 2 times larger, so the magnitude of the momentum becomes 2 times larger.

OQ9.9 Answer (c). The kinetic energy of a particle may be written as

$$KE = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

The ratio of the kinetic energies of two particles is then

$$\frac{(KE)_2}{(KE)_1} = \frac{p_2^2/2m_2}{p_1^2/2m_1} = \left(\frac{p_2}{p_1}\right)^2\left(\frac{m_1}{m_2}\right)$$

We see that, if the magnitudes of the momenta are equal ($p_2 = p_1$), the kinetic energies will be equal only if the masses are also equal. The correct response is then (c).

OQ9.10 Answer (d). Expressing the kinetic energy as $KE = p^2/2m$, we see that the ratio of the magnitudes of the momenta of two particles is

$$\frac{p_2}{p_1} = \frac{\sqrt{2m_2(KE)_2}}{\sqrt{2m_1(KE)_1}} = \sqrt{\left(\frac{m_2}{m_1}\right)\frac{(KE)_2}{(KE)_1}}$$

Thus, we see that if the particles have equal kinetic energies [$(KE)_2 = (KE)_1$], the magnitudes of their momenta are equal only if the masses are also equal. However, momentum is a *vector quantity* and we can say the two particles have equal momenta only it both the magnitudes and directions are equal, making choice (d) the correct answer.

OQ9.11 Answer (b). Before collision, the bullet, mass $m_1 = 10.0$ g, has speed $v_{1i} = v_b$, and the block, mass $m_2 = 200$ g, has speed $v_{2i} = 0$. After collision, the objects have a common speed (velocity) $v_{1f} = v_{2f} = v$. The collision of the bullet with the block is completely inelastic:

$$m_1v_{1i} + m_2v_2 = m_1v_{1f} + m_2v_{2f}$$

$$m_1v_b = (m_1 + m_2)v\,, \quad \text{so} \qquad v_b = v\left(\frac{m_1 + m_2}{m_1}\right)$$

The kinetic friction, $f_k = \mu_k n$, slows down the block with acceleration of magnitude $\mu_k g$. The block slides to a stop through a distance $d = 8.00$ m. Using $v_f^2 = v_i^2 + 2a(x_f - x_i)$, we find the speed of the block just after the collision:

$$v = \sqrt{2(0.400)(9.80 \text{ m/s}^2)(8.00 \text{ m})} = 7.92 \text{ m/s}.$$

Using the results above, the speed of the bullet before collision is

$$v_b = (7.92\,\text{m/s})\left(\frac{10 + 200}{10.0}\right) = 166 \text{ m/s}.$$

OQ9.12 Answer (c). The masses move through the same distance under the same force. Equal net work inputs imply equal kinetic energies.

OQ9.13 Answer (a). The same force gives the larger mass a smaller acceleration, so the larger mass takes a longer time interval to move through the same distance; therefore, the impulse given to the larger mass is larger, which means the larger mass will have a greater final momentum.

OQ9.14 Answer (d). Momentum of the ball-Earth system is conserved. Mutual gravitation brings the ball and the Earth together into one system. As the ball moves downward, the Earth moves upward, although with an acceleration on the order of 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate.

OQ9.15 Answer (d). Momentum is the same before and after the collision. Before the collision the momentum is

$$m_1 v_1 + m_2 v_2 = (3 \text{ kg})(+2 \text{ m/s}) + (2 \text{ kg})(-4 \text{ m/s}) = -2 \text{ kg}\cdot\text{m/s}$$

OQ9.16 Answer (a). The ball gives more rightward momentum to the block when the ball reverses its momentum.

OQ9.17 Answer (c). Assuming that the collision was head-on so that, after impact, the wreckage moves in the original direction of the car's motion, conservation of momentum during the impact gives

$$(m_c + m_t)v_f = m_c v_{0c} + m_t v_{0t} = m_c v + m_t(0)$$

or

$$v_f = \left(\frac{m_c}{m_c + m_t}\right)v = \left(\frac{m}{m + 2m}\right)v = \frac{v}{3}$$

OQ9.18 Answer (c). Billiard balls all have the same mass and collisions between them may be considered to be elastic. The dual requirements of conservation of kinetic energy and conservation of momentum in a one-dimensional, elastic collision are summarized by the two relations:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad \textbf{[1]}$$

and

$$v_{1i} - v_{2i} = \left(v_{1f} - v_{2f}\right) \qquad \textbf{[2]}$$

In this case, $m_1 = m_2$ and the masses cancel out of the first equation. Call the blue ball #1 and the red ball #2 so that $v_{1i} = -3v$, $v_{2i} = +v$, $v_{1f} = v_{\text{blue}}$, and $v_{2f} = v_{\text{red}}$. Then, the two equations become

$$-3v + v = v_{\text{blue}} + v_{\text{red}} \qquad \text{or} \qquad v_{\text{blue}} + v_{\text{red}} = v \qquad \textbf{[1]}$$

and

$$-3v - v = -\left(v_{\text{blue}} - v_{\text{red}}\right) \qquad \text{or} \qquad \left(v_{\text{blue}} - v_{\text{red}}\right) = 4v \qquad \textbf{[2]}$$

Adding the final versions of these equations yields $2v_{\text{blue}} = 2v$, or $v_{\text{blue}} = v$. Substituting this result into either [1] or [2] above then yields $v_{\text{red}} = -3v$.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ9.1 The passenger must undergo a certain momentum change in the collision. This means that a certain impulse must be exerted on the passenger by the steering wheel, the window, an air bag, or something. By increasing the distance over which the momentum change occurs, the time interval during which this change occurs is also increased, resulting in the force on the passenger being decreased.

CQ9.2 If the golfer does not "follow through," the club is slowed down by the golfer before it hits the ball, so the club has less momentum available to transfer to the ball during the collision.

CQ9.3 Its speed decreases as its mass increases. There are no external horizontal forces acting on the box, so its momentum cannot change as it moves along the horizontal surface. As the box slowly fills with water, its mass increases with time. Because the product mv must be constant, and because m is increasing, the speed of the box must decrease. Note that the vertically falling rain has no horizontal momentum of its own, so the box must "share" its momentum with the rain it catches.

CQ9.4 (a) It does not carry force, force requires another object on which to act.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

CQ9.5 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is much smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum. If we choose you as the system, momentum conservation is not violated because you are not an isolated system.

CQ9.6 The rifle has a much lower speed than the bullet and much less kinetic energy. Also, the butt distributes the recoil force over an area much larger than that of the bullet.

CQ9.7 The time interval over which the egg is stopped by the sheet (more for a faster missile) is much longer than the time interval over which the egg is stopped by a wall. For the same change in momentum, the longer the time interval, the smaller the force required to stop the egg. The sheet increases the time interval so that the stopping force is never too large.

CQ9.8 (a) Assuming that both hands are never in contact with a ball, and one hand is in contact with any one ball 20% of the time, the total contact time with the system of three balls is 3(20%) = 60% of the time. The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little closed loop with a parabolic top and likely a circular bottom, making three revolutions for every one revolution that one ball makes.

(b) On average, in one cycle of the system, the center of mass of the balls does not change position, so its average acceleration is zero (i.e., the average net force on the system is zero). Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J(0.60T) = 3F_gT$, and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

CQ9.9 (a) In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. The rocket body itself does accelerate as it blows exhaust containing momentum out the back.

(b) According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

CQ9.10 To generalize broadly, around 1740 the English favored position (a), the Germans position (b), and the French position (c). But in France Emilie de Chatelet translated Newton's *Principia* and argued for a more inclusive view. A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All the theories are equally correct. Each is useful for giving a mathematically simple and conceptually clear solution for some problems. There is another comprehensive mechanical theory, the angular impulse–angular momentum theorem, which we will glimpse in Chapter 11. It identifies the product of the torque of a force and the time it acts as the cause of a change in motion, and change in angular momentum as the effect.

We have here an example of how scientific theories are different from what people call a theory in everyday life. People who think that different theories are mutually exclusive should bring their thinking up to date to around 1750.

CQ9.11 No. Impulse, $\vec{\mathbf{F}}\Delta t$, depends on the force and the time interval during which it is applied.

CQ9.12 No. Work depends on the force and on the displacement over which it acts.

CQ9.13 (a) Linear momentum is conserved since there are no external forces acting on the system. The fragments go off in different directions and their vector momenta add to zero.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 9.1 Linear Momentum

P9.1 (a) The momentum is $p = mv$, so $v = p/m$ and the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$$

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$ so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

P9.2 $K = p^2/2m$, and hence, $p = \sqrt{2mK}$. Thus,

$$m = \frac{p^2}{2\cdot K} = \frac{(25.0\ \text{kg}\cdot\text{m/s})^2}{2(275\ \text{J})} = \boxed{1.14\ \text{kg}}$$

and

$$v = \frac{p}{m} = \frac{\sqrt{2m(K)}}{m} = \sqrt{\frac{2(K)}{m}} = \sqrt{\frac{2(275\ \text{J})}{1.14\ \text{kg}}} = \boxed{22.0\ \text{m/s}}$$

P9.3 We apply the impulse-momentum theorem to relate the change in the horizontal momentum of the sled to the horizontal force acting on it:

$$\Delta p_x = F_x\Delta t \rightarrow F_x = \frac{\Delta p_x}{\Delta t} = \frac{mv_{xf} - mv_{xi}}{\Delta t}$$

$$F_x = \frac{-(17.5\ \text{kg})(3.50\ \text{m/s})}{8.75\ \text{s}}$$

$$F_x = \boxed{7.00\ \text{N}}$$

***P9.4** We are given $m = 3.00$ kg and $\vec{\mathbf{v}} = (3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m/s.

(a) The vector momentum is then

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = (3.00\ \text{kg})\left[(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})\ \text{m/s}\right]$$
$$= (9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}})\ \text{kg}\cdot\text{m/s}$$

Thus, $\boxed{p_x = 9.00\ \text{kg}\cdot\text{m/s}}$ and $\boxed{p_y = -12.0\ \text{kg}\cdot\text{m/s}}$.

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00\ \text{kg}\cdot\text{m/s})^2 + (12.0\ \text{kg}\cdot\text{m/s})^2}$

$= \boxed{15.0\ \text{kg}\cdot\text{m/s}}$

at an angle of

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$$

P9.5 We apply the impulse-momentum theorem to find the average force the bat exerts on the baseball:

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t \rightarrow \vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = m\left(\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t}\right)$$

Choosing the direction toward home plate as the positive x direction, we have $\vec{\mathbf{v}}_i = (45.0\text{ m/s})\hat{\mathbf{i}}$, $\vec{\mathbf{v}}_f = (55.0\text{ m/s})\hat{\mathbf{j}}$, and $\Delta t = 2.00$ ms:

$$\vec{\mathbf{F}}_{\text{on ball}} = m\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t} = (0.145\text{ kg})\frac{(55.0\text{ m/s})\hat{\mathbf{j}} - (45.0\text{ m/s})\hat{\mathbf{i}}}{2.00 \times 10^{-3}\text{ s}}$$

$$\vec{\mathbf{F}}_{\text{on ball}} = \left(-3.26\hat{\mathbf{i}} + 3.99\hat{\mathbf{j}}\right)\text{ N}$$

By Newton's third law,

$$\vec{\mathbf{F}}_{\text{on bat}} = -\vec{\mathbf{F}}_{\text{on ball}} \quad \text{so} \quad \boxed{\vec{\mathbf{F}}_{\text{on bat}} = \left(+3.26\hat{\mathbf{i}} - 3.99\hat{\mathbf{j}}\right)\text{ N}}$$

Section 9.2 Analysis Model: Isolated system (Momentum)

P9.6 (a) The girl-plank system is isolated, so horizontal momentum is conserved.

We measure momentum relative to the ice: $\vec{\mathbf{p}}_{gi} + \vec{\mathbf{p}}_{pi} = \vec{\mathbf{p}}_{gf} + \vec{\mathbf{p}}_{pf}$.

The motion is in one dimension, so we can write,

$$v_{gi}\hat{\mathbf{i}} = v_{gp}\hat{\mathbf{i}} + v_{pi}\hat{\mathbf{i}} \rightarrow v_{gi} = v_{gp} + v_{pi}$$

where v_{gi} denotes the velocity of the girl relative to the ice, v_{gp} the velocity of the girl relative to the plank, and v_{pi} the velocity of the plank relative to the ice. The momentum equation becomes

$$0 = m_g v_{gi}\hat{\mathbf{i}} + m_p v_{pi}\hat{\mathbf{i}} \rightarrow 0 = m_g v_{gi} + m_p v_{pi}$$

$$0 = m_g\left(v_{gp} + v_{pi}\right) + m_p v_{pi}$$

$$0 = m_g v_{gp} + \left(m_g + m_p\right)v_{pi} \rightarrow v_{pi} = -\left(\frac{m_g}{m_g + m_p}\right)v_{gp}$$

solving for the velocity of the plank gives

$$v_{pi} = -\left(\frac{m_g}{m_g + m_p}\right) v_{gp} = -\left(\frac{45.0 \text{ kg}}{45.0 \text{ kg} + 150 \text{ kg}}\right)(1.50 \text{ m/s})$$

$$\boxed{v_{pi} = -0.346 \text{ m/s}}$$

(b) Using our result above, we find that

$$v_{gi} = v_{gp} + v_{pi} = (1.50 \text{ m/s}) + (-0.346 \text{ m/s})$$

$$\boxed{v_{gi} = 1.15 \text{ m/s}}$$

P9.7 (a) The girl-plank system is isolated, so horizontal momentum is conserved.

We measure momentum relative to the ice: $\vec{\mathbf{p}}_{gi} + \vec{\mathbf{p}}_{pi} = \vec{\mathbf{p}}_{gf} + \vec{\mathbf{p}}_{pf}$.

The motion is in one dimension, so we can write

$$v_{gi}\hat{\mathbf{i}} = v_{gp}\hat{\mathbf{i}} + v_{pi}\hat{\mathbf{i}} \rightarrow v_{gi} = v_{gp} + v_{pi}$$

where v_{gi} denotes the velocity of the girl relative to the ice, v_{gp} the velocity of the girl relative to the plank, and v_{pi} the velocity of the plank relative to the ice. The momentum equation becomes

$$0 = m_g v_{gi}\hat{\mathbf{i}} + m_p v_{pi}\hat{\mathbf{i}} \rightarrow 0 = m_g v_{gi} + m_p v_{pi}$$

$$0 = m_g\left(v_{gp} + v_{pi}\right) + m_p v_{pi}$$

$$0 = m_g v_{gp} + \left(m_g + m_p\right) v_{pi}$$

solving for the velocity of the plank gives

$$\boxed{v_{pi} = -\left(\frac{m_g}{m_g + m_p}\right) v_{gp}}$$

(b) Using our result above, we find that

$$v_{gi} = v_{gp} + v_{pi} = v_{gp}\frac{\left(m_g + m_p\right)}{m_g + m_p} - \frac{m_g}{m_g + m_p} v_{gp}$$

$$v_{gi} = \frac{\left(m_g + m_p\right) v_{gp} - m_g v_{gp}}{m_g + m_p}$$

$$v_{gi} = \frac{m_g v_{gp} + m_p v_{gp} - m_g v_{gp}}{m_g + m_p}$$

$$\boxed{v_{gi} = \left(\frac{m_p}{m_g + m_p}\right) v_{gp}}$$

P9.8 (a) Brother and sister exert equal-magnitude oppositely-directed forces on each other for the same time interval; therefore, the impulses acting on them are equal and opposite. Taking east as the positive direction, we have

impulse on boy: $I = F\Delta t = \Delta p = (65.0 \text{ kg})(-2.90 \text{ m/s}) = -189 \text{ N} \cdot \text{s}$

impulse on girl: $I = -F\Delta t = -\Delta p = +189 \text{ N} \cdot \text{s} = mv_f$

Her speed is then

$$v_f = \frac{I}{m} = \frac{189 \text{ N} \cdot \text{s}}{40.0 \text{ kg}} = 4.71 \text{ m/s}$$

meaning $\boxed{\text{she moves at 4.71 m/s east}}$.

(b) original chemical potential energy in girl's body = total final kinetic energy

$$\begin{aligned} U_{\text{chemical}} &= \frac{1}{2} m_{\text{boy}} v_{\text{boy}}^2 + \frac{1}{2} m_{\text{girl}} v_{\text{girl}}^2 \\ &= \frac{1}{2}(65.0 \text{ kg})(2.90 \text{ m/s})^2 + \frac{1}{2}(40.0 \text{ kg})(4.71 \text{ m/s})^2 \\ &= \boxed{717 \text{ J}} \end{aligned}$$

(c) $\boxed{\text{Yes.}}$ System momentum is conserved with the value zero.

(d) The forces on the two siblings are internal forces, which cannot change the momentum of the system— $\boxed{\text{the system is isolated}}$.

(e) $\boxed{\text{Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.}}$

***P9.9** We assume that the velocity of the blood is constant over the 0.160 s. Then the patient's body and pallet will have a constant velocity of $\dfrac{6\times10^{-5}\text{ m}}{0.160\text{ s}} = 3.75\times10^{-4}\text{ m/s}$ in the opposite direction. Momentum conservation gives

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}:$$

$$0 = m_{\text{blood}}(0.500\text{ m/s}) + (54.0\text{ kg})(-3.75\times10^{-4}\text{ m/s})$$

$$m_{\text{blood}} = 0.040\ 5\text{ kg} = \boxed{40.5\text{ g}}$$

P9.10 I have mass 72.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a\left(x_f - x_i\right): \qquad 0 - v_i^2 = 2\left(-9.80\text{ m/s}^2\right)(0.250\text{ m})$$

$$v_i = 2.20\text{ m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the planet down and myself up:

$$0 = \left(5.98\times10^{24}\text{ kg}\right)(-v_e) + (85.0\text{ kg})(2.20\text{ m/s})$$

$$v_e \sim \boxed{10^{-23}\text{ m/s}}$$

P9.11 (a) For the system of two blocks $\Delta p = 0$, or $p_i = p_f$. Therefore,

$$0 = mv_m + (3m)(2.00\text{ m/s})$$

Solving gives $v_m = \boxed{-6.00\text{ m/s}}$ (motion toward the left).

