23

Electric Fields

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ23.1 (i) Answer (c). The electron and proton have equal-magnitude charges.

(ii) Answer (b). The proton's mass is 1836 times larger than the electron's.

OQ23.2 Answer (e). The outer regions of the atoms in your body and the atoms making up the ground both contain negatively charged electrons. When your body is in close proximity to the ground, these negatively charged regions exert repulsive forces on each other. Since the atoms in the solid ground are rigidly locked in position and cannot move away from your body, this repulsive force prevents your body from penetrating the ground.

OQ23.3 Answer (b). To balance the weight of the ball, the magnitude of the upward electric force must equal the magnitude of the downward gravitational force, or $qE = mg$, which gives

$$E = \frac{mg}{q} = \frac{\left(5.0\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{4.0\times10^{-6}\ \text{C}} = 1.2\times10^{4}\ \text{N/C}$$

OQ23.4 Answer (a). The electric force is opposite to the field direction, so it is opposite to the velocity of the electron. From Newton's second law, the acceleration the electron will be

$$a_x = \frac{F_x}{m} = \frac{qE_x}{m} = \frac{\left(-1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{3}\ \text{N/C}\right)}{9.11\times10^{-31}\ \text{kg}}$$
$$= -1.76\times10^{14}\ \text{m/s}^2$$

The kinematics equation $v_x^2 = v_{0x}^2 + 2a_x(\Delta x)$, with $v_x = 0$, gives the stopping distance as

$$\Delta x = \frac{-v_{0x}^2}{2a_x} = \frac{-\left(3.00\times10^{6}\ \text{m/s}\right)^2}{2\left(-1.76\times10^{14}\ \text{m/s}^2\right)} = 2.56\times10^{-2}\ \text{m} = 2.56\ \text{cm}$$

OQ23.5 Answer (d). The displacement from the –4.00 nC charge at point (0, 1.00) m to the point (4.00, –2.00) m has components $r_x = \left(x_f - x_i\right) = +4.00\ \text{m}$ and $r_y = \left(y_f - y_i\right) = -3.00\ \text{m}$, so the magnitude of this displacement is $r = \sqrt{r_x^2 + r_y^2} = 5.00\ \text{m}$ and its direction is $\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = -36.9°$. The x component of the electric field at point (4.00, –2.00) m is then

$$E_x = E\cos\theta = \frac{k_e q}{r^2}\cos\theta$$
$$= \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-4.00\times10^{-9}\ \text{C}\right)}{(5.00\ \text{m})^2}\cos(-36.9°)$$
$$= -1.15\ \text{N/C}$$

OQ23.6 Answer (a). The equal-magnitude radially directed field contributions add to zero.

OQ23.7 Answer (b). When a charged insulator is brought near a metallic object, free charges within the metal move around, causing the metallic object to become polarized. Within the metallic object, the center of charge for the type of charge opposite to that on the insulator will be located closer to the charged insulator than will the center of charge for the same type of charge as that on the insulator.

This causes the attractive force between the charged insulator and the opposite type of charge in the metal to exceed the magnitude of the repulsive force between the insulator and the same type of charge in the metal. Thus, the net electric force between the insulator and an the metallic object is one of attraction.

OQ23.8 Answer (e). The magnitude of the electric field at distance r from a point charge q is $E = k_e q/r^2$, so

$$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{\left(5.11\times10^{-11}\ \text{m}\right)^2}$$
$$= 5.51\times10^{11}\ \text{N/C}\ \sim\ 10^{12}\ \text{N/C}$$

making (e) the best choice for this question.

OQ23.9 (i) Answer (d). Suppose the positive charge has the large value 1 μC. The object has lost some of its conduction electrons, in number

$$10^{-6}\ \text{C}\ (1\ \text{e}/1.60\times10^{-19}\ \text{C}) = 6.25\times10^{12}$$

and in mass

$$6.25\times10^{12}(9.11\times10^{-31}\ \text{kg}) = 5.69\times10^{-18}\ \text{kg}.$$

This is on the order of 10^{14} times smaller than the ~1 g mass of the coin, so it is an immeasurably small change.

(ii) Answer (b). The coin gains extra electrons, gaining mass on the order of 10^{-14} times its original mass for the charge –1 μC.

OQ23.10 Answer (c). Each charge produces a field as if it were alone in the Universe.

OQ23.11 (i) Answer (d). The charge at the upper left creates at the field point an electric field to the left, with magnitude we call E_1. The charge at lower right creates a downward electric field with an equal magnitude E_1. These two charges together create a field $\sqrt{2}E_1$ downward and to the left (at 45°). The positive charge has twice the charge but is $\sqrt{2}$ times farther from the field point, so it creates a field $2E_1\big/\left(\sqrt{2}\right)^2 = E_1$ upward and to the right. The fields from the two charges are opposite in direction, and the field from the negative charges is stronger, so the net field is then $\left(\sqrt{2}-1\right)E_1$, which is downward and to the left (at 45°).

(ii) Answer (a). With the positive charge removed, the magnitude of the field becomes $\sqrt{2}E_1$, larger than before.

OQ23.12 Answer (a). The magnitude of the electric force between charges Q_1 and Q_2, separated by distance r_i is $F = k_e Q_1 Q_2 / r^2$. If changes are made so $Q_1 \to Q_1/3$ and $r \to 2r$, the magnitude of the new force F' will be

$$F' = k_e \frac{(Q_1/3)Q_2}{(2r)^2} = \frac{1}{3(4)} k_e \frac{Q_1 Q_2}{r^2} = \frac{1}{12} k_e \frac{Q_1 Q_2}{r^2} = \frac{1}{12} F$$

OQ23.13 Answer (c). The charges nearer the center of the disk produce electric fields that make smaller angles with the central axis of the disk; therefore, these fields have smaller components perpendicular to the axis that cancel each other and larger components parallel to the axis which reinforce each other.

OQ23.14 Answer (b). A negative charge experiences a force opposite to the direction of the electric field.

OQ23.15 Answer (a). The magnitude of the electric force between two protons separated by distance r is $F = k_e e^2 / r^2$, so the distance of separation must be

$$r = \sqrt{\frac{k_e e^2}{F}} = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{2.30 \times 10^{-26} \text{ N}}} = 0.100 \text{ m}$$

ANSWERS TO CONCEPTUAL QUESTIONS

CQ23.1 No. Life would be no different if electrons were positively charged and protons were negatively charged. Opposite charges would still attract, and like charges would repel. The naming of positive and negative charge is merely a convention.

CQ23.2 The dry paper is initially neutral. The comb attracts the paper because its electric field causes the molecules of the paper to become polarized—the paper as a whole cannot be polarized because it is an insulator. Each molecule is polarized so that its unlike-charged side is closer to the charged comb than its like-charged side, so the molecule experiences a net attractive force toward the comb. Once the paper comes in contact with the comb, like charge can be transferred from the comb to the paper, and if enough of this charge is transferred, the like-charged paper is then repelled by the like-charged comb.

CQ23.3 The answer depends on whether the person is initially (a) uncharged or (b) charged.

(a) No. If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall.

(b) If the person carries a (small) charge q, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

CQ23.4 All of the constituents of air are nonpolar except for water. The polar water molecules in the air quite readily "steal" charge from a charged object, as any physics teacher trying to perform electrostatics demonstrations in humid weather well knows. As a result—it is difficult to accumulate large amounts of excess charge on an object in a humid climate. During a North American winter, the cold, dry air allows accumulation of significant excess charge, giving the potential (pun intended) for a shocking (pun also intended) introduction to static electricity sparks.

CQ23.5 No. Object A might have a charge opposite in sign to that of B, but it also might be neutral. In this latter case, object B causes object A (or the molecules of A if its material is an insulator) to be polarized, pulling unlike charge to the near face of A and pushing an equal amount of like charge to the far face. Then the force of attraction exerted by B on the induced unlike charge on the near side of A is slightly larger than the force of repulsion exerted by B on the induced like charge on the far side of A. Therefore, the net force on A is toward B.

CQ23.6 (a) Yes. The positive charges create electric fields that extend in all directions from those charges. The total field at point A is the vector sum of the individual fields produced by the charges at that point.

(b) No, because there are no field lines emanating from or converging on point A.

(c) No. There must be a charged object present to experience a force.

CQ23.7 The charge on the ground is negative because electric field lines produced by negative charge point toward their source.

CQ23.8 Conducting shoes are worn to avoid the build up of a static charge on them as the wearer walks. Rubber-soled shoes acquire a charge by friction with the floor and could discharge with a spark, possibly causing an explosive burning situation, where the burning is enhanced by the oxygen.

CQ23.9 (a) No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 23.4a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall.

(b) Polar water molecules in the air surrounding the balloon are attracted to the excess electrons on the balloon. The water molecules can pick up and transfer electrons from the balloon, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.

CQ23.10 (a) Yes. (b) The situation is similar to that of magnetic bar magnets, which can attract or repel each other depending on their orientation.

CQ23.11 Electrons have been removed from the glass object. Negative charge has been removed from the initially neutral rod, resulting in a net positive charge on the rod. The protons cannot be removed from the rod; protons are not mobile because they are within the nuclei of the atoms of the rod.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 23.1 Properties of Electric Charges

P23.1 (a) The charge due to loss of one electron is

$$0-1\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+1.60\times10^{-19}\text{ C}}$$

The mass of an average neutral hydrogen atom is 1.007 9 u. Losing one electron reduces its mass by a negligible amount, to

$$1.007\,9\left(1.660\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{1.67\times10^{-27}\text{ kg}}$$

(b) By similar logic, charge $=\boxed{+1.60\times10^{-19}\text{ C}}$

$$\text{mass}=22.99\left(1.66\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{3.82\times10^{-26}\text{ kg}}$$

(c) Gain of one electron: charge of $Cl^{-}=\boxed{1.60\times10^{-19}\text{ C}}$

$$\text{mass}=35.453\left(1.66\times10^{-27}\text{ kg}\right)+9.11\times10^{-31}\text{ kg}=\boxed{5.89\times10^{-26}\text{ kg}}$$

(d) Loss of two electrons: charge of $Ca^{++}=-2\left(-1.60\times10^{-19}\text{ C}\right)=$ $\boxed{+3.20\times10^{-19}\text{ C}}$

$$\text{mass}=40.078\left(1.66\times10^{-27}\text{ kg}\right)-2\left(9.11\times10^{-31}\text{ kg}\right)$$

$$=\boxed{6.65\times10^{-26}\text{ kg}}$$

(e) Gain of three electrons: charge of $N^{3-} = 3\left(-1.60\times10^{-19}\text{ C}\right) =$ $\boxed{-4.80\times10^{-19}\text{ C}}$

$$\text{mass} = 14.007\left(1.66\times10^{-27}\text{ kg}\right) + 3\left(9.11\times10^{-31}\text{ kg}\right)$$
$$= \boxed{2.33\times10^{-26}\text{ kg}}$$

(f) Loss of four electrons: charge of $N^{4+} = 4\left(1.60\times10^{-19}\text{ C}\right) =$ $\boxed{+6.40\times10^{-19}\text{ C}}$

$$\text{mass} = 14.007\left(1.66\times10^{-27}\text{ kg}\right) - 4\left(9.11\times10^{-31}\text{ kg}\right)$$
$$= \boxed{2.32\times10^{-26}\text{ kg}}$$

(g) We think of a nitrogen nucleus as a seven-times ionized nitrogen atom. Charge $= 7\left(1.60\times10^{-19}\text{ C}\right) = \boxed{1.12\times10^{-18}\text{ C}}$

$$\text{mass} = 14.007\left(1.66\times10^{-27}\text{ kg}\right) - 7\left(9.11\times10^{-31}\text{ kg}\right)$$
$$= \boxed{2.32\times10^{-26}\text{ kg}}$$

(h) Gain of one electron: charge $= \boxed{-1.60\times10^{-19}\text{ C}}$

$$\text{mass} = \left[2\left(1.007\,9\right) + 15.999\right]1.66\times10^{-27}\text{ kg} + 9.11\times10^{-31}\text{ kg}$$
$$= \boxed{2.99\times10^{-26}\text{ kg}}$$

P23.2 (a) $$N = \left(\frac{10.0\text{ grams}}{107.87\text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(47\ \frac{\text{electrons}}{\text{atom}}\right)$$
$$= \boxed{2.62\times10^{24}}$$

(b) $$\#\text{ electrons added} = \frac{Q}{e} = \frac{1.00\times10^{-3}\text{ C added}}{1.60\times10^{-19}\text{ C/electron}}$$
$$= 6.25\times10^{15}\text{ electrons added}$$

Thus,

$$\left(6.25\times10^{15}\text{ added}\right)\left(\frac{1}{2.62\times10^{24}\text{ present}}\right) = \left(\frac{2.38\text{ added}}{10^{9}\text{ present}}\right)$$

$\rightarrow$ 2.38 electrons for every 10^9 already present

Section 23.2 Charging Objects by Induction

Section 23.3 Coulomb's Law

***P23.3** The force on one proton is $\vec{\mathbf{F}} = \dfrac{k_e q_1 q_2}{r^2}$ away from the other proton. Its magnitude is

$$\left(8.99\times 10^9\ \text{N}\cdot\text{m/C}^2\right)\left(\frac{1.60\times 10^{-19}\ \text{C}}{2\times 10^{-15}\ \text{m}}\right)^2 = \boxed{57.5\ \text{N}}$$

***P23.4** In the first situation, $\vec{\mathbf{F}}_{A\text{ on }B,1} = \dfrac{k_e|q_A||q_B|}{r_1^2}\hat{\mathbf{i}}$. In the second situation, $|q_A|$ and $|q_B|$ are the same.

$$\vec{\mathbf{F}}_{B\text{ on }A,2} = -\vec{\mathbf{F}}_{A\text{ on }B} = \frac{k_e|q_A||q_B|}{r_2^2}\left(-\hat{\mathbf{i}}\right)$$

$$\frac{F_2}{F_1} = \frac{k_e|q_A||q_B|}{r_2^2}\frac{r_1^2}{k_e|q_A||q_B|}$$

$$F_2 = \frac{F_1 r_1^2}{r_2^2} = (2.62\ \mu\text{N})\left(\frac{13.7\ \text{mm}}{17.7\ \text{mm}}\right)^2 = 1.57\ \mu\text{N}$$

Then $\vec{\mathbf{F}}_{B\text{ on }A,2} = \boxed{1.57\ \mu\text{N to the left}}$.

***P23.5** The electric force is given by

$$F = k_e\frac{q_1 q_2}{(r_{12})^2} = \left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(+40\ \text{C})(-40\ \text{C})}{(2\ 000\ \text{m})^2}$$

$$= -3.60\times 10^6\ \text{N (attractive)} = \boxed{3.60\times 10^6\ \text{N downward}}$$

P23.6 (a) The two ions are both singly charged, $|q| = 1e$, one positive and one negative. Thus,

$$|F| = \frac{k_e|q_1||q_2|}{r^2} = \frac{k_e e^2}{r^2}$$

$$= \frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-19}\ \text{C}\right)^2}{\left(0.500\times 10^{-9}\ \text{m}\right)^2}$$

$$= \boxed{9.21\times 10^{-10}\ \text{N}}$$

(b) No. The electric force depends only on the magnitudes of the two charges and the distance between them.

P23.7 The end charges, of charge magnitude e, are distance $r = 2.17\ \mu$m apart. The spring stretches by $x = 0.010\,0r$, and the effective spring force balances the electrostatic attraction of the end charges:

$$kx = k_e\frac{e^2}{r^2} \rightarrow k = k_e\frac{e^2}{xr^2} = k_e\frac{e^2}{(0.010\,0r)r^2} = k_e\frac{e^2}{(0.010\,0)r^3}$$

$$k = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/C^2\right)\frac{\left(1.60\times10^{-19}\ \text{C}\right)^2}{(0.010\,0)\left(2.17\times10^{-6}\ \text{m}\right)^3}$$

$$= \boxed{2.25\times10^{-9}\ \text{N/m}}$$

P23.8 Suppose each person has mass 70 kg. In terms of elementary charges, each person consists of precisely equal numbers of protons and electrons and a nearly equal number of neutrons. The electrons comprise very little of the mass, so for each person we find the total number of protons and neutrons, taken together:

$$(70\ \text{kg})\left(\frac{1\ \text{u}}{1.66\times10^{-27}\ \text{kg}}\right) = 4\times10^{28}\ \text{u}$$

Of these, nearly one half, 2×10^{28}, are protons, and 1% of this is 2×10^{26}, constituting a charge of $(2\times10^{26})(1.60\times10^{-19}\ \text{C}) = 3\times10^7\ \text{C}$.

Thus, Feynman's force has magnitude

$$F = \frac{k_e q_1 q_1}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\times10^7\ \text{C})^2}{(0.5\ \text{m})^2} \sim \boxed{10^{26}\ \text{N}}$$

where we have used a half-meter arm's length. According to the particle in a gravitational field model, if the Earth were in an externally-produced uniform gravitational field of magnitude $9.80\ \text{m/s}^2$, it would weigh $F_g = mg = (6\times10^{24}\ \text{kg})(10\ \text{m/s}^2) \sim 10^{26}\ \text{N}$.

