24

Gauss's Law

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ24.1 (**i**) Answer (a). The field is cylindrically radial to the filament, and is nowhere zero at any face of the gaussian surface.

(**ii**) Answer (b). The flux is zero through the two faces pierced by the filament because the field is parallel to those surfaces.

OQ24.2 Answer (c). The outer wall of the conducting shell will become polarized to cancel out the external field. The interior field is the same as before.

OQ24.3 Answer (e). The symmetry of a charge distribution and of its field is the same. Gauss's law applies to these charge distributions because (a) has cylindrical symmetry, (b) has translational symmetry, (c) has spherical symmetry, and (d) has spherical symmetry.

OQ24.4 (**i**) Answer (c). Equal amounts of flux pass through each of the six faces of the cube.

(**ii**) Answer (b). Move the charge to very close below the center of one face, so that half the flux passes through that face and half the flux passes through the other five faces.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

OQ24.5 Answer (b). The electric flux through a closed surface equals q/ϵ_0, where q is the total charge contained within the surface:

$$q/\epsilon_0 = \left[(3.00 - 2.00 - 7.00 + 1.00)\times 10^{-9}\ \text{C}\right] \Big/ \left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)$$
$$= -5.65\times 10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}$$

OQ24.6 **(i)** Answer (e). The shell becomes polarized.

(ii) Answer (a). The net charge on the shell's inner and outer surfaces is zero.

(iii) Answer (c). The charge has been transferred to the outer surface of the conductor.

(iv) Answer (c). The charge has been transferred to the outer surface of the conductor.

(v) Answer (a). The charge has been transferred to the outer surface of the conductor.

OQ24.7 **(i)** Answer (c). Because the charge distributions are spherically symmetric, both spheres create equal fields at exterior points, like particles at the centers of the spheres.

(ii) Answer (e). The field within the conductor is zero. The field a distance r from the center of the insulator is proportional to r, so it is 4/5 of its value at the surface.

OQ24.8 Answer (c). The electric field inside a conductor is zero.

OQ24.9 (a) The ranking is A > B > D > C. Let q represent the charge of the insulating sphere. The field at A is $(4/5)^3 q/[4\pi(4\text{ cm})^2\epsilon_0]$. The field at B is $q/[4\pi(8\text{ cm})^2\epsilon_0]$. The field at C is zero. The field at D is $q/[4\pi(16\text{ cm})^2\epsilon_0]$.

(b) The ranking is B = D > A > C. The flux through the 4-cm sphere is $(4/5)^3 q/\epsilon_0$. The flux through the 8-cm sphere and through the 16-cm sphere is q/ϵ_0 because they enclose the same amount of charge. The flux through the 12-cm sphere is 0 because the field is zero inside the conductor.

OQ24.10 **(i)** Answer (a). The field is perpendicular to the sheet, and is nowhere zero at any face of the gaussian surface.

(ii) Answer (c). The flux is nonzero through the top and bottom faces because the field is perpendicular to them, and zero through the other four faces because the field is parallel to them.

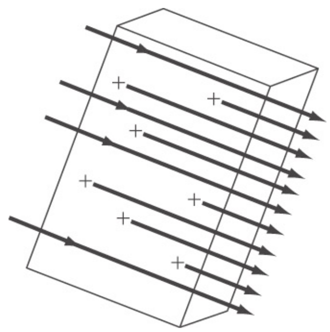
OQ24.11 The ranking is C > A = B > D. The total flux is proportional to the enclosed charge: $3Q > Q = Q > 0$.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ24.1 (a) If the volume charge density is nonzero, the field cannot be uniform in magnitude. Consider a gaussian surface in the shape of a rectangular box with two faces perpendicular to the direction of the field. It encloses some charge, so the net flux out of the box is nonzero. The field must be stronger on one side than on the other. The field cannot be uniform in magnitude.

(b) Now the volume contains no charge. The net flux out of the box is zero. The flux entering is equal to the flux exiting. The field must be uniform in magnitude along any line in the direction of the field. It can vary between points in a plane perpendicular to the field lines.



ANS. FIG. CQ24.1

CQ24.2 The electric flux through a closed surface is proportional to the total charge contained within the surface: (a) the flux is doubled because the charge is doubled, (b) the flux remains the same because the charge is the same, (c) the flux remains the same because the charge is the same, (d) the flux remains the same because the charge is the same, (e) the flux becomes zero because the charge inside the surface is zero.

CQ24.3 The net flux through any gaussian surface is zero. We can argue it two ways. Any surface contains zero charge, so Gauss's law says the total flux is zero. The field is uniform, so the field lines entering one side of the closed surface come out the other side and the net flux is zero.

CQ24.4 Gauss's law cannot be used to find the electric field at different points on a surface if the field is not constant over that surface. If the symmetry of an electric field allows us to say that $\int E\cos\theta\, dA = E\int \cos\theta\, dA$, where E is an unknown *constant* on the surface, then we can use Gauss's law. When electric field is a general unknown function $E(x, y, z)$, there can be no such simplification.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

CQ24.5 The electric flux is independent of the size and shape of the closed surface that contains the charge because all the field lines from the enclosed charge pass through the surface.

CQ24.6 The surface must enclose a positive total charge. Field lines emerge from positive charge and disappear into negative charge.

CQ24.7 (a) No. If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall.

(b) If the person carries a (small) charge q, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

CQ24.8 The sphere with large charge creates a strong field to polarize the other sphere. That means it pushes the excess like charge over to the far side, leaving charge of the opposite sign on the near side. This patch of opposite charge is smaller in amount but located in a stronger external field, so it can feel a force of attraction that is larger than the repelling force felt by the larger charge in the weaker field on the other side.

CQ24.9 There is zero force. The huge charged sheet creates a uniform field. The field can polarize the neutral sheet, creating in effect a film of opposite charge on the near face and a film with an equal amount of like charge on the far face of the neutral sheet. Since the field is uniform, the films of charge feel equal-magnitude forces of attraction and repulsion to the charged sheet. The forces add to zero.

CQ24.10 Inject some charge at arbitrary places within a conducting object. Every bit of the charge repels every other bit, so each bit runs away as far as it can, stopping only when it reaches the outer surface of the conductor.

CQ24.11 (a) The luminous flux on a given area is less when the sun is low in the sky, because the angle between the rays of the sun and the local area vector, $d\vec{\mathbf{A}}$, is greater than zero. The cosine of this angle is reduced.

(b) The decreased flux results, on the average, in colder weather.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 24.1 Electric Flux

P24.1 For a uniform electric field passing through a plane surface, $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta$, where θ is the angle between the electric field and the normal to the surface.

(a) The electric field is perpendicular to the surface, so $\theta = 0°$:

$$\Phi_E = \left(6.20 \times 10^5 \text{ N/C}\right)\left(3.20 \text{ m}^2\right)\cos 0°$$

$$\Phi_E = \boxed{1.98 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}}$$

(b) The electric field is parallel to the surface: $\theta = 90°$, so $\cos\theta = 0$, and the flux is $\boxed{\text{zero.}}$

P24.2 The electric flux through the bottom of the car is given by

$$\Phi_E = EA\cos\theta = \left(2.00 \times 10^4 \text{ N/C}\right)(3.00 \text{ m})(6.00 \text{ m})\cos 10.0°$$

$$= \boxed{355 \text{ kN} \cdot \text{m}^2/\text{C}}$$

P24.3 For a uniform field the flux is $\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta$.

The maximum value of the flux occurs when $\theta = 0$, or when the field is in the same direction as the area vector, which is defined as having the direction of the perpendicular to the area. Therefore, we can calculate the field strength at this point as

$$E = \frac{\Phi_{\text{max}}}{A} = \frac{\Phi_{\text{max}}}{\pi r^2}$$

$$E = \frac{5.20 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}}{\pi(0.200 \text{ m})^2} = 4.14 \times 10^6 \text{ N/C} = \boxed{4.14 \text{ MN/C}}$$

P24.4 (a) For the vertical rectangular surface, the area (shown as A′ in ANS FIG. P24.4) is

$$A' = (10.0 \text{ cm})(30.0 \text{ cm}) = 300 \text{ cm}^2 = 0.030\,0 \text{ m}^2$$

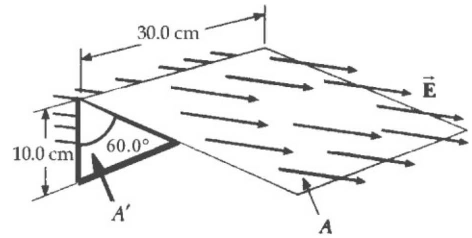
Since the electric field is perpendicular to the surface and is directed inward, $\theta = 180°$ and

$$\Phi_{E,A'} = EA'\cos\theta$$

$$\Phi_{E,A'} = \left(7.80 \times 10^4 \text{ N/C}\right)\left(0.030\,0 \text{ m}^2\right)\cos 180°$$

$$\Phi_{E,A'} = \boxed{-2.34 \text{ kN} \cdot \text{m}^2/\text{C}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.



ANS. FIG. P24.4

(b) To find the area A of the slanted surface, we note that the side for which dimensions are not given has length (10.0 cm) $= w\cos 60.0°$, so that

$$A = (30.0\text{ cm})(w) = (30.0\text{ cm})\left(\frac{10.0\text{ cm}}{\cos 60.0°}\right) = 600\text{ cm}^2$$
$$= 0.060\,0\text{ m}^2$$

The flux through this surface is then

$$\Phi_{E,A} = EA\cos\theta = \left(7.80\times10^4\right)(A)\cos 60.0°$$
$$= \left(7.80\times10^4\text{ N/C}\right)\left(0.060\,0\text{ m}^2\right)\cos 60.0°$$
$$= \boxed{+2.34\text{ kN}\cdot\text{m}^2/\text{C}}$$

(c) The bottom and the two triangular sides all lie *parallel* to $\vec{\mathbf{E}}$, so $\Phi_E = 0$ for each of these. Thus,

$$\Phi_{E,\text{ total}} = -2.34\text{ kN}\cdot\text{m}^2/\text{C} + 2.34\text{ kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$$

P24.5 For a uniform electric field passing through a plane surface, $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = EA\cos\theta$, where θ is the angle between the electric field and the normal to the surface.