(b)
$$\frac{1}{2}kx^2 = \frac{1}{2}mv_M^2 + \frac{1}{2}(3m)v_{3M}^2$$
$$= \frac{1}{2}(0.350\text{ kg})(-6.00\text{ m/s})^2 + \frac{3}{2}(0.350\text{ kg})(2.00\text{ m/s})^2$$
$$= \boxed{8.40\text{ J}}$$

(c) $\boxed{\text{The original energy is in the spring.}}$

(d) A force had to be exerted over a displacement to compress the spring, transferring energy into it by work.
$\boxed{\text{The cord exerts force, but over no displacement.}}$

(e) $\boxed{\text{System momentum is conserved with the value zero.}}$

(f) The forces on the two blocks are internal forces, which cannot change the momentum of the system— $\boxed{\text{the system is isolated.}}$

(g) $\boxed{\text{Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.}}$

Section 9.3 Analysis Model: Nonisolated system (Momentum)

P9.12 (a) $I = F_{\text{avg}}\Delta t$, where I is the impulse the man must deliver to the child:

$$I = F_{\text{avg}}\Delta t = \Delta p_{\text{child}} = m_{\text{child}}\left|v_f - v_i\right| \rightarrow F_{\text{avg}} = \frac{m_{\text{child}}\left|v_f - v_i\right|}{\Delta t}$$

Solving for the average force gives

$$F_{\text{avg}} = \frac{m_{\text{child}}\left|v_f - v_i\right|}{\Delta t} = \frac{(12.0\text{ kg})\left|0 - 60\text{ mi/h}\right|}{0.10\text{ s}}\left(\frac{0.447\text{ m/s}}{1\text{ mi/h}}\right)$$

$$= \boxed{3.22 \times 10^3\text{ N}}$$

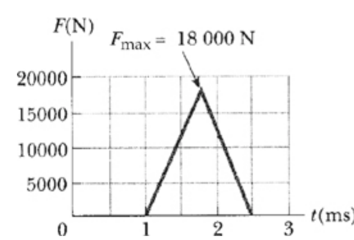
or

$$F_{\text{avg}} = \left(3.22 \times 10^3\text{ N}\right)\left(\frac{0.224\,8\text{ lb}}{1\text{ N}}\right) \approx \boxed{720\text{ lb}}$$

(b) $\boxed{\text{The man's claim is nonsense.}}$ He would not be able to exert a force of this magnitude on the child. In reality, the violent forces during the collision would tear the child from his arms.

(c) $\boxed{\text{These devices are essential for the safety of small children.}}$

P9.13 (a) The impulse delivered to the ball is equal to the area under the *F*-*t* graph. We have a triangle and so to get its area we multiply half its height times its width:



ANS. FIG. P9.13

$$I = \int F dt = \text{area under curve}$$

$$I = \frac{1}{2}\left(1.50 \times 10^{-3}\text{ s}\right)\left(18\,000\text{ N}\right) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $F = \dfrac{13.5\ \text{N}\cdot\text{s}}{1.50\times10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$

P9.14 (a) The impulse the floor exerts on the ball is equal to the change in momentum of the ball:

$$\begin{aligned}\Delta\vec{\mathbf{p}} &= m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right) = m\left(v_f - v_i\right)\hat{\mathbf{j}} \\ &= (0.300\ \text{kg})\left[(5.42\ \text{m/s}) - (-5.86\ \text{m/s})\right]\hat{\mathbf{j}} \\ &= \boxed{3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}}\end{aligned}$$

(b) Estimating the contact time interval to be 0.05 s, from the impulse-momentum theorem, we find

$$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}}{0.05\ \text{s}} \rightarrow \boxed{\vec{\mathbf{F}} = 7\times10^{2}\ \text{N}\ \hat{\mathbf{j}}}$$

P9.15 (a) The mechanical energy of the isolated spring-mass system is conserved:

$$\begin{aligned}K_i + U_{si} &= K_f + U_{sf} \\ 0 + \frac{1}{2}kx^2 &= \frac{1}{2}mv^2 + 0 \\ v &= x\sqrt{\frac{k}{m}}\end{aligned}$$

(b) $I = \left|\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i\right| = mv_f - 0 = mx\sqrt{\dfrac{k}{m}} = x\sqrt{km}$

(c) For the glider, $W = K_f - K_i = \dfrac{1}{2}mv^2 - 0 = \dfrac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

***P9.16** We take the x axis directed toward the pitcher.

(a) In the x direction, $p_{xi} + I_x = p_{xf}$:

$$\begin{aligned}I_x &= p_{xf} - p_{xi} \\ &= (0.200\ \text{kg})(40.0\ \text{m/s})\cos 30.0^\circ \\ &\qquad - (0.200\ \text{kg})(15.0\ \text{m/s})(-\cos 45.0^\circ) \\ &= 9.05\ \text{N}\cdot\text{s}\end{aligned}$$

In the y direction, $p_{yi} + I_y = p_{yf}$:

$$I_y = p_{yf} - p_{yi}$$
$$= (0.200 \text{ kg})(40.0 \text{ m/s})\sin 30.0°$$
$$- (0.200 \text{ kg})(15.0 \text{ m/s})(-\sin 45.0°)$$
$$= 6.12 \text{ N}\cdot\text{s}$$

Therefore, $\vec{\mathbf{I}} = \boxed{\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right) \text{ N}\cdot\text{s}}$

(b) $\vec{\mathbf{I}} = \frac{1}{2}\left(0 + \vec{\mathbf{F}}_m\right)(4.00 \text{ ms}) + \vec{\mathbf{F}}_m(20.0 \text{ ms}) + \frac{1}{2}\vec{\mathbf{F}}_m(4.00 \text{ ms})$

$$\vec{\mathbf{F}}_m \times 24.0 \times 10^{-3} \text{ s} = \left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right) \text{ N}\cdot\text{s}$$

$$\vec{\mathbf{F}}_m = \boxed{\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right) \text{ N}}$$

***P9.17** (a) From the kinematic equations,

$$\Delta t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_f + v_i} = \frac{2(1.20 \text{ m})}{0 + 25.0 \text{ m/s}} = \boxed{9.60 \times 10^{-2} \text{ s}}$$

(b) We find the average force from the momentum-impulse theorem:

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{(1\ 400 \text{ kg})(25.0 \text{ m/s} - 0)}{9.60 \times 10^{-2} \text{ s}} = \boxed{3.65 \times 10^5 \text{ N}}$$

(c) Using the particle under constant acceleration model,

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{25.0 \text{ m/s} - 0}{9.60 \times 10^{-2} \text{ s}} = \left(260 \text{ m/s}^2\right)\left(\frac{1\,g}{9.80 \text{ m/s}^2}\right) = \boxed{26.5g}$$

P9.18 We assume that the initial direction of the ball is in the $-x$ direction.

(a) The impulse delivered to the ball is given by

$$\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$$
$$= \left(0.060\ 0 \text{ kg}\right)(40.0 \text{ m/s})\hat{\mathbf{i}} - (0.060\ 0 \text{ m/s})(20.0 \text{ m/s})\left(-\hat{\mathbf{i}}\right)$$
$$= \boxed{3.60\hat{\mathbf{i}} \text{ N}\cdot\text{s}}$$

(b) We choose the tennis ball as a nonisolated system for energy. Let the time interval be from just before the ball is hit until just after. Equation 9.2 for conservation of energy becomes

$$\Delta K + \Delta E_{\text{int}} = T_{\text{MW}}$$

Solving for the energy sum $\Delta E_{\text{int}} - T_{\text{MW}}$ and substituting gives

$$\Delta E_{\text{int}} - T_{\text{MW}} = -\Delta K = -\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = \frac{1}{2}m\left(v_i^2 - v_f^2\right)$$

Substituting numerical values gives,

$$\Delta E_{\text{int}} - T_{\text{MW}} = -\frac{1}{2}(0.060\,0\text{ kg})\left[(20.0\text{ m/s})^2 - (40.0\text{ m/s})^2\right]$$
$$= 36.0\text{ J}$$

There is no way of knowing how the energy splits between ΔE_{int} and T_{MW} without more information.

P9.19 (a) The impulse is in the x direction and equal to the area under the F-t graph:

$$I = \left(\frac{0+4\text{ N}}{2}\right)(2\text{ s}-0) + (4\text{ N})(3\text{ s}-2\text{ s}) + \left(\frac{4\text{ N}+0}{2}\right)(5\text{ s}-3\text{ s})$$
$$= 12.0\text{ N}\cdot\text{s}$$

$$\boxed{\vec{\mathbf{I}} = 12.0\text{ N}\cdot\text{s }\hat{\mathbf{i}}}$$

(b) From the momentum-impulse theorem,

$$m\vec{\mathbf{v}}_i + \vec{\mathbf{F}}\Delta t = m\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}\Delta t}{m} = 0 + \frac{12.0\,\hat{\mathbf{i}}\text{ N}\cdot\text{s}}{2.50\text{ kg}} = \boxed{4.80\,\hat{\mathbf{i}}\text{ m/s}}$$

(c) From the same equation,

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}\Delta t}{m} = -2.00\,\hat{\mathbf{i}}\text{ m/s} + \frac{12.0\,\hat{\mathbf{i}}\text{ N}\cdot\text{s}}{2.50\text{ kg}} = \boxed{2.80\,\hat{\mathbf{i}}\text{ m/s}}$$

(d) $\vec{\mathbf{F}}_{\text{avg}}\Delta t = 12.0\hat{\mathbf{i}}\text{ N}\cdot\text{s} = \vec{\mathbf{F}}_{\text{avg}}(5.00\text{ s}) \rightarrow \vec{\mathbf{F}}_{\text{avg}} = \boxed{2.40\hat{\mathbf{i}}\text{ N}}$

P9.20 (a) A graph of the expression for force shows a parabola opening down, with the value zero at the beginning and end of the 0.800-s interval. We integrate the given force to find the impulse:

$$I = \int_0^{0.800\text{s}} F\,dt$$
$$= \int_0^{0.800\text{s}} (9\,200\,t\text{ N/s} - 11\,500\,t^2\text{ N/s}^2)\,dt$$
$$= \left[\frac{1}{2}(9\,200\text{ N/s})t^2 - \frac{1}{3}(11\,500\text{ N/s}^2)t^3\right]_0^{0.800\text{s}}$$
$$= \frac{1}{2}(9\,200\text{ N/s})(0.800\text{ s})^2 - \frac{1}{3}(11\,500\text{ N/s}^2)(0.800\text{ s})^3$$
$$= 2\,944\text{ N}\cdot\text{s} - 1\,963\text{ N}\cdot\text{s} = 981\text{ N}\cdot\text{s}$$

The athlete imparts a downward impulse to the platform, so the platform imparts to her an impulse of $\boxed{981\text{ N}\cdot\text{s, up.}}$

(b) We could find her impact speed as a free-fall calculation, but we choose to write it as a conservation-of-energy calculation:

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{impact}}^2$$

$$v_{\text{impact}} = \sqrt{2gy_{\text{top}}} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.600\ \text{m})}$$

$$= \boxed{3.43\ \text{m/s, down}}$$

(c) Gravity, as well as the platform, imparts impulse to her during the interaction with the platform

$$I = \Delta p$$

$$I_{\text{grav}} + I_{\text{platform}} = mv_f - mv_i$$

$$-mg\Delta t + I_{\text{platform}} = mv_f - mv_i$$

solving for the final velocity gives

$$v_f = v_i - mg\Delta t + \frac{I_{\text{platform}}}{m}$$

$$= (-3.43\ \text{m/s}) - \left(9.80\ \text{m/s}^2\right)(0.800\ \text{s}) + \frac{981\ \text{N}\cdot\text{s}}{65.0\ \text{kg}}$$

$$= \boxed{3.83\ \text{m/s, up}}$$

Note that the athlete is putting a lot of effort into jumping and does not exert any force "on herself." The usefulness of the force platform is to measure her effort by showing the force she exerts on the floor.

(d) Again energy is conserved in upward flight:

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{takeoff}}^2$$

which gives

$$y_{\text{top}} = \frac{v_{\text{takeoff}}^2}{2g} = \frac{(3.83\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{0.748\ \text{m}}$$

P9.21 After 3.00 s of pouring, the bucket contains

(3.00 s)(0.250 L/s) = 0.750 liter

of water, with mass (0.750 L)(1 kg/1 L) = 0.750 kg, and feeling gravitational force (0.750 kg)(9.80 m/s^2) = 7.35 N. The scale through the bucket must exert 7.35 N upward on this stationary water to support its weight. The scale must exert another 7.35 N to support the 0.750-kg bucket itself.

Water is entering the bucket with speed given by

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{impact}}^2$$

$$v_{\text{impact}} = \sqrt{2gy_{\text{top}}} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(2.60\text{ m}\right)}$$
$$= 7.14\text{ m/s, downward}$$

The scale exerts an extra upward force to stop the downward motion of this additional water, as described by

$$mv_{\text{impact}} + F_{\text{extra}}t = mv_f$$

The rate of change of momentum is the force itself:

$$\left(\frac{dm}{dt}\right)v_{\text{impact}} + F_{\text{extra}} = 0$$

which gives

$$F_{\text{extra}} = -\left(\frac{dm}{dt}\right)v_{\text{impact}} = -\left(0.250\text{ kg/s}\right)\left(-7.14\text{ m/s}\right) = 1.78\text{ N}$$

Altogether the scale must exert 7.35 N + 7.35 N + 1.78 N = $\boxed{16.5\text{ N}}$

Section 9.4 Collisions in One Dimension

P9.22 (a) Conservation of momentum gives

$$m_T v_{Tf} + m_C v_{Cf} = m_T v_{Ti} + m_C v_{Ci}$$

Solving for the final velocity of the truck gives

$$v_{Tf} = \frac{m_T v_{Ti} + m_C\left(v_{Ci} - v_{Cf}\right)}{m_T}$$
$$= \frac{\left(9\ 000\text{ kg}\right)\left(20.0\text{ m/s}\right) + \left(1\ 200\text{ kg}\right)\left[\left(25.0 - 18.0\right)\text{ m/s}\right]}{9\ 000\text{ kg}}$$

$$v_{Tf} = \boxed{20.9\text{ m/s East}}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\begin{aligned}
\Delta KE = KE_f - KE_i &= \left[\frac{1}{2}m_C v_{Cf}^2 + \frac{1}{2}m_T v_{Tf}^2\right] - \left[\frac{1}{2}m_C v_{Ci}^2 + \frac{1}{2}m_T v_{Ti}^2\right] \\
&= \frac{1}{2}\left[m_C\left(v_{Cf}^2 - v_{Ci}^2\right) + m_T\left(v_{Tf}^2 - v_{Ti}^2\right)\right] \\
&= \frac{1}{2}\left\{(1\ 200\ \text{kg})\left[(18.0\ \text{m/s})^2 - (25.0\ \text{m/s})^2\right]\right. \\
&\qquad \left. + (9\ 000\ \text{kg})\left[(20.9\ \text{m/s})^2 - (20.0\ \text{m/s})^2\right]\right\}
\end{aligned}$$

$$\Delta KE = \boxed{-8.68 \times 10^3\ \text{J}}$$

Note: If 20.9 m/s were used to determine the energy lost instead of 20.9333 as the answer to part (a), the answer would be very different. We have kept extra digits in all intermediate answers until the problem is complete.

(c) The mechanical energy of the car-truck system has decreased. Most of the energy was transformed to internal energy with some being carried away by sound.

P9.23 Momentum is conserved for the bullet-block system:

$$mv + 0 = (m + M)v_f$$

$$v = \left(\frac{m+M}{m}\right)v_f = \left(\frac{10.0\times 10^{-3}\ \text{kg} + 5.00\ \text{kg}}{10.0\times 10^{-3}\ \text{kg}}\right)(0.600\ \text{m/s})$$

$$= \boxed{301\ \text{m/s}}$$

P9.24 The collision is completely inelastic.

(a) Momentum is conserved by the collision:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$mv_1 + (2m)v_2 = mv_f + 2mv_f = 3mv_f$$

$$v_f = \frac{mv_1 + 2mv_2}{3m} \rightarrow \boxed{v_f = \frac{1}{3}(v_1 + 2v_2)}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\Delta K = K_f - K_i = \frac{1}{2}(3m)v_f^2 - \left[\frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2\right]$$

$$\Delta K = \frac{3m}{2}\left[\frac{1}{3}(v_1 + 2v_2)\right]^2 - \left[\frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2\right]$$

$$\Delta K = \frac{3m}{2}\left(\frac{v_1^2}{9} + \frac{4v_1v_2}{9} + \frac{4v_2^2}{9}\right) - \frac{mv_1^2}{2} - mv_2^2$$

$$= m\left(\frac{v_1^2}{6} + \frac{2v_1v_2}{3} + \frac{2v_2^2}{3} - \frac{v_1^2}{2} - v_2^2\right)$$

$$\Delta K = m\left(\frac{v_1^2}{6} + \frac{4v_1v_2}{6} + \frac{4v_2^2}{6} - \frac{3v_1^2}{6} - \frac{6v_2^2}{6}\right)$$

$$= m\left(-\frac{2v_1^2}{6} + \frac{4v_1v_2}{6} - \frac{2v_2^2}{6}\right)$$

$$\Delta K = \boxed{-\frac{m}{3}\left(v_1^2 + v_2^2 - 2v_1v_2\right)}$$

***P9.25** (a) We write the law of conservation of momentum as

$$mv_{1i} + 3mv_{2i} = 4mv_f$$

or $$v_f = \frac{4.00\text{ m/s} + 3(2.00\text{ m/s})}{4} = \boxed{2.50\text{ m/s}}$$

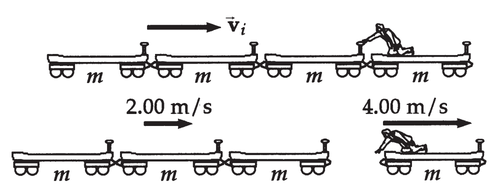
(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right]$$

$$= \frac{1}{2}\left(2.50\times 10^4\text{ kg}\right)[4(2.50\text{ m/s})^2$$

$$-(4.00\text{ m/s})^2 - 3(2.00\text{ m/s})^2]$$

$$= \boxed{-3.75\times 10^4\text{ J}}$$

***P9.26** (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor. Conservation of momentum gives



ANS. FIG. P9.26

$$(4m)v_i = (3m)(2.00\text{ m/s}) + m(4.00\text{ m/s})$$

$$v_i = \frac{6.00\text{ m/s} + 4.00\text{ m/s}}{4} = \boxed{2.50\text{ m/s}}$$

(b) $W_{\text{actor}} = K_f - K_i$

$$= \frac{1}{2}\left[(3m)(2.00 \text{ m/s})^2 + m(4.00 \text{ m/s})^2\right] - \frac{1}{2}(4m)(2.50 \text{ m/s})^2$$

$$W_{\text{actor}} = \frac{(2.50\times 10^4 \text{ kg})}{2}(12.0 + 16.0 - 25.0)(\text{m/s})^2 = \boxed{37.5 \text{ kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

P9.27 (a) From the text's analysis of a one-dimensional elastic collision with an originally stationary target, the x component of the neutron's velocity changes from v_i to $v_{1f} = (1 - 12)v_i/13 = -11v_i/13$. The x component of the target nucleus velocity is $v_{2f} = 2v_i/13$.