Thus, the forces are of the same order of magnitude.

P23.9 (a) $|F| = \dfrac{k_e|q_1||q_2|}{r^2}$

$$F = \frac{k_e e^2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.50\times10^{-9}\ \text{C}\right)\left(4.20\times10^{-9}\ \text{C}\right)}{(1.80\ \text{m})^2}$$

$$= \boxed{8.74\times10^{-8}\ \text{N}}$$

(b) The charges are like charges. $\boxed{\text{The force is repulsive.}}$

P23.10 (a) $F_e = \dfrac{k_e q_1 q_2}{r^2} = \dfrac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(3.80\times10^{-10}\ \text{m}\right)^2}$

$= \boxed{1.59\times10^{-9}\ \text{N}}\ \text{(repulsion)}$

(b) $F_g = \dfrac{Gm_1m_2}{r^2} = \dfrac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67\times10^{-27}\ \text{kg}\right)^2}{\left(3.80\times10^{-10}\ \text{m}\right)^2}$

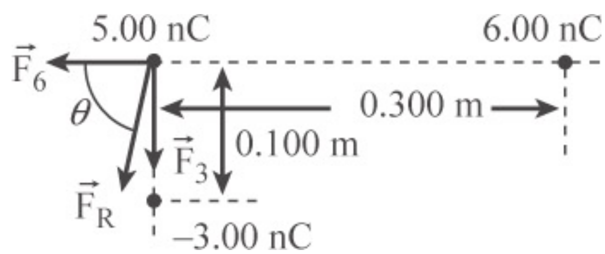
$= \boxed{1.29\times10^{-45}\ \text{N}}$

The electric force is $\boxed{\text{larger by } 1.24\times10^{36} \text{ times}}$.

(c) If $k_e\dfrac{q_1q_2}{r^2} = G\dfrac{m_1m_2}{r^2}$ with $q_1 = q_2 = q$ and $m_1 = m_2 = m$, then

$$\frac{q}{m} = \sqrt{\frac{G}{k_e}} = \sqrt{\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{8.61\times10^{-11}\ \text{C/kg}}$$

P23.11 The particle at the origin carries a positive charge of 5.00 nC. The electric force between this particle and the –3.00-nC particle located on the $-y$ axis will be attractive and point toward the $-y$ direction and is shown with $\vec{\mathbf{F}}_3$ in



ANS. FIG. P23.11

the diagram, while the electric force between this particle and the 6.00-nC particle located on the x axis will be repulsive and point toward the $-x$ direction, shown with $\vec{\mathbf{F}}_6$ in the diagram. The resultant force should point toward the third quadrant, as shown in the diagram with $\vec{\mathbf{F}}_R$. Although the charge on the x axis is greater in magnitude, its distance from the origin is three times larger than the –3.00-nC charge. We expect the resultant force to make a small angle with the $-y$ axis and be approximately equal in magnitude with F_3.

From the diagram in ANS. FIG. P23.11, the two forces are perpendicular, and the components of the resultant force are

$$F_x = -F_6 = -\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00\times10^{-9}\ \text{C}\right)\left(5.00\times10^{-9}\ \text{C}\right)}{(0.300\ \text{m})^2}$$

$$= -3.00\times10^{-6}\ \text{N} \quad \text{(to the left)}$$

$$F_y = -F_3 = -\left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(3.00 \times 10^{-9} \ \text{C}\right)\left(5.00 \times 10^{-9} \ \text{C}\right)}{(0.100 \ \text{m})^2}$$

$$= -1.35 \times 10^{-5} \ \text{N} \quad (\text{downward})$$

(a) The forces are perpendicular, so the magnitude of the resultant is

$$F_R = \sqrt{(F_6)^2 + (F_3)^2} = \boxed{1.38 \times 10^{-5} \ \text{N}}$$

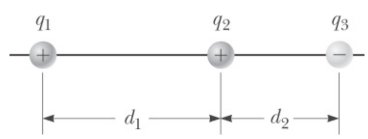
(b) The magnitude of the angle of the resultant is

$$\theta = \tan^{-1}\left(\frac{F_3}{F_6}\right) = 77.5°$$

The resultant force is in the third quadrant, so the direction is

$$\boxed{77.5° \text{ below } -x \text{ axis}}$$

P23.12 The forces are as shown in ANS. FIG. P23.12.



ANS. FIG. P23.12

$$F_1 = \frac{k_e q_1 q_2}{r_{12}^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(6.00 \times 10^{-6} \ \text{C}\right)\left(1.50 \times 10^{-6} \ \text{C}\right)}{\left(3.00 \times 10^{-2} \ \text{m}\right)^2}$$

$$= 89.9 \ \text{N}$$

$$F_2 = \frac{k_e q_1 |q_3|}{r_{13}^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(6.00 \times 10^{-6} \ \text{C}\right)\left(2.00 \times 10^{-6} \ \text{C}\right)}{\left(5.00 \times 10^{-2} \ \text{m}\right)^2}$$

$$= 43.2 \ \text{N}$$

$$F_3 = \frac{k_e q_2 |q_3|}{r_{23}^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(1.50 \times 10^{-6} \ \text{C}\right)\left(2.00 \times 10^{-6} \ \text{C}\right)}{\left(2.00 \times 10^{-2} \ \text{m}\right)^2}$$

$$= 67.4 \ \text{N}$$

(a) The net force on the 6 μC charge is

$$F_{(6\mu\text{C})} = F_1 - F_2 = \boxed{46.7 \ \text{N} \ \text{ to the left}}$$

(b) The net force on the 1.5 μC charge is

$$F_{(1.5\mu\text{C})} = F_1 + F_3 = \boxed{157 \ \text{N} \ \text{ to the right}}$$

(c) The net force on the –2 μC charge is

$$F_{(-2\mu\text{C})} = F_2 + F_3 = \boxed{111 \ \text{N} \ \text{ to the left}}$$

P23.13 (a) Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e(3q)Q}{x^2}\hat{\mathbf{i}} + \frac{k_e(q)Q}{(d-x)^2}(-\hat{\mathbf{i}}), \text{ where } d = 1.50 \text{ m}$$

The net force will be zero if $\frac{3}{x^2} = \frac{1}{(d-x)^2}$, or $d - x = \frac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of

$$x = 0.634d = 0.634(1.50 \text{ m}) = \boxed{0.951 \text{ m}}$$

(b) **[Yes, if the third bead has positive charge.]** The equilibrium would be stable because if charge Q were displaced either to the left or right on the rod, the new net force would be opposite to the direction Q has been displaced, causing it to be pushed back to its equilibrium position.

P23.14 (a) Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e q_1 Q}{x^2}\hat{\mathbf{i}} + \frac{k_e q_2 Q}{(d-x)^2}(-\hat{\mathbf{i}})$$

The net force will be zero if $\frac{q_1}{x^2} = \frac{q_2}{(d-x)^2}$:

$$\frac{q_1}{x^2} = \frac{q_2}{(d-x)^2} \quad \rightarrow \quad (d-x)^2 = x^2\left(\frac{q_2}{q_1}\right) \quad \rightarrow \quad d - x = x\sqrt{\frac{q_2}{q_1}}$$

because $d > x$. Thus,

$$d - x = x\sqrt{\frac{q_2}{q_1}} \quad \rightarrow \quad d = x + x\frac{\sqrt{q_2}}{\sqrt{q_1}} = x\left(\frac{\sqrt{q_1}+\sqrt{q_2}}{\sqrt{q_1}}\right)$$

$$\rightarrow \quad x = \boxed{\frac{\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}d}$$

(b) **[Yes, if the third bead has positive charge.]** The equilibrium would be stable because if charge Q were displaced either to the left or right on the rod, the new net force would be opposite to the direction Q has been displaced, causing it to be pushed back to its equilibrium position.

P23.15 The force exerted on the 7.00-μC charge by the 2.00-μC charge is

$$\vec{\mathbf{F}}_1 = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

$$= \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(2.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} \times(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}})$$

$$\vec{\mathbf{F}}_1 = (0.252\ \hat{\mathbf{i}} + 0.436\ \hat{\mathbf{j}})\ \mathrm{N}$$

Similarly, the force on the 7.00-μC charge by the –4.00-μC charge is

$$\mathbf{F}_2 = k_e \frac{q_1 q_3}{r^2}\hat{\mathbf{r}}$$

$$= -\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(-4.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} \times(\cos 60°\hat{\mathbf{i}} - \sin 60°\hat{\mathbf{j}})$$

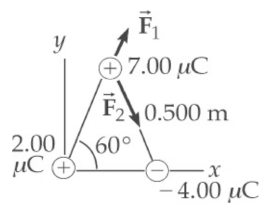
$$\vec{\mathbf{F}}_2 = (0.503\hat{\mathbf{i}} - 0.872\ \hat{\mathbf{j}})\ \mathrm{N}$$

Thus, the total force on the 7.00-μC charge is

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (0.755\ \hat{\mathbf{i}} - 0.436\ \hat{\mathbf{j}})\ \mathrm{N}$$

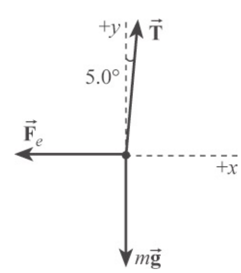
We can also write the total force as:

$$\vec{\mathbf{F}} = (0.755\ \mathrm{N})\hat{\mathbf{i}} - (0.436\ \mathrm{N})\hat{\mathbf{j}} = \boxed{0.872\ \mathrm{N\ at\ an\ angle\ of\ 330°}}$$



ANS. FIG. P23.15

P23.16 Consider the free-body diagram of one of the spheres shown in ANS. FIG. P23.16. Here, T is the tension in the string and F_e is the repulsive electrical force exerted by the other sphere.



ANS. FIG. P23.16

$$\sum F_y = 0 \ \Rightarrow\ T\cos 5.0° = mg$$

or $$T = \frac{mg}{\cos 5.0°}$$

$$\sum F_x = 0 \ \Rightarrow\ F_e = T\sin 5.0° = mg\tan 5.0°$$

At equilibrium, the distance separating the two spheres is $r = 2L\sin 5.0°$.

Thus, $F_e = mg\tan 5.0°$ becomes $\dfrac{k_e q^2}{(2L\sin 5.0°)^2} = mg\tan 5.0°$, which yields

$$L = \sqrt{\frac{k_e q^2}{mg\tan 5.0°(2\sin 5.0°)^2}}$$

$$= \sqrt{\frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.20\times 10^{-9}\ \text{C})^2}{(0.200\times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan 5.0°(2\sin 5.0°)^2}} = \boxed{0.299\ \text{m}}$$

P23.17 (a) $F = \dfrac{k_e e^2}{r^2} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times 10^{-19}\ \text{C})^2}{(0.529\times 10^{-10}\ \text{m})^2} = \boxed{8.22\times 10^{-8}\ \text{N}}$

toward the other particle.

(b) We have $F = \dfrac{mv^2}{r}$ from which

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{(8.22\times 10^{-8}\ \text{N})(0.529\times 10^{-10}\ \text{m})}{9.11\times 10^{-31}\ \text{kg}}}$$

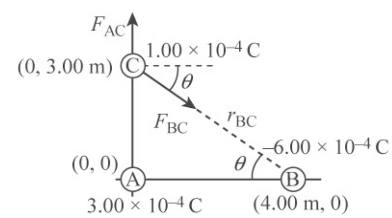
$$= \boxed{2.19\times 10^6\ \text{m/s}}$$

P23.18 Charge C is attracted to charge B and repelled by charge A, as shown in ANS. FIG. P23.18. In the sketch,

$$r_{BC} = \sqrt{(4.00\ \text{m})^2 + (3.00\ \text{m})^2} = 5.00\ \text{m}$$

and

$$\theta = \tan^{-1}\left(\frac{3.00\ \text{m}}{4.00\ \text{m}}\right) = 36.9°$$



ANS. FIG. P23.18

(a) $(F_{AC})_x = \boxed{0}$

(b) $(F_{AC})_y = |F_{AC}| = k_e \dfrac{|q_A||q_C|}{r_{AC}^2}$

$$(F_{AC})_y = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times 10^{-4}\ \text{C})(1.00\times 10^{-4}\ \text{C})}{(3.00\ \text{m})^2}$$

$$= \boxed{30.0\ \text{N}}$$

(c) $|F_{BC}| = k_e \dfrac{|q_B||q_C|}{r_{BC}^2}$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.00\times10^{-4}\ \text{C})(1.00\times10^{-4}\ \text{C})}{(5.00\ \text{m})^2}$$

$$= \boxed{21.6\ \text{N}}$$

(d) $(F_{BC})_x = |F_{BC}|\cos\theta = (21.6\ \text{N})\cos(36.9°) = \boxed{17.3\ \text{N}}$

(e) $(F_{BC})_y = -|F_{BC}|\sin\theta = -(21.6\ \text{N})\sin(36.9°) = \boxed{-13.0\ \text{N}}$

(f) $(F_R)_x = (F_{AC})_x + (F_{BC})_x = 0 + 17.3\ \text{N} = \boxed{17.3\ \text{N}}$

(g) $(F_R)_y = (F_{AC})_y + (F_{BC})_y = 30.0 - 13.0\ \text{N} = \boxed{17.0\ \text{N}}$

(h) $F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(17.3\ \text{N})^2 + (17.0\ \text{N})^2} = 24.3\ \text{N}$

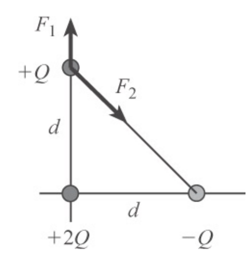
Both components are positive, placing the force in the first quadrant:

$$\varphi = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{17.0\ \text{N}}{17.3\ \text{N}}\right) = 44.5°$$

Therefore, $\vec{\mathbf{F}}_R = \boxed{24.3\ \text{N at } 44.5° \text{ above the } +x \text{ direction}}$.

P23.19 The force due to the first charge is given by

$$\vec{\mathbf{F}}_1 = \frac{k_e Q(2Q)}{d^2}\hat{\mathbf{j}} = \frac{k_e Q^2}{d^2}\left[2\hat{\mathbf{j}}\right]$$



ANS. FIG. P23.19

and the force due to the second charge is given by

$$\vec{\mathbf{F}}_2 = \frac{k_e Q(Q)}{(d^2+d^2)}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{\sqrt{2}}\right] = \frac{k_e Q^2}{d^2}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{2\sqrt{2}}\right]$$

thus the total force on the point charge +Q located at $x = 0$ and $y = d$ is

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{k_e Q^2}{d^2}\left[2\hat{\mathbf{j}}\right] + \frac{k_e Q^2}{d^2}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{2\sqrt{2}}\right] = \boxed{k_e\frac{Q^2}{d^2}\left[\frac{1}{2\sqrt{2}}\hat{\mathbf{i}} + \left(2 - \frac{1}{2\sqrt{2}}\right)\hat{\mathbf{j}}\right]}$$

P23.20 Each charge exerts a force of magnitude $\dfrac{k_e qQ}{(d/2)^2 + x^2}$ on the negative charge $-Q$: the top charge exerts its force directed upward and to the left, and bottom charge exerts its force directed downward and to the left, each at angle $\theta = \tan^{-1}\left(\frac{d}{2x}\right)$, respectively, above and below the x axis. The two positive charges together exert a net force:

$$\vec{\mathbf{F}} = -2\frac{k_e qQ}{(d/2)^2 + x^2}\cos\theta\hat{\mathbf{i}}$$

$$= -2\left[\frac{k_e qQ}{\left(d^2/4 + x^2\right)}\right]\left[\frac{x}{\left(d^2/4 + x^2\right)^{1/2}}\right]\hat{\mathbf{i}}$$

$$= \left[\frac{-2xk_e qQ}{\left(d^2/4 + x^2\right)^{3/2}}\right]\hat{\mathbf{i}} = m\vec{\mathbf{a}}$$

or for $x \ll \frac{d}{2}$, $\vec{\mathbf{a}} \approx -\left(\frac{2k_e qQ}{md^3/8}\right)\vec{\mathbf{x}} \quad \rightarrow \quad \vec{\mathbf{a}} \approx -\left(\frac{16k_e qQ}{md^3}\right)\vec{\mathbf{x}}$

(a) The acceleration of the charge is equal to a negative constant times its displacement from equilibrium, as in $\vec{\mathbf{a}} = -\omega^2\vec{\mathbf{x}}$, so we have Simple Harmonic Motion with $\omega^2 = \frac{16k_e qQ}{md^3}$.

(b) $\omega^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{16k_e qQ}{md^3} \rightarrow T = \frac{2\pi}{\omega} = \boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}}$, where m is the mass of the object with charge $-Q$.

