25

Electric Potential

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ25.1 Answer (b). Taken without reference to any other point, the potential could have any value.

OQ25.2 Answer (d). The potential is decreasing toward the bottom of the page, so the electric field is downward.

OQ25.3 **(i)** Answer (c). The two spheres come to the same potential, so Q/R is the same for both. If charge q moves from A to B, we find the charge on B:

$$\frac{Q_A}{R_A} = \frac{Q_B}{R_B} \rightarrow \frac{450\text{ nC} - q}{1.00\text{ cm}} = \frac{q}{2.00\text{ cm}} \rightarrow q = \frac{900\text{ nC}}{3} = 300\text{ nC}$$

Sphere A has charge 450 nC – 300 nC = 150 nC.

(ii) Answer (a). Contact between conductors allows all charge to flow to the exterior surface of sphere B.

OQ25.4 Answer (d).

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{\left(1.90\times 10^2\text{ V} - 1.20\times 10^2\text{ V}\right)}{(5.00\text{ m} - 3.00\text{ m})} = -35.0\text{ N/C}$$

OQ25.5 Ranking a > b = d > c. The potential energy of a system of two charges is $U = k_e q_1 q_2/r$. The potential energies are: (a) $U = 2k_e Q^2/r$, (b) $U = k_e Q^2/r$, (c) $U = -k_e Q^2/2r$, (d) $U = k_e Q^2/r$.

OQ25.6 (**i**) Answer (a). The particle feels an electric force in the negative x direction. An outside agent pushes it uphill against this force, increasing the potential energy.

(**ii**) Answer (c). The potential decreases in the direction of the electric field.

OQ25.7 Ranking $D > C > B > A$. Let L be length of a side of the square. The potentials are:

$$V_A = \frac{k_e Q}{L} + \frac{2k_e Q}{\sqrt{2}L} = \left(1+\sqrt{2}\right)\frac{k_e Q}{L}$$

$$V_B = \frac{2k_e Q}{L} + \frac{k_e Q}{\sqrt{2}L} = \left(2+\frac{1}{\sqrt{2}}\right)\frac{k_e Q}{L}$$

$$V_C = \frac{k_e Q}{\sqrt{2}L/2} + \frac{2k_e Q}{\sqrt{2}L/2} = 3\sqrt{2}\frac{k_e Q}{L}$$

$$V_D = \frac{k_e Q}{L/2} + \frac{2k_e Q}{L/2} = 6\frac{k_e Q}{L}$$

OQ25.8 Answer (a). The change in kinetic energy is the negative of the change in electric potential energy:

$$\Delta K = -q\Delta V = -(-e)V = e\left(1.00\times 10^4\text{ V}\right) = 1.00\times 10^4\text{ eV}$$

OQ25.9 Ranking c > a > d > b. We add the electric potential energies of all possible pairs. They are:

(a) $3\frac{k_e Q^2}{d}$

(b) $-2\frac{k_e Q^2}{d} + \frac{k_e Q^2}{d} = -\frac{k_e Q^2}{d}$

(c) $4\frac{k_e Q^2}{d} + 2\frac{k_e Q^2}{\sqrt{2}d} = \left(4+\sqrt{2}\right)\frac{k_e Q^2}{d}$

(d) $2\frac{k_e Q^2}{d} + \frac{k_e Q^2}{\sqrt{2}d} - 2\frac{k_e Q^2}{d} - \frac{k_e Q^2}{\sqrt{2}d} = 0$

OQ25.10 Answer (b). All charges are the same distance from the center. The potentials from the +1.50-μC, –1.00-μC, and –0.500-μC charges cancel.

OQ25.11 Answer (b). The work done on the proton equals the negative of the change in electric potential energy:

$$W = -q\Delta V \rightarrow q\Delta V = -W = -qEs\cos\theta$$
$$= -e\left(8.50\times 10^2 \text{ N/C}\right)(2.50 \text{ m})(1) = -3.40\times 10^{-16} \text{ J}$$

OQ25.12 (**i**) Answer (b). At points off the x axis the electric field has a nonzero y component. At points on the negative x axis the field is to the right and positive. At points to the right of x = 0.500 m the field is to the left and nonzero. The field is zero at one point between x = 0.250 m and x = 0.500 m.