The neutron started with kinetic energy $\frac{1}{2}m_1 v_i^2$.

The target nucleus ends up with kinetic energy $\frac{1}{2}(12m_1)\left(\frac{2v_i}{13}\right)^2$.

Then the fraction transferred is

$$\frac{\frac{1}{2}(12m_1)(2v_i/13)^2}{\frac{1}{2}m_1 v_i^2} = \frac{48}{169} = \boxed{0.284}$$

Because the collision is elastic, the other 71.6% of the original energy stays with the neutron. The carbon is functioning as a *moderator* in the reactor, slowing down neutrons to make them more likely to produce reactions in the fuel.

(b) The final kinetic energy of the neutron is

$$K_n = (0.716)(1.60\times 10^{-13} \text{ J}) = \boxed{1.15\times 10^{-13} \text{ J}}$$

and the final kinetic energy of the carbon nucleus is

$$K_C = (0.284)(1.60\times 10^{-13} \text{ J}) = \boxed{4.54\times 10^{-14} \text{ J}}$$

***P9.28** Let's first analyze the situation in which the wood block, of mass $m_w = 1.00$ kg, is held in a vise. The bullet of mass $m_b = 7.00$ g is initially moving with speed v_b and then comes to rest in the block due to the kinetic friction force f_k between the block and the bullet as the bullet deforms the wood fibers and moves them out of the way. The result is an increase in internal energy in the wood and the bullet. Identify the wood and the bullet as an isolated system for energy during the collision:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substituting for the energies:

$$\left(0 - \frac{1}{2}m_b v_b^2\right) + f_k d = 0 \qquad \textbf{[1]}$$

where d = 8.00 cm is the depth of penetration of the bullet in the wood.

Now consider the second situation, where the block is sitting on a frictionless surface and the bullet is fired into it. Identify the wood and the bullet as an isolated system for energy during the collision:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substituting for the energies:

$$\left[\frac{1}{2}(m_b + m_w)v_f^2 - \frac{1}{2}m_b v_b^2\right] + f_k d' = 0 \qquad \textbf{[2]}$$

where v_f is the speed with which the block and imbedded bullet slide across the table after the collision and d' is the depth of penetration of the bullet in this situation. Identify the wood and the bullet as an isolated system for momentum during the collision:

$$\Delta p = 0 \quad \rightarrow \quad p_i = p_f \quad \rightarrow \quad m_b v_b = (m_b + m_w)v_f \qquad \textbf{[3]}$$

Solving equation [3] for v_b, we obtain

$$v_b = \frac{(m_b + m_w)v_f}{m_b} \qquad \textbf{[4]}$$

Solving equation [1] for $f_k d$ and substituting for v_b from equation [4] above:

$$f_k d = \frac{1}{2}m_b v_b^2 = \frac{1}{2}m_b\left[\frac{(m_b + m_w)v_f}{m_b}\right]^2 = \frac{1}{2}\frac{(m_b + m_w)^2}{m_b}v_f^2 \qquad \textbf{[5]}$$

Solving equation [2] for $f_k d'$ and substituting for v_b from equation [4]:

$$f_k d' = -\left[\frac{1}{2}(m_b + m_w)v_f^2 - \frac{1}{2}m_b v_b^2\right]$$

$$= -\left[\frac{1}{2}(m_b + m_w)v_f^2 - \frac{1}{2}m_b\left[\frac{(m_b + m_w)v_f}{m_b}\right]^2\right]$$

$$f_k d' = \frac{1}{2}\left[\frac{m_w}{m_b}(m_b + m_w)\right]v_f^2 \qquad \textbf{[6]}$$

Dividing equation [6] by [5] gives

$$\frac{f_k d'}{f_k d} = \frac{d'}{d} = \frac{\frac{1}{2}\left[\frac{m_w}{m_b}(m_b + m_w)\right]v_f^2}{\frac{1}{2}\left[\frac{(m_b + m_w)^2}{m_b}\right]v_f^2} = \frac{m_w}{m_b + m_w}$$

Solving for d' and substituting numerical values gives

$$d' = \left(\frac{m_w}{m_b + m_w}\right)d = \left[\frac{1.00\text{ kg}}{0.007\ 00\text{ kg} + 1.00\text{ kg}}\right](8.00\text{ cm}) = \boxed{7.94\text{ cm}}$$

***P9.29** (a) The speed v of both balls just before the basketball reaches the ground may be found from $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ as

$$v = \sqrt{v_{yi}^2 + 2a_y\Delta y} = \sqrt{0 + 2(-g)(-h)} = \sqrt{2gh}$$

$$= \sqrt{2(9.80\text{ m/s}^2)(1.20\text{ m})} = \boxed{4.85\text{ m/s}}$$

(b) Immediately after the basketball rebounds from the floor, it and the tennis ball meet in an elastic collision. The velocities of the two balls just before collision are

for the tennis ball (subscript t): $v_{ti} = -v$

and for the basketball (subscript b): $v_{bi} = +v$

We determine the velocity of the tennis ball immediately after this elastic collision as follows:

Momentum conservation gives

$$m_t v_{tf} + m_b v_{bf} = m_t v_{ti} + m_b v_{bi}$$

or $$m_t v_{tf} + m_b v_{bf} = (m_b - m_t)v \qquad \textbf{[1]}$$

From the criteria for a perfectly elastic collision:

$$v_{ti} - v_{bi} = -\left(v_{tf} - v_{bf}\right)$$

or $$v_{bf} = v_{tf} + v_{ti} - v_{bi} = v_{tf} - 2v \qquad \textbf{[2]}$$

Substituting equation [2] into [1] gives

$$m_t v_{tf} + m_b\left(v_{tf} - 2v\right) = \left(m_b - m_t\right)v$$

or the upward speed of the tennis ball immediately after the collision is

$$v_{tf} = \left(\frac{3m_b - m_t}{m_t + m_b}\right)v = \left(\frac{3m_b - m_t}{m_t + m_b}\right)\sqrt{2gh}$$

The vertical displacement of the tennis ball during its rebound following the collision is given by $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ as

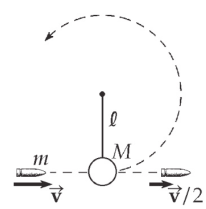
$$\Delta y = \frac{v_{yf}^2 - v_{yi}^2}{2a_y} = \frac{0 - v_{tf}^2}{2(-g)} = \left(\frac{1}{\cancel{2g}}\right)\left(\frac{3m_b - m_t}{m_t + m_b}\right)^2\left(\cancel{2g}h\right)$$

$$= \left(\frac{3m_b - m_t}{m_t + m_b}\right)^2 h$$

Substituting,

$$\Delta y = \left[\frac{3(590\text{ g}) - (57.0\text{ g})}{57.0\text{ g} + 590\text{ g}}\right]^2 (1.20\text{ m}) = \boxed{8.41\text{ m}}$$

P9.30 Energy is conserved for the bob-Earth system between bottom and top of the swing. At the top the stiff rod is in compression and the bob nearly at rest.



ANS. FIG. P9.30

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = 4g\ell \qquad \text{so} \qquad v_b = 2\sqrt{g\ell}$$

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \quad \rightarrow \quad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.31 The collision between the clay and the wooden block is completely inelastic. Momentum is conserved by the collision. Find the relation between the speed of the clay (C) just before impact and the speed of the clay+block (CB) just after impact:

$$\vec{\mathbf{p}}_{Bi} + \vec{\mathbf{p}}_{Ci} = \vec{\mathbf{p}}_{Bf} + \vec{\mathbf{p}}_{Cf} \rightarrow m_B v_{Bi} + m_C v_{Ci} = m_B v_{Bf} + m_C v_{Cf}$$

$$M(0) + mv_C = mv_{CB} + Mv_{CB} = (m+M)v_{CB}$$

$$v_C = \frac{(m+M)}{m} v_{CB}$$

Now use conservation of energy in the presence of friction forces to find the relation between the speed v_{CB} just after impact and the distance the block slides before stopping:

$$\Delta K + \Delta E_{\text{int}} = 0: \quad 0 - \frac{1}{2}(m+M)v_{CB}^2 - fd = 0$$

$$\text{and} \quad -fd = -\mu nd = -\mu(m+M)gd$$

$$\rightarrow \frac{1}{2}(m+M)v_{CB}^2 = \mu(m+M)gd \rightarrow v_{CB} = \sqrt{2\mu gd}$$

Combining our results, we have

$$v_C = \frac{(m+M)}{m}\sqrt{2\mu gd}$$

$$= \frac{(12.0\text{ g} + 100\text{ g})}{12.0\text{ g}}\sqrt{2(0.650)(9.80\text{ m/s}^2)(7.50\text{ m})}$$

$$\boxed{v_C = 91.2\text{ m/s}}$$

P9.32 The collision between the clay and the wooden block is completely inelastic. Momentum is conserved by the collision. Find the relation between the speed of the clay (C) just before impact and the speed of the clay+block (CB) just after impact:

$$\vec{\mathbf{p}}_{Bi} + \vec{\mathbf{p}}_{Ci} = \vec{\mathbf{p}}_{Bf} + \vec{\mathbf{p}}_{Cf} \rightarrow \mathrm{m}_B v_{Bi} + \mathrm{m}_C v_{Ci} = \mathrm{m}_B v_{Bf} + \mathrm{m}_C v_{Cf}$$

$$M(0) + mv_C = mv_{CB} + Mv_{CB} = (m+M)v_{CB}$$

$$v_C = \frac{(m+M)}{m} v_{CB}$$

Now use conservation of energy in the presence of friction forces to find the relation between the speed v_{CB} just after impact and the distance the block slides before stopping:

$$\Delta K + \Delta E_{\text{int}} = 0: \quad 0 - \frac{1}{2}(m+M)v_{CB}^2 - fd = 0$$

$$\text{and} \qquad -fd = -\mu nd = -\mu(m+M)gd$$

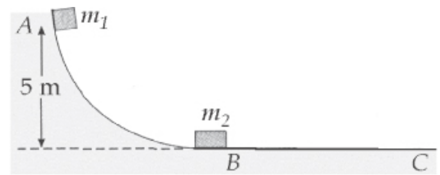
Then,

$$\frac{1}{2}(m+M)v_{CB}^2 = \mu(m+M)gd \rightarrow v_{CB} = \sqrt{2\mu gd}$$

Combining our results, we have

$$\boxed{v_C = \frac{(m+M)}{m}\sqrt{2\mu gd}}$$

P9.33 The mechanical energy of the isolated block-Earth system is conserved as the block of mass m_1 slides down the track. First we find v_1, the speed of m_1 at B before collision:



ANS. FIG. P9.33

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_1v_1^2 + 0 = 0 + m_1gh$$

$$v_1 = \sqrt{2(9.80 \text{ m/s}^2)(5.00 \text{ m})} = 9.90 \text{ m/s}$$

Now we use the text's analysis of one-dimensional elastic collisions to find v_{1f}, the speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_1 = -\frac{1}{3}(9.90) \text{ m/s} = -3.30 \text{ m/s}$$

Now the 5-kg block bounces back up to its highest point after collision according to

$$m_1gh_{max} = \frac{1}{2}m_1v_{1f}^2$$

which gives

$$h_{max} = \frac{v_{1f}^2}{2g} = \frac{(-3.30 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.556 \text{ m}}$$

P9.34 (a) Using conservation of momentum, $\left(\sum \vec{\mathbf{p}}\right)_{\text{before}} = \left(\sum \vec{\mathbf{p}}\right)_{\text{after}}$, gives

$$(4.00 \text{ kg})(5.00 \text{ m/s}) + (10.0 \text{ kg})(3.00 \text{ m/s}) + (3.00 \text{ kg})(-4.00 \text{ m/s}) = [(4.00 + 10.0 + 3.00) \text{ kg}]v$$

Therefore, $v = +2.24$ m/s, or $\boxed{\text{2.24 m/s toward the right}}$.

(b) $\boxed{\text{No.}}$ For example, if the 10.0-kg and 3.00-kg masses were to stick together first, they would move with a speed given by solving

$$(13.0\text{ kg})v_1 = (10.0\text{ kg})(3.00\text{ m/s}) + (3.00\text{ kg})(-4.00\text{ m/s})$$

or $v_1 = +1.38\text{ m/s}$

Then when this 13.0-kg combined mass collides with the 4.00-kg mass, we have

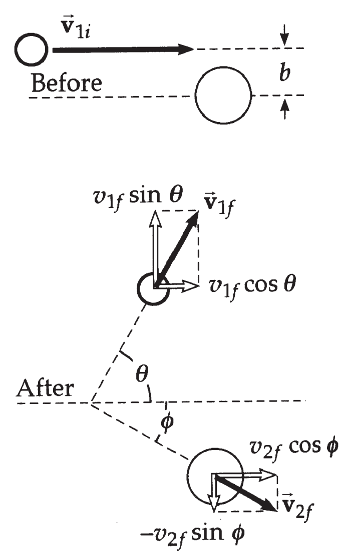
$$(17.0\text{ kg})v = (13.0\text{ kg})(1.38\text{ m/s}) + (4.00\text{ kg})(5.00\text{ m/s})$$

and $v = +2.24$ m/s, just as in part (a).

$\boxed{\text{Coupling order makes no difference to the final velocity.}}$

Section 9.5 Collisions in Two Dimensions

***P9.35** (a) We write equations expressing conservation of the x and y components of momentum, with reference to the figures on the right. Let the puck initially at rest be m_2. In the x direction,



ANS. FIG. P9.35

$$m_1v_{1i} = m_1v_{1f}\cos\theta + m_2v_{2f}\cos\phi$$

which gives

$$v_{2f}\cos\phi = \frac{m_1v_{1i} - m_1v_{1f}\cos\theta}{m_2}$$

or

$$v_{2f}\cos\phi = \left(\frac{1}{0.300\text{ kg}}\right)\left[(0.200\text{ kg})(2.00\text{ m/s}) - (0.200\text{ kg})(1.00\text{ m/s})\cos 53.0°\right]$$

In the y direction,

$$0 = m_1v_{1f}\sin\theta - m_2v_{2f}\sin\phi$$

which gives

$$v_{2f}\sin\phi = \frac{m_1v_{1f}\sin\theta}{m_2}$$

or

$$0 = (0.200 \text{ kg})(1.00 \text{ m/s})\sin 53.0° - (0.300 \text{ kg})(v_{2f} \sin\phi)$$

From these equations, we find

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{0.532}{0.932} = 0.571 \quad \text{or} \quad \boxed{\phi = 29.7°}$$

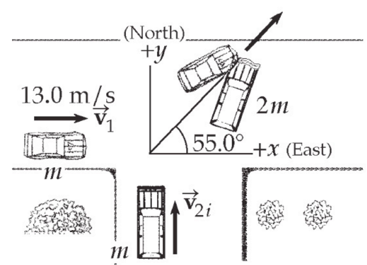
Then $v_{2f} = \dfrac{0.160 \text{ kg}\cdot\text{m/s}}{(0.300 \text{ kg})(\sin 29.7°)} = \boxed{1.07 \text{ m/s}}$

(b) $K_i = \dfrac{1}{2}(0.200 \text{ kg})(2.00 \text{ m/s})^2 = 0.400 \text{ J}$ and

$$K_f = \frac{1}{2}(0.200 \text{ kg})(1.00 \text{ m/s})^2 + \frac{1}{2}(0.300 \text{ kg})(1.07 \text{ m/s})^2 = 0.273 \text{ J}$$

$$f_{\text{lost}} = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{0.273 \text{ J} - 0.400 \text{ J}}{0.400 \text{ J}} = \boxed{-0.318}$$

P9.36 We use conservation of momentum for the system of two vehicles for both northward and eastward components, to find the original speed of car number 2.



ANS. FIG. P8.26

For the eastward direction:

$$m(13.0 \text{ m/s}) = 2mV_f \cos 55.0°$$

For the northward direction:

$$mv_{2i} = 2mV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0 \text{ m/s})\tan 55.0° = 18.6 \text{ m/s} = 41.5 \text{ mi/h}$$

Thus, the driver of the northbound car was untruthful. His original speed was more than 35 mi/h.

P9.37 We will use conservation of both the x component and the y component of momentum for the two-puck system, which we can write as a single vector equation.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

Both objects have the same final velocity, which we call $\vec{\mathbf{v}}_f$. Doing the algebra and substituting to solve for the one unknown gives

$$\vec{\mathbf{v}}_f = \frac{m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i}}{m_1 + m_2}$$
$$= \frac{(3.00\text{ kg})(5.00\hat{\mathbf{i}}\text{ m/s}) + (2.00\text{ kg})(-3.00\hat{\mathbf{j}}\text{ m/s})}{3.00\text{ kg} + 2.00\text{ kg}}$$

and calculating gives $\vec{\mathbf{v}}_f = \dfrac{15.0\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}{5.00}\text{ m/s} = \boxed{(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}})\text{ m/s}}$

P9.38 We write the conservation of momentum in the x direction, $p_{xf} = p_{xi}$, as

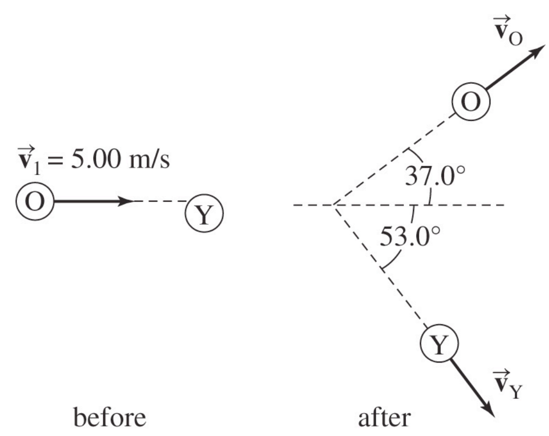
$$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00\text{ m/s})$$
$$0.799v_O + 0.602v_Y = 5.00\text{ m/s} \qquad \textbf{[1]}$$

and the conservation of momentum in the y direction, $p_{yf} = p_{yi}$, as

$$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$$
$$0.602v_O = 0.799v_Y \qquad \textbf{[2]}$$

Solving equations [1] and [2] simultaneously gives,

$\boxed{v_O = 3.99\text{ m/s}}$ and $\boxed{v_Y = 3.01\text{ m/s}}$



ANS. FIG. P9.38

P9.39 ANS. FIG. P9.38 illustrates the collision. We write the conservation of momentum in the x direction, $p_{xf} = p_{xi}$, as

$$mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$$
$$v_O \cos\theta + v_Y \sin\theta = v_i \qquad \textbf{[1]}$$

and the conservation of momentum in the y direction, $p_{yf} = p_{yi}$, as

$$mv_{\rm O}\sin\theta - mv_{\rm Y}\cos\left(90.0° - \theta\right) = 0$$

$$v_{\rm O}\sin\theta = v_{\rm Y}\cos\theta \qquad \textbf{[2]}$$

From equation [2],

$$v_{\rm O} = v_{\rm Y}\left(\frac{\cos\theta}{\sin\theta}\right) \qquad \textbf{[3]}$$

Substituting into equation [1],

$$v_{\rm Y}\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_{\rm Y}\sin\theta = v_i$$

so

$$v_{\rm Y}\left(\cos^2\theta + \sin^2\theta\right) = v_i\sin\theta, \text{ and } \boxed{v_{\rm Y} = v_i\sin\theta}$$

Then, from equation [3], $\boxed{v_{\rm O} = v_i\cos\theta}$.