(c) $v_{\text{max}} = \omega A = \boxed{4a\sqrt{\frac{k_e qQ}{md^3}}}$

P23.21 (a) The force is one of $\boxed{\text{attraction}}$. The distance r in Coulomb's law is the distance between the centers of the spheres. The magnitude of the force is

$$F = \frac{k_e q_1 q_2}{r^2}$$

$$= \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\frac{\left(12.0 \times 10^{-9} \text{ C}\right)\left(18.0 \times 10^{-9} \text{ C}\right)}{(0.300 \text{ m})^2}$$

$$= \boxed{2.16 \times 10^{-5} \text{ N}}$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of $\boxed{\text{repulsion}}$, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2}$$

$$= \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right) \frac{\left(3.00 \times 10^{-9} \text{ C}\right)\left(3.00 \times 10^{-9} \text{ C}\right)}{(0.300 \text{ m})^2}$$

$$= \boxed{8.99 \times 10^{-7} \text{ N}}$$

P23.22 Each of the dust particles is a particle in equilibrium. Express this mathematically for one of the particles:

$$\sum \vec{\mathbf{F}} = 0 \quad \rightarrow \quad F_e - F_g = 0 \quad \rightarrow \quad F_e = F_g$$

where we have recognized that the gravitational force is attractive and the electric force is repulsive, so the forces on one particle are in opposite directions. Substitute for the forces from Coulomb's law and Newton's law of universal gravitation, and solve for q, the unknown charge on each dust particle:

$$k_e \frac{q^2}{r^2} = G \frac{m^2}{r^2} \quad \rightarrow \quad q = \sqrt{\frac{G}{k_e}}\, m$$

Substitute numerical values:

$$q = \sqrt{\frac{6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2}{8.987\ 6 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} \left(1.00 \times 10^{-9} \text{ kg}\right)$$

$$= 8.61 \times 10^{-20} \text{ C}$$

This is about half of the smallest possible free charge, the charge of the electron. No such free charge exists. Therefore, the forces cannot balance. Even if the charge on each dust particle is due to one electron, the net force will be repulsive and the particles will move apart.

Section 23.4 Analysis Model: Particle in a Field (Electric)

***P23.23** For equilibrium, $\vec{\mathbf{F}}_e = -\vec{\mathbf{F}}_g$ or $q\vec{\mathbf{E}} = -mg\left(-\hat{\mathbf{j}}\right)$. Thus,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}}.$$

(a) For an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{(9.11\times10^{-31}\text{ kg})(9.80\text{ m/s}^2)}{-1.60\times10^{-19}\text{ C}}\hat{\mathbf{j}}$$

$$= \boxed{-(5.58\times10^{-11}\text{ N/C})\hat{\mathbf{j}}}$$

(b) For a proton, which is 1 836 times more massive than an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{(1.67\times10^{-27}\text{ kg})(9.80\text{ m/s}^2)}{-1.60\times10^{-19}\text{ C}}\hat{\mathbf{j}}$$

$$= \boxed{(1.02\times10^{-7}\text{ N/C})\hat{\mathbf{j}}}$$

P23.24 In order for the object to "float" in the electric field, the electric force exerted on the object by the field must be directed upward and have a magnitude equal to the weight of the object. Thus, $F_e = qE = mg$, and the magnitude of the electric field must be

$$E = \frac{mg}{|q|} = \frac{(3.80\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{18.0\times10^{-6}\text{ C}} = \boxed{2.07\times10^{3}\text{ N/C}}$$

The electric force on a negatively charged object is in the direction opposite to that of the electric field. Since the electric force must be directed upward, the electric field must be directed $\boxed{\text{downward}}$.

P23.25 We sum the electric fields from each of the other charges using Equation 23.7 for the definition of the electric field.

The field at charge q is given by

$$\vec{\mathbf{E}} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3$$

(a) Substituting for each of the charges gives

$$\vec{\mathbf{E}} = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

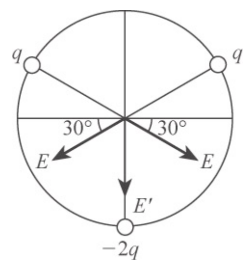
$$= \frac{k_e q}{a^2}\left[\left(2 + \frac{3}{2}\cos 45.0°\right)\hat{\mathbf{i}} + \left(\frac{3}{2}\sin 45.0° + 4\right)\hat{\mathbf{j}}\right]$$

$$= \boxed{\frac{k_e q}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

(b) The electric force on charge q is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \boxed{\frac{k_e q^2}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

P23.26 Call the fields $E = \frac{k_e q}{r^2}$ and $E' = \frac{k_e(2q)}{r^2} = 2E$ (see ANS. FIG. P23.26).



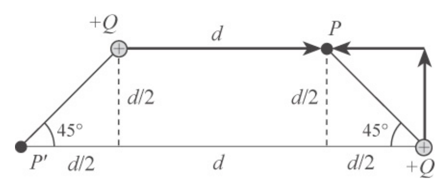
ANS. FIG. P23.26

The total field at the center of the circle has components

$$\begin{aligned}\vec{\mathbf{E}} &= (E\cos 30.0° - E\cos 30.0°)\hat{\mathbf{i}} - (E' + 2E\sin 30.0°)\hat{\mathbf{j}} \\ &= -(E' + 2E\sin 30.0°)\hat{\mathbf{j}} = -(2E + 2E\sin 30.0°)\hat{\mathbf{j}} \\ &= -2E(1 + \sin 30.0°)\hat{\mathbf{j}} \\ &= -2\frac{k_e q}{r^2}(1 + \sin 30.0°)\hat{\mathbf{j}} = -2\frac{k_e q}{r^2}(1.50)\hat{\mathbf{j}} = \boxed{-k_e \frac{3q}{r^2}\hat{\mathbf{j}}}\end{aligned}$$

P23.27 (a) See ANS. FIG. P23.27(a). The distance from the $+Q$ charge on the upper left is d, and the distance from the $+Q$ charge on the lower right to point P is

$$\sqrt{(d/2)^2 + (d/2)^2}$$

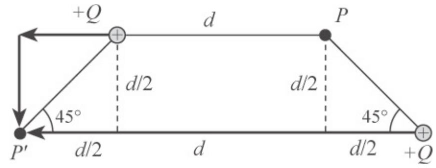


ANS. FIG. P23.27(a)

The total electric field at point P is then

$$\begin{aligned}\vec{\mathbf{E}}_P &= k_e \frac{Q}{d^2}\hat{\mathbf{i}} + k_e \frac{Q}{\left[(d/2)^2 + (d/2)^2\right]}\left(\frac{-\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) \\ &= k_e\left[\frac{Q}{d^2}\hat{\mathbf{i}} + \frac{Q}{d^2/2}\left(\frac{-\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)\right] \\ &= \boxed{k_e \frac{Q}{d^2}\left[\left(1 - \sqrt{2}\right)\hat{\mathbf{i}} + \sqrt{2}\hat{\mathbf{j}}\right]}\end{aligned}$$

(b) See ANS. FIG. P23.27(b). The distance from the +Q charge on the lower right to point P′ is $2d$, and the distance from the $+Q$ charge on the upper right to point P' is



ANS. FIG. P23.27(b)

$$\sqrt{(d/2)^2 + (d/2)^2}$$

The total electric field at point P' is then

$$\vec{\mathbf{E}}_{P'} = k_e \frac{Q}{\left[(d/2)^2 + (d/2)^2\right]}\left(\frac{-\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}\right) + k_e \frac{Q}{(2d)^2}\left(-\hat{\mathbf{i}}\right)$$

$$\vec{\mathbf{E}}_{P'} = -k_e \left[\frac{Q}{d^2/2}\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) + \frac{Q}{4d^2}\left(-\hat{\mathbf{i}}\right)\right]$$

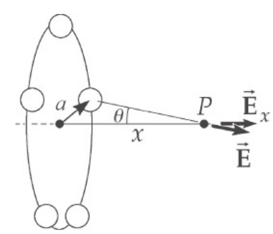
$$= -k_e \frac{Q}{4d^2}\left[\frac{8}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) + \left(\hat{\mathbf{i}}\right)\right]$$

$$\vec{\mathbf{E}}_{P'} = \boxed{-k_e \frac{Q}{4d^2}\left[\left(1 + 4\sqrt{2}\right)\hat{\mathbf{i}} + 4\sqrt{2}\hat{\mathbf{j}}\right]}$$

P23.28 (a) One of the charges creates at P a field

$$\vec{\mathbf{E}} = E_x \hat{\mathbf{i}} = \frac{\left(k_e Q/n\right)}{a^2 + x^2}\hat{\mathbf{i}}$$

at an angle θ to the x axis as shown in ANS. FIG. P23.28. When all the charges produce the field, for $n > 1$, by symmetry the components perpendicular to the x axis add to zero.



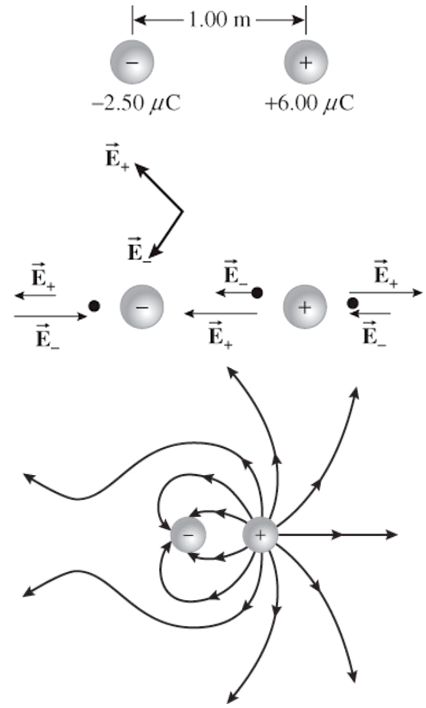
ANS. FIG. P23.28

The total field is then

$$\vec{\mathbf{E}} = nE_x\hat{\mathbf{i}} = n\left(\frac{k_e(Q/n)\hat{\mathbf{i}}}{a^2 + x^2}\cos\theta\right) = \boxed{\frac{k_e Q x \hat{\mathbf{i}}}{\left(a^2 + x^2\right)^{3/2}}}$$

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Because of the symmetrical arrangement of the charges, smearing the charge around the circle does not change its amount or its distance from the field point, so it does not change the field.

P23.29 The field of the positively-charged object is everywhere pointing radially away from its location. The object with negative charge creates everywhere a field pointing toward its different location. These two fields are directed along different lines at any point in the plane except for points along the extended line joining the particles; so the two fields cannot be oppositely-directed to add to zero except at some location along this line, which we take as the x axis. Observing the middle panel of ANS. FIG. P23.29, we see that at points to the left of the negatively-charged object, this particle creates field pointing to the right and the positive object creates field to the left. At some point along this segment the fields will add to zero. At locations in between the objects, both create fields pointing toward the left, so the total field is not zero. At points to the right of the positive 6-μC object, its field is directed to the right and is stronger than the leftward field of the –2.5-μC object, so the two fields cannot be equal in magnitude to add to zero. We have argued that only at a certain point straight to the left of both charges can the fields they separately produce be opposite in direction and equal in strength to add to zero.



ANS. FIG. P23.29

Let x represent the distance from the negatively-charged particle (charge q_-) to the zero-field point to its left. Then 1.00 m + x is the distance from the positive particle (of charge q_+) to this point. Each field is separately described by

$$\vec{\mathbf{E}} = k_e q\hat{\mathbf{r}}/x^2$$

so the equality in magnitude required for the two oppositely-directed vector fields to add to zero is described by

$$\frac{k_e|q_-|}{x^2} = \frac{k_e|q_+|}{(1\text{ m}+x)^2}$$

It is convenient to solve by taking the square root of both sides and cross-multiplying to clear of fractions:

$$|q_-|^{1/2}(1\text{ m}+x) = q_+^{1/2}x$$

$$1\text{ m} + x = \left(\frac{6.00}{2.50}\right)^{1/2} x = 1.55\,x$$

$$1\text{ m} = 0.549x$$

and $x = \boxed{1.82\text{ m}}$ to the left of the negatively-charged object.

P23.30 (a) Let s = 0.500 m be length of a side of the triangle. Call $q_1 = 7.00\ \mu\text{C}$ and $q_2 = |-4.00\ \mu\text{C}| = 4.00\ \mu\text{C}$. The electric field at the position of the 2.00- μC charge is the sum of the fields from the other two charges:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q_1}{r_1^2}\hat{\mathbf{r}}_1 + k_e \frac{q_2}{r_2^2}\hat{\mathbf{r}}_2$$

substituting,

$$\vec{\mathbf{E}} = k_e \frac{q_1}{s^2}\left(-\cos 60.0^\circ\,\hat{\mathbf{i}} - \sin 60.0^\circ\,\hat{\mathbf{j}}\right) + k_e \frac{q_2}{s^2}\hat{\mathbf{i}}$$

$$= \frac{k_e}{s^2}\left[\left(q_2 - q_1 \cos 60.0^\circ\right)\hat{\mathbf{i}} - q_1 \sin 60.0^\circ\,\hat{\mathbf{j}}\right]$$

substituting numerical values,

$$\vec{\mathbf{E}} = \left[\frac{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}{(0.500\text{ m})^2}\right] \times\left[4.00\times 10^{-6}\text{ C} - \left(7.00\times 10^{-6}\text{ C}\right)\cos 60.0^\circ\right]\hat{\mathbf{i}} - \left(7.00\times 10^{-6}\text{ C}\right)\sin 60.0^\circ\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}} = \left(1.80\times 10^4\ \text{N/C}\right)\hat{\mathbf{i}} - \left(2.18\times 10^5\ \text{N/C}\right)\hat{\mathbf{j}}$$

$$= \boxed{\left(18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right)\text{ kN/C}}$$

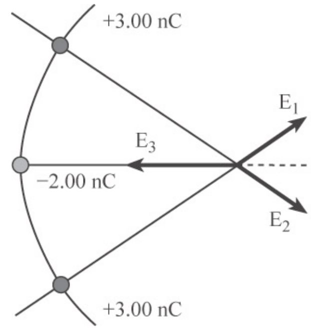
(b) The force on this charge is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(2.00\times 10^{-6}\text{ C}\right)\left(18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right)\text{ kN/C}$$

$$= \boxed{\left(0.0360\hat{\mathbf{i}} - 0.436\hat{\mathbf{j}}\right)\text{ N}}$$

P23.31 Call $Q = 3.00$ nC and $q = |-2.00 \text{ nC}| = 2.00$ nC, and $r = 4.00$ cm = 0.040 0 m. Then,

$$E_1 = E_2 = \frac{k_e Q}{r^2} \quad \text{and} \quad E_3 = \frac{k_e q}{r^2}$$



ANS. FIG. P23.31

Then,

$$E_y = 0$$

$$E_x = E_{total} = 2\frac{k_e Q}{r^2}\cos 30.0° - \frac{k_e q}{r^2}$$

$$E_x = \frac{k_e}{r^2}\left(2Q\cos 30.0° - q\right)$$

$$E_x = \left[\frac{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{(0.040\ 0 \text{ m})^2}\right] \times\left[2\left(3.00\times10^{-9} \text{ C}\right)\cos 30.0° - 2.00\times10^{-9} \text{ C}\right]$$

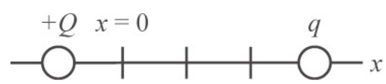
$$= 1.80\times10^4 \text{ N/C}$$

(a) $\boxed{1.80\times10^4 \text{ N/C to the right}}$

(b) The electric force on a point charge placed at point P is

$$F = qE = \left(-5.00\times10^{-9} \text{ C}\right)E = \boxed{-8.98\times10^{-5} \text{ N (to the left)}}$$

P23.32 The first charge creates at the origin a field $\frac{k_e Q}{a^2}$ to the right. Both charges are on the x axis, so the total field cannot have a vertical component, but it can be either to the right or to the left. If the total field at the origin is to the right, then q must be negative:



ANS. FIG. P23.32

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\hat{\mathbf{i}} \quad \rightarrow \quad q = -9Q$$

In the alternative, if the total field at the origin is to the left,

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\left(-\hat{\mathbf{i}}\right) \quad \rightarrow \quad q = +27Q$$

$\boxed{\text{The field at the origin can be to the right, if the unknown charge is } -9Q\text{, or the field can be to the left, if and only if the unknown charge is } +27Q.}$

***23.33** From the free-body diagram shown in ANS. FIG. P23.33,

$$\sum F_y = 0: \qquad T\cos 15.0° = 1.96\times 10^{-2}\ \text{N}$$

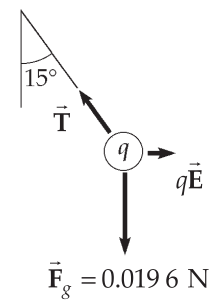
So $\qquad T = 2.03\times 10^{-2}\ \text{N}.$

From $\sum F_x = 0$, we have $qE = T\sin 15.0°$,

or

$$q = \frac{T\sin 15.0°}{E} = \frac{(2.03\times 10^{-2}\ \text{N})\sin 15.0°}{1.00\times 10^{3}\ \text{N/C}}$$

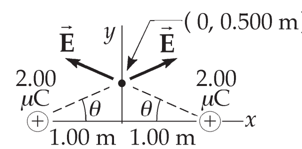
$$= 5.25\times 10^{-6}\ \text{C} = \boxed{5.25\ \mu\text{C}}$$



ANS. FIG. P23.33

***P23.34** (a) The distance from each charge to the point at $y = 0.500$ m is

$$d = \sqrt{(1.00\ \text{m})^2 + (0.500\ \text{m})^2} = 1.12\ \text{m}$$



ANS. FIG. P23.34

the magnitude of the electric field from each charge at that point is then given by

$$E = \frac{k_e q}{r^2} = \frac{(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times 10^{-6}\ \text{C})}{(1.12\ \text{m})^2} = 14\ 400\ \text{N/C}$$

The x components of the two fields cancel and the y components add, giving

$$E_x = 0 \text{ and } \qquad E_y = 2(14\ 400\ \text{N/C})\sin 26.6° = 1.29\times 10^{4}\ \text{N/C}$$

so $\qquad \boxed{\vec{\mathbf{E}} = 1.29\times 10^{4}\,\hat{\mathbf{j}}\ \text{N/C}}$.