(a) The electric field is perpendicular to the surface, so $\theta = 0°$:

$$\Phi_E = \left(3.50\times10^3\text{ N/C}\right)\left[(0.350\text{ m})(0.700\text{ m})\right]\cos 0°$$
$$= \boxed{858\text{ N}\cdot\text{m}^2/\text{C}}$$

(b) The electric field is parallel to the surface: $\theta = 90°$, so $\cos\theta = 0$, and the flux is $\boxed{\text{zero}}$.

(c) For the specified plane,

$$\Phi_E = \left(3.50\times10^3\text{ N/C}\right)\left[(0.350\text{ m})(0.700\text{ m})\right]\cos 40.0°$$
$$= \boxed{657\text{ N}\cdot\text{m}^2/\text{C}}$$

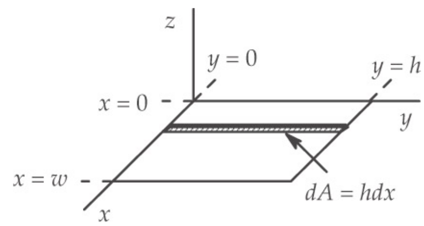
© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.6 We are given an electric field in the general form

$$\vec{\mathbf{E}} = ay\hat{\mathbf{i}} + bz\hat{\mathbf{j}} + cx\hat{\mathbf{k}}$$

In the xy plane, $z = 0$ so that the electric field reduces to

$$\vec{\mathbf{E}} = ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}$$



ANS. FIG. P24.6

To obtain the flux, we integrate (see ANS. FIG. P24.6 for the definition of dA):

$$\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int \left(ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}\right) \cdot \hat{\mathbf{k}}\, dA$$

$$\Phi_E = ch \int_{x=0}^{w} x\, dx = ch \left.\frac{x^2}{2}\right|_{x=0}^{w} = \boxed{\frac{chw^2}{2}}$$

Where the $\hat{\mathbf{k}}$ term was eliminated since $\hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 0$.

Section 24.2 Gauss's Law

P24.7 The electric flux through the hole is given by Gauss's Law (Equation 24.6) as

$$\Phi_{E,\,\text{hole}} = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}_{\text{hole}} = \left(\frac{k_e Q}{R^2}\right)\left(\pi r^2\right)$$

$$= \left(\frac{\left(8.99 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\left(10.0 \times 10^{-6}\ \text{C}\right)}{(0.100\ \text{m})^2}\right) \times \pi\left(1.00 \times 10^{-3}\ \text{m}\right)^2$$

$$= \boxed{28.2\ \text{N} \cdot \text{m}^2/\text{C}}$$

P24.8 The gaussian surface encloses the +1.00-nC and –3.00-nC charges, but not the +2.00-nC charge. The electric flux is therefore

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\left(1.00 \times 10^{-9}\ \text{C} - 3.00 \times 10^{-9}\ \text{C}\right)}{8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = \boxed{-226\ \text{N} \cdot \text{m}^2/\text{C}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.9 The total charge within the closed surface is

$$5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C} = -61.0\ \mu\text{C}$$

(a) So, from Equation 24.6, the total electric flux is

$$\Phi_E = \frac{q}{\epsilon_0} = \frac{-61.0 \times 10^{-6}\ \text{C}}{\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right)} = \boxed{-6.89\ \text{MN} \cdot \text{m}^2/\text{C}}$$

(b) $\boxed{\text{Since the net electric flux is negative, more lines enter than leave the surface.}}$

P24.10 (a) From $E = \dfrac{k_e Q}{r^2}$,

$$Q = \frac{Er^2}{k_e} = \frac{\left(8.90 \times 10^2\ \text{N/C}\right)(0.750\ \text{m})^2}{\left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)} = 5.57 \times 10^{-8}\ \text{C}$$

But Q is negative since $\vec{\mathbf{E}}$ points inward, so

$$Q = -5.57 \times 10^{-8}\ \text{C} = \boxed{-55.7\ \text{nC}}$$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution, concentric with the spherical shell.

P24.11 The electric flux through each of the surfaces is given by $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$.

Flux through S_1: $\quad \Phi_E = \dfrac{-2Q + Q}{\epsilon_0} = \boxed{-\dfrac{Q}{\epsilon_0}}$

Flux through S_2: $\quad \Phi_E = \dfrac{+Q - Q}{\epsilon_0} = \boxed{0}$

Flux through S_3: $\quad \Phi_E = \dfrac{-2Q + Q - Q}{\epsilon_0} = \boxed{-\dfrac{2Q}{\epsilon_0}}$

Flux through S_4: $\quad \Phi_E = \boxed{0}$

P24.12 The total flux through the surface of the cube is

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 1.92 \times 10^7\ \text{N} \cdot \text{m}^2/\text{C}$$

(a) By symmetry, the flux through each face of the cube is the same.

$$\left(\Phi_E\right)_{\text{one face}} = \frac{1}{6}\Phi_E = \frac{1}{6}\frac{q_{\text{in}}}{\epsilon_0}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

$$(\Phi_E)_{\text{one face}} = \frac{1}{6}\left(\frac{170\times 10^{-6}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)$$
$$= \boxed{3.20\times 10^{6}\ \text{N}\cdot\text{m}^2/\text{C}}$$

(b) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \left(\dfrac{170\times 10^{-6}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right) = \boxed{1.92\times 10^{7}\ \text{N}\cdot\text{m}^2/\text{C}}$

(c) The answer to part (a) would change because the charge could now be at different distances from each face of the cube. The answer to part (b) would be unchanged because the flux through the entire closed surface depends only on the total charge inside the surface.

P24.13 Consider as a gaussian surface a box with horizontal area A, lying between 500 and 600 m elevation. From Gauss's Law,

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}:$$

$$(+120\ \text{N/C})A + (-100\ \text{N/C})A = \frac{\rho A(100\ \text{m})}{\epsilon_0}$$

$$\rho = \frac{(20.0\ \text{N/C})(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{100\ \text{m}} = \boxed{1.77\times 10^{-12}\ \text{C/m}^3}$$

The charge is $\boxed{\text{positive}}$, to produce the net outward flux of electric field.

P24.14 (a) The total electric flux through the surface of the shell is

$$\Phi_{E,\text{ shell}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{12.0\times 10^{-6}}{8.85\times 10^{-12}} = 1.36\times 10^{6}\ \text{N}\cdot\text{m}^2/\text{C}$$
$$= \boxed{1.36\ \text{MN}\cdot\text{m}^2/\text{C}}$$

(b) Through any hemispherical urface of the shell, by symmetry,

$$\Phi_{E,\text{ half shell}} = \frac{1}{2}(1.36\times 10^{6}\ \text{N}\cdot\text{m}^2/\text{C}) = 6.78\times 10^{5}\ \text{N}\cdot\text{m}^2/\text{C}$$
$$= \boxed{678\ \text{kN}\cdot\text{m}^2/\text{C}}$$

(c) $\boxed{\text{No}}$, the same number of field lines will pass through each surface, no matter how the radius changes.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.15 (a) The gaussian surface encloses a charge of +3.00 nC.

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{3.00\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 339\ \text{N}\cdot\text{m}^2/\text{C}$$

(b) $\boxed{\text{No. The electric field is not uniform on this surface. Gauss's law is only practical to use when all portions of the surface satisfy one or more of the conditions listed in Section 24.3.}}$

P24.16 (a) One-half of the total flux created by the charge q goes through the plane. Thus,

$$\Phi_{E,\text{ plane}} = \frac{1}{2}\Phi_{E,\text{ total}} = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \boxed{\frac{q}{2\epsilon_0}}$$

(b) The square looks like an infinite plane to a charge *very close* to the surface. Hence,

$$\Phi_{E,\text{ square}} \approx \Phi_{E,\text{ plane}} = \boxed{\frac{q}{2\epsilon_0}}$$

(c) $\boxed{\text{The plane and the square look the same to the charge.}}$

P24.17 (a) If $R \le d$, the sphere encloses no charge and $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \boxed{0}$.

(b) If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$

so $\Phi_E = \boxed{\dfrac{2\lambda\sqrt{R^2 - d^2}}{\epsilon_0}}$

P24.18 (a) $\boxed{\text{The net flux is zero through the sphere because the number of field lines entering the sphere equals the number of lines leaving the sphere.}}$

(b) The electric field through the curved side of the cylinder is zero because the field lines are parallel to that surface and do not pass through it. The electric field lines pass outward through the ends of the cylinder, so both have a positive flux. Because the field is uniform, the flux is $\pi R^2 E$ for each end.

$\boxed{\text{The net flux is } 2\pi R^2 E \text{ through the cylinder.}}$

(c) The net flux is positive, so the charge in the cylinder is positive. To be a uniform field, the field lines must originate from a plane of charge. $\boxed{\text{The net charge inside the cylinder is positive and is distributed on a plane parallel to the ends of the cylinder.}}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.19 The total charge is $Q-6|q|$. The total outward flux from the cube is $\dfrac{Q-6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \frac{Q-6|q|}{6\epsilon_0}$$

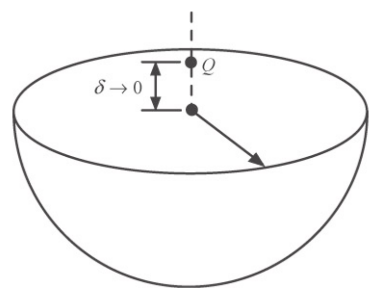
$$(\Phi_E)_{\text{one face}} = \frac{Q-6|q|}{6\epsilon_0} = \frac{(5.00-6.00)\times10^{-6}\ \text{C}\cdot\text{N}\cdot\text{m}^2}{6\times8.85\times10^{-12}\ \text{C}^2}$$

$$= \boxed{-18.8\ \text{kN}\cdot\text{m}^2/\text{C}}$$

P24.20 The total charge is $Q-6|q|$. The total outward flux from the cube is $\dfrac{Q-6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q-6|q|}{6\epsilon_0}}$$

P24.21 (a) With δ very small, all points on the hemisphere are nearly at a distance R from the charge, so the field everywhere on the curved surface is $\dfrac{k_eQ}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:



ANS. FIG. P24.21

$$\Phi_{\text{curved}} = \int\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E_{\text{local}}A_{\text{hemisphere}}$$

$$\Phi_{\text{curved}} = \left(k_e\frac{Q}{R^2}\right)\left(\frac{1}{2}4\pi R^2\right) = \frac{1}{4\pi\epsilon_0}Q(2\pi) = \boxed{\frac{+Q}{2\epsilon_0}}$$

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \quad \text{or} \quad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \boxed{\frac{-Q}{2\epsilon_0}}$$

P24.22 For uniform electric field lines passing through a flat surface, the electric flux is $\Phi_E = EA\cos\theta$, where θ is the angle between the electric field vector and the normal to the surface.