(**ii**) Answer (c). The electric potential is negative at this and at all points because both charges are negative.

(**iii**) Answer (d). The potential cannot be zero at a finite distance because both charges are negative.

OQ25.13 Answer (b). The same charges at the same distance away create the same contribution to the total potential.

OQ25.14 The ranking is e > d > a = c > b. The change in kinetic energy is the negative of the change in electric potential energy, so we work out $-q\Delta V = -q\left(V_f - V_i\right)$ in each case.

(a) $-(-e)(60\text{ V} - 40\text{ V}) = +20\text{ eV}$

(b) $-(-e)(20\text{ V} - 40\text{ V}) = -20\text{ eV}$

(c) $-(e)(20\text{ V} - 40\text{ V}) = +20\text{ eV}$

(d) $-(e)(10\text{ V} - 40\text{ V}) = +30\text{ eV}$

(e) $-(-2e)(60\text{ V} - 40\text{ V}) = +40\text{ eV}$

OQ25.15 Answer (b). The change in kinetic energy is the negative of the change in electric potential energy:

$$\Delta K = -q\Delta V \rightarrow K_B - K_A = q\left(V_A - V_B\right)$$

$$\frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 + (2e)\left(V_A - V_B\right)$$

Solving for the speed gives

$$v_B = \sqrt{v_A^2 + \frac{4e\left(V_A - V_B\right)}{m}}$$
$$= \sqrt{\left(6.20\times 10^5 \text{m/s}\right)^2 + \frac{4\left(1.60\times 10^{-19}\text{C}\right)\left(1.50\times 10^3\text{V} - 4.00\times 10^3\text{V}\right)}{6.63\times 10^{-27}\text{kg}}}$$
$$= 3.78\times 10^5 \text{ m/s}$$

ANSWERS TO CONCEPTUAL QUESTIONS

CQ25.1 The main factor is the radius of the dome. One often overlooked aspect is also the humidity of the air—drier air has a larger dielectric breakdown strength, resulting in a higher attainable -electric potential. If other grounded objects are nearby, the maximum potential might be reduced.

CQ25.2 (a) The proton accelerates in the direction of the electric field, (b) its kinetic energy increases as (c) the electric potential energy of the system decreases.

CQ25.3 To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system by an outside agent. Hence energy is stored, and potential energy is positive. As charges with opposite signs move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.

CQ25.4 (a) The grounding wire can be touched equally well to any point on the sphere. Electrons will drain away into the ground.

(b) The sphere will be left positively charged. The ground, wire, and sphere are all conducting. They together form an equipotential volume at zero volts during the contact. However close the grounding wire is to the negative charge, electrons have no difficulty in moving within the metal through the grounding wire to ground. The ground can act as an infinite source or sink of electrons. In this case, it is an electron sink.

CQ25.5 When one object B with electric charge is immersed in the electric field of another charge or charges A, the system possesses electric potential energy. The energy can be measured by seeing how much work the field does on the charge B as it moves to a reference location. We choose not to visualize A's effect on B as an action-at-a-distance, but as the result of a two-step process: Charge A creates electric potential throughout the surrounding space. Then the potential acts on B to inject the system with energy.

CQ25.6 (a) The electric field is cylindrically radial. The equipotential surfaces are nesting coaxial cylinders around an infinite line of charge.

(b) The electric field is spherically radial. The equipotential surfaces are nesting concentric spheres around a uniformly charged sphere.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 25.1 Electric Potential and Potential Difference

Section 25.2 Potential Difference in a Uniform Electric Field

***P25.1** (a) From Equation 25.6,

$$E = \frac{|\Delta V|}{d} = \frac{600 \text{ J/C}}{5.33 \times 10^{-3} \text{ m}} = \boxed{1.13 \times 10^{5} \text{ N/C}}$$

(b) The force on an electron is given by

$$F = |q|E = \left(1.60 \times 10^{-19} \text{ C}\right)\left(1.13 \times 10^{5} \text{ N/C}\right) = \boxed{1.80 \times 10^{-14} \text{ N}}$$