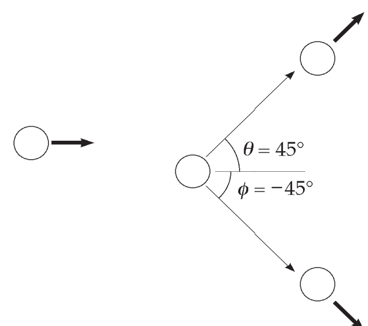
We did not need to write down an equation expressing conservation of mechanical energy. In this situation, the requirement on perpendicular final velocities is equivalent to the condition of elasticity.

***P9.40** (a) The vector expression for conservation of momentum, $\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$ gives $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$.

$$mv_i = mv\cos\theta + mv\cos\phi \qquad \textbf{[1]}$$

$$0 = mv\sin\theta + mv\sin\phi \qquad \textbf{[2]}$$

From [2], $\sin\theta = -\sin\phi$ so $\theta = -\phi$.



ANS. FIG. P9.40

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so

$$\boxed{v = \frac{v_i}{\sqrt{2}}}$$

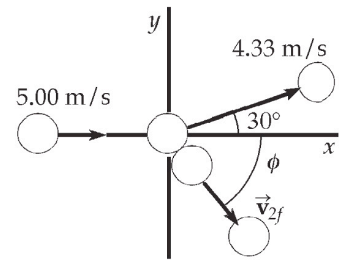
(b) Hence, [1] gives

$$v_i = \frac{2v_i \cos\theta}{\sqrt{2}}$$

with $\theta = \boxed{45.0°}$ and $\phi = \boxed{-45.0°}$.

P9.41 By conservation of momentum for the system of the two billiard balls (with all masses equal), in the x and y directions separately,

$$5.00\text{ m/s} + 0 = (4.33\text{ m/s})\cos 30.0° + v_{2fx}$$
$$v_{2fx} = 1.25\text{ m/s}$$
$$0 = (4.33\text{ m/s})\sin 30.0° + v_{2fy}$$
$$v_{2fy} = -2.16\text{ m/s}$$
$$\vec{\mathbf{v}}_{2f} = \boxed{2.50\text{ m/s at } -60.0°}$$



ANS. FIG. P9.41

Note that we did not need to explicitly use the fact that the collision is perfectly elastic.

P9.42 (a) $\boxed{\text{The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is perfectly inelastic.}}$

(b) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback):

$$(90.0\text{ kg})(5.00\text{ m/s}) + 0 = (185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta = 2.43\text{ m/s} \qquad \textbf{[1]}$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent):

$$(95.0\text{ kg})(3.00\text{ m/s}) + 0 = (185\text{ kg})V\sin\theta$$

which gives

$$V\sin\theta = 1.54\text{ m/s} \qquad \textbf{[2]}$$

Divide equation [2] by [1]:

$$\tan\theta = \frac{1.54}{2.43} = 0.633$$

From which, $\boxed{\theta = 32.3°}$.

Then, either [1] or [2] gives $V = \boxed{2.88\text{ m/s}}$.

(c) $K_i = \frac{1}{2}(90.0\text{ kg})(5.00\text{ m/s})^2 + \frac{1}{2}(95.0\text{ kg})(3.00\text{ m/s})^2 = 1.55 \times 10^3\text{ J}$

$K_f = \frac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2 = 7.67 \times 10^2\text{ J}$

Thus, the kinetic energy lost is $\boxed{\text{786 J into internal energy}}$.

P9.43 (a) With three particles, the total final momentum of the system is $m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} + m_3\vec{\mathbf{v}}_{3f}$ and it must be zero to equal the original momentum. The mass of the third particle is

$$m_3 = (17.0 - 5.00 - 8.40) \times 10^{-27}\text{ kg}$$

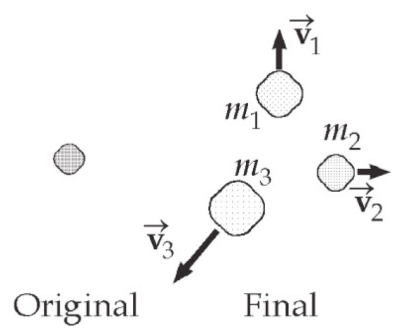
or $m_3 = 3.60 \times 10^{-27}\text{ kg}$

Solving $m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} + m_3\vec{\mathbf{v}}_{3f} = 0$ for $\vec{\mathbf{v}}_{3f}$ gives

$$\vec{\mathbf{v}}_{3f} = -\frac{m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}}{m_3}$$

$$\vec{\mathbf{v}}_{3f} = -\frac{(3.36\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}) \times 10^{-20}\text{ kg}\cdot\text{m/s}}{3.60 \times 10^{-27}\text{ kg}}$$

$$= \boxed{(-9.33 \times 10^6\,\hat{\mathbf{i}} - 8.33 \times 10^6\,\hat{\mathbf{j}})\text{ m/s}}$$



ANS. FIG. P9.43

(b) The original kinetic energy of the system is zero.

The final kinetic energy is $K = K_{1f} + K_{2f} + K_{3f}$.

The terms are

$$K_{1f} = \frac{1}{2}(5.00 \times 10^{-27}\text{ kg})(6.00 \times 10^6\text{ m/s})^2 = 9.00 \times 10^{-14}\text{ J}$$

$$K_{2f} = \frac{1}{2}(8.40\times10^{-27}\ \text{kg})(4.00\times10^{6}\ \text{m/s})^2 = 6.72\times10^{-14}\ \text{J}$$

$$\begin{aligned} K_{3f} &= \frac{1}{2}(3.60\times10^{-27}\ \text{kg}) \\ &\qquad \times\left[(-9.33\times10^{6}\ \text{m/s})^2+(-8.33\times10^{6}\ \text{m/s})^2\right] \\ &= 28.2\times10^{-14}\ \text{J} \end{aligned}$$

Then the system kinetic energy is

$$\begin{aligned} K &= 9.00\times10^{-14}\ \text{J} + 6.72\times10^{-14}\ \text{J} + 28.2\times10^{-14}\ \text{J} \\ &= \boxed{4.39\times10^{-13}\ \text{J}} \end{aligned}$$

P9.44 The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0\ \text{m/s})$$

and $\quad v_{Bi} = 8.33\ \text{m/s}$

From conservation of energy,

$$K_i = \frac{1}{2}m(10.0\ \text{m/s})^2 + \frac{1}{2}(1.20m)(8.33\ \text{m/s})^2 = \frac{1}{2}m(183\ \text{m}^2/\text{s}^2)$$

$$K_f = \frac{1}{2}m(v_G)^2 + \frac{1}{2}(1.20m)(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}m(183\ \text{m}^2/\text{s}^2)\right)$$

or $\quad v_G^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2 \qquad$ **[1]**

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or $\quad v_G = 1.20v_B \qquad$ **[2]**

Solving [1] and [2] simultaneously, we find

$$(1.20v_B)^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2$$

$$v_B = (91.7\ \text{m}^2/\text{s}^2/2.64)^{1/2}$$

which gives

$\boxed{v_B = 5.89\ \text{m/s}}$ (speed of blue puck after collision)

and $\quad \boxed{v_G = 7.07\ \text{m/s}}$ (speed of green puck after collision)

Section 9.6 The Center of Mass

P9.45 The x coordinate of the center of mass is

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{2.00\text{ kg} + 3.00\text{ kg} + 2.50\text{ kg} + 4.00\text{ kg}} = 0$$

and the y coordinate of the center of mass is

$$\begin{aligned} y_{\text{CM}} &= \frac{\sum m_i y_i}{\sum m_i} \\ &= \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg} + 2.50\text{ kg} + 4.00\text{ kg}}\right) \\ &\qquad \times [(2.00\text{ kg})(3.00\text{ m}) + (3.00\text{ kg})(2.50\text{ m}) \\ &\qquad\quad + (2.50\text{ kg})(0) + (4.00\text{ kg})(-0.500\text{ m})] \\ y_{\text{CM}} &= 1.00\text{ m} \end{aligned}$$

Then $\boxed{\vec{\mathbf{r}}_{\text{CM}} = \left(0\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right)\text{ m}}$

P9.46 Let the x axis start at the Earth's center and point toward the Moon.

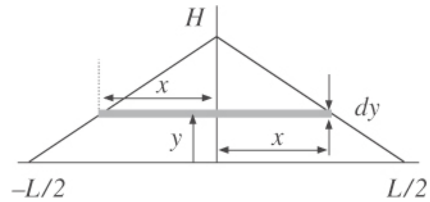
$$\begin{aligned} x_{\text{CM}} &= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \\ &= \frac{\left(5.97\times10^{24}\text{ kg}\right)(0) + \left(7.35\times10^{22}\text{ kg}\right)\left(3.84\times10^{8}\text{ m}\right)}{6.05\times10^{24}\text{ kg}} \\ &= \boxed{4.66\times10^{6}\text{ m from the Earth's center}} \end{aligned}$$

The center of mass is within the Earth, which has radius 6.37×10^6 m. It is 1.7 Mm below the point on the Earth's surface where the Moon is straight overhead.

P9.47 The volume of the monument is that of a thick triangle of base $L = 64.8$ m, height $H = 15.7$ m, and width $W = 3.60$ m: $V = \frac{1}{2} LHW = 1.83 \times 10^3\text{ m}^3$. The monument has mass $M = \rho V = (3\,800\text{ kg/m}^3)V = 6.96 \times 10^6$ kg. The height of the center of mass (CM) is $y_{\text{CM}} = H/3$ (derived below). The amount of work done on the blocks is

$$\begin{aligned} U_g &= Mgy_{\text{CM}} \\ &= Mg\frac{H}{3} = \left(6.96\times10^{6}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(\frac{15.7\text{ m}}{3}\right) \\ &= \boxed{3.57\times10^{8}\text{ J}} \end{aligned}$$

We derive $y_{CM} = H/3$ here:



ANS. FIG. P9.47

We model the monument with the figure shown above. Consider the monument to be composed of slabs of infinitesimal thickness dy stacked on top of each other. A slab at height y has a infinitesimal volume element $dV = 2xWdy$, where W is the width of the monument and x is a function of height y.

The equation of the sloping side of the monument is

$$y = H - \frac{H}{L/2}x \to y = H - \frac{2H}{L}x \to y = H\left(1 - \frac{2}{L}x\right)$$

where x ranges from 0 to $+L/2$. Therefore,

$$x = \frac{L}{2}\left(1 - \frac{y}{H}\right)$$

where y ranges from 0 to H. The infinitesimal volume of a slab at height y is then

$$dV = 2xWdy = LW\left(1 - \frac{y}{H}\right)dy.$$

The mass contained in a volume element is $dm = \rho dV$.

Because of the symmetry of the monument, its CM lies above the origin of the coordinate axes at position y_{CM}:

$$y_{CM} = \frac{1}{M}\int_0^M y\,dm = \frac{1}{M}\int_0^V y\rho\,dV = \frac{1}{M}\int_0^H y\rho LW\left(1 - \frac{y}{H}\right)dy$$

$$y_{CM} = \frac{\rho LW}{M}\int_0^H\left(y - \frac{y^2}{H}\right)dy = \frac{\rho LW}{M}\left(\frac{y^2}{2} - \frac{y^3}{3H}\right)\Bigg|_0^H$$

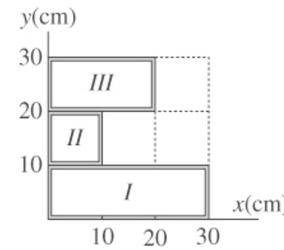
$$= \frac{\rho LW}{M}\left(\frac{H^2}{2} - \frac{H^3}{3H}\right)$$

$$y_{CM} = \frac{\rho LWH^2}{M}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6}\frac{\rho LWH^2}{\left(\frac{1}{2}\rho LWH\right)} = \left(\frac{2}{1}\right)\frac{H}{6}$$

$$y_{CM} = \frac{H}{3}$$

where we have used $M = \rho\left(\frac{1}{2}LHW\right)$.

P9.48 We could analyze the object as nine squares, each represented by an equal-mass particle at its center. But we will have less writing to do if we think of the sheet as composed of three sections, and consider the mass of each section to be at the geometric center of that section. Define the mass per unit area to be σ, and number the rectangles as shown. We can then calculate the mass and identify the center of mass of each section.



ANS. FIG. P9.48

$$m_I = (30.0\text{ cm})(10.0\text{ cm})\sigma \quad \text{with} \quad CM_I = (15.0\text{ cm}, 5.00\text{ cm})$$

$$m_{II} = (10.0\text{ cm})(20.0\text{ cm})\sigma \quad \text{with} \quad CM_{II} = (5.00\text{ cm}, 20.0\text{ cm})$$

$$m_{III} = (10.0\text{ cm})(10.0\text{ cm})\sigma \quad \text{with} \quad CM_{III} = (15.0\text{ cm}, 25.0\text{ cm})$$

The overall center of mass is at a point defined by the vector equation:

$$\vec{\mathbf{r}}_{\text{CM}} \equiv \left(\sum m_i \vec{\mathbf{r}}_i\right) / \sum m_i$$

Substituting the appropriate values, $\vec{\mathbf{r}}_{\text{CM}}$ is calculated to be:

$$\vec{\mathbf{r}}_{\text{CM}} = \left(\frac{1}{\sigma\left(300\text{ cm}^2 + 200\text{ cm}^2 + 100\text{ cm}^2\right)}\right) \times \left\{\sigma[(300)(15.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}) + (200)(5.00\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}}) + (100)(15.0\hat{\mathbf{i}} + 25.0\hat{\mathbf{j}})]\text{ cm}^3\right\}$$

Calculating,

$$\vec{\mathbf{r}}_{\text{CM}} = \frac{4\,500\hat{\mathbf{i}} + 1\,500\hat{\mathbf{j}} + 1\,000\hat{\mathbf{i}} + 4\,000\hat{\mathbf{j}} + 1\,500\hat{\mathbf{i}} + 2\,500\hat{\mathbf{j}}}{600}\text{ cm}$$

and evaluating, $\vec{\mathbf{r}}_{\text{CM}} = \boxed{(11.7\hat{\mathbf{i}} + 13.3\hat{\mathbf{j}})\text{ cm}}$

P9.49 This object can be made by wrapping tape around a light, stiff, uniform rod.

(a) $$M = \int_0^{0.300\text{ m}} \lambda\, dx = \int_0^{0.300\text{ m}} [50.0 + 20.0x]\, dx$$

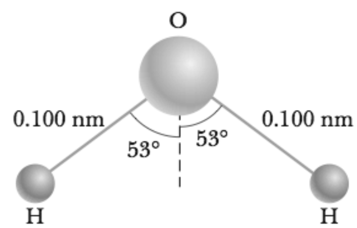
$$M = \left[50.0x + 10.0x^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b) $$x_{CM} = \frac{\int_{\text{all mass}} x\,dm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda\, x\,dx = \frac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x + 20.0x^2\right]dx$$

$$x_{CM} = \frac{1}{15.9\text{ g}}\left[25.0x^2 + \frac{20x^3}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***P9.50** We use a coordinate system centered in the oxygen (O) atom, with the x axis to the right and the y axis upward. Then, from symmetry,

$$\boxed{x_{CM} = 0}$$



ANS. FIG. P9.50

and

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$$

$$= \left(\frac{1}{15.999\text{ u} + 1.008\text{ u} + 1.008\text{ u}}\right)$$
$$\times[0 - (1.008\text{ u})(0.100\text{ nm})\cos 53.0°$$
$$-(1.008\text{ u})(0.100\text{ nm})\cos 53.0°]$$

The center of mass of the molecule lies on the dotted line shown in ANS. FIG. P9.50, 0.006 73 nm below the center of the O atom.