(a) $(\Phi_E)_{\text{face 1}} = \boxed{EA\cos\theta}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

(b) The normal points to the right; the angle between the electric field and the normal is 90° + θ:

$$(\Phi_E)_{\text{face 2}} = EA\cos(90° + \theta) = \boxed{-EA\sin\theta}$$

(c) The normal points downward in the figure, the angle between the electric field and the normal is 180° – θ:

$$(\Phi_E)_{\text{face 3}} = EA\cos(180° - \theta) = \boxed{-EA\cos\theta}$$

(d) The normal points to the left; the angle between the electric field and the normal is 90° – θ:

$$(\Phi_E)_{\text{face 4}} = EA\cos(90° - \theta) = \boxed{EA\sin\theta}$$

(e) The normal points in or out of the page; the angle between the electric field and the normal is 90°:

$$(\Phi_E)_{\text{top or bottom}} = EA\cos(90°) = \boxed{0}$$

(f) $\Phi_E = \sum(\Phi_E)_{\text{faces}} = EA\cos\theta - EA\sin\theta - EA\cos\theta + EA\sin\theta + 0 + 0 = \boxed{0}$

(g) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} \to q_{\text{in}} = \boxed{0}$

Section 24.3 Application of Gauss's Law to Various Charge Distributions

***P24.23** The distance between centers is $2 \times 5.90 \times 10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F = \frac{k_e q_1 q_2}{r^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(46)^2(1.60 \times 10^{-19} \text{ C})^2}{(2 \times 5.90 \times 10^{-15} \text{ m})^2}$$

$$= 3.50 \times 10^3 \text{ N} = \boxed{3.50 \text{ kN}}$$

P24.24 Note that the electric field in each case is directed radially inward, toward the filament. We use $E = \dfrac{2k_e\lambda}{r}$ and substitute numerical values.

(a) At r = 10.0 cm = 0.100 m,

$$E = \frac{2k_e\lambda}{r} = \frac{2(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(90.0 \times 10^{-6} \text{ C/m})}{0.100 \text{ m}}$$

$$= \boxed{16.2 \text{ MN/C}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

(b) At r = 20.0 cm = 0.200 m,

$$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C/m}\right)}{0.200\ \text{m}}$$
$$= \boxed{8.09\ \text{MN/C}}$$

(c) At r = 100 cm = 1.00 m,

$$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C/m}\right)}{1.00\ \text{m}}$$
$$= \boxed{1.62\ \text{MN/C}}$$

P24.25 The charge per unit area of the plastic sheet must be sufficiently large to result in an upward electric force on the Styrofoam that cancels the downward gravitational force:

$$mg = qE = q\left(\frac{\sigma}{2\,\epsilon_0}\right) = q\left(\frac{Q/A}{2\,\epsilon_0}\right)$$

Solving for the charge per unit area gives

$$\frac{Q}{A} = \frac{2\,\epsilon_0\,mg}{q}$$
$$= \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(10.0\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{\left|-0.700\times10^{-6}\ \text{C}\right|}$$
$$= \boxed{2.48\ \mu\text{C/m}^2}$$

P24.26 The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = \dfrac{k_e q}{r^2}$.

$$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(82\times1.60\times10^{-19}\ \text{C}\right)}{\left[(208)^{1/3}\left(1.20\times10^{-15}\ \text{m}\right)\right]^2}$$

$$E = \boxed{2.33\times10^{21}\ \text{N/C}}\ \text{away from the nucleus}$$

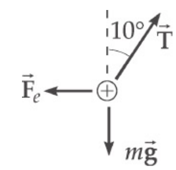
P24.27 For a large uniformly charged sheet, $\vec{\mathbf{E}}$ will be perpendicular to the sheet, and will have a magnitude of

$$E = \frac{\sigma}{2\,\epsilon_0} = 2\pi k_e\sigma$$
$$= (2\pi)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(9.00\times10^{-6}\ \text{C/m}^2\right)$$

so $\qquad \vec{\mathbf{E}} = 5.08\times10^5\ \text{N/C}\ \hat{\mathbf{j}}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.28 Consider two balloons of diameter 0.200 m, each with mass 1.00 g, hanging apart with a 0.050 0 m separation on the ends of strings making angles of 10.0° with the vertical.



ANS. FIG. P24.28

(a) $\sum F_y = T\cos 10° - mg = 0 \Rightarrow T = \dfrac{mg}{\cos 10°}$

$\sum F_x = T\sin 10° - F_e = 0 \Rightarrow F_e = T\sin 10°$

so $F_e = \left(\dfrac{mg}{\cos 10.0°}\right)\sin 10.0° = mg\tan 10,0°$

$= (0.001\ 00\ \text{kg})(9.80\ \text{m/s}^2)\tan 10.0°$

$F_e \approx 2\times 10^{-3}\ \text{N}\ \boxed{\sim 10^{-3}\ \text{N or 1 mN}}$

(b) The charge on each balloon can be found from $F_e = \dfrac{k_e q^2}{r^2}$:

$$q = \sqrt{\frac{F_e r^2}{k_e}} \approx \sqrt{\frac{(2\times 10^{-3}\ \text{N})(0.25\ \text{m})^2}{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}}$$

$\approx 1.2\times 10^{-7}\ \text{C}\ \boxed{\sim 10^{-7}\ \text{C or 100 nC}}$

(c) $$E = \frac{k_e q}{r^2} \approx \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.2\times 10^{-7}\ \text{C})}{(0.25\ \text{m})^2} \approx 1.7\times 10^4\ \text{N/C}$$

$\boxed{\sim 10\,\text{kN/C}}$

(d) The electric flux created by each balloon is

$$\Phi_E = \frac{q}{\epsilon_0} \approx \frac{1.2\times 10^{-7}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.4\times 10^4\ \text{N}\cdot\text{m}^2/\text{C}$$

$\boxed{\sim 10\,\text{kN}\cdot\text{m}^2/\text{C}}$

P24.29 (a) Consider the spherical symmetry of the situation. A gaussian sphere concentric wth the shell, with radius 10.0 cm, encloses 0 charge. Then at the surface of this sphere, inside the charged shell, we have $\vec{\mathbf{E}} = \boxed{0}$.

(b) For a gaussian sphere of radius 20.0 cm, we apply $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \dfrac{q_{in}}{\epsilon_0}$.

The field is radially outward, and $4\pi r^2 E = q/\epsilon_0$:

$$E = \frac{k_e q}{r^2} = \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(32.0\times 10^{-6}\ \text{C})}{(0.200\ \text{m})^2}$$

$= 7.19\times 10^6\ \text{N/C}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

so $\vec{\mathbf{E}} = \boxed{7.19 \text{ MN/C radially outward}}$

P24.30 (a) The charge per unit area of the wall is

$$\sigma = \left(8.60 \times 10^{-6} \text{ C/cm}^2\right)\left(\frac{100 \text{ cm}}{\text{m}}\right)^2 = 8.60 \times 10^{-2} \text{ C/m}^2$$

The electric field at a distance of 2.00 cm is then

$$E = \frac{\sigma}{2 \in_0} = \frac{8.60 \times 10^{-2} \text{ C/m}^2}{2\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)}$$
$$= \boxed{4.86 \times 10^9 \text{N/C away from the wall}}$$

(b) $\boxed{\text{So long as the distance from the wall is small compared to the width and height of the wall, the distance does not affect the field.}}$

P24.31 The approximation in this case is that the filament length is so large when compared to the cylinder length that the "infinite line" of charge can be assumed.

(a) We have

$$E = \frac{2k_e\lambda}{r}$$

where the linear charge density is

$$\lambda = \frac{2.00 \times 10^{-6} \text{ C}}{7.00 \text{ m}} = 2.86 \times 10^{-7} \text{ C/m}$$

so

$$E = \frac{(2)(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C})(2.86 \times 10^{-7} \text{ C/m})}{0.100 \text{ m}}$$
$$= \boxed{51.4 \text{ kN/C radially outward}}$$

(b) We can find the flux by multiplying the field and the lateral surface area of the cylinder:

$$\Phi_E = 2\pi rLE = 2\pi rL\left(\frac{2k_e\lambda}{r}\right) = 4\pi k_e\lambda L$$

so

$$\Phi_E = 4\pi(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.86 \times 10^{-7} \text{ C/m})(0.020\,0 \text{ m})$$
$$= \boxed{6.46 \times 10^2 \text{ N} \cdot \text{m}^2/\text{C}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.32 (a) The area of each face is $A = 1.00\ \text{m}^2$.