(b) The electric force at this point is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (-3.00\times 10^{-6}\ \text{C})(1.29\times 10^{4}\ \text{N/C}\,\hat{\mathbf{j}})$$

$$= \boxed{-3.86\times 10^{-2}\,\hat{\mathbf{j}}\ \text{N}}$$

***P23.35** (a) The electric field at the origin due to each of the charges is given by

$$\vec{\mathbf{E}}_1 = \frac{k_e|q_1|}{r_1^2}(-\hat{\mathbf{j}})$$

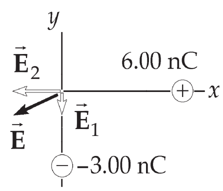
$$= \frac{(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times 10^{-9}\ \text{C})}{(0.100\ \text{m})^2}(-\hat{\mathbf{j}})$$

$$= -(2.70\times 10^{3}\ \text{N/C})\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_2 = \frac{k_e|q_2|}{r_2^2}\left(-\hat{\mathbf{i}}\right)$$
$$= \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(6.00\times10^{-9}\ \mathrm{C}\right)}{(0.300\ \mathrm{m})^2}\left(-\hat{\mathbf{i}}\right)$$
$$= -\left(5.99\times10^2\ \ \mathrm{N/C}\right)\hat{\mathbf{i}}$$

and their sum is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_1 = \boxed{-\left(5.99\times10^2\ \ \mathrm{N/C}\right)\hat{\mathbf{i}} - \left(2.70\times10^3\ \ \mathrm{N/C}\right)\hat{\mathbf{j}}}$$



ANS. FIG. P23.35

(b) The vector electric force is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(5.00\times10^{-9}\ \mathrm{C}\right)\left(-599\hat{\mathbf{i}} - 2\,700\hat{\mathbf{j}}\right)\ \mathrm{N/C}$$

$$\vec{\mathbf{F}} = \left(-3.00\times10^{-6}\hat{\mathbf{i}} - 13.5\times10^{-6}\hat{\mathbf{j}}\right)\ \mathrm{N} = \boxed{\left(-3.00\hat{\mathbf{i}} - 13.5\hat{\mathbf{j}}\right)\ \mu\mathrm{N}}$$

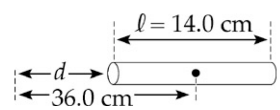
***P23.36** The electric field at any point x is

$$E = \frac{k_e q}{(x-a)^2} - \frac{k_e q}{[x-(-a)]^2} = \frac{k_e q(4ax)}{\left(x^2-a^2\right)^2}$$

When x is much, much greater than a, we find $E \approx \boxed{\dfrac{4a\left(k_e q\right)}{x^3}}$.

Section 23.5 Electric Field of a Continuous Charge Distribution

P23.37 (a) From Example 23.7, the magnitude of the electric field produced by the rod is



ANS. FIG. P23.37

$$|E| = \frac{k_e\lambda\ell}{d(\ell+d)} = \frac{k_e(Q/\ell)\ell}{d(\ell+d)} = \frac{k_e Q}{d(\ell+d)}$$
$$= \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(22.0\times10^{-6}\ \mathrm{C}\right)}{(0.290\ \mathrm{m})(0.140\ \mathrm{m}+0.290\ \mathrm{m})}$$

$$E = \boxed{1.59\times10^6\ \mathrm{N/C}}$$

(b) The charge is negative, so the electric field is directed towards its source, $\boxed{\text{to the right}}$.

P23.38 The electric field for the disk is given by

$$E = 2\pi k_e \sigma \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

in the positive x direction (away from the disk). Substituting,

$$E = 2\pi\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(7.90 \times 10^{-3} \text{ C/m}^2\right) \times \left(1 - \frac{x}{\sqrt{x^2 + (0.350)^2}}\right)$$

$$= \left(4.46 \times 10^8 \text{ N/C}\right)\left(1 - \frac{x}{\sqrt{x^2 + 0.123}}\right)$$

(a) At $x = 0.050\,0$ m,

$$E = 3.83 \times 10^8 \text{ N/C} = \boxed{383 \text{ MN/C}}$$

(b) At $x = 0.100$ m,

$$E = 3.24 \times 10^8 \text{ N/C} = \boxed{324 \text{ MN/C}}$$

(c) At $x = 0.500$ m,

$$E = 8.07 \times 10^7 \text{ N/C} = \boxed{80.7 \text{ MN/C}}$$

(d) At $x = 2.000$ m,

$$E = 6.68 \times 10^8 \text{ N/C} = \boxed{6.68 \text{ MN/C}}$$

P23.39 We may particularize the result of Example 23.8 to

$$|E| = \frac{k_e x Q}{\left(x^2 + a^2\right)^{3/2}} = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(75.0 \times 10^{-6} \text{ C/m}^2\right)x}{\left(x^2 + 0.100^2\right)^{3/2}}$$

$$= \frac{6.74 \times 10^5 x}{\left(x^2 + 0.010\,0\right)^{3/2}}$$

where we choose the x axis along the axis of the ring. The field is parallel to the axis, directed away from the center of the ring above and below it.

(a) At $x = 0.010\,0$ m, $\vec{\mathbf{E}} = 6.64 \times 10^6 \,\hat{\mathbf{i}}$ N/C $= \boxed{6.64\hat{\mathbf{i}} \text{ MN/C}}$

(b) At $x = 0.050\,0$ m, $\vec{\mathbf{E}} = 2.41\times10^{7}\,\hat{\mathbf{i}}$ N/C $= \boxed{24.1\hat{\mathbf{i}}\ \text{MN/C}}$

(c) At $x = 0.300$ m, $\vec{\mathbf{E}} = 6.40\times10^{6}\,\hat{\mathbf{i}}$ N/C $= \boxed{6.40\hat{\mathbf{i}}\ \text{MN/C}}$

(d) At $x = 1.00$ m, $\vec{\mathbf{E}} = 6.64\times10^{5}\,\hat{\mathbf{i}}$ N/C $= \boxed{0.664\hat{\mathbf{i}}\ \text{MN/C}}$

P23.40 The electric field at a distance x is $E_x = 2\pi k_e\sigma\left[1-\dfrac{x}{\sqrt{x^2+R^2}}\right]$

This is equivalent to $E_x = 2\pi k_e\sigma\left[1-\dfrac{1}{\sqrt{1+R^2/x^2}}\right]$

For large x, $\dfrac{R^2}{x^2} << 1$ and $\sqrt{1+\dfrac{R^2}{x^2}} \approx 1+\dfrac{R^2}{2x^2}$

so $$E_x = 2\pi k_e\sigma\left(1-\frac{1}{\left[1+R^2/(2x^2)\right]}\right) = 2\pi k_e\sigma\frac{\left(1+R^2/(2x^2)-1\right)}{\left[1+R^2/(2x^2)\right]}$$

Substitute $\sigma = \dfrac{Q}{\pi R^2}$,

$$E_x = \frac{k_eQ(1/x^2)}{\left[1+R^2/(2x^2)\right]} = \frac{k_eQ}{x^2+R^2/2}$$

But for $x >> R$, $\dfrac{1}{x^2+R^2/2} \approx \dfrac{1}{x^2}$, so

$$\boxed{E_x \approx \frac{k_eQ}{x^2} \text{ for a disk at large distances}}$$

P23.41 (a) From Example 23.9,

$$E = 2\pi k_e\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

here,

$$\sigma = \frac{Q}{\pi R^2} = \frac{5.20\times10^{-6}}{\pi(0.0300)^2} = 1.84\times10^{-3}\ \text{C/m}^2$$

the electric field is then

$$E = 2\pi k_e \sigma \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

$$E = 2\pi\left(8.99\times10^9\right)\left(1.84\times10^{-3}\right)$$

$$\times\left(1 - \frac{0.00300}{\sqrt{(0.00300)^2 + (0.0300)^2}}\right)$$

$$E = \left(1.04\times10^8\ \text{N/C}\right)\left(1 - \frac{0.00300}{\sqrt{(0.00300)^2 + (0.0300)^2}}\right)$$

$$= \boxed{9.36\times10^7\ \text{N/C}}$$

(b) The near-field approximation gives:

$$E = 2\pi k_e \sigma = \boxed{1.04\times10^8\ \text{N/C}\ (\text{about } 11\%\ \text{high})}$$

(c) The electric field at this point is

$$E = \left(1.04\times10^8\ \text{N/C}\right)\left(1 - \frac{0.300}{\sqrt{(0.300)^2 + (0.0300)^2}}\right)$$

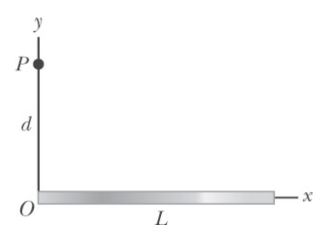
$$= \boxed{5.16\times10^5\ \text{N/C}}$$

(d) With this approximation, suppressing units,

$$E = k_e\frac{Q}{r^2} = \left(8.99\times10^9\right)\left[\frac{5.20\times10^{-6}}{(0.30)^2}\right]$$

$$= \boxed{5.19\times10^5\ \text{N/C}\ (\text{about } 0.6\%\ \text{high})}$$

P23.42 (a) The electric field at point P due to each element of length dx is $dE = \frac{k_e dq}{x^2 + d^2}$ and is directed along the line joining the element to point P. The charge element $dq = Qdx/\text{L}$. The x and y components are



ANS. FIG. P23.42

$$E_x = \int dE_x = \int dE\sin\theta$$

where $\sin\theta = \frac{x}{\sqrt{d^2 + x^2}}$

and

$$E_y = \int dE_y = \int dE\cos\theta \quad \text{where} \quad \cos\theta = \frac{d}{\sqrt{d^2 + x^2}}$$

Therefore,

$$E_x = -k_e \frac{Q}{L}\int_0^L \frac{x\,dx}{\left(d^2 + x^2\right)^{3/2}} = -k_e \frac{Q}{L}\left[\frac{-1}{\left(d^2 + x^2\right)^{1/2}}\right]_0^L$$

$$E_x = -k_e \frac{Q}{L}\left[\frac{-1}{\left(d^2 + L^2\right)^{1/2}} - \frac{-1}{\left(d^2 + 0\right)^{1/2}}\right]$$

$$E_x = \boxed{-k_e \frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2 + L^2\right)^{1/2}}\right]}$$

and

$$E_y = k_e \frac{Qd}{L}\int_0^L \frac{dx}{\left(d^2 + x^2\right)^{3/2}} = k_e \frac{Qd}{L}\left[\frac{x}{d^2\left(d^2 + x^2\right)^{1/2}}\right]_0^L$$

$$E_y = k_e \frac{Q}{\not{L}d}\left[\frac{\not{L}}{\left(d^2 + L^2\right)^{1/2}} - 0\right] \quad \rightarrow \quad E_y = \boxed{k_e \frac{Q}{d}\frac{1}{\left(d^2 + L^2\right)^{1/2}}}$$

(b) When $d >> L$,

$$E_x = -k_e \frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2 + L^2\right)^{1/2}}\right] \rightarrow -k_e \frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2\right)^{1/2}}\right] \rightarrow \boxed{E_x \approx 0}$$

and

$$E_y = k_e \frac{Q}{d}\frac{1}{\left(d^2 + L^2\right)^{1/2}} \rightarrow k_e \frac{Q}{d}\frac{1}{\left(d^2\right)^{1/2}} \rightarrow \boxed{E_y \approx k_e \frac{Q}{d^2}}$$

which is the field of a point charge Q at a distance d along the y axis above the charge.

P23.43 (a) Magnitude $|E| = \int \frac{k_e dq}{x^2}$, where $dq = \lambda_0 dx$

$$E = k_e\lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e\lambda_0\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e\lambda_0}{x_0}}$$

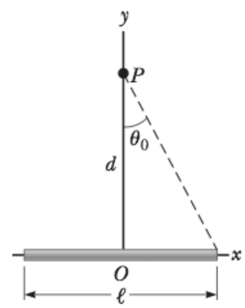
(b) The charge is positive, so the electric field points away from its source, $\boxed{\text{to the left}}$.

P23.44 (a) The electric field at point P, due to each element of length dx, is $dE = \dfrac{k_e dq}{x^2 + d^2}$ and is directed along the line joining the element to point P. By symmetry,

$$E_x = \int dE_x = 0$$

and since $dq = \lambda dx$,

$$E = E_y = \int dE_y = \int dE \cos\theta$$



ANS. FIG. P23.44

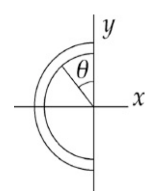
where $\cos\theta = \dfrac{y}{\sqrt{x^2 + d^2}}$.

Therefore, $$E = 2k_e \lambda d \int_0^{\ell/2} \frac{dx}{\left(x^2 + d^2\right)^{3/2}} = \boxed{\frac{2k_e \lambda \sin\theta_0}{d}}$$

with $\sin\theta_0 = \dfrac{\ell/2}{\sqrt{\left(\ell/2\right)^2 + d^2}}$.

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\dfrac{2k_e \lambda}{d}}$.

P23.45 Due to symmetry, $E_y = \int dE_y = 0$, and $E_x = -\int dE \sin\theta = -k_e \int \dfrac{dq \sin\theta}{r^2}$ where $dq = \lambda ds = \lambda r d\theta$; the component E_x is negative because charge $q = -7.50\ \mu\text{C}$, causing the net electric field to be directed to the left.



ANS. FIG. P23.45

$$E_x = -\frac{k_e \lambda}{r} \int_0^{\pi} \sin\theta d\theta = -\frac{k_e \lambda}{r}\left(-\cos\theta\right)\Big|_0^{\pi} = -\frac{2k_e \lambda}{r}$$

where $\lambda = \dfrac{|q|}{L}$ and $r = \dfrac{L}{\pi}$. Thus,

$$E_x = -\frac{2k_e |q| \pi}{L^2} = -\frac{2\left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\left(7.50 \times 10^{-6}\ \text{C}\right)\pi}{(0.140\ \text{m})^2}$$

$$E_x = -2.16 \times 10^7\ \text{N/C}$$

(a) magnitude $E = \boxed{2.16 \times 10^7\ \text{N/C}}$

(b) $\boxed{\text{to the left}}$

P23.46 (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge $\dfrac{Qdx}{h}$ and produces, at the chosen point, a field

$$d\vec{\mathbf{E}} = \frac{k_e x}{\left(x^2 + R^2\right)^{3/2}} \frac{Qdx}{h} \hat{\mathbf{i}}$$

The total field is

$$\vec{\mathbf{E}} = \int_{\text{all charge}} d\mathbf{E} = \int_d^{d+h} \frac{k_e Qx dx}{h\left(x^2 + R^2\right)^{3/2}} \hat{\mathbf{i}}$$

$$= \frac{k_e Q\hat{\mathbf{i}}}{2h} \int_{x=d}^{d+h} \left(x^2 + R^2\right)^{-3/2} 2x\,dx$$

integrating,

$$\vec{\mathbf{E}} = \frac{k_e Q\hat{\mathbf{i}}}{2h} \left. \frac{\left(x^2 + R^2\right)^{-1/2}}{(-1/2)} \right|_{x=d}^{d+h}$$

$$= \boxed{\frac{k_e Q\hat{\mathbf{i}}}{h} \left[\frac{1}{\left(d^2 + R^2\right)^{1/2}} - \frac{1}{\left((d+h)^2 + R^2\right)^{1/2}} \right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $\dfrac{Qdx}{h}$, and charge-per-area $\sigma = \dfrac{Qdx}{\pi R^2 h}$. One disk produces a field

$$d\vec{\mathbf{E}} = \frac{2\pi k_e Qdx}{\pi R^2 h} \left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}} \right) \hat{\mathbf{i}}$$

So, $$\vec{\mathbf{E}} = \int_{\text{all charge}} d\vec{\mathbf{E}} = \int_{x=d}^{d+h} \frac{2k_e Qdx}{R^2 h} \left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}} \right) \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h} \left[\int_d^{d+h} dx - \frac{1}{2} \int_{x=d}^{d+h} \left(x^2 + R^2\right)^{-1/2} 2x\,dx \right]$$

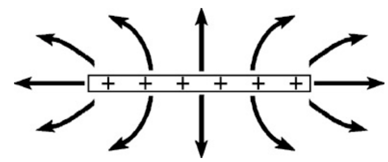
$$= \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h} \left[x\Big|_d^{d+h} - \frac{1}{2} \left. \frac{\left(x^2 + R^2\right)^{1/2}}{1/2} \right|_d^{d+h} \right]$$

$$\vec{\mathbf{E}} = \frac{2k_eQ\hat{\mathbf{i}}}{R^2h}\left[d+h-d-\left((d+h)^2+R^2\right)^{1/2}+\left(d^2+R^2\right)^{1/2}\right]$$

$$\vec{\mathbf{E}} = \boxed{\frac{2k_eQ\hat{\mathbf{i}}}{R^2h}\left[h+\left(d^2+R^2\right)^{1/2}-\left((d+h)^2+R^2\right)^{1/2}\right]}$$

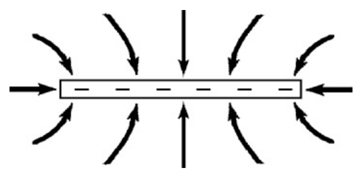
Section 23.6 Electric Field Lines

P23.47 The field lines are shown in ANS. FIG. P23.47, where we've followed the rules for drawing field lines where field lines point toward negative charge, meeting the rod perpendicularly and ending there.