Section 9.7 Systems of Many Particles

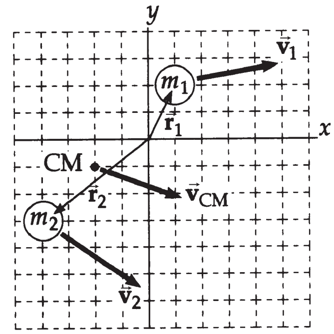
P9.51 (a) $$\vec{\mathbf{v}}_{CM} = \frac{\sum m_i \vec{\mathbf{v}}_i}{M} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{M}$$

$$= \left(\frac{1}{5.00\text{ kg}}\right)[(2.00\text{ kg})(2.00\hat{\mathbf{i}}\text{ m/s} - 3.00\hat{\mathbf{j}}\text{ m/s})$$
$$+ (3.00\text{ kg})(1.00\hat{\mathbf{i}}\text{ m/s} + 6.00\hat{\mathbf{j}}\text{ m/s})]$$

$$\vec{\mathbf{v}}_{CM} = \boxed{\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) $$\vec{\mathbf{p}} = M\vec{\mathbf{v}}_{CM} = (5.00\text{ kg})(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}})\text{ m/s} = \boxed{(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}}$$

***P9.52** (a) ANS. FIG. P9.52 shows the position vectors and velocities of the particles.



ANS. FIG. P9.52

(b) Using the definition of the position vector at the center of mass,

$$\vec{\mathbf{r}}_{CM} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2}$$

$$\vec{\mathbf{r}}_{CM} = \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\right)[(2.00\text{ kg})(1.00\text{ m},\ 2.00\text{ m}) + (3.00\text{ kg})(-4.00\text{ m},\ -3.00\text{ m})]$$

$$\vec{\mathbf{r}}_{CM} = \boxed{\left(-2.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ m}}$$

(c) The velocity of the center of mass is

$$\vec{\mathbf{v}}_{CM} = \frac{\vec{\mathbf{P}}}{M} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$$

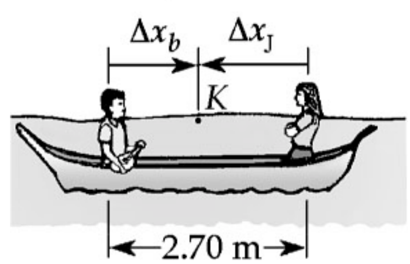
$$= \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\right)[(2.00\text{ kg})(3.00\text{ m/s},\ 0.50\text{ m/s}) + (3.00\text{ kg})(3.00\text{ m/s},\ -2.00\text{ m/s})]$$

$$\vec{\mathbf{v}}_{CM} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\vec{\mathbf{P}} = M\vec{\mathbf{v}}_{CM}$ or as $\vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$. Either gives

$$\vec{\mathbf{P}} = \boxed{\left(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$$

P9.53 No outside forces act on the boat-plus-lovers system, so its momentum is conserved at zero and the center of mass of the boat-passengers system stays fixed:



ANS. FIG. P9.53

$$x_{CM,i} = x_{CM,f}$$

Define K to be the point where they kiss, and Δx_J and Δx_b as shown in the figure. Since Romeo moves with the boat (and thus $\Delta x_{Romeo} = \Delta x_b$), let m_b be the combined mass of Romeo and the boat. The front of the boat and the shore are to the right in this picture, and we take the positive x direction to the right. Then,

$$m_J \Delta x_J + m_b \Delta x_b = 0$$

Choosing the x axis to point toward the shore,

$$(55.0 \text{ kg})\Delta x_J + (77.0 \text{ kg} + 80.0 \text{ kg})\Delta x_b = 0$$

and $\quad \Delta x_J = -2.85\Delta x_b$

As Juliet moves away from shore, the boat and Romeo glide toward the shore until the original 2.70-m gap between them is closed. We describe the relative motion with the equation

$$|\Delta x_J| + \Delta x_b = 2.70 \text{ m}$$

Here the first term needs absolute value signs because Juliet's change in position is toward the left. An equivalent equation is then

$$-\Delta x_J + \Delta x_b = 2.70 \text{ m}$$

Substituting, we find

$$+2.85\Delta x_b + \Delta x_b = 2.70 \text{ m}$$

so $\quad \Delta x_b = \dfrac{2.70 \text{ m}}{3.85} = \boxed{0.700 \text{ m}}$ towards the shore

P9.54 The vector position of the center of mass is (suppressing units)

$$\vec{r}_{CM} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2} = \frac{3.5\left[\left(3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)t + 2\hat{\mathbf{j}}t^2\right] + 5.5\left[3\hat{\mathbf{i}} - 2\hat{\mathbf{i}}t^2 + 6\hat{\mathbf{j}}t\right]}{3.5 + 5.5}$$

$$= \left(1.83 + 1.17t - 1.22t^2\right)\hat{\mathbf{i}} + \left(-2.5t + 0.778t^2\right)\hat{\mathbf{j}}$$

(a) At $t = 2.50$ s,

$$\vec{\mathbf{r}}_{CM} = (1.83 + 1.17\cdot 2.5 - 1.22\cdot 6.25)\hat{\mathbf{i}} + (-2.5\cdot 2.5 + 0.778\cdot 6.25)\hat{\mathbf{j}}$$

$$= \boxed{\left(-2.89\hat{\mathbf{i}} - 1.39\hat{\mathbf{j}}\right) \text{ cm}}$$

(b) The velocity of the center of mass is obtained by differentiating the expression for the vector position of the center of mass with respect to time:

$$\vec{\mathbf{v}}_{CM} = \frac{d\vec{\mathbf{r}}_{CM}}{dt} = (1.17 - 2.44t)\hat{\mathbf{i}} + (-2.5 + 1.56t)\hat{\mathbf{j}}$$

At $t = 2.50$ s,

$$\vec{\mathbf{v}}_{CM} = (1.17 - 2.44\cdot 2.5)\hat{\mathbf{i}} + (-2.5 + 1.56\cdot 2.5)\hat{\mathbf{j}}$$

$$= (-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}) \text{ cm/s}$$

Now, the total linear momentum is the total mass times the velocity of the center of mass.

$$\vec{\mathbf{p}} = (9.00\ \text{g})(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}})\ \text{cm/s}$$

$$= \boxed{(-44.5\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}})\ \text{g}\cdot\text{cm/s}}$$

(c) As was shown in part (b), $\boxed{\left(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}\right)\ \text{cm/s}}$

(d) Differentiating again, $\vec{\mathbf{a}}_{\text{CM}} = \dfrac{d\vec{\mathbf{v}}_{CM}}{dt} = (-2.44)\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}$

The center of mass acceleration is $\boxed{\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)\ \text{cm/s}^2}$ at t = 2.50 s and at all times.

(e) The net force on the system is equal to the total mass times the acceleration of the center of mass:

$$\vec{\mathbf{F}}_{\text{net}} = (9.00\ \text{g})\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)\ \text{cm/s}^2 = \boxed{\left(-220\hat{\mathbf{i}} + 140\hat{\mathbf{j}}\right)\ \mu\text{N}}$$

P9.55 (a) Conservation of momentum for the two-ball system gives us:

$$(0.200\ \text{kg})(1.50\ \text{m/s}) + (0.300\ \text{kg})(-0.400\ \text{m/s})$$
$$= (0.200\ \text{kg})v_{1f} + (0.300\ \text{kg})v_{2f}$$

Relative velocity equation:

$$v_{2f} - v_{1f} = 1.90\ \text{m/s}$$

Then, suppressing units, we have

$$0.300 - 0.120 = 0.200v_{1f} + 0.300(1.90 + v_{1f})$$

$$v_{1f} = -0.780\ \text{m/s} \qquad v_{2f} = 1.12\ \text{m/s}$$

$$\boxed{\vec{\mathbf{v}}_{1f} = -0.780\hat{\mathbf{i}}\ \text{m/s}} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 1.12\hat{\mathbf{i}}\ \text{m/s}}$$

(b) Before, $\vec{\mathbf{v}}_{\text{CM}} = \dfrac{(0.200\ \text{kg})(1.50\ \text{m/s})\hat{\mathbf{i}} + (0.300\ \text{kg})(-0.400\ \text{m/s})\hat{\mathbf{i}}}{0.500\ \text{kg}}$

$$\boxed{\vec{\mathbf{v}}_{\text{CM}} = (0.360\ \text{m/s})\hat{\mathbf{i}}}$$

Afterwards, the center of mass must move at the same velocity, because the momentum of the system is conserved.

Section 9.8 Deformable Systems

P9.56 (a) $\boxed{\text{Yes}}$ The only horizontal force on the vehicle is the frictional force exerted by the floor, so it gives the vehicle all of its final momentum, $(6.00\text{ kg})(3.00\hat{\mathbf{i}}\text{ m/s}) = \boxed{18.0\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}}$.

(b) $\boxed{\text{No. The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work.}}$

(c) $\boxed{\text{Yes, we could say that the final momentum of the cart came from the floor or from the Earth through the floor.}}$

(d) $\boxed{\text{No. The kinetic energy came from the original gravitational potential energy of the Earth-elevated load system, in the amount } KE = \left(\frac{1}{2}\right)(6.00\text{ kg})(3.00\text{ m/s})^2 = 27.0\text{ J.}}$

(e) $\boxed{\text{Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the wheels from slipping backward.}}$

P9.57 (a) When the cart hits the bumper it immediately stops, and the hanging particle keeps moving with its original speed v_i. The particle swings up as a pendulum on a fixed pivot, keeping constant energy. Measure elevations from the pivot:

$$\frac{1}{2}mv_i^2 + mg(-L) = 0 + mg(-L\cos\theta)$$

Then $v_i = \boxed{\sqrt{2gL(1-\cos\theta)}}$

(b) The bumper continues to exert a force to the left until the particle has swung down to $\boxed{\text{its lowest point}}$. This leftward force is necessary to reverse the rightward motion of the particle and accelerate it to the left.

P9.58 (a) $\boxed{\text{Yes}}$ The floor exerts a force, larger than the person's weight over time as he is taking off.

(b) $\boxed{\text{No}}$ The work by the floor on the person is zero because the force exerted by the floor acts over zero distance.

(c) He leaves the floor with a speed given by $\frac{1}{2}mv^2 = mgy_f$, or

$$v = \sqrt{2gy_f} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(0.150 \text{ m}\right)} = 1.71 \text{ m/s}$$

so his momentum immediately after he leaves the floor is

$$p = mv = \left(60.0 \text{ kg}\right)\left(1.71 \text{ m/s up}\right) = \boxed{103 \text{ kg}\cdot\text{m/s up}}$$

(d) Yes. You could say that it came from the planet, that gained momentum 103 kg · m/s down, but it came through the force exerted by the floor over a time interval on the person, so it came through the floor or from the floor through direct contact.

(e) His kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(60.0 \text{ kg}\right)\left(1.71 \text{ m/s}\right)^2 = \boxed{88.2 \text{ J}}$$

(f) No. The energy came from chemical energy in the person's leg muscles. The floor did no work on the person.

P9.59 Consider the motion of the center of mass (CM) of the system of the two pucks. Because the pucks have equal mass m, the CM lies at the midpoint of the line connecting the pucks.

(a) The force F accelerates the CM to the right at the rate

$$a_{\text{CM}} = \frac{F}{2m}$$

According to Figure P9.59, when the force has moved through distance d, the CM has moves through distance $D_{\text{CM}} = d - \frac{1}{2}\ell$. We can find the speed of the CM, which is the same as the speed v of the pucks when they meet and stick together:

$$v_f^2 = v_i^2 + 2a_{\text{CM}}\left(x_f - x_i\right)$$

$$v_{\text{CM}}^2 = 0 + 2\left(\frac{F}{2m}\right)\left(d - \frac{1}{2}\ell\right) \rightarrow \qquad v = v_{\text{CM}} = \boxed{\sqrt{\frac{F\left(2d - \ell\right)}{2m}}}$$

(b) The force F does work on the system through distance d, the work done is $W = Fd$. Relate this work to the change in kinetic energy and internal energy:

$$\Delta K + \Delta E_{\text{int}} = W$$

$$\text{where } \Delta K = \frac{1}{2}(2m)v_{\text{CM}}^2 = m\left[\frac{F(2d-\ell)}{2m}\right] = \frac{F(2d-\ell)}{2}$$

$$\left[\frac{F(2d-\ell)}{2}\right] + \Delta E_{\text{int}} = Fd \to \Delta E_{\text{int}} = Fd - \left[\frac{F(2d-\ell)}{2}\right]$$

$$\Delta E_{\text{int}} = Fd - Fd + \frac{F\ell}{2}$$

$$\Delta E_{\text{int}} = \boxed{\frac{F\ell}{2}}$$

Section 9.9 Rocket Propulsion

P9.60 (a) The fuel burns at a rate given by

$$\frac{dM}{dt} = \frac{12.7 \text{ g}}{1.90 \text{ s}} = 6.68 \times 10^{-3} \text{ kg/s}$$

From the rocket thrust equation,

$$\text{Thrust} = v_e \frac{dM}{dt}: \; 5.26 \text{ N} = v_e\left(6.68 \times 10^{-3} \text{ kg/s}\right)$$

$$v_e = \boxed{787 \text{ m/s}}$$

(b) $v_f - v_i = v_e \ln\left(\dfrac{M_i}{M_f}\right)$:

$$v_f - 0 = (787 \text{ m/s})\ln\left(\frac{53.5 \text{ g} + 25.5 \text{ g}}{53.5 \text{ g} + 25.5 \text{ g} - 12.7 \text{ g}}\right)$$

$$v_f = \boxed{138 \text{ m/s}}$$

***P9.61** The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\ \text{kg})(25.0\ \text{m/s}) - 0}{1.00\ \text{s}}$$
$$= \boxed{15.0\ \text{N}}$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0-N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

P9.62 (a) The thrust, F, is equal to the time rate of change of momentum as fuel is exhausted from the rocket.

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv_e)$$

Since the exhaust velocity v_e is a constant,

$$F = v_e(dm/dt),\ \text{where}\ dm/dt = 1.50 \times 10^4\ \text{kg/s}$$

and $v_e = 2.60 \times 10^3\ \text{m/s}$.

Then $F = (2.60 \times 10^3\,\text{m/s})(1.50 \times 10^4\,\text{kg/s}) = \boxed{3.90 \times 10^7\ \text{N}}$

(b) Applying $\sum F = ma$ gives

$$\sum F_y = \text{Thrust} - Mg = Ma:$$
$$3.90 \times 10^7\ \text{N} - (3.00 \times 10^6\ \text{kg})(9.80\ \text{m/s}^2) = (3.00 \times 10^6\ \text{kg})a$$

$$a = \boxed{3.20\ \text{m/s}^2}$$

P9.63 In $v = v_e \ln \frac{M_i}{M_f}$ we solve for M_i.

(a) $M_i = e^{v/v_e} M_f \quad \to \quad M_i = e^5 (3.00 \times 10^3\ \text{kg}) = 4.45 \times 10^5\ \text{kg}$

The mass of fuel and oxidizer is

$$\Delta M = M_i - M_f = (445 - 3.00) \times 10^3\,\text{kg}$$
$$= \boxed{442\ \text{metric tons}}$$

(b) $\Delta M = e^2 (3.00\ \text{metric tons}) - 3.00\ \text{metric tons} = \boxed{19.2\ \text{metric tons}}$

(c) This is much less than the suggested value of 442/2.5. Mathematically, the logarithm in the rocket propulsion equation is not a linear function. Physically, a higher exhaust speed has an extra-large cumulative effect on the rocket body's final speed, by counting again and again in the speed the body attains second after second during its burn.

P9.64 (a) From the equation for rocket propulsion in the text,

$$v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i}t\right)$

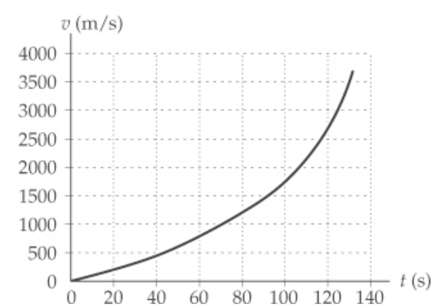
With the definition, $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

(b) With, $v_e = 1\,500$ m/s, and $T_p = 144$ s,

$$v = -(1\,500 \text{ m/s})\ln\left(1 - \frac{t}{144 \text{ s}}\right)$$

t (s)	v (m/s)
0	0
20	224
40	488
60	808
80	1 220
100	1 780
120	2 690
132	3 730



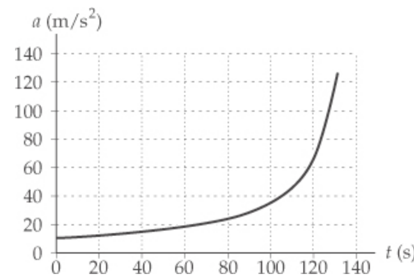
ANS. FIG. P9.64(b)

(c) $$a(t) = \frac{dv}{dt} = \frac{d\left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right]}{dt} = -v_e\left(\frac{1}{1 - \frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right) = \left(\frac{v_e}{T_p}\right)\left(\frac{1}{1 - \frac{t}{T_p}}\right),$$

or $a(t) = \boxed{\dfrac{v_e}{T_p - t}}$

(d) With, $v_e = 1\,500$ m/s, and $T_p = 144$ s, $a = \dfrac{1\,500 \text{ m/s}}{144 \text{ s} - t}$.

t (s)	a (m/s^2)
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125



ANS. FIG. P9.64(d)

(e) $$x(t) = 0 + \int_0^t v\,dt = \int_0^t \left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right]dt = v_e T_p \int_0^t \ln\left[1 - \frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$$

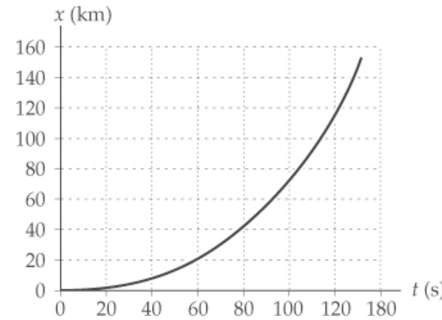
$$x(t) = v_e T_p \left[\left(1 - \frac{t}{T_p}\right)\ln\left(1 - \frac{t}{T_p}\right) - \left(1 - \frac{t}{T_p}\right)\right]_0^t$$

$$x(t) = \boxed{v_e\left(T_p - t\right)\ln\left(1 - \frac{t}{T_p}\right) + v_e t}$$

(f) With, $v_e = 1.500$ m/s $= 1.50$ km/s, and $T_p = 144$ s,

$$x = 1.50(144 - t)\ln\left(1 - \frac{t}{144}\right) + 1.50t$$

t (s)	x (m)
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153



ANS. FIG. P9.64(f)

Additional Problems

P9.65 (a) At the highest point, the velocity of the ball is zero, so momentum is also $\boxed{\text{zero}}$.

(b) Use $v_{yf}^2 = v_{yi}^2 + 2a(y_f - y_i)$ to find the maximum height $H_{\max}$:

$$0 = v_i + 2(-g)H_{\max}$$

or $$H_{\max} = \frac{v_i^2}{2g}$$

Now, find the speed of the ball for $(y_f - y_i) = \frac{1}{2}H_{\max}$:

$$v_f^2 = v_i^2 = 2(-g)\left(\frac{1}{2}H_{\max}\right)$$

$$= v_i^2 - 2g\left(\frac{1}{2}\right)\left(\frac{v_i^2}{2g}\right) = v_i^2 - \frac{1}{2}v_i^2 = \frac{1}{2}v_i^2$$

which gives $v_f = \dfrac{v_i}{\sqrt{2}}$

Then, $p_f = mv_f = \boxed{\dfrac{mv_i}{\sqrt{2}}\text{, upward}}$

P9.66 (a) The system is isolated because the skater is on frictionless ice — if it were otherwise, she would be able to move. Initially, the horizontal momentum of the system is zero, and this quantity is conserved; so when she throws the gloves in one direction, she will move in the opposite direction because the total momentum will remain zero. The system has total mass M. After the skater throws the gloves, the mass of the gloves, m, is moving with velocity $\vec{\mathbf{v}}_{\text{gloves}}$ and the mass of the skater less the gloves, $M - m$, is moving with velocity $\vec{\mathbf{v}}_{\text{girl}}$:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$

$$0 = (M - m)\vec{\mathbf{v}}_{\text{girl}} + m\vec{\mathbf{v}}_{\text{gloves}} \rightarrow \vec{\mathbf{v}}_{\text{girl}} = \boxed{-\left(\frac{m}{M-m}\right)\vec{\mathbf{v}}_{\text{gloves}}}$$

The term $M - m$ is the total mass less the mass of the gloves.