For the left face, the angle between the electric field and the normal is 0°:

$$\begin{aligned}(\Phi_E)_{\text{left face}} &= EA\cos\theta = (20.0\ \text{N/C})(1.00\ \text{m}^2)\cos 0° \\ &= 20.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the right face, the angle between the electric field and the normal is 180°:

$$\begin{aligned}(\Phi_E)_{\text{right face}} &= EA\cos\theta = (35.0\ \text{N/C})(1.00\ \text{m}^2)\cos 180° \\ &= -35.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the top face, the angle between the electric field and the normal is 180°:

$$\begin{aligned}(\Phi_E)_{\text{top face}} &= EA\cos\theta = (25.0\ \text{N/C})(1.00\ \text{m}^2)\cos 180° \\ &= -25.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the bottom face, the angle between the electric field and the normal is 0°:

$$\begin{aligned}(\Phi_E)_{\text{bottom face}} &= EA\cos\theta = (15.0\ \text{N/C})(1.00\ \text{m}^2)\cos 0° \\ &= 15.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the front face, the angle between the electric field and the normal is 0°:

$$\begin{aligned}(\Phi_E)_{\text{front face}} &= EA\cos\theta = (20.0\ \text{N/C})(1.00\ \text{m}^2)\cos 0° \\ &= 20.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the back face, the angle between the electric field and the normal is 0°:

$$\begin{aligned}(\Phi_E)_{\text{back face}} &= EA\cos\theta = (20.0\ \text{N/C})(1.00\ \text{m}^2)\cos 0° \\ &= 20.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

The total flux is then

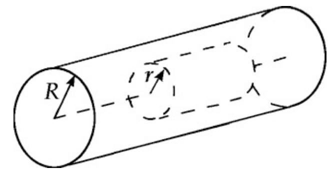
$$\begin{aligned}\Phi_E &= (20.0 - 35.0 - 25.0 + 15.0 + 20.0 + 20.0)\ \text{N}\cdot\text{m}^2/\text{C} \\ &= \boxed{15.0\ \text{N}\cdot\text{m}^2/\text{C}}\end{aligned}$$

(b) $$\begin{aligned}\Phi_E &= \frac{q_{\text{in}}}{\epsilon_0} \to q_{\text{in}} = \epsilon_0\,\Phi_E = (8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(15.0\ \text{N}\cdot\text{m}^2/\text{C}) \\ &= \boxed{1.33\times 10^{-10}\ \text{C}}\end{aligned}$$

(c) $\boxed{\text{No; fields on the faces would not be uniform.}}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.33 If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = EdA\cos 90.0° = 0$. The curved surface has $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.



ANS. FIG. P24.33

Gauss's law, $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \dfrac{q}{\epsilon_0}$, becomes $E\displaystyle\int_{\substack{\text{Curved}\\\text{Surface}}} dA = \dfrac{\rho\pi r^2 L}{\epsilon_0}$.

Now the lateral surface area of the cylinder is $2\pi rL$:

$$E(2\pi r)L = \frac{\rho\pi r^2 L}{\epsilon_0}$$

Thus, $$\vec{\mathbf{E}} = \boxed{\frac{\rho r}{2\,\epsilon_0}\text{ radially away from the cylinder axis}}.$$

P24.34 (a) The electric field is given by

$$E = \frac{2k_e\lambda}{r} = \frac{2k_e(Q/\ell)}{r}$$

Solving for the charge Q gives

$$Q = \frac{Er\ell}{2k_e} = \frac{(3.60\times10^4\text{ N/C})(0.190\text{ m})(2.40\text{ m})}{2(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C})} =$$

$$Q = +9.13\times10^{-7}\text{ C} = \boxed{+913\text{ nC}}$$

(b) Since the charge is uniformly distributed on the surface of the cylindrical shell, a gaussian surface in the shape of a cylinder of 4.00 cm in radius encloses no charge, and $\vec{\mathbf{E}} = \boxed{0}$.

P24.35 (a) At the center of the sphere, the total charge is zero, so

$$E = \frac{k_e Qr}{a^3} = \boxed{0}$$

(b) At a distance of 10.0 cm = 0.100 m from the center,

$$E = \frac{k_e Qr}{a^3} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C})(26.0\times10^{-6}\text{ C})(0.100\text{ m})}{(0.400\text{ m})^3}$$

$$= \boxed{365\text{ kN/C}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

(c) At a distance of 40.0 cm = 0.400 m from the center, all of the charge is enclosed, so

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}\right)\left(26.0\times 10^{-6}\ \text{C}\right)}{(0.400\ \text{m})^2}$$
$$= \boxed{1.46\ \text{MN/C}}$$

(d) At a distance of 60.0 cm = 0.600 m from the center,

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}\right)\left(26.0\times 10^{-6}\ \text{C}\right)}{(0.600\ \text{m})^2}$$
$$= \boxed{649\ \text{kN/C}}$$

The direction for each electric field is $\boxed{\text{radially outward}}$.

P24.36 The volume of the spherical shell is

$$\frac{4}{3}\pi\left[(0.25\ \text{m})^3 - (0.20\ \text{m})^3\right] = 3.19\times 10^{-2}\ \text{m}^3$$

Its charge is

$$\rho V = \left(-1.33\times 10^{-6}\ \text{C/m}^3\right)\left(3.19\times 10^{-2}\ \text{m}^3\right) = -4.25\times 10^{-8}\ \text{C}$$

The net charge inside a sphere containing the proton's path as its equator is

$$-60\times 10^{-9}\ \text{C} - 4.25\times 10^{-8}\ \text{C} = -1.02\times 10^{-7}\ \text{C}$$

The electric field is radially inward with magnitude

$$E = \frac{k_e|q|}{r^2} = \frac{|q|}{\epsilon_0\, 4\pi r^2} = \frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.02\times 10^{-7}\ \text{C}\right)}{(0.250\ \text{m})^2}$$
$$= 1.47\times 10^4\ \text{N/C}$$

For the proton, Newton's second law gives

$$\sum F = ma: \qquad eE = \frac{mv^2}{r}$$

solving for the proton's speed then gives

$$v = \left(\frac{eEr}{m}\right)^{1/2} = \left[\frac{\left(1.60\times 10^{-19}\ \text{C}\right)\left(1.47\times 10^4\ \text{N/C}\right)(0.250\ \text{m})}{1.67\times 10^{-27}\ \text{kg}}\right]^{1/2}$$
$$= \boxed{5.94\times 10^5\ \text{m/s}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

Section 24.4 Conductors in Electrostatic Equilibrium

P24.37 $\oint E\,dA = E(2\pi rl) = \dfrac{q_{\text{in}}}{\epsilon_0}$ $\quad E = \dfrac{q_{\text{in}}/l}{2\pi \epsilon_0 r} = \dfrac{\lambda}{2\pi \epsilon_0 r}$ for the field outside the metal rod.

(a) At r = 3.00 cm, $\quad \vec{\mathbf{E}} = \boxed{0}$

(b) At r = 10.0 cm,

$$\vec{\mathbf{E}} = \frac{30.0\times 10^{-9}\ \text{C}}{2\pi\left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.100\ \text{m})}$$
$$= \boxed{5\,400\ \text{N/C, outward}}$$

(c) At r = 100 cm,

$$\vec{\mathbf{E}} = \frac{30.0\times 10^{-9}\ \text{C}}{2\pi\left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(1.00\ \text{m})}$$
$$= \boxed{540\ \text{N/C, outward}}$$

P24.38 Let's calculate the electric field just outside the surface:

$$E = k_e\frac{q}{r^2} = \left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{40.0\times 10^{-9}\ \text{C}}{(0.15\ \text{m})^2}\right]$$
$$= 1.60\times 10^4\ \text{N} = 16.0\ \text{kN/C}$$

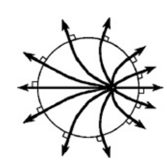
This should be the value of the electric field at the peak of the curve in Figure P24.38. We see, however, that the peak in the figure occurs at about 6.5 kN/C. Therefore, it is not possible that this figure represents the electric field for the given situation.

P24.39 The surface area is $A = 4\pi a^2$. The field is then

$$E = \frac{k_e Q}{a^2} = \frac{Q}{4\pi \epsilon_0 a^2} = \frac{Q}{A\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$$

It is not equal to $\sigma/2\epsilon_0$. At a point just outside, the uniformly charged surface looks just like a uniform flat sheet of charge. The distance to the field point is negligible compared to the radius of curvature of the surface.

P24.40 An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.



ANS. FIG. P24.40

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.41 The fields are equal. The equation $E = \dfrac{\sigma_{\text{conductor}}}{\epsilon_0}$ suggested in the chapter for the field outside the aluminum looks different from the equation $E = \dfrac{\sigma_{\text{insulator}}}{2\epsilon_0}$ for the field around glass. But its charge will spread out to cover both sides of the aluminum plate, so the density is $\sigma_{\text{conductor}} = \dfrac{Q}{2A}$. The glass carries charge only on area A, with $\sigma_{\text{insulator}} = \dfrac{Q}{A}$. The two fields are $\dfrac{Q}{2A\epsilon_0}$, the same in magnitude, and both are perpendicular to the plates, vertically upward if Q is positive.

P24.42 (a) Let a flat box have face area A perpendicular to its thickness dx. The flux at $x = 0.3$ m is into the box is

$$\Phi_E = -EA = -(6\ 000\ \text{N/C}\cdot\text{m}^2)(0.3\ \text{m})^2\ A = -(540\ \text{N/C})\ A$$

The flux at $x = 0.3$ m + dx is out of the box is

$$\begin{aligned}\Phi_E &= +EA = +(6\ 000\ \text{N/C}\cdot\text{m}^2)(0.3\ \text{m} + dx)^2\ A \\ &= +(540\ \text{N/C})\ A + (3\ 600\ \text{N/C}\cdot\text{m})\ dx\ A\end{aligned}$$

(The term in $(dx)^2$ is negligible.) The charge in the box is $\rho A\ dx$ where ρ is the unknown. Applying Gauss's law, $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$, we obtain

$$\begin{aligned}&-(540\ \text{N/C})\ A + (540\ \text{N/C})\ A \\ &\qquad + (3\ 600\ \text{N/C}\cdot\text{m})\ dx\ A = \rho A\ dx/\epsilon_0\end{aligned}$$

Solving for ρ gives

$$\begin{aligned}\rho &= (3\ 600\ \text{N/C}\cdot\text{m})\epsilon_0 \\ &= (3\ 600\ \text{N/C}\cdot\text{m})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) \\ &= \boxed{31.9\ \text{nC/m}^3}\end{aligned}$$

(b) $\boxed{\text{No; then the field would have to be zero.}}$

P24.43 The charge divides equally between the identical spheres, with charge $Q/2$ on each. Then, they repel like point charges at their centers:

$$\begin{aligned}F &= \frac{k_e(Q/2)(Q/2)}{(L+R+R)^2} = \frac{k_eQ^2}{4(L+2R)^2} \\ &= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(60.0\times10^{-6}\ \text{C})^2}{4(2.01\ \text{m})^2} = \boxed{2.00\ \text{N}}\end{aligned}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.44 (a) $E = \dfrac{\sigma}{\epsilon_0}$, so

$$\sigma = \left(8.00 \times 10^4\ \text{N/C}\right)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right)$$
$$= 7.08 \times 10^{-7}\ \text{C/m}^2$$

$\sigma = \boxed{708\ \text{nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma = \dfrac{Q}{A}$, so

$$Q = \sigma A = \left(7.08 \times 10^{-7}\ \text{C/m}^2\right)(0.500\ \text{m})^2$$
$$= 1.77 \times 10^{-7}\ \text{C} = \boxed{177\ \text{nC}}$$

positive on one face and negative on the other.