ANS. FIG. P23.47

P23.48 For the positively charged disk, a side view of the field lines, pointing into the disk, is shown in ANS. FIG. P23.48.



ANS. FIG. P23.48

P23.49 Field lines emerge from positive charge and enter negative charge.

(a) The number of field lines emerging from positive q_2 and entering negative charge q_1 is proportional to their charges:

$$\frac{q_1}{q_2} = \frac{-6}{18} = \boxed{-\frac{1}{3}}$$

(b) From above, $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$.

P23.50 (a) The electric field has the general appearance shown in ANS. FIG. P23.50 below.

(b) It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second panel of ANS. FIG. P23.50 indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

(c) You may need to review vector addition in Chapter 1. The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$.

The electric field from a point charge is

$\vec{\mathbf{E}} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$.

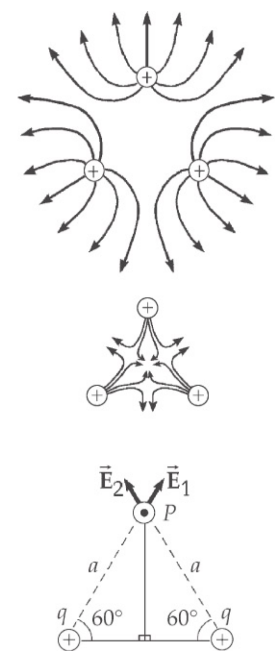
As shown in the bottom panel of ANS. FIG. P23.50,

$$\vec{\mathbf{E}}_1 = k_e \frac{q}{a^2}$$

to the right and upward at 60°, and

$$\vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}$$

to the left and upward at 60°. So,



ANS. FIG. P23.50

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right) + \left(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right)\right]$$

$$= k_e \frac{q}{a^2}\left[2\left(\sin 60°\hat{\mathbf{j}}\right)\right] = \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

Section 23.7 Motion of a Charged Particle in a Uniform Electric Field

P23.51 (a) We obtain the acceleration of the proton from the particle under a net force model, with $F = qE$ representing the electric force:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{\left(1.602 \times 10^{-19}\ \text{C}\right)\left(640\ \text{N/C}\right)}{1.67 \times 10^{-27}\ \text{kg}} = \boxed{6.14 \times 10^{10}\ \text{m/s}^2}$$

(b) The particle under constant acceleration model gives us $v_f = v_i + at$, from which we obtain

$$t = \frac{v_f - 0}{a} = \frac{1.20 \times 10^6\ \text{m/s}}{6.14 \times 10^{10}\ \text{m/s}^2} = \boxed{19.5\ \mu\text{s}}$$

(c) Again, from the particle under constant acceleration model,

$$\Delta x = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(6.14 \times 10^{10}\ \text{m/s}^2)(19.5 \times 10^{-6}\ \text{s})^2$$

$$= \boxed{11.7\ \text{m}}$$

(d) The final kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(1.20\times10^{6}\text{ m/s})^2 = \boxed{1.20\times10^{-15}\text{ J}}$$

P23.52 (a) $|a| = \dfrac{qE}{m} = \dfrac{\left(1.602\times10^{-19}\text{ C}\right)\left(6.00\times10^{5}\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 5.76\times10^{13}\text{ m/s,}$

so $\vec{\mathbf{a}} = \boxed{-5.76\times10^{13}\hat{\mathbf{i}}\text{ m/s}^2}$.

(b) $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$

$0 = v_i^2 + 2\left(-5.76\times10^{13}\text{ m/s}^2\right)\left(0.070\,0\text{ m}\right)$

$\vec{\mathbf{v}}_i = \boxed{2.84\times10^{6}\hat{\mathbf{i}}\text{ m/s}}$

(c) $v_f = v_i + at$

$0 = 2.84\times10^{6}\text{ m/s} + \left(-5.76\times10^{13}\text{ m/s}^2\right)t \quad\rightarrow\quad t = \boxed{4.93\times10^{-8}\text{ s}}$

P23.53 We use $v_f = v_i + at$, where $v_i = 0$, $t = 48.0 \times 10^{-9}$ s, and $a = F/m = eE/m$.

For the electron, $m = m_e = 9.11\times10^{-31}\text{ kg}$

and for the proton, $m = m_p = 1.67\times10^{-27}\text{ kg}$

The electric force on both particles is given by

$$F = eE = \left(1.60\times10^{-19}\text{ C}\right)\left(5.20\times10^{2}\text{ N/C}\right) = 8.32\times10^{-17}\text{ N}$$

Then, for the electron,

$$v_{fe} = v_{ie} + at = 0 + \left(\frac{eE}{m_e}\right)t = \left(\frac{8.32\times10^{-17}\text{ N}}{9.11\times10^{-31}\text{ kg}}\right)\left(48.0\times10^{-9}\text{ s}\right)$$
$$= \boxed{4.38\times10^{6}\text{ m/s}}$$

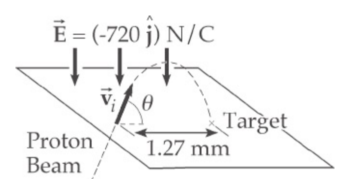
and for the proton,

$$v_{fp} = v_{ip} + at = 0 + \left(\frac{eE}{m_p}\right)t = \left(\frac{8.32\times10^{-17}\text{ N}}{1.67\times10^{-27}\text{ kg}}\right)\left(48.0\times10^{-9}\text{ s}\right)$$
$$= \boxed{2.39\times10^{3}\text{ m/s}}$$

P23.54 (a) $\boxed{\text{Particle under constant velocity}}$

(b) $\boxed{\text{Particle under constant acceleration}}$

(c) The vertical acceleration caused by the



ANS. FIG. P23.54

electric force is constant and downward;
therefore, **the proton moves in a parabolic path just like a projectile in a gravitational field.**

(d) We may neglect the effect of the acceleration of gravity on the proton because the magnitude of the vertical acceleration caused by the electric force is

$$a_y = \frac{eE}{m_p} = \frac{(1.60\times10^{-19}\ \text{C})(720\ \text{N/C})}{1.67\times10^{-27}\ \text{kg}} = 6.90\times10^{10}\ \text{m/s}^2$$

which is much greater than that of gravity.

Replacing acceleration g in Equation 4.13 with eE/m_p, we have

$$R = \frac{v_i^2 \sin 2\theta}{eE/m_p} = \boxed{\frac{m_p v_i^2 \sin 2\theta}{eE}}$$

(e) $R = \dfrac{m_p v_i^2 \sin 2\theta}{eE} = \dfrac{(1.67\times10^{-27}\ \text{kg})(9.55\times10^3\ \text{m/s})^2 \sin 2\theta}{(1.60\times10^{-19}\ \text{C})(720\ \text{N/C})}$

$= 1.27\times10^{-3}\ \text{m}$

which gives $\sin 2\theta = 0.961$, or

$$\theta = \boxed{36.9°} \quad \text{or} \quad 90.0° - \theta = \boxed{53.1°}$$

(f) $\Delta t = \dfrac{R}{v_{ix}} = \dfrac{R}{v_i \cos\theta}$

If $\theta = 36.9°$, $\Delta t = \boxed{166\ \text{ns}}$. If $\theta = 53.1°$, $\Delta t = \boxed{221\ \text{ns}}$.

P23.55 The work done on the charge is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}} = q\vec{\mathbf{E}}\cdot\vec{\mathbf{d}}$ and the kinetic energy changes according to $W = K_f - K_i = 0 - K$.

Assuming $\vec{\mathbf{v}}$ is in the $+x$ direction, we have $(-e)\vec{\mathbf{E}}\cdot d\hat{\mathbf{i}} = -K$.

Then, $e\vec{\mathbf{E}}\cdot(d\hat{\mathbf{i}}) = K$, and

$$\vec{\mathbf{E}} = \frac{K}{ed}\hat{\mathbf{i}}$$

(a) $E = \boxed{\dfrac{K}{ed}}$

(b) Because a negative charge experiences an electric force opposite to the direction of an electric field, the required electric field will be **in the direction of motion**.

P23.56 (a) The positive charge experiences a constant downward force (in the direction of the electric field):

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(1.00\times10^{-6}\text{ C}\right)\left(2\,000\text{ N/C}\right)\left(-\hat{\mathbf{j}}\right) = 2.00\times10^{-3}\left(-\hat{\mathbf{j}}\right)\text{ N}$$

and moves with acceleration:

$$\vec{\mathbf{a}} = \frac{\sum\vec{\mathbf{F}}}{m} = \frac{\left(2.00\times10^{-3}\text{ N}\right)\left(-\hat{\mathbf{j}}\right)}{2.00\times10^{-16}\text{ kg}} = 1.00\times10^{13}\left(-\hat{\mathbf{j}}\right)\text{ m/s}^2$$

Note that the gravitational acceleration is on the order of a trillion times smaller than the electrical acceleration of the particle. Thus, its trajectory is $\boxed{\text{a parabola}}$ opening downward.

(b) The maximum height the charge attains above the bottom negative plate is described by

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

Solving for the height gives

$$y_f - y_i = \frac{v_{yf}^2 - v_{yi}^2}{2a_y} = \frac{0 - \left[\left(1.00\times10^5\text{ m/s}\right)\sin 37.0^\circ\right]^2}{2\left(1.00\times10^{13}\text{ m/s}^2\right)}$$

$$= 1.81\times10^{-4}\text{ m} = 0.181\text{ mm}$$

Since this height is less than the 1.00 cm separation of the plates, the charge passes through its highest point and returns to strike $\boxed{\text{the negative plate}}$.

(c) The particle's x-component of velocity is constant at

$$(1.00\times10^5\text{ m/s})\cos 37^\circ = 7.99\times10^4\text{ m/s}$$

Starting at time $t = 0$, we find the time t when the particle returns to the negative plate from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2$$

Substituting numerical values,

$$0 = 0 + \left[\left(1.00\times10^5\text{ m/s}\right)\sin 37.0^\circ\right]t + \frac{1}{2}\left(-1.00\times10^{13}\text{ m/s}^2\right)t^2$$

since $t > 0$, the only valid solution to this quadratic equation is $t = 1.20\times10^{-8}$ s. The particle's range is then

$$x_f = x_i + v_xt = 0 + \left(7.99\times10^4\text{ m/s}\right)\left(1.20\times10^{-8}\text{ s}\right)$$

$$= 9.61\times10^{-4}\text{ m}$$

The particle strikes the negative plate after moving a horizontal distance of 0.961 mm.

P23.57 $\vec{\mathbf{E}}$ is directed along the y direction; therefore, $a_x = 0$ and $x = v_{xi}t$.

(a) $t = \dfrac{x}{v_{xi}} = \dfrac{0.050\,0\text{ m}}{4.50\times 10^{5}\text{ s}} = 1.11\times 10^{-7}\text{ s} = \boxed{111\text{ ns}}$

(b) $a_y = \dfrac{qE}{m} = \dfrac{(1.602\times 10^{-19}\text{ C})(9.60\times 10^{3}\text{ N/C})}{1.67\times 10^{-27}\text{ kg}} = 9.21\times 10^{11}\text{ m/s}^2$

$y_f - y_i = v_{yi}t + \dfrac{1}{2}a_y t^2$:

$y_f = \dfrac{1}{2}(9.21\times 10^{11}\text{ m/s}^2)(1.11\times 10^{-7}\text{ s})^2$

$= 5.68\times 10^{-3}\text{ m} = \boxed{5.67\text{ mm}}$

(c) $v_x = 4.50\times 10^{5}\text{ m/s}$

$v_{yf} = v_{yi} + a_y t = (9.21\times 10^{11}\text{ m/s}^2)(1.11\times 10^{-7}\text{ s}) = 1.02\times 10^{5}\text{ m/s}$

$\vec{\mathbf{v}} = \boxed{(450\hat{\mathbf{i}} + 102\hat{\mathbf{j}})\text{ km/s}}$

Additional Problems

P23.58 (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$Q = \sigma A$

$= (15.0\times 10^{-9}\text{ C/m}^2)2\pi(0.025\,0\text{ m})[0.025\,0\text{ m} + 0.060\,0\text{ m}]$

$= \boxed{2.00\times 10^{-10}\text{ C}}$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$Q = \sigma A = (15.0\times 10^{-9}\text{ C/m}^2)[2\pi(0.025\,0\text{ m})(0.060\,0\text{ m})]$

$= \boxed{1.41\times 10^{-10}\text{ C}}$

(c) $Q = \rho V = \rho\pi r^2 L = (500\times 10^{-9}\text{ C/m}^3)[\pi(0.025\,0\text{ m})^2(0.060\,0\text{ m})]$

$= \boxed{5.89\times 10^{-11}\text{ C}}$

P23.59 The electric field is given by the sum of the fields due to each of the *n* particles:

$$\vec{\mathbf{E}} = \sum \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{k_e q}{a^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(2a)^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) + \cdots$$

$$= \frac{-k_e q\hat{\mathbf{i}}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$$

$$= \boxed{-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}}$$

P23.60 The positive charge, call it q, is 50.0 cm – 20.9 cm = 29.1 cm from charge Q. The force on q from the –3.00 nC charge balances the force on q from the $-Q$ charge:

$$\frac{k_e(3.00\text{ nC})q}{(0.209\text{ m})^2} = \frac{k_e Q q}{(0.291\text{ m})^2}$$

which then gives

$$Q = (3.00\text{ nC})\left(\frac{0.291\text{ m}}{0.209\text{ m}}\right)^2 = \boxed{5.82\text{ nC}}$$

P23.61 (a) Take up the incline as the positive x direction. Newton's second law along the incline gives

$$\sum F_x = -mg\sin\theta + |Q|E = 0$$

solving for the electric field gives

$$E = \boxed{\frac{mg}{|Q|}\sin\theta}$$

(b) The electric force must be up the incline, so the electric field must point down the incline because the charge is negative.

$$E = \frac{mg}{|Q|}\sin\theta = \frac{\left(5.40\times10^{-3}\right)(9.80)}{\left|7.00\times10^{-6}\right|}\sin 25.0°$$

$$= \boxed{3.19\times10^3\text{ N/C, down the incline}}$$

P23.62 The downward electric force on the 0.800 μC charge is balanced by the upward spring force:

$$\frac{k_e q_1 q_2}{r^2} = kx$$

solving for the spring constant gives

$$k = \frac{k_e q_1 q_2}{x r^2}$$

$$= \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(0.800\times10^{-6}\ \text{C}\right)\left(0.600\times10^{-6}\ \text{C}\right)}{(0.0350\ \text{m})(0.0500\ \text{m})^2}$$

$$= \boxed{49.3\ \text{N/m}}$$

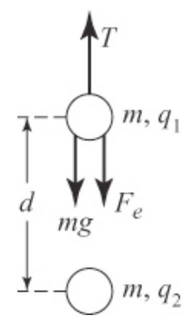
P23.63 We integrate the expression for the incremental electric field to obtain