(b) As she throws the gloves and exerts a force on them, the gloves exert an equal and opposite force on her (Newton's third law) that causes her to accelerate from rest to reach the velocity $\vec{\mathbf{v}}_{\text{girl}}$.

P9.67 In $\vec{\mathbf{F}}\Delta t = \Delta(m\vec{\mathbf{v}})$, one component gives

$$\Delta p_y = m\left(v_{yf} - v_{yi}\right) = m(v\cos 60.0° - v\cos 60.0°) = 0$$

So the wall does not exert a force on the ball in the y direction. The other component gives

$$\begin{aligned}\Delta p_x &= m\left(v_{xf} - v_{xi}\right) = m(-v\sin 60.0° - v\sin 60.0°)\\ &= -2mv\sin 60.0° = -2(3.00\text{ kg})(10.0\text{ m/s})\sin 60.0°\\ &= -52.0\text{ kg}\cdot\text{m/s}\end{aligned}$$

So $$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{\Delta p_x\hat{\mathbf{i}}}{\Delta t} = \frac{-52.0\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}}{0.200\text{ s}} = -260\hat{\mathbf{i}}\text{ N}$$

P9.68 (a) In the same symbols as in the text's Example, the original kinetic energy is

$$K_A = \frac{1}{2}m_1 v_{1A}^2$$

The example shows that the kinetic energy immediately after latching together is

$$K_B = \frac{1}{2}\left(\frac{m_1 v_{1A}^2}{m_1 + m_2}\right)$$

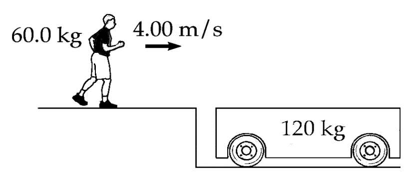
so the fraction of kinetic energy remaining as kinetic energy is

$$K_B/K_A = \boxed{m_1/(m_1+m_2)}$$

(b) Momentum is conserved in the collision so momentum after divided by momentum before is $\boxed{1.00}$.

(c) Energy is an entirely different thing from momentum. A comparison: When a photographer's single-use flashbulb flashes, a magnesium filament oxidizes. Chemical energy disappears. (Internal energy appears and light carries some energy away.) The measured mass of the flashbulb is the same before and after. It can be the same in spite of the 100% energy conversion, because energy and mass are totally different things in classical physics. In the ballistic pendulum, conversion of energy from mechanical into internal does not upset conservation of mass or conservation of momentum.

***P9.69** (a) Conservation of momentum for this totally inelastic collision gives



ANS. FIG. P9.69

$$m_p v_i = (m_p + m_c)v_f$$

$$(60.0\text{ kg})(4.00\text{ m/s}) = (120\text{ kg} + 60.0\text{ kg})v_f$$

$$\vec{\mathbf{v}}_f = \boxed{1.33\hat{\mathbf{i}}\text{ m/s}}$$

(b) To obtain the force of friction, we first consider Newton's second law in the *y* direction, $\sum F_y = 0$, which gives

$$n - (60.0\text{ kg})(9.80\text{ m/s}) = 0$$

or n = 588 N. The force of friction is then

$$f_k = \mu_k n = (0.400)(588\text{ N}) = 235\text{ N}$$

$$\vec{\mathbf{f}}_k = \boxed{-235\hat{\mathbf{i}}\text{ N}}$$

(c) The change in the person's momentum equals the impulse, or

$$p_i + I = p_f$$

$$mv_i + Ft = mv_f$$

$$(60.0\text{ kg})(4.00\text{ m/s}) - (235\text{ N})t = (60.0\text{ kg})(1.33\text{ m/s})$$

$$t = \boxed{0.680\text{ s}}$$

(d) The change in momentum of the person is

$$m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i = (60.0\text{ kg})(1.33 - 4.00)\hat{\mathbf{i}}\text{ m/s} = \boxed{-160\hat{\mathbf{i}}\text{ N}\cdot\text{s}}$$

The change in momentum of the cart is

$$(120\text{ kg})(1.33\text{ m/s}) - 0 = \boxed{+160\hat{\mathbf{i}}\text{ N}\cdot\text{s}}$$

(e) $$x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33)\text{ m/s}](0.680\text{ s}) = \boxed{1.81\text{ m}}$$

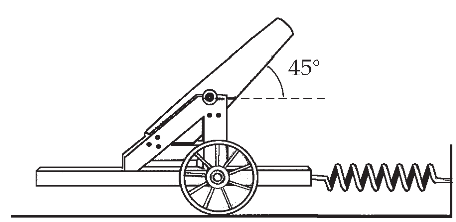
(f) $$x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33\text{ m/s})(0.680\text{ s}) = \boxed{0.454\text{ m}}$$

(g) $$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(60.0\text{ kg})(1.33\text{ m/s})^2 - \frac{1}{2}(60.0\text{ kg})(4.00\text{ m/s})^2 = \boxed{-427\text{ J}}$$

(h) $$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(120\text{ kg})(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$$

(i) The force exerted by the person on the cart must be equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about *why*. The distance moved by the cart is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, –320 J, becomes +320 J of additional internal energy in this perfectly inelastic collision.

***P9.70** (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing:



ANS. FIG. P9.70

$$p_{xf} = p_{xi}$$

$$m_{\text{shell}} v_{\text{shell}} \cos 45.0° + m_{\text{cannon}} v_{\text{recoil}} = 0$$

$$(200 \text{ kg})(125 \text{ m/s})\cos 45.0° + (5\,000 \text{ kg}) v_{\text{recoil}} = 0$$

or $\quad v_{\text{recoil}} = \boxed{-3.54 \text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$$

$$0 + 0 + \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv_{\text{recoil}}^2 + 0 + 0$$

$$x_{\text{max}} = \sqrt{\frac{mv_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000 \text{ kg})(-3.54 \text{ m/s})^2}{2.00 \times 10^4 \text{ N/m}}} = \boxed{1.77 \text{ m}}$$

(c) $\left|F_{s,\text{ max}}\right| = kx_{\text{max}}$

$$\left|F_{s,\text{ max}}\right| = (2.00 \times 10^4 \text{ N/m})(1.77 \text{ m}) = \boxed{3.54 \times 10^4 \text{ N}}$$

(d) $\boxed{\text{No.}}$ The spring exerts a force on the system during the firing. The force represents an impulse, so the momentum of the system is not conserved in the horizontal direction. Consider the vertical direction. There are two vertical forces on the system: the normal force from the ground and the gravitational force. During the firing, the normal force is larger than the gravitational force. Therefore, there is a net impulse on the system in the upward direction. The impulse accounts for the initial vertical momentum component of the projectile.

P9.71 (a) Momentum of the bullet-block system is conserved in the collision, so you can relate the speed of the block and bullet right after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet-block-Earth system to relate the speed after the collision to the maximum height.

(b) Momentum is conserved by the collision. Find the relation between the speed of the bullet v_i just before impact and the speed of the bullet + block v just after impact:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2i}$$

$$mv_i + M(0) = mv + Mv = (m + M)v$$

$$\rightarrow \qquad v_i = \frac{(m + M)}{m} v$$

For the bullet-block-Earth system, total energy is conserved. Find the relation between the speed of the bullet-block v and the height h the block climbs to:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m+M)v^2 + 0 = (m+M)gh \rightarrow v = \sqrt{2gh}$$

Combining our results, we find

$$v_i = \frac{m+M}{m}\sqrt{2gh} = \left(\frac{1.255\text{ kg}}{0.005\ 00\text{ kg}}\right)\sqrt{2(9.80\text{ m/s}^2)(0.220\text{ m})}$$

$$v_i = \boxed{521\text{ m/s}}$$

P9.72 (a) Momentum of the bullet-block system is conserved in the collision, so you can relate the speed of the block and bullet right after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet-block-Earth system to relate the speed after the collision to the maximum height.

(b) Momentum is conserved by the collision. Find the relation between the speed of the bullet v_i just before impact and the speed of the bullet + block v just after impact:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2i}$$

$$mv_i + M(0) = mv + Mv = (m+M)v$$

$$\rightarrow \qquad v_i = \frac{(m+M)}{m}v$$

For the bullet-block-Earth system, total energy is conserved. Find the relation between the speed of the bullet-block v and the height h the block climbs to:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m+M)v^2 + 0 = (m+M)gh \rightarrow v = \sqrt{2gh}$$

Combining our results, we find $\boxed{v_i = \frac{m+M}{m}\sqrt{2gh}}$.

***P9.73** Momentum conservation for the system of the two objects can be written as

$$3mv_i - mv_i = mv_{1f} + 3mv_{2f}$$

The relative velocity equation then gives

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f}$$

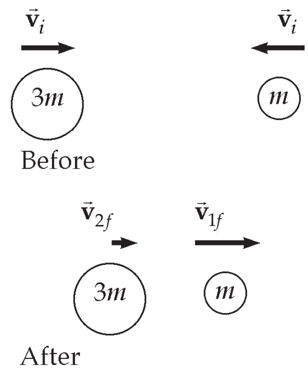
or

$$-v_i - v_i = -v_{1f} + v_{2f}$$
$$2v_i = v_{1f} + 3v_{2f}$$

Which gives

$$0 = 4v_{2f}$$

or $v_{1f} = \boxed{2v_i}$ and $v_{2f} = \boxed{0}$.



ANS. FIG. P9.73

P9.74 (a) The mass of the sleigh plus you is 270 kg. Your velocity is 7.50 m/s in the x direction. You unbolt a 15.0-kg seat and throw it back at the ravening wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the sleigh afterward, and the velocity of the seat relative to the ground.

(b) We substitute $v_{1f} = 8.00\text{ m/s} - v_{2f}$:

$$(270\text{ kg})(7.50\text{ m/s}) = (15.0\text{ kg})(-8.00\text{ m/s} + v_{2f}) + (255\text{ kg})v_{2f}$$
$$2\ 025\text{ kg}\cdot\text{m/s} = -120\text{ kg}\cdot\text{m/s} + (270\text{ kg})v_{2f}$$
$$v_{2f} = \frac{2\ 145\text{ m/s}}{270} = 7.94\text{ m/s}$$
$$v_{1f} = 8.00\text{ m/s} - 7.94\text{ m/s} = 0.055\ 6\text{ m/s}$$

The final velocity of the seat is $-0.055\,6\hat{\mathbf{i}}$ m/s. That of the sleigh is $7.94\hat{\mathbf{i}}$ m/s.

(c) You transform potential energy stored in your body into kinetic energy of the system:

$$\Delta K + \Delta U_{\text{body}} = 0$$
$$\Delta U_{\text{body}} = -\Delta K = K_i - K_f$$

$$\Delta U_{\text{body}} = \frac{1}{2}(270\text{ kg})(7.50\text{ m/s})^2 - \left[\frac{1}{2}(15.0\text{ kg})(0.0556\text{ m/s})^2 + \frac{1}{2}(255\text{ kg})(7.94\text{ m/s})^2\right]$$

$$\Delta U_{\text{body}} = 7\ 594\text{ J} - [0.023\ 1\text{ J} + 8\ 047\text{ J}]$$

$$\Delta U_{\text{body}} = \boxed{-453\text{ J}}$$

P9.75 (a) When the spring is fully compressed, each cart moves with same velocity v. Apply conservation of momentum for the system of two gliders

$$p_i = p_f: \qquad m_1v_1 + m_2v_2 = (m_1 + m_2)v \rightarrow \boxed{v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}}$$

(b) Only conservative forces act; therefore, $\Delta E = 0$.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \left(\frac{1}{k(m_1 + m_2)}\right)[(m_1 + m_2)m_1v_1^2 + (m_1 + m_2)m_2v_2^2 - (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2]$$

$$x_m = \sqrt{\frac{m_1m_2(v_1^2 + v_2^2 - 2v_1v_2)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}}$$

(c) $m_1v_1 + m_2v_2 = m_1v_{1f} + m_2v_{2f}$

Conservation of momentum: $m_1(v_1 - v_{1f}) = m_2(v_{2f} - v_2)$ **[1]**

Conservation of energy: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $m_1(v_1^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_2^2)$

Factoring gives

$$m_1(v_1 - v_{1f})(v_1 + v_{1f}) = m_2(v_{2f} - v_2)\cdot(v_{2f} + v_2)$$

and with the use of the momentum equation (equation [1]),

this reduces to $$v_1 + v_{1f} = v_{2f} + v_2$$

or $$v_{1f} = v_{2f} + v_2 - v_1 \qquad \textbf{[2]}$$

Substituting equation [2] into equation [1] and simplifying yields

$$\boxed{v_{2f} = \frac{2m_1v_1 + (m_2 - m_1)v_2}{m_1 + m_2}}$$

Upon substitution of this expression for into equation [2], one finds

$$\boxed{v_{1f} = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}}$$

Observe that these results are the same as two equations given in the chapter text for the situation of a perfectly elastic collision in one dimension. Whatever the details of how the spring behaves, this collision ends up being just such a perfectly elastic collision in one dimension.

P9.76 We hope the momentum of the equipment provides enough recoil so that the astronaut can reach the ship before he loses life support! But can he do it?

Relative to the spacecraft, the astronaut has a momentum p = (150 kg)(20 m/s) = 3 000 kg · m/s away from the spacecraft. He must throw enough equipment away so that his momentum is reduced to at least zero relative to the spacecraft, so the equipment must have momentum of at least 3 000 kg · m/s relative to the spacecraft. If he throws the equipment at 5.00 m/s relative to himself in a direction away from the spacecraft, the velocity of the equipment will be 25.0 m/s away from the spacecraft. How much mass travelling at 25.0 m/s is necessary to equate to a momentum of 3 000 kg · m/s?

$$p = 3\ 000 \text{ kg} \cdot \text{m/s} = m(25.0 \text{ m/s})$$

which gives

$$m = \frac{3\ 000 \text{ kg} \cdot \text{m/s}}{25.0 \text{ m/s}} = 120 \text{ kg}$$

$$\boxed{\text{In order for his motion to reverse under these condition, the final mass of the astronaut and space suit is 30 kg, much less than is reasonable.}}$$

P9.77 Use conservation of mechanical energy for a block-Earth system in which the block slides down a frictionless surface from a height h:

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f \rightarrow \frac{1}{2}mv^2+0=0+mgh \rightarrow v=\sqrt{2gh}$$

Note this also applies in reverse, a mass travelling at speed v will climb to a height h on a frictionless surface: $h=\dfrac{v^2}{2g}$.

From above, we see that because each block starts from the same height h, each block has the same speed v when it meets the other block:

$$v_1=v_2=v=\sqrt{2\left(9.80\text{ m/s}^2\right)\left(5.00\text{ m}\right)}=9.90\text{ m/s}$$

Apply conservation of momentum to the two-block system:

$$m_1v_{1f}+m_2v_{2f}=m_1v+m_2(-v)$$

$$m_1v_{1f}+m_2v_{2f}=(m_1-m_2)v \qquad \textbf{[1]}$$

For an elastic, head-on collision:

$$v_{1i}-v_{2i}=v_{1f}-v_{2f}$$

$$v-(-v)=v_{2f}-v_{1f}$$

$$v_{2f}=v_{1f}+2v \qquad \textbf{[2]}$$

Substituting equation [2] into [1] gives

$$m_1v_{1f}+m_2\left(v_{1f}+2v\right)=(m_1-m_2)v$$

$$(m_1+m_2)v_{1f}=(m_1-m_2)v-2m_2v$$

$$v_{1f}=\left(\frac{m_1-3m_2}{m_1+m_2}\right)v=\left[\frac{2.00\text{ kg}-3(4.00\text{ kg})}{2.00\text{ kg}+4.00\text{ kg}}\right](9.90\text{ m/s})$$

$$=-16.5\text{ m/s}$$

Using this result and equation [2], we have

$$v_{2f}=v_{1f}+2v=\left(\frac{m_1-3m_2}{m_1+m_2}\right)v+2v$$

$$v_{2f}=\left(\frac{3m_1-m_2}{m_1+m_2}\right)v=\left[\frac{3(2.00\text{ kg})-4.00\text{ kg}}{2.00\text{ kg}+4.00\text{ kg}}\right](9.90\text{ m/s})$$

$$=3.30\text{ m/s}$$

Using our result above, we find the height that each block rises to:

$$h_1 = \frac{v_{1f}^2}{2g} = \frac{(-16.5\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{13.9\text{ m}}$$

and $$h_2 = \frac{v_{2f}^2}{2g} = \frac{(3.30\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{0.556\text{ m}}$$

P9.78 (a) Proceeding step by step, we find the stone's speed just before collision, using energy conservation for the stone-Earth system:

$$m_a g y_i = \frac{1}{2} m_a v_i^2$$

which gives

$$v_i = \sqrt{2gh} = [2(9.80\text{ m/s}^2)(1.80\text{ m})]^{1/2} = 5.94\text{ m/s}$$

Now for the elastic collision with the stationary cannonball, we use the specialized Equation 9.22 from the chapter text, with $m_1 = 80.0$ kg and $m_2 = m$:

$$v_{\text{cannonball}} = v_{2f} = \frac{2m_1 v_{1i}}{m_1 + m_2} = \frac{2(80.0\text{ kg})(5.94\text{ m/s})}{80.0\text{ kg} + m}$$
$$= \frac{950\text{ kg}\cdot\text{m/s}}{80.0\text{ kg} + m}$$

The time for the cannonball's fall into the ocean is given by

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2 \rightarrow -36.0 = \frac{1}{2}(-9.80)t^2 \rightarrow t = 2.71\text{ s}$$

so its horizontal range is

$$R = v_{2f}t = (2.71\text{ s})\left(\frac{950\text{ kg}\cdot\text{m/s}}{80.0\text{ kg} + m}\right)$$
$$= \boxed{\frac{2.58\times10^3\text{ kg}\cdot\text{m}}{80.0\text{ kg} + m}}$$

(b) The maximum value for R occurs for $m \rightarrow 0$, and is

$$R = \frac{2.58\times10^3\text{ kg}\cdot\text{m}}{80.0\text{ kg} + m} = \frac{2.58\times10^3\text{ kg}\cdot\text{m}}{80.0\text{ kg} + 0} = \boxed{32.2\text{ m}}$$

(c) As indicated in part (b), the maximum range corresponds to $\boxed{m \rightarrow 0}$

(d) $\boxed{\text{Yes, until the cannonball splashes down. No; the kinetic energy of the system is split between the stone and the cannonball after the collision and we don't know how it is split without using the conservation of momentum principle.}}$

(e) The range is equal to the product of $v_{\text{cannonball}}$, the speed of the cannonball after the collision, and t, the time at which the cannonball reaches the ocean. But $v_{\text{cannonball}}$ is proportional to v_i, the speed of the stone just before striking the cannonball, which is, in turn, proportional to the square root of g. The time t at which the cannonball strikes the ocean is inversely proportional to the square root of g. Therefore, the product $R = (v_{\text{cannonball}})t$ is *independent* of g. At a location with weaker gravity, the stone would be moving more slowly before the collision, but the cannonball would follow the same trajectory, moving more slowly over a longer time interval.