P24.45 (a) Inside surface: consider a cylindrical gaussian surface of arbitrary length ℓ within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0} \quad \rightarrow \quad 0 = \frac{\left(\lambda + \lambda_{\text{inner}}\right)\ell}{\epsilon_0}$$

so $\lambda_{\text{inner}} = \boxed{-\lambda}$.

(b) Outside surface: consider a cylindrical gaussian surface of arbitrary length ℓ outside the metal. The total charge within the gaussian surface is

$$q_{\text{wire}} + q_{\text{cylinder}} = q_{\text{wire}} + \left(q_{\text{inner surface}} + q_{\text{outer surface}}\right)$$
$$\lambda\ell + 2\lambda\ell = \lambda\ell + \left(-\lambda\ell + \lambda_{\text{outer}}\ell\right) \quad \rightarrow \quad \lambda_{\text{outer}} = \boxed{3\lambda}$$

(c) Gauss's law:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E 2\pi r\ell = \frac{3\lambda\ell}{\epsilon_0} \quad \rightarrow \quad E = 2\frac{3\lambda}{4\pi \epsilon_0 r} = \boxed{6k_e \frac{\lambda}{r}\text{, radially outward}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.46 (a) We ignore "edge" effects and assume that the total charge distributes itself uniformly over each side of the plate, with one half the total charge on each side. The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2}\left(\frac{q}{A}\right) = \frac{1}{2}\frac{(4.00 \times 10^{-8}\ \text{C})}{(0.500\ \text{m})^2} = 8.00 \times 10^{-8}\ \text{C/m}^2$$
$$= \boxed{80.0\ \text{nC/m}^2}$$

(b) Just above the plate,

$$\vec{\mathbf{E}} = \left(\frac{\sigma}{\epsilon_0}\right)\hat{\mathbf{k}} = \left(\frac{8.00 \times 10^{-8}\ \text{C/m}^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)\hat{\mathbf{k}} = \boxed{(9.04\ \text{kN/C})\hat{\mathbf{k}}}$$

(c) Just below the plate, $\vec{\mathbf{E}} = \boxed{(-9.04\ \text{kN/C})\hat{\mathbf{k}}}$.

***P24.47** (a) $\vec{\mathbf{E}} = \boxed{0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.00 \times 10^{-6}\ \text{C})}{(0.030\,0\ \text{m})^2} = 7.99 \times 10^7\ \text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{79.9\ \text{MN/C radially outward}}$$

(c) $\vec{\mathbf{E}} = \boxed{0}$

(d) $$E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.00 \times 10^{-6}\ \text{C})}{(0.070\,0\ \text{m})^2} = 7.34 \times 10^6\ \text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{7.34\ \text{MN/C radially outward}}$$

Additional Problems

P24.48 The electric field makes an angle of 70.0° to the normal. The square has side d = 5.00 cm.

$$\Phi_E = EA\cos\theta = Ed^2\cos\theta$$
$$\rightarrow E = \frac{\Phi_E}{d^2\cos\theta} = \frac{6.00\ \text{N}\cdot\text{m}^2/\text{C}}{(0.150\ \text{m})^2\cos 70.0°} = \boxed{780\ \text{N/C}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.49 The electric field makes an angle of 60.0° with to the normal. The square has side d = 5.00 cm.

$$\Phi_E = EA\cos\theta = \left(3.50\times10^2\ \text{N/C}\right)\left(5.00\times10^{-2}\ \text{m}\right)^2\cos 60.0°$$
$$= \boxed{0.438\ \text{N}\cdot\text{m}^2/\text{C}}$$

P24.50 (a) The field is zero within the metal of the shell. The exterior electric field lines end at equally spaced points on the outer surface because the surface of the conductor is an equipotential surface. The charge on the outer surface is distributed uniformly. Its amount is given by

$$EA = Q/\epsilon_0$$

Solving for the charge Q gives

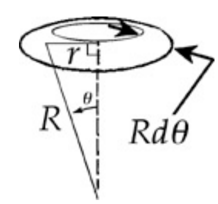
$$Q = -(890\ \text{N/C})\ 4\pi\ (0.750\ \text{m})^2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)$$
$$= -55.7\ \text{nC}$$

$\boxed{\text{The charge on the exterior surface is } -55.7\text{ nC distributed uniformly.}}$

(b) For the net charge of the shell to be zero, the shell must carry +55.7 nC on its inner surface, induced there by –55.7 nC in the cavity within the shell. The charge in the cavity could have any distribution and give any corresponding distribution to the charge on the inner surface of the shell. $\boxed{\text{The charge on the interior surface is +55.7 nC. It can have any distribution.}}$ For example, a large positive charge might be within the cavity close to its topmost point, and a slightly larger negative charge near its easternmost point. The inner surface of the shell would then have plenty of negative charge near the top and even more positive charge centered on the eastern side.

(c) See the comments in (b). $\boxed{\text{The charge within the shell is –55.7 nC. It can have any distribution.}}$ For example, the charge could be distributed on the surface of an insulator of arbitrary shape.

P24.51 The $\vec{\mathbf{E}}$ field due to the point charge is uniform and points radially outward, so $\Phi_E = EA$. The arc length of a small ring-shaped element of the sphere is $ds = Rd\theta$, and its circumference is $2\pi r = 2\pi R\sin\theta$.



ANS. FIG. P24.51

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

The area of the circular cap is

$$A = \int 2\pi r\, ds = \int_0^\theta (2\pi R \sin\theta) R\, d\theta = 2\pi R^2 \int_0^\theta \sin\theta\, d\theta$$
$$A = 2\pi R^2 (-\cos\theta)\Big|_0^\theta = 2\pi R^2 (1-\cos\theta)$$

The flux is then

$$\Phi_E = EA = \left(\frac{1}{4\pi \in_0}\right)\frac{Q}{R^2}\cdot(2\pi R^2)(1-\cos\theta)$$
$$= \left(\frac{Q}{2\in_0}\right)(1-\cos\theta)$$
$$\Phi_E = \left[\frac{50.0\times 10^{-6}\ \text{C}}{2(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\right](1-\cos 45.0^\circ)$$
$$= \boxed{8.27\times 10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

P24.52 Refer to ANS. FIG. P24.51 above. The $\vec{\mathbf{E}}$ field due to the point charge is uniform and points radially outward, so $\Phi_E = EA$. The arc length of a small ring-shaped element of the sphere is $ds = Rd\theta$, and its circumference is $2\pi r = 2\pi R \sin\theta$.

The area of the circular cap is

$$A = \int 2\pi r\, ds = \int_0^\theta (2\pi R \sin\theta) R\, d\theta = 2\pi R^2 \int_0^\theta \sin\theta\, d\theta$$
$$A = 2\pi R^2 (-\cos\theta)\Big|_0^\theta = 2\pi R^2 (1-\cos\theta)$$

The flux is then

$$\Phi_E = EA = \left(\frac{1}{4\pi \in_0}\right)\frac{Q}{R^2}\cdot(2\pi R^2)(1-\cos\theta)$$
$$= \boxed{\left(\frac{Q}{2\in_0}\right)(1-\cos\theta)}$$

***P24.53** Please review Example 23.9 in your textbook, emphasizing the Finalize section. There, it is shown that the electric field due to a nonconducting plane sheet of charge has a constant magnitude given by $E_z = \dfrac{|\sigma_{\text{sheet}}|}{2\in_0}$, where σ_{sheet} is the uniform charge per unit area on the sheet. This field is everywhere perpendicular to the xy plane, is directed away from the sheet if it has a positive charge density, and is directed toward the sheet if it has a negative charge density.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

In this problem, we have two plane sheets of charge, both parallel to the xy plane and separated by a distance of z_0. The upper sheet has charge density $\sigma_{\text{sheet}} = -2\sigma$, while the lower sheet has $\sigma_{\text{sheet}} = +\sigma$. Taking upward as the positive z-direction, the fields due to each of the sheets in the three regions of interest are:

	Lower sheet (at $z = 0$)	**Upper sheet (at $z = z_0$)**
Region	Electric Field	Electric Field
$z < 0$	$E_z = -\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = -\dfrac{\sigma}{2\epsilon_0}$	$E_z = +\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{\epsilon_0}$
$0 < z < z_0$	$E_z = +\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{2\epsilon_0}$	$E_z = +\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{\epsilon_0}$
$z > z_0$	$E_z = +\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{2\epsilon_0}$	$E_z = -\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = -\dfrac{\sigma}{\epsilon_0}$

When both plane sheets of charge are present, the resultant electric field in each region is the vector sum of the fields due to the individual sheets for that region.

(a) For $z < 0$,

$$E_z = E_{z,\,\text{lower}} + E_{z,\,\text{upper}} = -\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{\epsilon_0} = \boxed{+\frac{\sigma}{2\epsilon_0}}$$

(b) For $0 < z < z_0$,

$$E_z = E_{z,\,\text{lower}} + E_{z,\,\text{upper}} = +\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{\epsilon_0} = \boxed{+\frac{3\sigma}{2\epsilon_0}}$$

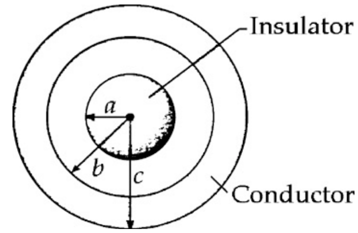
(c) For $z > z_0$,

$$E_z = E_{z,\,\text{lower}} + E_{z,\,\text{upper}} = +\frac{\sigma}{2\epsilon_0} - \frac{\sigma}{\epsilon_0} = \boxed{-\frac{\sigma}{2\epsilon_0}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.54 Choose as each gaussian surface a concentric sphere of radius r. The electric field will be perpendicular to its surface, and will be uniform in strength over its surface. The density of charge in the insulating sphere is

$$\rho = Q / \left(\tfrac{4}{3}\pi a^3\right)$$



ANS. FIG. P24.54

(a) The sphere of radius $r < a$ encloses charge

$$q_{\text{in}} = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = \boxed{Q\left(\frac{r}{R}\right)^3}$$

(b) Applying Gauss's law to this sphere reveals the magnitude of the field at its surface.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}\left(\frac{r}{a}\right)^3 \rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Qr}{a^3} = \boxed{k_e\frac{Qr}{a^3}}$$

(c) For a sphere of radius r with $a < r < b$, the whole insulating sphere is enclosed, so the charge within is Q: $q_{\text{in}} = \boxed{Q}$.