$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}} = \int_{x_0}^{\infty}\left[\frac{k_e\lambda_0 x_0 dx\left(-\hat{\mathbf{i}}\right)}{x^3}\right] = -k_e\lambda_0 x_0\hat{\mathbf{i}}\int_{x_0}^{\infty} x^{-3}\,dx$$

$$= -k_e\lambda_0 x_0\hat{\mathbf{i}}\left(-\frac{1}{2x^2}\bigg|_{x_0}^{\infty}\right)$$

$$= \boxed{\frac{k_e\lambda_0}{2x_0}\left(-\hat{\mathbf{i}}\right)}$$

***P23.64** (a) The gravitational force exerted on the upper sphere by the lower one is negligible in comparison to the gravitational force exerted by the Earth and the downward electrical force exerted by the lower sphere. Therefore,



ANS. FIG. P23.64

$$\sum F_y = 0 \quad \rightarrow \quad T - mg - F_e = 0$$

or $$T = mg + \frac{k_e|q_1||q_2|}{d^2}$$

substituting numerical values,

$$T = \left(7.50\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) + \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(32.0\times10^{-9}\ \text{C}\right)\left(58.0\times10^{-9}\ \text{C}\right)}{\left(2.00\times10^{-2}\ \text{m}\right)^2}$$

$$= \boxed{0.115\ \text{N}}$$

(b) Once again, from the particle under a net force model,

$$\sum F_y = 0 \quad \rightarrow \quad T - mg - F_e = 0$$

or $$\frac{k_e|q_1||q_2|}{d^2} = T - mg$$

solving for the distance d then gives

$$d = \sqrt{\frac{k_e|q_1||q_2|}{T - mg}}$$

substituting numerical values, with $T = 0.180$ N,

$$d = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(32.0\times10^{-9}\ \text{C})(58.0\times10^{-9}\ \text{C})}{0.180\ \text{N} - (7.50\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}}$$

$$= 1.25\times10^{-2}\ \text{m} = \boxed{1.25\ \text{cm}}$$

P23.65 The proton moves with acceleration

$$|a_p| = \frac{qE}{m} = \frac{(1.60\times10^{-19}\ \text{C})(640\ \text{N/C})}{1.673\times10^{-27}\ \text{kg}} = 6.13\times10^{10}\ \text{m/s}^2$$

while the electron has acceleration

$$|a_e| = \frac{(1.60\times10^{-19}\ \text{C})(640\ \text{N/C})}{9.110\times10^{-31}\ \text{kg}} = 1.12\times10^{14}\ \text{m/s}^2 = 1\,836a_p$$

(a) We want to find the distance traveled by the proton (i.e., $d = \frac{1}{2}a_p t^2$), knowing:

$$4.00\ \text{cm} = \frac{1}{2}a_p t^2 + \frac{1}{2}a_e t^2 = (1\,837)\left(\frac{1}{2}a_p t^2\right)$$

Thus,

$$d = \frac{1}{2}a_p t^2 = \frac{4.00\ \text{cm}}{1\,837} = \boxed{2.18\times10^{-5}\ \text{m}}$$

(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{\text{Na}} = \frac{1}{2}a_{\text{Na}}t^2$). This is found from:

$$4.00\ \text{cm} = \frac{1}{2}a_{\text{Na}}t^2 + \frac{1}{2}a_{\text{Cl}}t^2:$$

$$4.00\ \text{cm} = \frac{1}{2}\left(\frac{eE}{22.99\ \text{u}}\right)t^2 + \frac{1}{2}\left(\frac{eE}{35.45\ \text{u}}\right)t^2$$

This may be written as

$$4.00\ \text{cm} = \frac{1}{2}a_{\text{Na}}t^2 + \frac{1}{2}(0.649a_{\text{Na}})t^2 = 1.65\left(\frac{1}{2}a_{\text{Na}}t^2\right)$$

so $$d_{\text{Na}} = \frac{1}{2}a_{\text{Na}}t^2 = \frac{4.00\ \text{cm}}{1.65} = \boxed{2.43\ \text{cm}}$$

P23.66 We find the equal-magnitude charges on both spheres:

$$F = k_e \frac{q_1 q_2}{r^2} = k_e \frac{q^2}{r^2}$$

so $$q = r\sqrt{\frac{F}{k_e}} = (1.00\text{ m})\sqrt{\frac{1.00\times 10^4\text{ N}}{8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times 10^{-3}\text{ C}$$

The number of electrons transferred is then

$$N_{\text{xfer}} = \frac{1.05\times 10^{-3}\text{ C}}{1.60\times 10^{-19}\text{ C}/e^-} = 6.59\times 10^{15}\text{ electrons}$$

The whole number of electrons in each sphere is

$$N_{\text{tot}} = \left(\frac{10.0\text{ g}}{107.87\text{ g/mol}}\right)(6.02\times 10^{23}\text{ atoms/mol})(47\ e^-/\text{atom})$$
$$= 2.62\times 10^{24}\ e^-$$

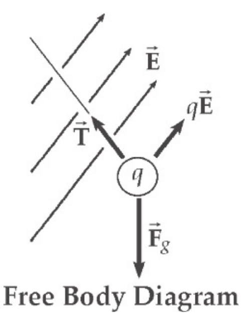
The fraction transferred is then

$$f = \frac{N_{\text{xfer}}}{N_{\text{tot}}} = \left(\frac{6.59\times 10^{15}}{2.62\times 10^{24}}\right) = \boxed{2.51\times 10^{-9}}$$

or 2.51 charges in every billion.

P23.67 ANS. FIG. P23.67 shows the free-body diagram for Newton's second law gives

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{T}} + q\vec{\mathbf{E}} + \vec{\mathbf{F}}_g = 0$$



ANS. FIG. P23.67

We are given

$$E_x = 3.00\times 10^5\text{ N/C}$$

and $$E_y = 5.00\times 10^5\text{ N/C}$$

Applying Newton's second law or the first condition for equilibrium in the x and y directions,

$$\sum F_x = qE_x - T\sin 37.0° = 0 \quad \textbf{[1]}$$

$$\sum F_y = qE_y + T\cos 37.0° - mg = 0 \quad \textbf{[2]}$$

(a) We solve for T from equation [1]:

$$T = \frac{qE_x}{\sin 37.0°}$$

and substitute into equation [2] to obtain

$$q = \frac{mg}{E_y + \dfrac{E_x}{\tan 37.0^\circ}}$$

$$= \frac{(1.00 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{5.00 \times 10^5\ \text{N/C} + \left(\dfrac{3.00 \times 10^5\ \text{N/C}}{\tan 37.0^\circ}\right)}$$

$$q = \boxed{1.09 \times 10^{-8}\ \text{C}}$$

(b) Using the above result for q in equation [1], we find that the tension is

$$T = \frac{qE_x}{\sin 37.0^\circ} = \frac{(1.09 \times 10^{-8}\ \text{C})(3.00 \times 10^5\ \text{N/C})}{\sin 37.0^\circ}$$

$$= \boxed{5.44 \times 10^{-3}\ \text{N}}$$

P23.68 This is the general version of the preceding problem . The known quantities are A, B, m, g, and θ. The unknowns are q and T.

Refer to ANS. FIG. P23.67 above. The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 51.

Again, from Newton's second law,

$$\sum F_x = -T \sin\theta + qA = 0 \qquad \textbf{[1]}$$

and

$$\sum F_y = +T \cos\theta + qB - mg = 0 \qquad \textbf{[2]}$$

(a) Substituting $T = \dfrac{qA}{\sin\theta}$ into equation [2], we obtain

$$\frac{qA\cos\theta}{\sin\theta} + qB = mg$$

Isolating q on the left,

$$\boxed{q = \frac{mg}{(A\cot\theta + B)}}$$

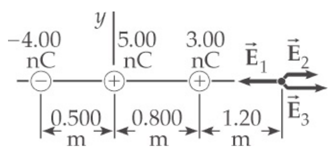
(b) Substituting this value into equation [1], we obtain

$$\boxed{T = \frac{mgA}{(A\cos\theta + B\sin\theta)}}$$

If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 51. If you find this problem more difficult than problem 51, the little list at the first step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the analysis step, and for recognizing when we have an answer.

P23.69 (a) Refer to ANS. FIG. P23.69(a). The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-4.00\times10^{-9}\ \text{C}\right)}{(2.50\ \text{m})^2}\hat{\mathbf{i}}$$
$$= -5.75\hat{\mathbf{i}}\ \text{N/C}$$



ANS. FIG. P23.69(a)

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge, are

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-9}\ \text{C}\right)}{(2.00\ \text{m})^2}\hat{\mathbf{i}}$$
$$= 11.2\ \text{N/C}\ \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_3 = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times10^{-9}\ \text{C}\right)}{(1.20\ \text{m})^2}\hat{\mathbf{i}} = 18.7\ \text{N/C}\ \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_R = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 = \boxed{24.2\ \text{N/C in } +x \text{ direction}}$$

(b) In this case, referring to ANS. FIG. P23.69(b),

$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (-8.46\ \text{N/C})\left(0.243\hat{\mathbf{i}} + 0.970\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (11.2\ \text{N/C})\left(+\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{E}}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (5.81\ \text{N/C})\left(-0.371\hat{\mathbf{i}} + 0.928\hat{\mathbf{j}}\right)$$

The components of the resultant electric field are

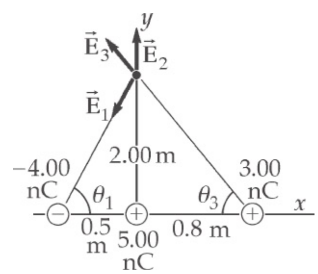
$$E_x = E_{1x} + E_{3x} = -4.21\hat{\mathbf{i}}\ \text{N/C} \quad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\hat{\mathbf{j}}\ \text{N/C}$$

then, the magnitude of the resultant electric field is

$$E_R = \boxed{9.42 \text{ N/C}}$$

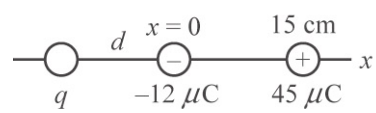
and is directed at

$$\theta = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}\left(\frac{8.43 \text{ N/C}}{4.21 \text{ N/C}}\right) = \boxed{63.4° \text{ above } -x \text{ axis}}$$



ANS. FIG. P23.69(b)

P23.70 (a) The two given charges exert equal-size forces of attraction on each other. If a third charge, positive or negative, were placed between them they could not be in equilibrium. If the third charge were at a point $x > 15.0$ cm, it would exert a stronger force on the 45.0-μC charge than on the –12.0-μC charge, and could not produce equilibrium for both. Thus the third charge must be at $x = -d < 0$.



ANS. FIG. P23.70

It is possible in just one way.

(b) The equilibrium of the third charge requires

$$\frac{k_e q(12.0\ \mu\text{C})}{d^2} = \frac{k_e q(45.0\ \mu\text{C})}{(15.0 \text{ cm} + d)^2} \rightarrow \left(\frac{15.0 \text{ cm} + d}{d}\right)^2 = \frac{45.0}{12.0} = 3.75$$

Solving,

$$15.0 \text{ cm} + d = 1.94d \qquad \rightarrow \qquad d = 16.0 \text{ cm}$$

The third charge is at $\boxed{x = -16.0 \text{ cm}}$.

(c) The equilibrium of the –12.0-μC charge requires

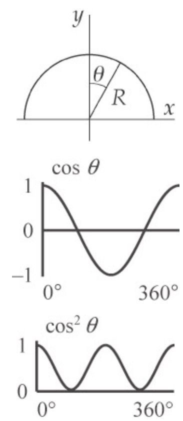
$$\frac{k_e q(12.0\ \mu\text{C})}{(16.0 \text{ cm})^2} = \frac{k_e(45.0\ \mu\text{C})(12.0\ \mu\text{C})}{(15.0 \text{ cm})^2}$$

solving,

$$q = \boxed{+51.3\ \mu\text{C}}$$

All six individual forces are now equal in magnitude, so we have equilibrium as required, and this is the only solution.

P23.71 To find the force on the test charge at point P, we first determine the charge per unit length on the semicircle:



ANS. FIG. P23.71

$$Q = \int \lambda\, d\ell = \int_{-90.0°}^{90.0°} \lambda_0 \cos\theta R\, d\theta = \lambda_0 R \sin\theta \Big|_{-90.0°}^{90.0°}$$

$$= \lambda_0 R[1-(-1)] = 2\lambda_0 R$$

or $\quad Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\ \text{m},$

which gives $\lambda_0 = 10.0\ \mu\text{C/m}$.

The force on the charge from each incremental section of the semicircle is

$$dF_y = \frac{k_e q(\lambda d\ell)\cos\theta}{R^2} = \frac{k_e q\left(\lambda_0 \cos^2\theta R d\theta\right)}{R^2}$$

Integrating,

$$F_y = \int_{-90.0°}^{90.0°} \frac{k_e q \lambda_0}{R}\cos^2\theta\, d\theta = \frac{k_e q\lambda_0}{R}\int_{-\pi/2}^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta$$

$$F_y = \frac{k_e q\lambda_0}{R}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\Bigg|_{-\pi/2}^{\pi/2} = \frac{k_e q\lambda_0}{R}\left[\left(\frac{\pi}{4}+0\right)-\left(\frac{-\pi}{4}+0\right)\right]$$

$$F_y = \frac{k_e q\lambda_0}{R}\left(\frac{\pi}{2}\right)$$

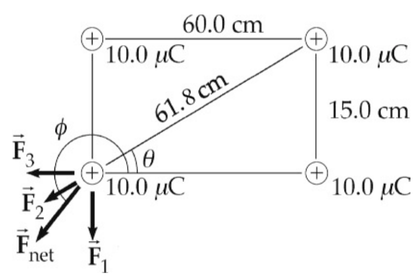
$$F_y = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times10^{-6}\ \text{C}\right)\left(10.0\times10^{-6}\ \text{C/m}\right)\left(\dfrac{\pi}{2}\right)}{(0.600\ \text{m})}$$

$$F_y = 0.706\ \text{N, downward} = \boxed{-0.706\hat{\mathbf{i}}\ \text{N}}$$

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

P23.72 The magnitude of the electric force is given by $F = \dfrac{k_e q_1 q_2}{r^2}$. The angle θ in ANS. FIG. P23.72 is found from

$$\theta = \tan^{-1}\left(\frac{15.0}{60.0}\right) = 14.0°$$



ANS. FIG. P23.72

$$F_1 = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times10^{-6}\ \text{C}\right)^2}{(0.150\ \text{m})^2}$$
$$= 40.0\ \text{N}$$

$$F_2 = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times10^{-6}\ \text{C}\right)^2}{(0.618\ \text{m})^2} = 2.35\ \text{N}$$

$$F_3 = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times10^{-6}\ \text{C}\right)^2}{(0.600\ \text{m})^2} = 2.50\ \text{N}$$

$$F_x = -F_3 - F_2\cos14.0° = -2.50 - 2.35\cos14.0° = -4.78\ \text{N}$$
$$F_y = -F_1 - F_2\sin14.0° = -40.0 - 2.35\sin14.0° = -40.5\ \text{N}$$

(a) $F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{(-4.78\ \text{N})^2 + (-40.5\ \text{N})^2} = \boxed{40.8\ \text{N}}$

(b) $\tan\phi = \dfrac{F_y}{F_x} = \dfrac{-40.5\ \text{N}}{-4.78\ \text{N}} \rightarrow \phi = \boxed{263°}$

P23.73 We model the spheres as particles. They have different charges. They exert on each other forces of equal magnitude. They have equal masses, so their strings make equal angles θ with the vertical. We define r as the distance between the centers of the two spheres. We find r from

$$\sin\theta = \frac{r/2}{40.0\ \text{cm}}$$

from which we obtain

$$r = (80.0\ \text{cm})\sin\theta$$

Now let T represent the string tension. We have, from the particle under a net force model,

$$\sum F_x = 0: \quad \frac{k_e q_1 q_2}{r^2} - T\sin\theta = 0 \quad \rightarrow \quad \frac{k_e q_1 q_2}{r^2} = T\sin\theta \qquad \textbf{[1]}$$

$$\sum F_y = 0: \quad T\cos\theta - mg = 0 \quad \rightarrow \quad mg = T\cos\theta \qquad \textbf{[2]}$$

Dividing equation [1] by [2] to eliminate T gives

$$\frac{k_e q_1 q_2}{r^2 mg} = \tan\theta = \frac{r/2}{\sqrt{(40.0\ \text{cm})^2 - r^2/4}}$$

Clearing the fractions,

$$k_e q_1 q_2\sqrt{(80.0\ \text{cm})^2 - r^2} = mgr^3$$

Substituting numerical values gives

$$\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(200\times10^{-9}\ \mathrm{C}\right)\left(300\times10^{-9}\ \mathrm{C}\right)$$
$$\times\sqrt{(0.800\ \mathrm{m})^2-r^2}=\left(2.40\times10^{-3}\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right)r^3$$

Suppressing units,

$$(0.800)^2-r^2=1\,901\,r^6$$

Instead of attempting to solve this equation, we instead home in on a solution by trying values, tabulated below:

r	$0.640-r^2-1901\,r^6$
0	+0.64
0.5	–29.3
0.2	+0.48
0.3	–0.84
0.24	+0.22
0.27	–0.17
0.258	+0.013
0.259	–0.001

Thus the distance to three digits is 0.259 m = $\boxed{2.59\text{ cm.}}$

P23.74 Use Figure 23.24 for guidance on the physical setup of this problem. Let the electron enter at the origin of coordinates at the left end and just under the upper plate, which we choose to be negative so that the electron accelerates downward. The electron is a particle under constant velocity in the horizontal direction:

$$x_f=v_{xi}t$$

The electron is a particle under constant acceleration in the vertical direction:

$$y_f=\tfrac{1}{2}a_yt^2$$

Eliminate t between the equations:

$$y_f=\tfrac{1}{2}a_y\left(\frac{x_f}{v_{xi}}\right)^2\quad\rightarrow\quad y_f=\left(\frac{a_y}{2v_{xi}{}^2}\right)x_f{}^2$$

Substitute for the acceleration of the particle in terms of the electric force:

$$y_f = \left(\frac{-eE}{2v_{xi}^{\ 2}\ m_e}\right)x_f^{\ 2}$$

Substitute numerical values, letting the final horizontal position be at the right end of the plates:

$$y_f = \left[\frac{-\left(1.60\times10^{-19}\ \text{C}\right)(200\ \text{N/C})}{2\left(3.00\times10^{6}\ \text{m/s}\right)^2\left(9.11\times10^{-31}\ \text{kg}\right)}\right](0.200\ \text{m})^2$$
$$= -0.078\ 1\ \ \text{m}$$

Therefore, when the electron leaves the plates, its final position is well below that of the lower plate, which is at position y = –1.50 cm = –0.015 m. Consequently, because we have let the electron enter the field at as high a position as possible, the electron will strike the lower plate long before it reaches the end, regardless of where it enters the field.