(c) Because the electron is repelled by the negative plate, the force used to move the electron must be applied in the direction of the electron's displacement. The work done to move the electron is

$$W = F \cdot s \cos\theta = \left(1.80 \times 10^{-14} \text{ N}\right)\left[(5.33 - 2.00) \times 10^{-3} \text{ m}\right]\cos 0°$$
$$= \boxed{4.37 \times 10^{-17} \text{ J}}$$

***P25.2** (a) We follow the path from (0, 0) to (20.0 cm, 0) to (20.0 cm, 50.0 cm).

$\Delta U = -$ (work done)

$\Delta U = -$ [work from origin to (20.0 cm, 0)]

$-$ [work from (20.0 cm, 0) to (20.0 cm, 50.0 cm)]

Note that the last term is equal to 0 because the force is perpendicular to the displacement.

$$\Delta U = -(qE_x)\Delta x = -\left(12.0 \times 10^{-6} \text{ C}\right)(250 \text{ V/m})(0.200 \text{ m})$$
$$= \boxed{-6.00 \times 10^{-4} \text{ J}}$$

(b) $$\Delta V = \frac{\Delta U}{q} = -\frac{6.00 \times 10^{-4} \text{ J}}{12.0 \times 10^{-6} \text{ C}} = -50.0 \text{ J/C} = \boxed{-50.0 \text{ V}}$$

P25.3 (a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$K_i + U_i = K_f + U_f: \qquad 0 + qV = \frac{1}{2}mv_p^2 + 0$$

$$\left(1.60 \times 10^{-19} \text{ C}\right)(120 \text{ V})\left(\frac{1 \text{ J}}{1 \text{ V} \cdot \text{C}}\right) = \frac{1}{2}\left(1.67 \times 10^{-27} \text{ kg}\right)v_p^2$$

$$v_p = \boxed{1.52 \times 10^{5} \text{ m/s}}$$

(b) The electron will gain speed in moving the other way, from $V_i = 0$ to $V_f = 120$ V: $K_i + U_i = K_f + U_f$

$$0 + 0 = \frac{1}{2}mv_e^2 + qV$$

$$0 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)v_e^2 + \left(-1.60\times10^{-19}\text{ C}\right)(120\text{ J/C})$$

$$v_e = \boxed{6.49\times10^{6}\text{ m/s}}$$

P25.4 The potential difference is

$$\Delta V = V_f - V_i = -5.00\text{ V} - 9.00\text{ V} = -14.0\text{ V}$$

and the total charge to be moved is

$$Q = -N_A e = -\left(6.02\times10^{23}\right)\left(1.60\times10^{-19}\text{ C}\right) = -9.63\times10^{4}\text{ C}$$

Now, from $\Delta V = \dfrac{W}{Q}$, we obtain

$$W = Q\Delta V = (-9.63\times10^{4}\text{ C})(-14.0\text{ J/C}) = \boxed{1.35\text{ MJ}}$$

P25.5 The electric field is uniform. By Equation 25.3,

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\int_A^C \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} - \int_C^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

$$V_B - V_A = (-E\cos 180°)\int_{-0.300}^{0.500} dy - (E\cos 90.0°)\int_{-0.200}^{0.400} dx$$

$$V_B - V_A = (325\text{ V/m})(0.800\text{ m}) = \boxed{+260\text{ V}}$$

P25.6 Assume the opposite. Then at some point A on some equipotential surface the electric field has a nonzero component E_p in the plane of the surface. Let a test charge start from point A and move some distance on the surface in the direction of the field component. Then $\Delta V = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ is nonzero. The electric potential charges across the surface and it is not an equipotential surface. The contradiction shows that our assumption is false, that $E_p = 0$, and that the field is perpendicular to the equipotential surface.

P25.7 We use the energy version of the isolated system model to equate the energy of the electron-field system when the electron is at $x = 0$ to the energy when the electron is at $x = 2.00$ cm. The unknown will be the difference in potential $V_f - V_i$. Thus, $K_i + U_i = K_f + U_f$ becomes

$$\frac{1}{2}mv_i^2 + qV_i = \frac{1}{2}mv_f^2 + qV_f$$

or $$\frac{1}{2}m\left(v_i^2 - v_f^2\right) = q\left(V_f - V_i\right),$$

so $$V_f - V_i = \Delta V = \frac{m\left(v_i^2 - v_f^2\right)}{2q}.$$

(a) Noting that the electron's charge is negative, and evaluating the potential difference, we have