(d) Gauss's law for this sphere becomes:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \boxed{k_e\frac{Q}{r^2}}$$

(e) For $b \le r \le c$, $\boxed{E = 0}$ because there is no electric field inside a conductor.

(f) For $b \le r \le c$, we know $E = 0$. Assume the inner surface of the hollow sphere holds charge Q_{inner}. By Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$0 = \frac{Q + Q_{\text{inner}}}{\epsilon_0} \rightarrow Q_{\text{inner}} = \boxed{-Q}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

(g) The total charge on the hollow sphere is zero; therefore, charge on the outer surface is opposite to that on the inner surface:

$$Q_{\text{outer}} = -Q_{\text{inner}} = \boxed{+Q}$$

(h) A surface of area A holding charge Q has surface charge $\sigma = q/A$. The solid, insulating sphere has small surface charge because its total charge Q is uniformly distributed throughout its volume. The $\boxed{\text{inner surface of radius } b}$ has smaller surface area, and therefore larger surface charge, than the outer surface of radius c.

P24.55 The electric field has these values (consult the solution to P24.54(a)–(e) for details). Suppressing units,

For $0 < r < a$, $$E = k_e \frac{Qr}{a^3} = \left(8.99\times10^9\right)\frac{3.00\times10^{-6}}{(0.050\ 0)^3}r$$

For $a < r < b$, $$E = k_e \frac{Q}{r^2} = \left(8.99\times10^9\right)\frac{3.00\times10^{-6}}{r^2}$$

For b $< r < c$, $E = 0$ (inside conductor)

For $r > c$, from Gauss's law (suppressing units):

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0} \rightarrow E\left(4\pi r^2\right) = \frac{Q+q}{\epsilon_0}$$

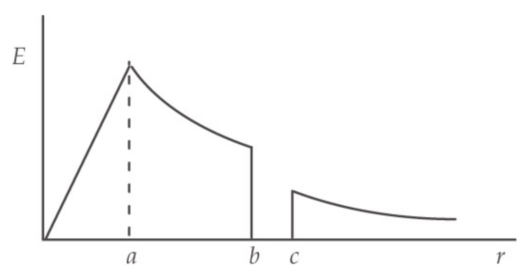
$$\rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{Q+q}{r^2} = k_e\frac{Q+q}{r^2}$$

$$= \left(8.99\times10^9\right)\frac{3.00\times10^{-6} - 1.00\times10^{-6}}{r^2}$$

$$E = \left(8.99\times10^9\right)\frac{2.00\times10^{-6}}{r^2}$$

where r is in meters and E in N/C. The field is radially outward.

The graph appears in ANS. FIG. P24.55 below, with a = 0.050 0 m, b = 0.100 m, and c = 0.150 m.

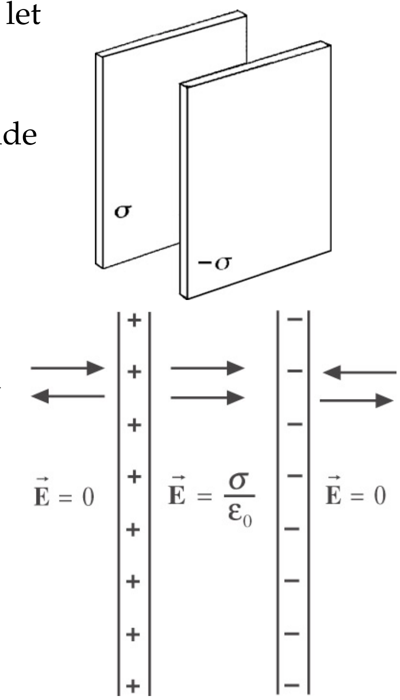


ANS. FIG. P24.55

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.56 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by the textbook equation

$$|E_+| = |E_-| = \frac{\sigma}{2\,\epsilon_0}$$



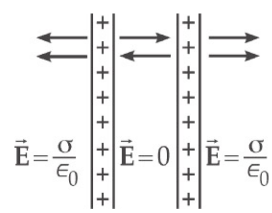
ANS. FIG. P24.56(a-c)

(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\vec{\mathbf{E}} = \boxed{0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\vec{\mathbf{E}} = \boxed{0}$.



ANS. FIG. P24.56(d)

(d) Now, both sheets are positively charged. We find that

(1) To the left of both sheets, both fields are directed toward the left:

$$\vec{\mathbf{E}} = \boxed{2\frac{\sigma}{\epsilon_0} \text{ to the left}}$$

(2) Between the sheets, the fields cancel because they are opposite to each other: $\vec{\mathbf{E}} = \boxed{0}$.

(3) To the right of both sheets, both fields are directed toward the right:

$$\vec{\mathbf{E}} = \boxed{2\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.57 We have

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{q_{\text{in}}}{\epsilon_0}$$

(a) Solving for the charge Q on the insulating sphere, we write, for the region $a < r < b$,

$$\begin{aligned} Q &= \epsilon_0 E\left(4\pi r^2\right) \\ &= \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(-3.60\times10^3\ \text{N/C}\right)4\pi(0.100\ \text{m})^2 \\ &= -4.00\times10^{-9}\ \text{C} = \boxed{-4.00\ \text{nC}} \end{aligned}$$

(b) We take Q' to be the net charge on the hollow sphere. For $r > c$,

$$\begin{aligned} Q + Q' &= \epsilon_0 E\left(4\pi r^2\right) \\ &= \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.00\times10^2\ \text{N/C}\right) \\ &\qquad \times 4\pi(0.500\ \text{m})^2 \\ &= 5.56\times10^{-9}\ \text{C} \end{aligned}$$

so

$$Q' = +9.56\times10^{-9}\ \text{C} = \boxed{+9.56\ \text{nC}}$$

(c) For $b < r < c$, $E = 0$; therefore, $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = q_{\text{in}}/\epsilon_0 = 0$ implies $q_{\text{in}} = Q + Q_1 = 0$, where Q_1 is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00\ \text{nC}}$.

(d) Let Q_2 be the total charge on the outer surface of the hollow sphere; then,

$$Q' = Q_1 + Q_2 \rightarrow Q_2 = Q' - Q_1 = 9.56\ \text{nC} - 4.00\ \text{nC} = \boxed{+5.56\ \text{nC}}$$

P24.58 The charge density is determined by $Q = \frac{4}{3}\pi a^3 \rho$. Solving gives

$$\rho = \frac{3Q}{4\pi a^3}$$

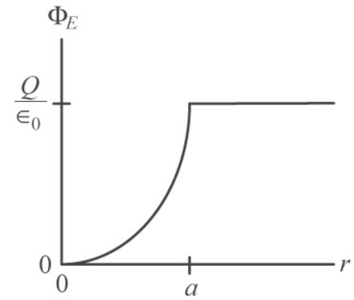
(a) The flux is that created by the enclosed charge within radius r:

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{4\pi r^3 \rho}{3\epsilon_0} = \frac{4\pi r^3 3Q}{3\epsilon_0 4\pi a^3} = \boxed{\frac{Qr^3}{\epsilon_0 a^3}}$$

(b) $\Phi_E = \boxed{\frac{Q}{\epsilon_0}}$. Note that the answers to parts (a) and (b) agree at $r = a$.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

(c) ANS. FIG. P24.58(c) plots the flux vs. r.



ANS. FIG. P24.58(c)

P24.59 Consider the charge distribution to be an unbroken charged spherical shell with uniform charge density σ and a circular disk with charge per area $-\sigma$. The total field is that due to the whole sphere,

$$E_{\text{sphere}} = \frac{Q}{4\pi \epsilon_0 R^2} = \frac{4\pi R^2 \sigma}{4\pi \epsilon_0 R^2} = \frac{\sigma}{\epsilon_0} \text{ outward}$$

plus the field of the disk

$$E_{\text{disk}} = -\frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0} \text{ radially inward}$$

The total field is

$$E_{\text{sphere}} + E_{\text{disk}} = \frac{\sigma}{\epsilon_0} - \frac{\sigma}{2\epsilon_0} = \boxed{\frac{\sigma}{2\epsilon_0} \text{ radially outward}}$$

P24.60 The cylindrical symmetry of the charge distribution implies that the field direction is radially outward perpendicular to the axis. The field strength depends on r but not on the other cylindrical coordinates θ or z. Choose a gaussian cylinder of radius r and length L; the electric field is normal to this surface. Recalling that $k_e = \frac{1}{4\pi\epsilon_0} \rightarrow \frac{1}{\epsilon_0} = 4\pi k_e$, we have $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = 4\pi k_e q_{\text{in}}$.

(a) If $r < a$, we have

$$\Phi_E = 4\pi k_e q_{\text{in}}$$

$$E(2\pi rL) = (4\pi k_e)\lambda L \rightarrow E = \boxed{2k_e \frac{\lambda}{r}, \text{ outward}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

(b) If $a < r < b$, we have

$$\Phi_E = 4\pi k_e q_{\text{in}}$$
$$E(2\pi rL) = (4\pi k_e)\left[\lambda L + \rho\pi\left(r^2 - a^2\right)L\right] \rightarrow$$
$$E = \boxed{\frac{2k_e}{r}\left[\lambda + \rho\pi\left(r^2 - a^2\right)\right], \text{ outward}}$$

(c) If $r > b$, we have

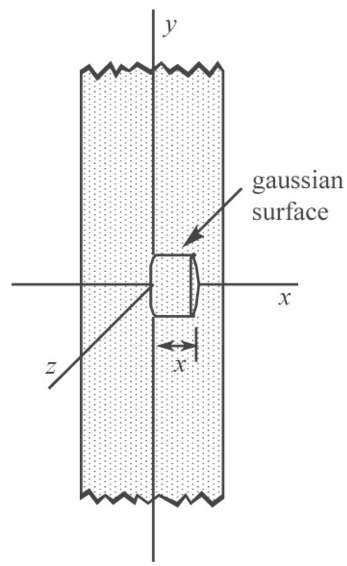
$$\Phi_E = 4\pi k_e q_{\text{in}}$$
$$E(2\pi rL) = (4\pi k_e)\left[\lambda L + \rho\pi\left(b^2 - a^2\right)L\right]$$
$$E = \boxed{\frac{2k_e}{r}\left[\lambda + \rho\pi\left(b^2 - a^2\right)\right], \text{ outward}}$$

Challenge Problems

P24.61 (a) Consider a cylindrical shaped gaussian surface perpendicular to the yz plane with its left end in the yz plane and its right end at distance x, as shown in ANS. FIG. P24.61.