P9.79 We will use the subscript 1 for the blue bead and the subscript 2 for the green bead. Conservation of mechanical energy for the blue bead-Earth system, $K_i + U_i = K_f + U_f$, can be written as

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh$$

where v_1 is the speed of the blue bead at point B just before it collides with the green bead. Solving for v_1 gives

$$v_1 = \sqrt{2gh} = \sqrt{2\left(9.80 \text{ m/s}^2\right)(1.50 \text{ m})} = 5.42 \text{ m/s}$$

Now recall Equations 9.21 and 9.22 for an elastic collision:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$
$$v_{2f} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}$$

For this collision, the green bead is at rest, so $v_{2i} = 0$, and Equation 9.22 simplifies to

$$v_{2f} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i}$$

Plugging in gives

$$v_{2f} = \left(\frac{2(0.400\text{ kg})}{0.400\text{ kg} + 0.600\text{ kg}}\right)(5.42\text{ m/s}) = 4.34\text{ m/s}$$

Now, we use conservation of the mechanical energy of the green bead after collision to find the maximum height the ball will reach. This gives

$$0 + m_2 g\, y_{\max} = \frac{1}{2} m_2 v_{2f}^2 + 0$$

Solving for $y_{\max}$ gives

$$y_{\max} = \frac{v_{2f}^2}{2g} = \frac{(4.34\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{0.960\text{ m}}$$

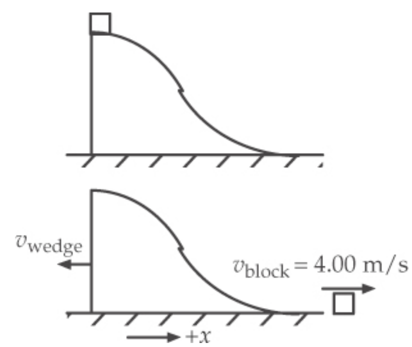
P9.80 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or

$$(3.00\text{ kg})v_{\text{wedge}} + (0.500\text{ kg})(+4.00\text{ m/s}) = 0$$

so $v_{\text{wedge}} = \boxed{-0.667\text{ m/s}}$



ANS. FIG. P9.80

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

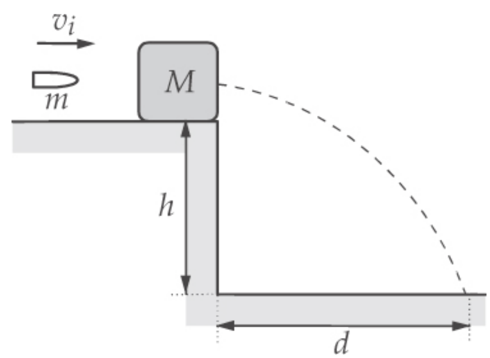
or $$\left[0 + m_1 g h\right] + 0 = \left[\frac{1}{2} m_1 (4.00\text{ m/s})^2 + 0\right] + \frac{1}{2} m_2 (-0.667\text{ m/s})^2$$

which gives $\boxed{h = 0.952\text{ m}}$

***P9.81** Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M+m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad \textbf{[1]}$$



ANS. FIG. P9.81

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus, $$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into [1] from above gives

$$v_i = \left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}} = \left(\frac{250\text{ g} + 8.00\text{ g}}{8.00\text{ g}}\right)\sqrt{\frac{(9.80\text{ m/s}^2)(2.00\text{ m})^2}{2(1.00\text{ m})}}$$
$$= \boxed{143\text{ m/s}}$$

P9.82 Refer to ANS. FIG. P9.81. Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M+m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad \textbf{[1]}$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus, $$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into [1] from above gives $v_i = \boxed{\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

P9.83 (a) From conservation of momentum,

$$\vec{\mathbf{p}}_{1i}+\vec{\mathbf{p}}_{2i}=\vec{\mathbf{p}}_{1f}+\vec{\mathbf{p}}_{2f}\rightarrow m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2i}$$

$$\begin{aligned}&(0.500\text{ kg})\left(2.00\hat{\mathbf{i}}-3.00\hat{\mathbf{j}}+1.00\hat{\mathbf{k}}\right)\text{ m/s}\\ &\qquad+(1.50\text{ kg})\left(-1.00\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}\right)\text{ m/s}\\ &\qquad\qquad=(0.500\text{ kg})\left(-1.00\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}-8.00\hat{\mathbf{k}}\right)\text{ m/s}\\ &\qquad\qquad\qquad+(1.50\text{ kg})\vec{\mathbf{v}}_{2f}\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{v}}_{2f}&=\left(\frac{1}{1.50\text{ kg}}\right)\left[\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\right.\\ &\qquad\left.+\left(0.500\hat{\mathbf{i}}-1.50\hat{\mathbf{j}}+4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\right]\\ &=\boxed{0}\end{aligned}$$

The original kinetic energy is

$$\begin{aligned}&\frac{1}{2}(0.500\text{ kg})\left(2^2+3^2+1^2\right)\text{ m}^2/\text{s}^2\\ &\qquad+\frac{1}{2}(1.50\text{ kg})\left(1^2+2^2+3^2\right)\text{m}^2/\text{s}^2=14.0\text{ J}\end{aligned}$$

The final kinetic energy is

$$\frac{1}{2}(0.500\text{ kg})\left(1^2+3^2+8^2\right)\text{ m}^2/\text{s}^2+0=18.5\text{ J}$$

different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\begin{aligned}&\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\\ &\qquad=(0.500\text{ kg})\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{ m/s}\\ &\qquad\qquad+(1.50\text{ kg})\vec{\mathbf{v}}_{2f}\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{v}}_{2f}&=\left(\frac{1}{1.50\text{ kg}}\right)\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m/s}\\ &\qquad+\left(0.125\hat{\mathbf{i}}-0.375\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m/s}\\ &=\boxed{\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{m/s}}\end{aligned}$$

We see $\vec{\mathbf{v}}_{2f}=\vec{\mathbf{v}}_{1f}$ so the collision is $\boxed{\text{perfectly inelastic}}$.

(c) Again, from conservation of momentum,

$$\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\ \mathrm{kg}\cdot\mathrm{m/s}$$
$$=\left(0.500\ \mathrm{kg}\right)\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\mathrm{m/s}+\left(1.50\ \mathrm{kg}\right)\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f}=\left(\frac{1}{1.50\ \mathrm{kg}}\right)\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\ \mathrm{kg}\cdot\mathrm{m/s}$$
$$+\left(0.500\hat{\mathbf{i}}-1.50\hat{\mathbf{j}}-0.500a\hat{\mathbf{k}}\right)\ \mathrm{kg}\cdot\mathrm{m/s}$$
$$=\boxed{(-2.67-0.333a)\hat{\mathbf{k}}\ \mathrm{m/s}}$$

Then, from conservation of energy:

$$14.0\ \mathrm{J}=\frac{1}{2}\left(0.500\ \mathrm{kg}\right)\left(1^2+3^2+a^2\right)\ \mathrm{m^2/s^2}$$
$$+\frac{1}{2}\left(1.50\ \mathrm{kg}\right)(2.67+0.333a)^2\ \mathrm{m^2/s^2}$$
$$=2.50\ \mathrm{J}+0.250a^2+5.33\ \mathrm{J}+1.33a+0.0833a^2$$

This gives, suppressing units, a quadratic equation in a,

$$0=0.333a^2+1.33a-6.167=0$$

which solves to give

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

With $\boxed{a=2.74}$,

$$\vec{\mathbf{v}}_{2f}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\ \mathrm{m/s}=\boxed{-3.58\hat{\mathbf{k}}\ \mathrm{m/s}}$$

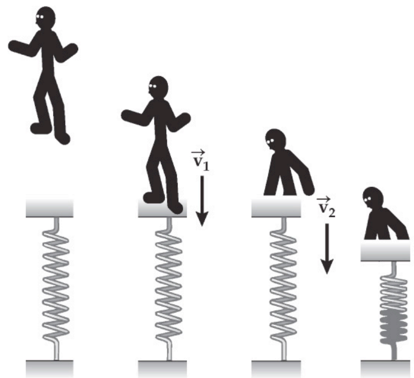
With $\boxed{a=-6.74}$,

$$\vec{\mathbf{v}}_{2f}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\ \mathrm{m/s}=\boxed{-0.419\hat{\mathbf{k}}\ \mathrm{m/s}}$$

P9.84 Consider the motion of the firefighter during the three intervals: (1) before, (2) during, and (3) after collision with the platform.

(a) While falling a height of 4.00 m, her speed changes from $v_i = 0$ to v_1 as found from

$$\Delta E=\left(K_f+U_f\right)-\left(K_i-U_i\right)$$
$$K_f=\Delta E-U_f+K_i+U_i$$



ANS FIG. P9.84

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2 = fh\cos(180°) - 0 + 0 + mgh$$

Solving for v_1 gives

$$\begin{aligned} v_1 &= \sqrt{\frac{2(-fh+mgh)}{m}} \\ &= \sqrt{\frac{2\left[-(300\text{ N})(4.00\text{ m})+(75.0\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m})\right]}{75.0\text{ kg}}} \\ &= \boxed{6.81\text{ m/s}} \end{aligned}$$

(b) During the inelastic collision, momentum of the firefighter-platform system is conserved; and if v_2 is the speed of the firefighter and platform just after collision, we have $mv_1 = (m + M)v_2$, or

$$v_2 = \frac{m_1v_1}{m+M} = \frac{(75.0\text{ kg})(6.81\text{ m/s})}{75.0\text{ kg}+20.0\text{ kg}} = 5.38\text{ m/s}$$

Following the collision and again solving for the work done by nonconservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform)

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}$$

or $$-fs = 0 + (m+M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m+M)v^2 - 0 - 0$$

This results in a quadratic equation in s:

$2\,000s^2 - (931)s + 300s - 1\,375 = 0$

with solution $\boxed{s = 1.00 \text{ m}}$

P9.85 Each primate swings down according to

$$mgR = \frac{1}{2}mv_1^2 \quad \text{and} \quad MgR = \frac{1}{2}Mv_1^2 \quad \rightarrow \quad v_1 = \sqrt{2gR}$$

For the collision,

$$-mv_1 + Mv_1 = +(m+M)v_2$$

$$v_2 = \frac{M-m}{M+m}v_1$$

While the primates are swinging up,

$$\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$$

$$v_2 = \sqrt{2gR(1-\cos 35.0°)}$$

$$\sqrt{2gR(1-\cos 35.0°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

which gives

$$\boxed{\frac{m}{M} = 0.403}$$

P9.86 (a) We can obtain the initial speed of the projectile by utilizing conservation of momentum:

$$m_1 v_{1A} + 0 = (m_1 + m_2)v_B$$

Solving for v_{1A} gives

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29 \text{ m/s}}$$

(b) We begin with the kinematic equations in the x and y direction:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

And simplify by plugging in $x_0 = y_0 = 0$, $v_{y0} = 0$, $v_{x0} = v_{1A}$, $a_x = 0$, and $a_y = g$:

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1At}$$

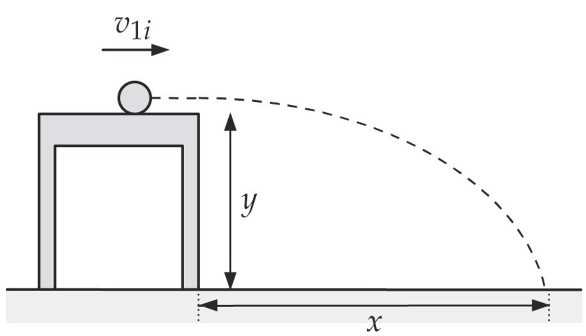
Combining them gives

$$v_{1A} = \frac{x}{\sqrt{2y/g}} = x\sqrt{\frac{g}{2y}}$$

Substituting the numerical values from the problem statement gives

$$v_{1A} = x\sqrt{\frac{g}{2y}} = (2.57\text{ m})\sqrt{\frac{9.80\text{ m/s}^2}{2(0.853\text{ m})}} = \boxed{6.16\text{ m/s}}$$

(c) $\boxed{\text{Most of the 2\% difference between the values for speed could be accounted for by air resistance.}}$



ANS. FIG. P9.86

P9.87 The force exerted by the spring on each block is in magnitude.

$$|F_s| = kx = (3.85\text{ N/m})(0.08\text{ m}) = 0.308\text{ N}$$

(a) With no friction, the elastic energy in the spring becomes kinetic energy of the blocks, which have momenta of equal magnitude in opposite directions. The blocks move with constant speed after they leave the spring. From conservation of energy,

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$\frac{1}{2}(3.85 \text{ N/m})(0.080\ 0 \text{ m})^2$$
$$= \frac{1}{2}(0.250 \text{ kg})v_{1f}^2 + \frac{1}{2}(0.500 \text{ kg})v_{2f}^2 \qquad \textbf{[1]}$$

And from conservation of linear momentum,

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$
$$0 = (0.250 \text{ kg})v_{1f}(-\hat{\mathbf{i}}) + (0.500 \text{ kg})v_{2f}\hat{\mathbf{i}}$$
$$v_{1f} = 2v_{2f}$$

Substituting this into [1] gives

$$0.012\ 3 \text{ J} = \frac{1}{2}(0.250 \text{ kg})\left(2v_{2f}\right)^2 + \frac{1}{2}(0.500 \text{ kg})v_{2f}^2$$
$$= \frac{1}{2}(1.50 \text{ kg})v_{2f}^2$$

Solving,

$$v_{2f} = \left(\frac{0.012\ 3 \text{ J}}{0.750 \text{ kg}}\right)^{1/2} = 0.128 \text{ m/s} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 0.128\hat{\mathbf{i}} \text{ m/s}}$$
$$v_{1f} = 2(0.128 \text{ m/s}) = 0.256 \text{ m/s} \qquad \boxed{\vec{\mathbf{v}}_{1f} = -0.256\hat{\mathbf{i}} \text{ m/s}}$$

(b) For the lighter block,

$$\sum F_y = ma_y, \quad n - 0.250 \text{ kg}\left(9.80 \text{ m/s}^2\right) = 0, \quad n = 2.45 \text{ N},$$
$$f_k = \mu_k n = 0.1\left(2.45 \text{ N}\right) = 0.245 \text{ N}.$$

We assume that the maximum force of static friction is a similar size. Since 0.308 N is larger than 0.245 N, this block moves. For the heavier block, the normal force and the frictional force are twice as large: f_k = 0.490 N. Since 0.308 N is less than this, the heavier block stands still. In this case, the frictional forces exerted by the floor change the momentum of the two-block system. The lighter block will gain speed as long as the spring force is larger than the friction force: that is until the spring compression becomes x_f given by

$$\left|F_s\right| = kx,\ 0.245 \text{ N} = (3.85 \text{ N·m})x_f,\ 0.063\ 6 \text{ m} = x_f$$

Now for the energy of the lighter block as it moves to this maximum-speed point, we have

$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + 0.012\,3\text{ J} - (0.245\text{ N})(0.08 - 0.063\,6\text{ m})$$
$$= \frac{1}{2}(0.250\text{ kg})v_f^2 + \frac{1}{2}(3.85\text{ N/m})(0.063\,6\text{ m})^2$$

$$0.012\,3\text{ J} - 0.004\,01\text{ J} = \frac{1}{2}(0.250\text{ kg})v_f^2 + 0.007\,80\text{ J}$$

$$\left(\frac{2(0.000\,515\text{ J})}{0.250\text{ kg}}\right)^{1/2} = v_f = 0.064\ 2\text{ m/s}$$

Thus for the heavier block the maximum velocity is $\boxed{0}$ and for the lighter block, $\boxed{-0.064\ 2\hat{\mathbf{i}}\text{ m/s}}$.

(c) For the lighter block, $f_k = 0.462(2.45\text{ N}) = 1.13\text{ N}$. The force of static friction must be at least as large. The 0.308-N spring force is too small to produce motion of either block. Each has $\boxed{0}$ maximum speed.