P23.75 Charge Q resides on each of the blocks, which repel as point charges:

$$F = \frac{k_e Q^2}{L^2} = k\left(L - L_i\right)$$

Solving for Q, we find

$$Q = L\sqrt{\frac{k\left(L-L_i\right)}{k_e}} = (0.500\ \text{m})\sqrt{\frac{(100\ \text{N/m})(0.500\ \text{m} - 0.400\ \text{m})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}}$$
$$= \boxed{1.67\times10^{-5}\text{C}}$$

P23.76 Charge Q resides on each of the blocks, which repel as point charges:

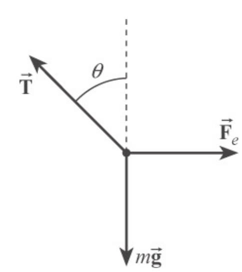
$$F = \frac{k_e Q^2}{L^2} = k\left(L - L_i\right)$$

Solving for Q, we find

$$Q = \boxed{L\sqrt{\frac{k\left(L-L_i\right)}{k_e}}}$$

P23.77 Consider the free-body diagram of the rightmost charge given in ANS. FIG. P23.77. Newton's second law then gives

$$\sum F_y = 0 \ \Rightarrow\ T\cos\theta = mg \quad \text{or} \quad T = \frac{mg}{\cos\theta}$$



ANS. FIG. P23.77

and

$$\sum F_x = 0$$

$$\Rightarrow\ F_e = T\sin\theta = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta$$

But,

$$F_e = \frac{k_e q^2}{r_1^2} + \frac{k_e q^2}{r_2^2} = \frac{k_e q^2}{(L\sin\theta)^2} + \frac{k_e q^2}{(2L\sin\theta)^2} = \frac{5k_e q^2}{4L^2\sin^2\theta}$$

Thus,

$$\frac{5k_e q^2}{4L^2\sin^2\theta} = mg\tan\theta \text{ or } q = \sqrt{\frac{4L^2 mg\sin^2\theta\tan\theta}{5k_e}}$$

If $\theta = 45°$, $m = 0.100$ kg, and $L = 0.300$ m, then

$$q = \sqrt{\frac{4(0.300\text{ m})^2(0.100\text{ kg})(9.80\text{ m/s}^2)\sin^2(45.0°)\tan(45.0°)}{5(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}}$$

or $$q = 1.98\times10^{-6}\text{ C} = \boxed{1.98\ \mu\text{C}}$$

P23.78 From Example 23.8, the electric field due to a uniformly charged ring is given by

$$E = \frac{k_e Qx}{(x^2+a^2)^{3/2}}$$

For a maximum, we differentiate E with respect to x and set the result equal to zero:

$$\frac{dE}{dx} = Qk_e\left[\frac{1}{(x^2+a^2)^{3/2}} - \frac{3x^2}{(x^2+a^2)^{5/2}}\right] = 0$$

solving for x gives

$$x^2 + a^2 - 3x^2 = 0 \quad\text{or}\quad x = \frac{a}{\sqrt{2}}$$

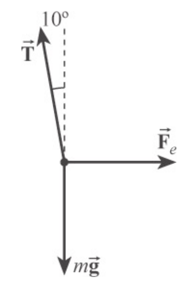
Substituting into the expression for E gives

$$E = \frac{k_e Qa}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}} = \frac{k_e Q}{3\frac{\sqrt{3}}{2}a^2}$$

$$= \boxed{\frac{2k_e Q}{3\sqrt{3}a^2}} = \boxed{\frac{Q}{6\sqrt{3}\pi\epsilon_0 a^2}}$$

P23.79 The charges are q and $2q$. The magnitude of the repulsive force that one charge exerts on the other is

$$F_e = 2k_e \frac{q^2}{r^2}$$



ANS. FIG. P23.79

From Figure P23.79 in the textbook, observe that the distance separating the two spheres is

$$r = d + 2L\sin 10°$$

From the free-body diagram of one sphere given in ANS. FIG. P23.79, observe that

$$\sum F_y = 0 \Rightarrow T\cos 10° = mg \quad \text{or} \quad T = mg/\cos 10°$$

and

$$\sum F_x = 0 \Rightarrow F_e = T\sin 10° = \left(\frac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10°$$

Thus,

$$2k_e \frac{q^2}{r^2} = mg\tan 10° \quad \rightarrow \quad 2k_e \frac{q^2}{\left(d + 2L\sin 10°\right)^2} = mg\tan 10°$$

or

$$q = \sqrt{\frac{mg\left(d + 2L\sin\theta\right)^2 \tan 10°}{2k_e}}$$

$$= \sqrt{\frac{(0.015\text{ kg})(9.80\text{ m/s}^2)\left[0.0300\text{ m} + 2(0.0500\text{ m})\sin 10°\right]^2 \tan 10°}{2\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)}}$$

$$= 5.69\times 10^{-8}\text{ C}$$

giving $\boxed{1.14\times 10^{-7}\text{ C on one sphere and } 5.69\times 10^{-8}\text{ C on the other.}}$

P23.80 (a) The bowl exerts a normal force on each bead, directed along the radius line at angle θ above the horizontal. Consider the free-body diagram shown in ANS. FIG. P23.80 for the bead on the left side of the bowl:

$$\sum F_y = n\sin\theta - mg = 0 \quad \rightarrow \quad n = \frac{mg}{\sin\theta}$$

Also,

$$\sum F_x = -F_e + n\cos\theta = 0$$

which gives

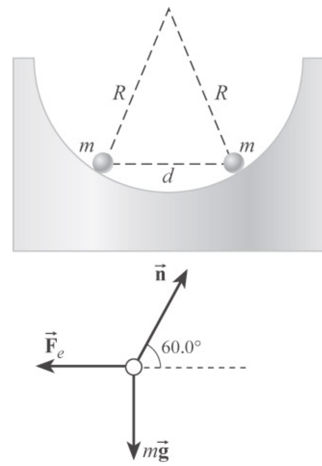
$$F_e = n\cos\theta = \left(\frac{mg}{\sin\theta}\right)\cos\theta = \frac{mg}{\tan\theta}$$

The electric force is

$$F_e = \frac{k_e q^2}{d^2}$$

And from ANS. FIG. P23.80,

$$\tan\theta = \frac{\sqrt{R^2 - (d/2)^2}}{(d/2)} = \frac{\sqrt{4R^2 - d^2}}{d}$$



ANS. FIG. P23.80

Therefore,

$$F_e = \frac{k_e q^2}{d^2} = \frac{mg}{\tan\theta} = \frac{mg}{\sqrt{4R^2 - d^2}/d} \quad \rightarrow \quad q = \boxed{\left(\frac{mgd^3}{k_e\sqrt{4R^2 - d^2}}\right)^{1/2}}$$

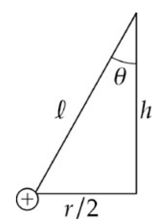
(b) As $d \rightarrow 2R$, $\sqrt{4R^2 - d^2} \rightarrow 0$; therefore, $\boxed{q \rightarrow \infty}$.

P23.81 (a) From the $2Q$ charge we have

$$F_e - T_2\sin\theta_2 = 0 \text{ and } mg - T_2\cos\theta_2 = 0$$

Combining these we find

$$\frac{F_e}{mg} = \frac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$$



ANS. FIG. P23.81

From the Q charge we have

$$F_e = T_1\sin\theta_1 = 0 \text{ and } mg - T_1\cos\theta_1 = 0$$

Combining these we find

$$\frac{F_e}{mg} = \frac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1 \text{ or } \boxed{\theta_2 = \theta_1}$$

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_e Q^2}{r^2}$ · If we assume θ is small then $\tan\theta \approx \dfrac{r/2}{\ell}$.

Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r. $\dfrac{F_e}{mg} = \tan\theta$, then $\dfrac{2k_e Q^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell}$ and solving for r we find $r \approx \left(\dfrac{4k_e Q^2 \ell}{mg}\right)^{1/3}$.

P23.82 The field on the axis of the ring is calculated in Example 19.6 in the chapter text as

$$E = E_x = \frac{k_e x Q}{\left(x^2 + a^2\right)^{3/2}}$$

The force experienced by a charge $-q$ placed along the axis of the ring is

$$F = -k_e Qq\left[\frac{x}{\left(x^2 + a^2\right)^{3/2}}\right]$$

and when $x \ll a$, this becomes

$$F = -\left(\frac{k_e Qq}{a^3}\right)x$$

This expression for the force is in the form of Hooke's law, with an effective spring constant of

$$k = \frac{k_e Qq}{a^3}$$

Since $\omega = 2\pi f = \sqrt{\frac{k}{m}}$, we have

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{k_e Qq}{ma^3}}}$$

P23.83 (a) The total non-contact force on the cork ball is:

$$F = qE + mg = m\left(g + \frac{qE}{m}\right)$$

which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{g + qE/m}} \\ &= 2\pi\sqrt{\frac{0.500\text{ m}}{9.80\text{ m/s}^2 + \left[\frac{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N/C}\right)}{1.00\times10^{-3}\text{ kg}}\right]}} \\ &= \boxed{0.307\text{ s}} \end{aligned}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get

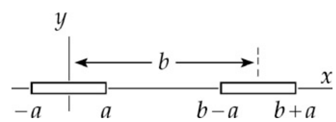
$$T = 2\pi\sqrt{\frac{L}{qE/m}}$$

$$T = 2\pi\sqrt{\frac{0.500\text{ m}}{\left[\frac{(2.00\times10^{-6}\text{ C})(1.00\times10^{5}\text{ N/C})}{1.00\times10^{-3}\text{ kg}}\right]}} = 0.314\text{ s}$$

(a 2.28% difference).

Challenge Problems

P23.84 According to the result of Example 23.7 in the textbook, the left-hand rod creates this field at a distance d from its right-hand end:



ANS FIG. P23.84

$$E = \frac{k_eQ}{d(2a+d)}$$

The force per unit length exerted by the left-hand rod on the right-hand rod is then

$$dF = \frac{k_eQQ}{2a}\frac{dx}{d(d+2a)}$$

Integrating,

$$F = \frac{k_eQ^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x+2a)} = \frac{k_eQ^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a+x}{x}\right)\Bigg|_{b-2a}^{b}$$

$$= \frac{+k_eQ^2}{4a^2}\left(-\ln\frac{2a+b}{b}+\ln\frac{b}{b-2a}\right) = \frac{k_eQ^2}{4a^2}\ln\frac{b^2}{(b-2a)(b+2a)}$$

$$= \boxed{\left(\frac{k_eQ^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2-4a^2}\right)}$$

P23.85 First, we use unit vectors to find the total electric field at point A produced by the 7 other charges.

source charge	vector field components	equivalent field
(1) lower left, front:	$\vec{\mathbf{E}}_1 = \frac{k_eq}{r_1^2}\hat{\mathbf{r}}_1 = \frac{k_eq}{s^2+s^2}\frac{\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_eq}{s^2}\left(\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$

(2) lower right, front:	$\vec{\mathbf{E}}_2 = \frac{k_e q}{r_2^2}\hat{\mathbf{r}}_2 = \frac{k_e q}{s^2}\hat{\mathbf{k}}$	$\frac{k_e q}{s^2}\hat{\mathbf{k}}$
(3) lower right, back:	$\vec{\mathbf{E}}_3 = \frac{k_e q}{r_3^2}\hat{\mathbf{r}}_3 = \frac{k_e q}{s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{k}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{k}}\right)$
(4) lower left, back:	$\vec{\mathbf{E}}_4 = \frac{k_e q}{r_4^2}\hat{\mathbf{r}}_4 = \frac{k_e q}{s^2+s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$	$\left(\frac{1}{3\sqrt{3}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$
(5) upper right, back	$\vec{\mathbf{E}}_5 = \frac{k_e q}{r_5^2}\hat{\mathbf{r}}_5 = \frac{k_e q}{s^2}\hat{\mathbf{i}}$	$\frac{k_e q}{s^2}\hat{\mathbf{i}}$
(6) upper left, back	$\vec{\mathbf{E}}_6 = \frac{k_e q}{r_6^2}\hat{\mathbf{r}}_6 = \frac{k_e q}{s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$
(7) upper right, front	$\vec{\mathbf{E}}_7 = \frac{k_e q}{r_7^2}\hat{\mathbf{r}}_7 = \frac{k_e q}{s^2}\hat{\mathbf{j}}$	$\frac{k_e q}{s^2}\hat{\mathbf{j}}$
total field $\vec{\mathbf{E}}_{\text{total}} = \frac{k_e q}{s^2}\left[1+\frac{2}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$		

Notice that because of symmetry, the components of the field have the same magnitude.