Use Gauss's law: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$

By symmetry, the electric field is zero in the yz plane and is perpendicular to $d\vec{\mathbf{A}}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap.



ANS. FIG. P24.61

For $x > \frac{d}{2}$,

$$dq = \rho\, dV = \rho A dx = CAx^2 dx$$

$$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\epsilon_0}\int dq$$

$$EA = \frac{CA}{\epsilon_0}\int_0^{d/2} x^2\, dx = \frac{1}{3}\left(\frac{CA}{\epsilon_0}\right)\left(\frac{d^3}{8}\right)$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

Then

$$E = \frac{Cd^3}{24\in_0}$$

or $$\boxed{\vec{\mathbf{E}} = \frac{Cd^3}{24\in_0}\hat{\mathbf{i}} \text{ for } x > \frac{d}{2}; \quad \vec{\mathbf{E}} = -\frac{Cd^3}{24\in_0}\hat{\mathbf{i}} \text{ for } x < -\frac{d}{2}}$$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$,

$$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\in_0}\int dq = \frac{CA}{\in_0}\int_0^x x^2\,dx = \frac{CAx^3}{3\in_0}$$

$$\boxed{\vec{\mathbf{E}} = \frac{Cx^3}{3\in_0}\hat{\mathbf{i}} \text{ for } x > 0; \quad \vec{\mathbf{E}} = -\frac{Cx^3}{3\in_0}\hat{\mathbf{i}} \text{ for } x < 0}$$

P24.62 First, consider the field at distance $r < R$ from the center of a uniform sphere of positive charge ($Q = +e$) with radius R. From Gauss's law,

$$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\in_0}$$

$$\left(4\pi r^2\right)E = \frac{q_{\text{in}}}{\in_0} = \frac{1}{\in_0}\rho V = \frac{1}{\in_0}\left(\frac{+e}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi r^3$$

$$\rightarrow \left(4\pi r^2\right)E = \left(\frac{e}{\in_0 R^3}\right)r^3$$

$$\rightarrow E = \left(\frac{e}{4\pi\in_0 R^3}\right)r, \text{ directed outward}$$

(a) The force exerted on a point charge $q = -e$ located at distance r from the center is then

$$F = qE = -e\left(\frac{e}{4\pi\in_0 R^3}\right)r = -\left(\frac{e^2}{4\pi\in_0 R^3}\right)r = \boxed{-Kr}$$

(b) From (a),

$$K = \frac{e^2}{4\pi\in_0 R^3} = \boxed{\frac{k_e e^2}{R^3}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

(c) $F_r = m_e a_r = -\left(\frac{k_e e^2}{R^3}\right) r$, so $a_r = -\left(\frac{k_e e^2}{m_e R^3}\right) r = -\omega^2 r$

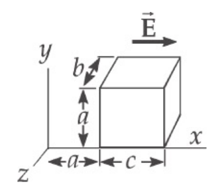
Thus, the motion is simple harmonic with frequency

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{k_e e^2}{m_e R^3}}}$$

(d) $f = 2.47 \times 10^{15}\ \text{Hz} = \frac{1}{2\pi}\sqrt{\frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19}\ \text{C})^2}{(9.11\times 10^{-31}\ \text{kg})R^3}}$

which yields $R^3 = 1.05 \times 10^{-30}\ \text{m}^3$, or $R = \boxed{1.02\times 10^{-10}\ \text{m}}$

P24.63 (a) The electric field throughout the region is directed along x; therefore, $\vec{\mathbf{E}}$ will be perpendicular to normal dA over the four faces of the surface which are perpendicular to the yz plane, and E will be parallel to normal dA over the two faces which are parallel to the yz plane. Therefore,



ANS. FIG. P24.63

$$\Phi_E = -\left(E_x\big|_{x=a}\right)A + \left(E_x\big|_{x=a+c}\right)A$$

$$\Phi_E = -\left(3+2a^2\right)ab + \left[3+2(a+c)^2\right]ab$$

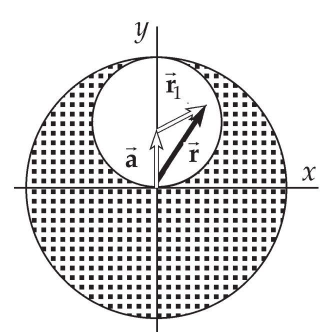
$$\Phi_E = 2abc(2a+c)$$

Substituting the given values for a, b, and c, and noting that the units of electric flux are N · m/C, we find

$$\Phi_E = \boxed{0.269\ \text{N}\cdot\text{m}^2/\text{C}}$$

(b) $\Phi_E = \frac{q_{in}}{\epsilon_0} \rightarrow q_{in} = \epsilon_0 \Phi_E = \boxed{2.38\times 10^{-12}\ \text{C}}$

P24.64 The resultant field within the cavity is the superposition of two fields, one $\vec{\mathbf{E}}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\vec{\mathbf{E}}_-$ due to a sphere of negative charge of radius a centered within the cavity.



ANS. FIG. P24.64

$$\frac{4}{3}\left(\frac{\pi r^3 \rho}{\epsilon_0}\right) = 4\pi r^2 E_+$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

so $$\vec{\mathbf{E}}_+ = \frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}} = \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0}$$

$$-\frac{4}{3}\left(\frac{\pi r_1^3\rho}{\epsilon_0}\right) = 4\pi r_1^2 E_-$$

so $$\vec{\mathbf{E}}_- = \frac{\rho r_1}{3\epsilon_0}(-\hat{\mathbf{r}}_1) = \frac{-\rho}{3\epsilon_0}\vec{\mathbf{r}}_1$$

Substituting $\vec{\mathbf{r}} = \vec{\mathbf{a}} + \vec{\mathbf{r}}_1$ gives

$$\vec{\mathbf{E}}_- = \frac{-\rho(\vec{\mathbf{r}} - \vec{\mathbf{a}})}{3\epsilon_0}$$

Adding the fields gives

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0} - \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0} + \frac{\rho\vec{\mathbf{a}}}{3\epsilon_0} = \frac{\rho\vec{\mathbf{a}}}{3\epsilon_0} = 0\hat{\mathbf{i}} + \frac{\rho a}{3\epsilon_0}\hat{\mathbf{j}}$$

Thus, $\boxed{E_x = 0}$ and $\boxed{E_y = \dfrac{\rho a}{3\epsilon_0}}$ at all points within the cavity.

P24.65 By symmetry, the electric field is radial and, therefore, uniform over the gaussian surface:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{1}{\epsilon_0}\int_0^r \rho\, dV = \frac{1}{\epsilon_0}\int_0^r \left(\frac{a}{r}\right) 4\pi r^2\, dr = \frac{a}{\epsilon_0}\int_0^r 4\pi r\, dr$$

$$E\left(4\pi r^2\right) = \frac{2\pi a}{\epsilon_0} r^2$$

$$E = \frac{a}{2\epsilon_0},\ \boxed{\text{radially outward}}\ \text{(if } a \text{ is positive)}$$

P24.66 (a) We call the constant A', reserving the symbol A to denote area. The whole charge of the ball is

$$Q = \int_{\text{ball}} dQ = \int_{\text{ball}} \rho dV = \int_{r=0}^{R} A' r^2 4\pi r^2 dr = 4\pi A' \left.\frac{r^5}{5}\right|_0^R = \frac{4\pi A' R^5}{5}$$

To find the electric field, consider as gaussian surface a concentric sphere of radius r outside the ball of charge:

In this case, $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \dfrac{Q}{\epsilon_0}$ reads $EA\cos 0° = \dfrac{Q}{\epsilon_0}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

Solving, $E(4\pi r^2) = \dfrac{4\pi A' R^5}{5\epsilon_0}$

and the electric field is $E = \boxed{\dfrac{A'R^5}{5\epsilon_0 r^2}}$

(b) Let the gaussian sphere lie inside the ball of charge:

$$\oint_{\substack{\text{spherical surface,}\\ \text{radius } r}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{spherical volume,}\\ \text{radius } r}} dQ / \epsilon_0$$

Now the integrals become

$$E(\cos 0)\oint dA = \int \frac{\rho dV}{\epsilon_0} \quad \text{or} \quad EA = \int_0^r \frac{A' r^2 \left(4\pi r^2\right) dr}{\epsilon_0}$$

Performing the integration,

$$E(4\pi r^2) = \left(\frac{A' 4\pi}{\epsilon_0}\right)\left(\frac{r^5}{5}\right)\Bigg|_0^r = \frac{A' 4\pi r^5}{5\epsilon_0}$$

and the field is $E = \boxed{\dfrac{A' r^3}{5\epsilon_0}}$

P24.67 In this case the charge density is *not uniform*, and Gauss's law is written as $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \dfrac{1}{\epsilon_0}\int \rho\, dV$. We use a gaussian surface which is a cylinder of radius *r*, length ℓ, and is coaxial with the charge distribution.