$$\Delta V = \frac{\left(9.11\times 10^{-31}\text{ kg}\right)\left[(3.70\times 10^{6}\text{ m/s})^2 - (1.40\times 10^{5}\text{ m/s})^2\right]}{2\left(-1.60\times 10^{-19}\text{ C}\right)}$$

$$= \boxed{-38.9\text{ V}}$$

(b) The negative sign means that the 2.00-cm location is lower in potential than the origin:

$$\boxed{\text{The origin is at the higher potential.}}$$

P25.8 (a) The electron-electric field is an isolated system:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_e v_i^2 + (-e)V_i = 0 + (-e)V_f$$

$$e\left(V_f - V_i\right) = -\frac{1}{2}m_e v_i^2$$

The potential difference is then

$$\Delta V_e = -\frac{m_e v_i^2}{2e} = -\frac{\left(9.11\times 10^{-31}\text{ kg}\right)\left(2.85\times 10^{7}\text{ m/s}\right)^2}{2\left(1.60\times 10^{-19}\text{ C}\right)}$$

$$= -2.31\times 10^{3}\text{ V} = \boxed{-2.31\text{ kV}}$$

(b) From (a), we see that the stopping potential is proportional to the kinetic energy of the particle.

Because a proton is more massive than an electron, a proton traveling at the same speed as an electron has more initial kinetic energy and requires a greater magnitude stopping potential.

(c) The proton-electric field is an isolated system:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_p v_i^2 + eV_i = 0 + eV_f$$

$$e\left(V_f - V_i\right) = \frac{1}{2}m_p v_i^2$$

The potential difference is

$$\Delta V_p = \frac{m_p v_i^2}{2e}$$

Therefore, from (a),

$$\frac{\Delta V_p}{\Delta V_e} = \frac{m_p v_i^2/2e}{-m_e v_i^2/2e} \quad \rightarrow \quad \boxed{\Delta V_p/\Delta V_e = -m_p/m_e}$$

P25.9 Arbitrarily take $V = 0$ at point P. Then the potential at the original position of the charge is (by Equation 25.3)

$$\Delta V = V - 0 = V = -\vec{\mathbf{E}} \cdot \vec{\mathbf{s}} = -EL\cos\theta \quad \text{(relative to } P\text{)}$$

At the final point a,

$$V = -EL \qquad \text{(relative to } P\text{)}$$

Because the table is frictionless and the particle-field system is isolated, we have

$$(K + U)_i = (K + U)_f$$

or

$$0 - qEL\cos\theta = \frac{1}{2}mv^2 - qEL$$

solving for the speed gives

$$v = \sqrt{\frac{2qEL(1-\cos\theta)}{m}}$$

$$= \sqrt{\frac{2\left(2.00\times10^{-6}\text{ C}\right)(300\text{ N/C})(1.50\text{ m})(1-\cos 60.0°)}{0.010\,0\text{ kg}}}$$

$$= \boxed{0.300\text{ m/s}}$$

P25.10 (a) The system consisting of the mass-spring-electric field is $\boxed{\text{isolated.}}$

(b) The system has both $\boxed{\text{electric potential energy and elastic potential energy}}$: U_e and U_{sp}.

(c) Taking the electric potential to be zero at the initial configuration, after the block has stretched the spring a distance x, the final electric potential is (from equation 25.3)

$$\Delta V = V = -\vec{\mathbf{E}}\cdot\vec{\mathbf{s}} = -Ex$$

By energy conservation within the system,

$$\left(K + U_{\text{sp}} + U_e\right)_i = \left(K + U_{\text{sp}} + U_e\right)_f$$

$$0 + 0 + 0 = 0 + \frac{1}{2}kx^2 + QV$$

$$0 = \frac{1}{2}kx^2 + Q(-Ex) \quad \to \quad x = \boxed{\frac{2QE}{k}}$$

(d) $\boxed{\text{Particle in equilibrium}}$

(e) $\sum F = 0 \quad \to \quad -kx_0 + QE = 0 \quad \to \quad x_0 = \boxed{\dfrac{QE}{k}}$

(f) The particle is no longer in equilibrium; therefore, the force equation becomes

$$\sum F = ma \quad \to \quad -kx + QE = m\frac{d^2x}{dt^2}$$

$$-k\left(x - \frac{QE}{k}\right) = m\frac{d^2x}{dt^2}$$

Defining $x' = x - x_0$, we have $\dfrac{d^2x'}{dt^2} = \dfrac{d^2(x - x_0)}{dt^2} = \dfrac{d^2x}{dt^2}$.