(a) At point A,

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}_{\text{total}} = \frac{k_e q^2}{s^2}\left[1+\frac{2}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$$

$$= \frac{k_e q^2}{s^2}(1.90)\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$$

$$\rightarrow \boxed{F_x = F_y = F_z = 1.90 k_e \frac{q^2}{s^2}}$$

(b) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\frac{k_e q^2}{s^2}}$

(c) away from the origin

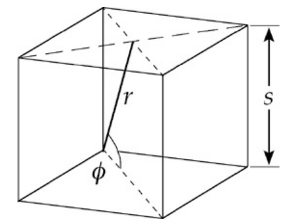
P23.86 (a) Zero contribution from the same face due to symmetry, opposite face contributes

$$E = 4\left(\frac{k_e q}{r^2}\sin\phi\right)$$

where

$$r = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2 + s^2} = \sqrt{1.5}s = 1.22s$$

$$\sin\phi = \frac{s}{r}, \qquad E = 4\frac{k_e q s}{r^3} = \frac{4}{(1.22)^3}\frac{k_e q}{s^2} = \boxed{2.18\frac{k_e q}{s^2}}$$



ANS. FIG. P23.86

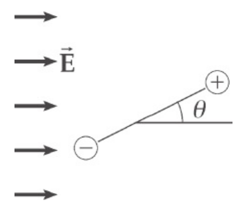
(b) At the top face, $\boxed{\text{the electric field is in the } \hat{\mathbf{k}} \text{ direction.}}$

P23.87 (a) The electrostatic forces exerted on the two charges result in a net torque

$$\tau = -2Fa\sin\theta = -2Eqa\sin\theta$$

For small θ, $\sin\theta \approx \theta$ and using $p = 2qa$, we have

$$\tau = -Ep\theta$$



ANS. FIG. P23.87

The torque produces an angular acceleration given by

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

where the moment of inertia of the dipole is $I = 2ma^2$

Combining the two expressions for torque, we have

$$\frac{d^2\theta}{dt^2} = -\left(\frac{Ep}{I}\right)\theta$$

This equation can be written in the form $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ which is the standard equation characterizing simple harmonic motion, with

$$\omega^2 = \frac{Ep}{I} = \frac{E(2qa)}{2ma^2} = \frac{qE}{ma}$$

The frequency of oscillation is $f = \omega/2\pi$, so

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{qE}{ma}}}$$

(b) If the masses are unequal, the dipole will oscillate about its center of mass (CM). Assume mass m_2 is greater than mass m_1, and treat the *center* of the dipole as being at the origin of an x axis, so that mass m_1 is at $x = -a$ and mass m_2 is at $x = +a$. The coordinate of the CM of the dipole is then

$$x_{\text{cm}} = \frac{m_2 a - m_1 a}{m_1 + m_2} = a\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$

relative to the center of the dipole. Notice that the moment of inertia of the dipole about its *center* is

$$I = m_1 a^2 + m_2 a^2$$

but its center is a distance x_{cm} from its CM. By the parallel-axis theorem, the moment of inertia of the dipole about its *center* is related to its moment about its CM thus:

$$I = m_1 a^2 + m_2 a^2 = I_{\text{CM}} + \left(m_1 + m_2\right) x_{\text{cm}}{}^2$$

therefore,

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - \left(m_1 + m_2\right) x_{\text{cm}}{}^2$$

The moment of inertia of the dipole about its CM is then

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - \left(m_1 + m_2\right) a^2 \left(\frac{m_2 - m_1}{m_1 + m_2}\right)^2$$

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - a^2 \frac{\left(m_2 - m_1\right)^2}{\left(m_1 + m_2\right)}$$

$$I_{\text{CM}} = \frac{\left(m_1 + m_2\right)\left(m_1 a^2 + m_2 a^2\right) - \left(m_2{}^2 a^2 - 2m_1 m_2 a^2 + m_1{}^2 a^2\right)}{\left(m_1 + m_2\right)}$$

$$I_{\text{CM}} = \frac{\left(m_1{}^2 a^2 + 2m_1 m_2 a^2 + m_2{}^2 a^2\right) - \left(m_2{}^2 a^2 - 2m_1 m_2 a^2 + m_1{}^2 a^2\right)}{\left(m_1 + m_2\right)}$$

$$I_{\text{CM}} = \frac{4m_1 m_2 a^2}{\left(m_1 + m_2\right)}$$

Therefore, from part (a),

$$\omega^2 = \frac{Ep}{I_{\text{CM}}} = \frac{E\left(2qa\right)}{\left[\dfrac{4m_1 m_2 a^2}{\left(m_1 + m_2\right)}\right]} = \frac{qE\left(m_1 + m_2\right)}{2m_1 m_2 a} = \left(2\pi f\right)^2$$

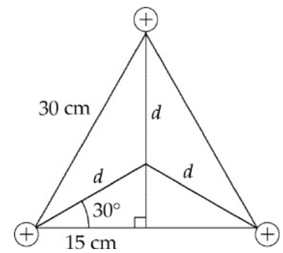
and

$$f = \frac{1}{2\pi}\sqrt{\frac{qE(m_1 + m_2)}{2m_1 m_2 a}}$$

P23.88 From ANS. FIG. P23.88(a) we have

$$d\cos 30.0° = 15.0 \text{ cm}$$

or $$d = \frac{15.0 \text{ cm}}{\cos 30.0°}.$$



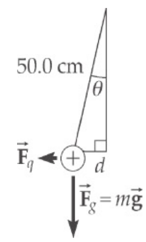
ANS. FIG. P23.88(a)

From ANS. FIG. P23.88(b) we have

$$\theta = \sin^{-1}\left(\frac{d}{50.0 \text{ cm}}\right)$$

$$\theta = \sin^{-1}\left(\frac{15.0 \text{ cm}}{(50.0 \text{ cm})(\cos 30.0°)}\right) = 20.3°$$

$$\frac{F_q}{mg} = \tan\theta \quad \text{or} \quad F_q = mg\tan 20.3° \qquad \textbf{[1]}$$

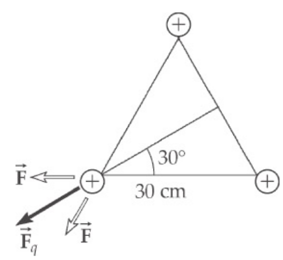


ANS. FIG. P23.88(b)

From ANS. FIG. P23.88(c) we have

$$F_q = 2F\cos 30.0°$$

$$F_q = 2\left[\frac{k_e q^2}{(0.300 \text{ m})^2}\right]\cos 30.0° \qquad \textbf{[2]}$$



ANS. FIG. P23.88(c)

Combining equations [1] and [2],

$$2\left[\frac{k_e q^2}{(0.300 \text{ m})^2}\right]\cos 30.0° = mg\tan 20.3°$$

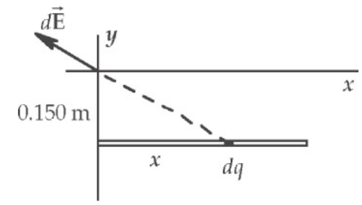
$$q^2 = \frac{mg(0.300 \text{ m})^2\tan 20.3°}{2k_e\cos 30.0°}$$

$$q^2 = \frac{(2.00\times10^{-3} \text{ kg})(9.80 \text{ m/s}^2)(0.300 \text{ m})^2\tan 20.3°}{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\cos 30.0°}$$

$$q = \sqrt{4.20\times10^{-14} \text{ C}^2} = 2.05\times10^{-7} \text{ C} = \boxed{0.205 \ \mu\text{C}}$$

P23.89 $$d\vec{\mathbf{E}} = \frac{k_e dq}{x^2 + (0.150\text{ m})^2}\left(\frac{-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right)$$

$$= \frac{k_e \lambda\left(-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$



ANS. FIG. P23.89

$$\vec{\mathbf{E}} = \int_{\text{all charge}} d\vec{\mathbf{E}} = k_e\lambda \int_{x=0}^{0.400\text{ m}} \frac{\left(-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$

$$\vec{\mathbf{E}} = k_e\lambda\left[\left.\frac{+\hat{\mathbf{i}}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\hat{\mathbf{j}}x}{(0.150\text{ m})^2\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

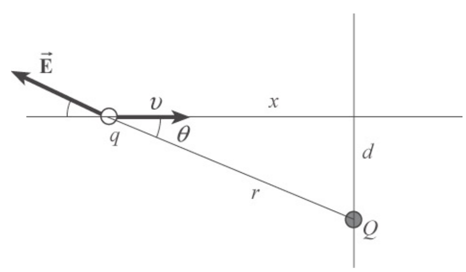
$$\vec{\mathbf{E}} = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times10^{-9}\text{ C/m}\right)$$

$$\times\left[\hat{\mathbf{i}}(2.34 - 6.67)\text{ m}^{-1} + \hat{\mathbf{j}}(6.24 - 0)\text{ m}^{-1}\right]$$

$$\vec{\mathbf{E}} = \left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)\times10^3\text{ N/C} = \boxed{\left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)\text{ kN/C}}$$

P23.90 We work under the assumption that v_x has the nearly constant value v. Initially, with the particle nearly at infinity, $v_x = v$ and $v_y = 0$. As the moving charge travels toward and passes the fixed charge Q, the velocity component v_y increases according to

$$m\frac{dv_y}{dt} = F_y$$



or $$m\frac{dv_y}{dx}\frac{dx}{dt} = qE_y$$

ANS. FIG. P23.90

Now $\dfrac{dx}{dt} = v_x$ has the nearly constant value v; therefore, we have

$$dv_y = \frac{q}{mv}E_y dx \quad\rightarrow\quad v_y = \int_0^{v_y} dv_y = \frac{q}{mv}\int_{-\infty}^{\infty} E_y dx$$

The radially outward component of the electric field varies along the x axis. We assume that the distance r between charges is does not depend significantly on y.

From the figure, we see that $E = \frac{k_e Q}{r^2}$, $r \approx \sqrt{d^2 + x^2}$, $E_y = E\sin\theta$, and $\sin\theta \approx \frac{d}{\sqrt{d^2 + x^2}}$. We evaluate the integral from above:

$$\int_{-\infty}^{\infty} E_y\,dx = \int_{-\infty}^{\infty} E\sin\theta\,dx \approx \int_{-\infty}^{\infty} \frac{k_e Q}{(d^2 + x^2)} \frac{d}{\sqrt{d^2 + x^2}}dx$$

$$= k_e Q d \int_{-\infty}^{\infty} \frac{dx}{(d^2 + x^2)^{3/2}}$$

$$= (k_e Q d)\left(\frac{x}{d^2(d^2 + x^2)^{1/2}}\right)\Bigg|_{-\infty}^{\infty} = \frac{k_e Q d}{d^2}[1 - (-1)] = \frac{2k_e Q}{d}$$

So, the v_y is

$$v_y = \frac{q}{mv}\int_{-\infty}^{\infty} E_y\,dx = \frac{q}{mv}\left(\frac{2k_e Q}{d}\right) = \frac{2k_e qQ}{mvd}$$

The angle of deflection is described by

$$\tan\theta = \frac{v_y}{v_x} \approx \frac{v_y}{v} = \frac{2k_e qQ}{mv^2 d} \quad \rightarrow \quad \theta = \boxed{\tan^{-1}\frac{2k_e qQ}{mv^2 d}}$$

P23.91 (a) The two charges create fields of equal magnitude, both with outward components along the x axis and with upward and downward y components that add to zero. The net field is then

$$\vec{\mathbf{E}} = \frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} + \frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} = 2\frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}}$$

$$= \frac{2(8.99\times10^{9})(52\times10^{-9})x\ \hat{\mathbf{i}}}{\left[(0.25)^2 + x^2\right]^{3/2}}$$

$\boxed{\vec{\mathbf{E}} = \frac{935x}{(0.0625 + x^2)^{3/2}}\hat{\mathbf{i}}}$ where $\vec{\mathbf{E}}$ is in newtons per coulomb and x is in meters.

(b) At $x = 0.36$ m,

$$\vec{\mathbf{E}} = \frac{935(0.36)\ \hat{\mathbf{i}}}{(0.0625 + (0.36)^2)^{3/2}} = \boxed{4.00\text{ kN/C }\hat{\mathbf{i}}}$$

(c) We solve $1\,000 = (935\,x)(0.0625 + x^2)^{-3/2}$ by tabulating values for the field function:

x	$(935\,x)(0.0625 + x^2)^{-3/2}$
0	0
0.01	597
0.02	1 185
0.1	4 789
0.2	5 698
0.36	4 000
0.9	1 032
1	854
∞	0

We see that there are two points where $E = 1\,000$ N/C. We home in on them to determine their coordinates as (to three digits)

$\boxed{x = 0.016\,8 \text{ m and } x = 0.916 \text{ m.}}$

(d) The table in part (c) shows that

$\boxed{\text{nowhere is the field so large as } 16\,000 \text{ N/C.}}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P23.2 (a) 2.62×10^{24}; (b) 2.38 electrons for every 10^9 already present

P23.4 1.57 μN to the left

P23.6 (a) 9.21×10^{-10} N; (b) No. The electric force depends only on the magnitudes of the two charges and the distance between them.

P23.8 $\sim 10^{26}$ N

P23.10 (a) 1.59×10^{-9} N; (b) 1.29×10^{-45} N, larger by 1.24×10^{36} times; (c) 8.61×10^{-11} C/kg

P23.12 (a) 46.7 N to the left; (b) 157 N to the right; (c) 111 N to the left

P23.14 (a) $\dfrac{\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}d$; (b) Yes, if the third bead has a positive charge.

P23.16 0.229 m

P23.18 (a) 0; (b) 30.0 N; (c) 21.6 N; (d) 17.3 N; (e) –13.0 N; (f) 17.3 N; (g) 17.0 N; (h) 24.3 N at 44.5° above the $+x$ direction

P23.20 (a) The acceleration of the charge is equal to a negative constant times its displacement from equilibrium, as in $\vec{\mathbf{a}} = -\omega^2\vec{\mathbf{x}}$, so we have Simple Harmonic Motion with $\omega^2 = \dfrac{16k_e qQ}{md^3}$; (b) $\dfrac{\pi}{2}\sqrt{\dfrac{md^3}{k_e qQ}}$; (c) $4a\sqrt{\dfrac{k_e qQ}{md^3}}$

P23.22 The unknown charge on each dust particle is about half of the smallest possible free charge, the charge of the electron. No such free charge exists. Therefore, the forces cannot balance.

P23.24 2.07×10^3 N/C; down

P23.26 $-k_e \dfrac{3q}{r^2}\hat{\mathbf{j}}$

P23.28 (a) $\dfrac{k_e Qx\hat{\mathbf{i}}}{\left(a^2+x^2\right)^{3/2}}$; (b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Because of the symmetrical arrangement of the charges, smearing the charge around the circle does not change its amount or its distance from the field point, so it does not change the field.

P23.30 (a) $\left(18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right)$kN/C; (b) $\left(0.0360\hat{\mathbf{i}} - 0.436\hat{\mathbf{j}}\right)$N

P23.32 The field at the origin can be to the right, if the unknown charge is –9*Q*, or the field can be to the left, if and only if the unknown charge is +27*Q*.

P23.34 (a) $1.29\times 10^{4}\,\hat{\mathbf{j}}\ \text{N/C}$; (b) $-3.86\times 10^{-2}\,\hat{\mathbf{j}}\ \text{N}$

P23.36 $\dfrac{4a(k_e q)}{x^3}$

P23.38 (a) 383 MN/C; (b) 324 MN/C; (c) 80.7 MN/C; (d) 6.68 MN/C

P23.40 $E_x \approx \dfrac{k_e Q}{x^2}$ for a disk at large distances

P23.42 (a) $-k_e \dfrac{Q}{L}\left[\dfrac{1}{d} - \dfrac{1}{\left(d^2+L^2\right)^{1/2}}\right]$ and $k_e \dfrac{Q}{d}\dfrac{1}{\left(d^2+L^2\right)^{1/2}}$; (b) $E_x \approx 0$ and $E_y \approx k_e \dfrac{Q}{d^2}$ which is the field of a point charge *Q* at a distance *d* along the *y* axis above the charge.

P23.44 (a) $\dfrac{2k_e\lambda\sin\theta_0}{d}$; (b) $\dfrac{2k_e\lambda}{d}$

P23.46 (a) $\dfrac{k_e Q\hat{\mathbf{i}}}{h}\left[\dfrac{1}{\left(d^2+R^2\right)^{1/2}} - \dfrac{1}{\left((d+h)^2+R^2\right)^{1/2}}\right]$

(b) $\dfrac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h+\left(d^2+R^2\right)^{1/2} - \left((d+h)^2+R^2\right)^{1/2}\right]$

P23.48 See ANS. FIG. P23.48.

P23.50 (a) See ANS. FIG. P23.50; (b) At the center; (c) $1.73k_e \dfrac{q}{a^2}\hat{\mathbf{j}}$

P23.52 (a) $-5.76\times 10^{13}\,\hat{\mathbf{i}}\ \text{m/s}^2$; (b) $\vec{v}_i = 2.84\times 10^{6}\,\hat{\mathbf{i}}\ \text{m/s}$; (c) 4.93×10^{-8} s

P23.54 (a) Particle under constant velocity; (b) Particle under constant acceleration; (c) the proton moves in a parabolic path just like a projectile in a gravitational field; (d) $\dfrac{m_p v_i^2 \sin 2\theta}{eE}$; (e) 36.9° or 53.1°; (f) 166 ns or 221 ns

P23.56 (a) a parabola; (b) the negative plate; (c) The particle strikes the negative plate after moving a horizontal distance of 0.961 mm.

P23.58 (a) 2.00×10^{-10} C; (b) 1.41×10^{-10} C; (c) 5.89×10^{-11} C

P23.60 5.81 nC

P23.62 49.3 N/m

P23.64 (a) 0.115 N; (b) 1.25 cm

P23.66 2.51×10^{-9}

P23.68 (a) $q = \dfrac{mg}{(A\cot\theta + B)}$; (b) $T = \dfrac{mgA}{(A\cos\theta + B\sin\theta)}$

P23.70 (a) It is possible in just one way; (b) $x = -16.0$ cm; (c) $+51.3\mu$C

P23.72 (a) 40.9 N; (b) 263°

P23.74 See P23.74 for complete solution

P23.76 $L\sqrt{\dfrac{k(L-L_i)}{k_e}}$

P23.78 $\dfrac{2k_eQ}{3\sqrt{3}a^2} = \dfrac{Q}{6\sqrt{3}\pi \in_0 a^2}$

P23.80 (a) $\left(\dfrac{mgd^3}{k_e\sqrt{4R^2 - d^2}}\right)^{1/2}$; (b) $q \to \infty$

P23.82 $\dfrac{1}{2\pi}\sqrt{\dfrac{k_eQq}{ma^3}}$

P23.84 See P23.84 for full solution

P23.86 (a) $2.18\dfrac{k_eq}{s^2}$; (b) the direction is the $\hat{\mathbf{k}}$ direction

P23.88 $0.205\mu C$

P23.90 $\theta = \tan^{-1}\left(\dfrac{2k_eqQ}{mv^2d}\right)$