(a) When $r < R$, this becomes $E\left(2\pi r \ell\right) = \dfrac{\rho_0}{\epsilon_0}\displaystyle\int_0^r \left(a - \frac{r}{b}\right) dV$. The element of volume is a cylindrical shell of radius *r*, length ℓ, and thickness *dr* so that $dV = 2\pi r \ell dr$.

$$E\left(2\pi r\ell\right) = \left(\frac{2\pi r^2 \ell \rho_0}{\epsilon_0}\right)\left(\frac{a}{2} - \frac{r}{3b}\right) \text{ so inside the cylinder,}$$

$$E = \boxed{\frac{\rho_0 r}{2\epsilon_0}\left(a - \frac{2r}{3b}\right)}$$

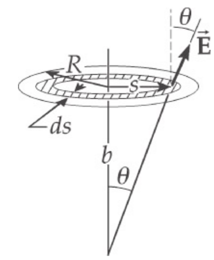
(b) When $r > R$, Gauss's law becomes $E\left(2\pi r\ell\right) = \dfrac{\rho_0}{\epsilon_0}\displaystyle\int_0^R \left(a - \frac{r}{b}\right)\left(2\pi r \ell dr\right)$

or outside the cylinder, $E = \boxed{\dfrac{\rho_0 R^2}{2\epsilon_0 r}\left(a - \dfrac{2R}{3b}\right)}$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.68 The total flux through a surface enclosing the charge Q is $\frac{Q}{\epsilon_0}$. The flux through the disk is

$$\Phi_{\text{disk}} = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



ANS. FIG. P24.68

where the integration covers the area of the disk. We must evaluate this integral and set it equal to $\frac{1}{4}\frac{Q}{\epsilon_0}$ to find how b and R are related. In the figure, take $d\vec{\mathbf{A}}$ to be the area of an annular ring of radius s and width ds. The flux through $d\vec{\mathbf{A}}$ is $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EdA\cos\theta = E(2\pi s ds)\cos\theta$.

The magnitude of the electric field has the same value at all points within the annular ring,

$$E = \frac{1}{4\pi \epsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi \epsilon_0}\frac{Q}{s^2+b^2} \quad \text{and} \quad \cos\theta = \frac{b}{r} = \frac{b}{\left(s^2+b^2\right)^{1/2}}$$

Integrating from $s = 0$ to $s = R$ to get the flux through the entire disk,

$$\Phi_{E,\text{ disk}} = \frac{Qb}{2\epsilon_0}\int_0^R \frac{s ds}{\left(s^2+b^2\right)^{3/2}} = \frac{Qb}{2\epsilon_0}\left[-\left(s^2+b^2\right)^{-1/2}\right]\Bigg|_0^R$$

$$= \frac{Q}{2\epsilon_0}\left[1 - \frac{b}{\left(R^2+b^2\right)^{1/2}}\right]$$

The flux through the disk equals $\frac{Q}{4\epsilon_0}$ provided that $\frac{b}{\left(R^2+b^2\right)^{1/2}} = \frac{1}{2}$.

This is satisfied if $\boxed{R = \sqrt{3}b}$.

P24.69 (a) The slab has left-to-right symmetry, so its field must be equal in strength at x and at $-x$. The field points everywhere away from the central plane. Take as gaussian surface a rectangular box of thickness $2x$ and height and width L, centered on the $x = 0$ plane. The gaussian surface, shown shaded in the second panel of ANS. FIG. P24.69, lies inside the slab. The charge the surface contains is $\rho V = \rho(2xL^2)$. The total flux leaving it is EL^2 through the right face, EL^2 through the left face, and zero through each of the other four sides.

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

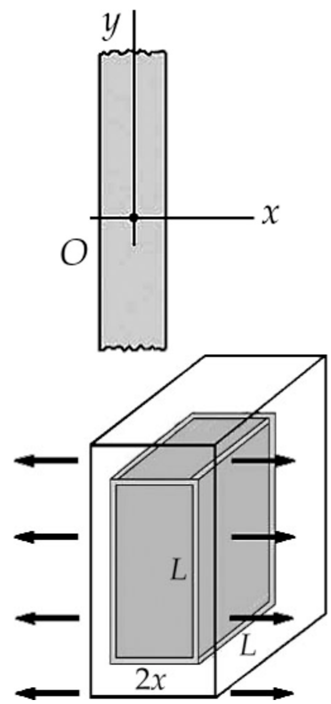
Thus Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$$

becomes

$$2EL^2 = \frac{\rho 2xL^2}{\epsilon_0}$$

so the field is $\quad E = \boxed{\dfrac{\rho x}{\epsilon_0}}$



ANS. FIG. P24.69

(b) The electron experiences a force opposite to $\vec{\mathbf{E}}$. When displaced to $x > 0$, it experiences a restoring force to the left. For the electron, Newton's second law gives

$$\sum \vec{\mathbf{F}} = m_e \vec{\mathbf{a}}:$$

$$q\vec{\mathbf{E}} = m_e \vec{\mathbf{a}} \quad \text{or} \quad \frac{-e\rho x\hat{\mathbf{i}}}{\epsilon_0} = m_e \vec{\mathbf{a}}$$

Solving for the acceleration,

$$\vec{\mathbf{a}} = -\left(\frac{e\rho}{m_e \epsilon_0}\right)x\hat{\mathbf{i}} \quad \text{or} \quad \vec{\mathbf{a}} = -\omega^2 x\hat{\mathbf{i}}$$

That is, its acceleration is proportional to its displacement and oppositely directed, as is required for simple harmonic motion.

Solving for the frequency, $\omega^2 = \dfrac{e\rho}{m_e \epsilon_0}$ and

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{e\rho}{m_e \epsilon_0}}}$$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P24.2 $355\ \text{kN}\cdot\text{m}^2/\text{C}$

P24.4 (a) $-2.34\ \text{kN}\cdot\text{m}^2/\text{C}$; (b) $+2.34\ \text{kN}\cdot\text{m}^2/\text{C}$; (c) 0

P24.6 $chw^2/2$

P24.8 $-226\ \text{N}\cdot\text{m}^2/\text{C}$

P24.10 (a) –55.7 nC; (b) negative, spherically symmetric

P24.12 (a) $3.20\times 10^6\ \text{N}\cdot\text{m}^2/\text{C}$; (b) $1.92\times 10^7\ \text{N}\cdot\text{m}^2/\text{C}$; (c) The answer to part (a) would change because the charge could now be at different distances from each face of the cube. The answer to part (b) would be unchanged because the flux through the entire closed surface depends only on the total charge inside the surface.

P24.14 (a) $1.36\ \text{MN}\cdot\text{m}^2/\text{C}$; (b) $678\ \text{kN}\cdot\text{m}^2/\text{C}$; (c) no

P24.16 (a) $\dfrac{q}{2\in_0}$; (b) $\dfrac{q}{2\in_0}$; (c) The plane and the square look the same to the charge.

P24.18 (a) The net flux is zero through the sphere because the number of field lines entering the sphere equals the number of lines leaving the sphere; (b) The net flux is $2\pi R^2E$ through the cylinder; (c) The net charge inside the cylinder is positive and is distributed on a plane parallel to the ends of the cylinder.

P24.20 $\dfrac{Q-6|q|}{6\in_0}$

P24.22 (a) $EA\cos\theta$; (b) $-EA\sin\theta$; (c) $-EA\cos\theta$; (d) $EA\sin\theta$; (e) 0; (f) 0; (g) 0

P24.24 (a) 16.2 MN/C; (b) 8.09 MN/C; (c) 1.62 MN/C

P24.26 2.33×10^{21} N/C

P24.28 (a) $\sim 10^{-3}$ N or 1 mN; (b) $\sim 10^{-7}$ C or 100 nC; (c) ~10 kN/C; (d) $\sim 10\ \text{kN}\cdot\text{m}^2/\text{C}$

P24.30 (a) 4.86×10^9 N/C away from the wall; (b) So long as the distance from the wall is small compared to the width and height of the wall, the distance does not affect the field.

P24.32 (a) $15.0\ \text{N}\cdot\text{m}^2/\text{C}$; (b) 1.33×10^{-10} C; (c) No; fields on the faces would not be uniform.

P24.34 (a) +913 nC; (b) 0

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.

P24.36 5.94×10^5 m/s

P24.38 The electric field just outside the surface occurs at 16.0 kN/C. The peak in the figure occurs at about 6.5 kN/C. Therefore, it is not possible that this figure represents the electric field for the given situation.

P24.40 See ANS. FIG. P24.40.

P24.42 (a) 31.9 nC/m^3; (b) No; then the field would have to be zero.

P24.44 (a) 708 nC/m^2; (b) 177 nC

P24.46 (a) 80.0 nC/m^2; (b) $(9.04 \text{ kN/C})\hat{\mathbf{k}}$; (c) $(-9.04 \text{ kN/C})\hat{\mathbf{k}}$

P24.48 780 N/C

P24.50 (a) The charge on the exterior surface is –55.7 nC distributed uniformly; (b) The charge on the interior surface is +55.7 nC. It can have any distribution; (c) The charge within the shell is –55.7 nC. It can have any distribution.

P24.52 $\dfrac{Q}{2\epsilon_0}(1-\cos\theta)$

P24.54 (a) $Q\left(\dfrac{r}{R}\right)^3$; (b) $k_e\dfrac{Qr}{a^3}$; (c) Q; (d) $k_e\dfrac{Q}{r^2}$; (e) $E = 0$; (f) $-Q$; (g) $+Q$; (h) inner surface of radius b

P24.56 (a) 0; (b) $\dfrac{\sigma}{\epsilon_0}$ to the right; (c) 0; (d) (1) $2\dfrac{\sigma}{\epsilon_0}$ to the left; (2) 0; (3) $2\dfrac{\sigma}{\epsilon_0}$ to the right

P24.58 (a) $\dfrac{Qr^3}{\epsilon_0 a^3}$; (b) $\dfrac{Q}{\epsilon_0}$; (c) See ANS. FIG. P24.58(c).

P24.60 (a) $2k_e\dfrac{\lambda}{r}$, outward; (b) $\dfrac{2k_e}{r}\left[\lambda+\rho\pi\left(r^2-a^2\right)\right]$, outward; (c) $\dfrac{2k_e}{r}\left[\lambda+\rho\pi\left(b^2-a^2\right)\right]$, outward

P24.62 (a) $-Kr$; (b) $\dfrac{k_e e^2}{R^3}$; (c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}$; (d) 1.02×10^{-10} m

P24.64 $E_x = 0$ and $E_y = \dfrac{\rho a}{3\epsilon_0}$

P24.66 (a) $AR^5/5\epsilon_0 r^2$; (b) $AR^5/5\epsilon_0$

P24.68 $R = \sqrt{3}b$

© 2014 Cengage Learning. All Rights Reserved. May not be scanned, copied or duplicated, or posted to a publicly accessible website, in whole or in part.