P9.88 The orbital speed of the Earth is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi\left(1.496\times10^{11}\text{ m}\right)}{3.156\times10^{7}\text{ s}} = 2.98\times10^{4}\text{ m/s}$$

In six months the Earth reverses its direction, to undergo momentum change

$$m_E\left|\Delta\vec{\mathbf{v}}_E\right| = 2m_E v_E = 2\left(5.98\times10^{24}\text{ kg}\right)\left(2.98\times10^{4}\text{ m/s}\right)$$
$$= 3.56\times10^{29}\text{ kg}\cdot\text{m/s}$$

Relative to the center of mass, the Sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S\left|\Delta\vec{\mathbf{v}}_S\right| = 3.56\times10^{29}\text{ kg}\cdot\text{m/s}$

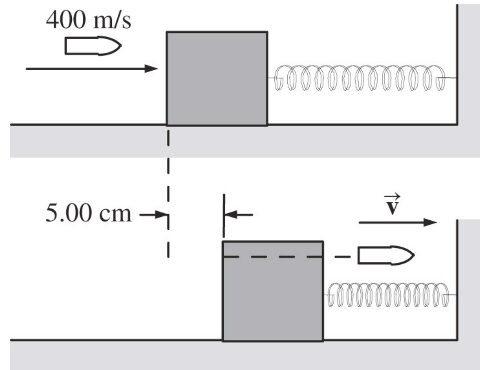
Then $$\left|\Delta\vec{\mathbf{v}}_S\right| = \frac{3.56\times10^{29}\text{ kg}\cdot\text{m/s}}{1.991\times10^{30}\text{ kg}} = \boxed{0.179\text{ m/s}}$$

ANS. FIG. P9.88

P9.89 (a) We find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops. After the collision, the mechanical energy is conserved in the block-spring system:



ANS. FIG. P9.89

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\text{ N/m})(5.00\times10^{-2}\text{ m})^2}{1.00\text{ kg}}} = 1.50\text{ m/s}$$

$$v = \frac{mv_i - MV_i}{m}$$

$$= \frac{(5.00\times10^{-3}\text{ kg})(400\text{ m/s}) - (1.00\text{ kg})(1.50\text{ m/s})}{5.00\times10^{-3}\text{ kg}}$$

$$v = \boxed{100\text{ m/s}}$$

(b) Identifying the system as the block and the bullet and the time interval from just before the collision to just after the collision,

$$\Delta K + \Delta E_{\text{int}} = 0 \quad \text{gives}$$

$$\Delta E_{\text{int}} = -\Delta K = -\left(\frac{1}{2}mv^2 + \frac{1}{2}MV_i^2 - \frac{1}{2}mv_i^2\right)$$

Then

$$\Delta E_{\text{int}} = -\left[\frac{1}{2}(0.005\ 00\text{ kg})(100\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(1.50\text{ m/s})^2 - \frac{1}{2}(0.005\ 00\text{ kg})(400\text{ m/s})^2\right]$$

$$= \boxed{374\text{ J}}$$

P9.90 (a) We have, from the impulse-momentum theorem, $\vec{\mathbf{p}}_i + \vec{\mathbf{F}}t = \vec{\mathbf{p}}_f$:

$$(3.00\text{ kg})(7.00\text{ m/s})\hat{\mathbf{j}} + \left(12.0\hat{\mathbf{i}}\text{ N}\right)(5.00\text{ s}) = (3.00\text{ kg})\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \boxed{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) The particle's acceleration is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t} = \frac{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}}\right)\text{ m/s}}{5.00\text{ s}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$$

(c) From Newton's second law,

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{12.0\hat{\mathbf{i}}\text{ N}}{3.00\text{ kg}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$$

(d) The vector displacement of the particle is

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$= \left(7.00\text{ m/s}\,\hat{\mathbf{j}}\right)(5.00\text{ s}) + \frac{1}{2}\left(4.00\text{ m/s}^2\hat{\mathbf{i}}\right)(5.00\text{ s})^2$$

$$\Delta\vec{\mathbf{r}} = \boxed{\left(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}}\right)\text{ m}}$$

(e) Now, from the work-kinetic energy theorem, the work done on the particle is

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = \left(12.0\hat{\mathbf{i}}\text{ N}\right)\left(50.0\hat{\mathbf{i}}\text{ m} + 35.0\hat{\mathbf{j}}\text{ m}\right) = \boxed{600\text{ J}}$$

(f) The final kinetic energy of the particle is

$$\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\cdot\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m}^2/\text{s}^2$$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})\left(449\text{ m}^2/\text{s}^2\right) = \boxed{674\text{ J}}$$

(g) The final kinetic energy of the particle is

$$\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$$

(h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

P9.91 We note that the initial velocity of the target particle is zero (that is, $v_{2i} = 0$). Then, from conservation of momentum,

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0 \qquad \textbf{[1]}$$

For head-on elastic collisions, $v_{1i} - v_{2i} = \left(v_{1f} - v_{2f}\right)$, and with $v_{2i} = 0$, this gives

$$v_{2f} = v_{1i} + v_{1f} \qquad \textbf{[2]}$$

Substituting equation [2] into [1] yields

$$m_1 v_{1f} + m_2\left(v_{1i} + v_{1f}\right) = m_1 v_{1i}$$

or

$$\left(m_1 + m_2\right) v_{1f} = \left(m_1 - m_2\right) v_{1i}$$

which gives

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad \textbf{[3]}$$

Now, we substitute equation [3] into [2] to obtain

$$v_{2f} = v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} \qquad \textbf{[4]}$$

Equations [3] and [4] can now be used to answer both parts (a) and (b).

(a) If $m_1 = 2.00\text{ g}, m_2 = 1.00\text{ g}$, and $v_{1i} = 8.00\text{ m/s}$, then

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{2.00\text{ g} - 1.00\text{ g}}{2.00\text{ g} + 1.00\text{ g}}\right)(8.00\text{ m/s}) = \boxed{2.67\text{ m/s}}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = \left[\frac{2(2.00\text{ g})}{2.00\text{ g} + 1.00\text{ g}}\right](8.00\text{ m/s}) = \boxed{10.7\text{ m/s}}$$

(b) If $m_1 = 2.00\text{ g}, m_2 = 10.0\text{ g}$, and $v_{1i} = 8.00\text{ m/s}$, we find

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{2.00\text{ g} - 10.0\text{ g}}{2.00\text{ g} + 10.0\text{ g}}\right)(8.00\text{ m/s}) = \boxed{5.33\text{ m/s}}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = \left[\frac{2(2.00\text{ g})}{2.00\text{ g} + 10.0\text{ g}}\right](8.00\text{ m/s}) = \boxed{2.67\text{ m/s}}$$

(c) The final kinetic energy of the 2.00-g particle in each case is:

Case (a):

$$KE_{1f} = \frac{1}{2}m_1 v_{1f}^2 = \frac{1}{2}\left(2.00\times10^{-3}\text{ kg}\right)(2.67\text{ m/s})^2 = \boxed{7.11\times10^{-3}\text{ J}}$$

Case (b):

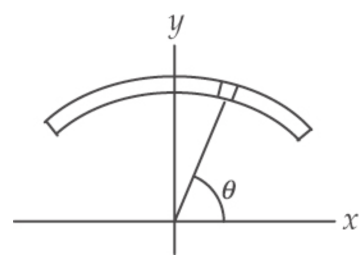
$$KE_{1f} = \frac{1}{2}m_1 v_{1f}^2 = \frac{1}{2}\left(2.00\times10^{-3}\text{ kg}\right)(5.33\text{ m/s})^2 = \boxed{2.84\times10^{-2}\text{ J}}$$

Since the incident kinetic energy is the same in cases (a) and (b), we observe that

$$\boxed{\begin{array}{l}\text{the incident particle loses more kinetic energy in case (a),}\\ \text{in which the target mass is 1.00 g.}\end{array}}$$

Challenge Problems

P9.92 Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r,\ r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the x axis has mass given by $\frac{dm}{rd\theta} = \frac{m}{L},\ dm = \frac{mr}{L}d\theta,$ where we have used the definition of radian measure. Now



ANS. FIG. P9.92

$$y_{\text{CM}} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45°}^{135°} r\sin\theta\frac{Mr}{L}d\theta = \frac{r^2}{L}\int_{45°}^{135°}\sin\theta\,d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2\frac{1}{L}(-\cos\theta)\Big|_{45°}^{135°} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by

$$\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1-\frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\ 5\,L}$$

P9.93 The x component of momentum for the system of the two objects is

$$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$$

$$-mv_i + 3mv_i = 0 + 3mv_{2x}$$

The y component of momentum of the system is

$$0 + 0 = -mv_{1y} + 3mv_{2y}$$

By conservation of energy of the system,

$$+\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$$

we have $v_{2x} = \dfrac{2v_i}{3}$

also $v_{1y} = 3v_{2y}$

So the energy equation becomes

$$4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $v_{2y} = \dfrac{\sqrt{2}v_i}{3}$

(a) The object of mass m has final speed

$$v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$$

and the object of mass $3m$ moves at

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right)$

$$\theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$$

P9.94 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\dfrac{dp}{dt} = \dfrac{d(mv)}{dt} = v\dfrac{dm}{dt} = (0.750\ \text{m/s})(5.00\ \text{kg/s}) = \boxed{3.75\ \text{N}}$

(b) The only horizontal force on the sand is belt friction, which causes the momentum of the sand to change: $F = \dfrac{dp}{dt} = \boxed{3.75\ \text{N}}$ as above.

(c) The belt is in equilibrium:

$$\sum F_x = ma_x: \qquad +F_{\text{ext}} - f = 0 \quad \text{and} \quad F_{\text{ext}} = \boxed{3.75\ \text{N}}$$

(d) $W = F\Delta r\cos\theta = (3.75\ \text{N})(0.750\ \text{m})\cos 0° = \boxed{2.81\ \text{J}}$

(e) $\dfrac{dK}{dt} = \dfrac{d\left(\frac{1}{2}mv^2\right)}{dt} = \dfrac{1}{2}v^2\dfrac{dm}{dt} = \dfrac{1}{2}(0.750\ \text{m/s})^2(5.00\ \text{kg/s}) = \boxed{1.41\ \text{J/s}}$

(f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

P9.95 Depending on the length of the cord and the time interval Δt for which the force is applied, the sphere may have moved very little when the force is removed, or we may have x_1 and x_2 nearly equal, or the sphere may have swung back, or it may have swung back and forth several times. Our solution applies equally to all of these cases.

(a) The applied force is constant, so the center of mass of the glider-sphere system moves with constant acceleration. It starts, we define, from $x = 0$ and moves to $(x_1 + x_2)/2$. Let v_1 and v_2 represent the horizontal components of velocity of glider and sphere at the moment the force stops. Then the velocity of the center of mass is $v_{CM} = (v_1 + v_2)/2$, and because the acceleration is constant we have

$$\frac{x_1 + x_2}{2} = \left(\frac{v_1 + v_2}{2}\right)\left(\frac{\Delta t}{2}\right)$$

which gives

$$\Delta t = 2\left(\frac{x_1 + x_2}{v_1 + v_2}\right)$$

The impulse-momentum theorem for the glider-sphere system is

$$F\Delta t = mv_1 + mv_2$$

or

$$2F\left(\frac{x_1 + x_2}{v_1 + v_2}\right) = m(v_1 + v_2)$$

$$2F(x_1 + x_2) = m(v_1 + v_2)^2$$

Dividing both sides by 4m and rearranging gives

$$\frac{2F(x_1 + x_2)}{4m} = \frac{m(v_1 + v_2)^2}{4m}$$

$$\frac{F(x_1 + x_2)}{2m} = \frac{(v_1 + v_2)^2}{4} = v_{\text{CM}}^2$$

or

$$v_{\text{CM}} = \boxed{\sqrt{\frac{F(x_1 + x_2)}{2m}}}$$

(b) The applied force does work that becomes, after the force is removed, kinetic energy of the constant-velocity center-of-mass motion plus kinetic energy of the vibration of the glider and sphere relative to their center of mass. The applied force acts only on the glider, so the work-energy theorem for the pushing process is

$$Fx_1 = \frac{1}{2}(2m)v_{\text{CM}}^2 + E_{\text{vib}}$$

Substitution gives

$$Fx_1 = \frac{1}{2}(2m)\left[\frac{F(x_1 + x_2)}{2m}\right] + E_{\text{vib}} = \frac{1}{2}Fx_1 + \frac{1}{2}Fx_2 + E_{\text{vib}}$$

Then,

$$E_{\text{vib}} = \frac{1}{2}Fx_1 - \frac{1}{2}Fx_2$$

When the cord makes its largest angle with the vertical, the vibrational motion is turning around. No kinetic energy is associated with the vibration at this moment, but only gravitational energy:

$$mgL(1 - \cos\theta) = F(x_1 - x_2)/2$$

Solving gives

$$\boxed{\theta = \cos^{-1}[1 - F(x_1 - x_2)/2mgL]}$$

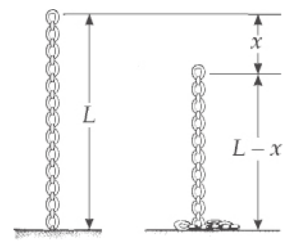
P9.96 The force exerted by the table is equal to the change in momentum of each of the links in the chain. By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:



ANS. FIG. P9.96

$$dm = \frac{M}{L}dx$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

ANSWERS TO EVEN-NUMBERED PROBLEMS

P9.2 1.14 kg; 22.0 m/s

P9.4 (a) $p_x = 9.00\ \text{kg}\cdot\text{m/s}$, $p_y = -12.0\ \text{kg}\cdot\text{m/s}$; (b) 15.0 kg · m/s

P9.6 (a) $v_{pi} = -0.346\ \text{m/s}$; (b) $v_{gi} = 1.15\ \text{m/s}$

P9.8 (a) 4.71 m/s East; (b) 717 J

P9.10 10^{-23} m/s

P9.12 (a) 3.22×10^3 N, 720 lb; (b) not valid; (c) These devices are essential for the safety of small children.

P9.14 (a) $\Delta\vec{\mathbf{p}} = 3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}$; (b) $\vec{\mathbf{F}} = 7 \times 10^2\ \text{N}\hat{\mathbf{j}}$

P9.16 (a) $\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\ \text{N}\cdot\text{s}$; (b) $\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right)\ \text{N}$

P9.18 (a) $3.60\hat{\mathbf{i}}\ \text{N}\cdot\text{s}$ away from the racket; (b) −36.0 **J**

P9.20 (a) $981\ \text{N}\cdot\text{s}$, up; (b) 3.43 m/s, down; (c) 3.83 m/s, up; (d) 0.748 m

P9.22 (a) 20.9 m/s East; (b) −8.68 × 103 J; (c) Most of the energy was transformed to internal energy with some being carried away by sound.

P9.24 (a) $v_f = \dfrac{1}{3}\left(v_1 + 2v_2\right)$; (b) $\Delta K = -\dfrac{m}{3}\left(v_1^2 + v_2^2 - 2v_1v_2\right)$

P9.26 (a) 2.50 m/s; (b) 37.5 kJ; (c) The event considered in this problem is the time reversal of the perfectly inelastic collision in Problem 9.25. The same momentum conservation equation describes both processes.

P9.28 7.94 cm

P9.30 $v = \dfrac{4M}{m}\sqrt{g\ell}$

P9.32 $v_c = \dfrac{\left(m + M\right)}{m}\sqrt{2\mu gd}$

P9.34 (a) 2.24 m/s toward the right; (b) No. Coupling order makes no difference to the final velocity.

P9.36 The driver of the northbound car was untruthful. His original speed was more than 35 mi/h.

P9.38 $v_\text{O} = 3.99\ \text{m/s}$ and $v_\text{Y} = 3.01\ \text{m/s}$

P9.40 $v = \dfrac{v_i}{\sqrt{2}}$, 45.0°, –45.0°

P9.42 The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction; (b) $\theta = 32.3°$, 2.88 m/s; (c) 786 J into internal energy

P9.44 $v_B = 5.89$ m/s; $v_G = 7.07$ m/s

P9.46 4.67×10^6 m from the Earth's center

P9.48 11.7 cm; 13.3 cm

P9.50 The center of mass of the molecule lies on the dotted line shown in ANS. FIG. P9.50, 0.006 73 nm below the center of the O atom.

P9.52 (a) See ANS. FIG. P8.42; (b) $\left(-2.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)$ m; (c) $\left(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)$ m/s; (d) $\left(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)$ kg · m/s

P9.54 (a) $\left(-2.89\hat{\mathbf{i}} - 1.39\hat{\mathbf{j}}\right)$cm; (b) $\left(-44.5\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}}\right)$g · cm/s; (c) $\left(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}\right)$cm/s; (d) $\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)$cm/s^2; (e) $\left(-220\hat{\mathbf{i}} + 140\hat{\mathbf{j}}\right)\mu$N

P9.56 (a) Yes. $18.0\hat{\mathbf{i}}$ kg · m/s; (b) No. The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work; (c) Yes, we could say that the final momentum of the card came from the floor or from the Earth through the floor; (d) No. The kinetic energy came from the original gravitational potential energy of the Earth-elevated load system, in the amount 27.0 J; (e) Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the wheels from slipping backward.

P9.58 (a) yes; (b) no; (c) 103 kg·m/s, up; (d) yes; (e) 88.2 J; (f) no, the energy came from chemical energy in the person's leg muscles

P9.60 (a) 787 m/s; (b) 138 m/s

P9.62 (a) 3.90×10^7 N; (b) 3.20 m/s^2

P9.64 (a) $-v_e \ln\left(1 - \dfrac{t}{T_p}\right)$; (b) See ANS. FIG. P9.64(b); (c) $\dfrac{v_e}{T_p - t}$; (d) See ANS. FIG. P9.64(d); (e) $v_e\left(T_p - t\right)\ln\left(1 - \dfrac{t}{T_p}\right) + v_e t$; (f) See ANS. FIG. P9.64(f)

P9.66 (a) $-\left(\dfrac{m}{M-m}\right)\vec{\mathbf{v}}_{\text{gloves}}$; (b) As she throws the gloves and exerts a force on them, the gloves exert an equal and opposite force on her that causes her to accelerate from rest to reach the velocity $\vec{\mathbf{v}}_{\text{girl}}$.

P9.68 (a) $K_E/K_A = m_1/(m_1+m_2)$; (b) 1.00; (c) See P9.68(c) for argument.

P9.70 (a) –3.54 m/s; (b) 1.77 m; (c) 3.54 × 10^4 N; (d) No

P9.72 (a) See P9.72(a) for description; (b) $v_i = \dfrac{m+M}{m}\sqrt{2gh}$

P9.74 (a) See P9.74 for complete statement; (b) The final velocity of the seat is $-0.055\,6\,\hat{\mathbf{i}}$ m/s. That of the sleigh is $7.94\,\hat{\mathbf{i}}$ m/s; (c) −453 J

P9.76 In order for his motion to reverse under these conditions, the final mass of the astronaut and space suit is 30 kg, much less than is reasonable.

P9.78 (a) $2.58\times10^3\ \text{kg}\cdot\text{m}/(80\ \text{kg}+m)$; (b) 32.2 m; (c) $m \to 0$; (d) See P9.78(d) for complete answer; (e) See P9.78(e) for complete answer.

P9.80 (a) −0.667 m/s; (b) h = 0.952 m

P9.82 $\left(\dfrac{M+m}{m}\right)\sqrt{\dfrac{gd^2}{2h}}$

P9.84 (a) 6.81 m/s; (b) s = 1.00 m

P9.86 (a) 6.29 m/s; (b) 6.16 m/s; (c) Most of the 2% difference between the values for speed could be accounted for by air resistance.

P9.88 0.179 m/s

P9.90 (a) $(20.0\hat{\mathbf{i}}+7.00\hat{\mathbf{j}})$ m/s; (b) $4.00\hat{\mathbf{i}}$ m/s^2; (c) $4.00\hat{\mathbf{i}}$ m/s^2;

(d) $(50.0\hat{\mathbf{i}}+35.0\hat{\mathbf{j}})$ m; (e) 600 J; (f) 674 J; (g) 674 J; (h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

P9.92 0.063 5L

P9.94 (a) 3.75 N; (b) 3.75 N; (c) 3.75 N; (d) 2.81 J; (e) 1.41 J/s; (f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

P9.96 $\dfrac{3Mgx}{L}$