Substitute $x' = x - x_0$ into the force equation:

$$-k\left(x - \frac{QE}{k}\right) = m\frac{d^2x}{dt^2} \quad \to \quad -kx' = m\frac{d^2x'}{dt^2}$$

$$\to \quad \boxed{\frac{d^2x'}{dt^2} = -\frac{kx'}{m}}$$

(g) The result of part (f) is the equation for simple harmonic motion $a_{x'} = -\omega^2 x'$ with

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \quad \to \quad T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{k}}}$$

(h) The period does not depend on the electric field. The electric field just shifts the equilibrium point for the spring, just like a gravitational field does for an object hanging from a vertical spring.

P25.11 Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length, $V = -Ed$ and $U_e = -\lambda LEd$.

(a) The rod-field system is isolated:

$$K_i + U_i = K_f + U_f$$

$$0 + 0 = \frac{1}{2}m_{\text{rod}}v^2 - qV$$

$$0 = \frac{1}{2}\mu L v^2 - \lambda LEd$$

$$\frac{1}{2}\mu L v^2 = \lambda LEd$$

Solving for the speed gives

$$v = \sqrt{\frac{2\lambda Ed}{\mu}} = \sqrt{\frac{2(40.0\times10^{-6}\text{ C/m})(100\text{ N/C})(2.00\text{ m})}{(0.100\text{ kg/m})}}$$

$$= \boxed{0.400\text{ m/s}}$$

(b) $\boxed{\text{The same.}}$ Each bit of the rod feels a force of the same size as before.

Section 25.3 Electric Potential and Potential Energy Due to Point Charges

P25.12 (a) At a distance of 0.250 cm from an electron, the electric potential is

$$V = k_e\frac{q}{r} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-1.60\times10^{-19}\text{ C}}{0.250\times10^{-2}\text{ m}}\right)$$

$$= \boxed{-5.76\times10^{-7}\text{ V}}$$

(b) The difference in potential between the two points is given by

$$|\Delta V| = \left|k_e\frac{q}{r_2} - k_e\frac{q}{r_1}\right| = k_e q\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Substituting numerical values,

$$|\Delta V| = \left|(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})\times\left(\frac{1}{0.250\times10^{-2}\text{ m}} - \frac{1}{0.750\times10^{-2}\text{ m}}\right)\right|$$

$$|\Delta V| = \boxed{3.84\times10^{-7}\text{ V}}$$

(c) Because the charge of the proton has the same magnitude as that of the electron, only the sign of the answer to part (a) would change.

P25.13 The total electric potential is the sum of the potentials from the individual charges,

$$V = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

(a) The 4.50-μC and –2.24-μC charges are distances 1.25 cm and 1.80 cm, respectively, from the origin. The electric potential is then

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{4.50\times10^{-6}\ \text{C}}{1.25\times10^{-2}\ \text{m}} + \frac{-2.24\times10^{-6}\ \text{C}}{1.80\times10^{-2}\ \text{m}}\right]$$

$$V = \boxed{2.12\times10^6\ \text{V}}$$

(b) The distance of the 4.50-μC charge to the point is

$$r_1 = \sqrt{(0.0150\ \text{m})^2 + (0.0125\ \text{m})^2} = 0.0195\ \text{m},$$

and the distance of the –2.24-μC charge to the point is

$$r_2 = \sqrt{(0.0150\ \text{m})^2 + (0.0180\ \text{m})^2} = 0.0234\ \text{m}$$

The potential is

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{4.50\times10^{-6}\ \text{C}}{r_1} + \frac{-2.24\times10^{-6}\ \text{C}}{r_2}\right]$$

$$V = \boxed{1.21\times10^6\ \text{V}}$$

P25.14 The potential due to the two charges is given by $V = k_e \sum_i \frac{q_i}{r_i}$.

(a) The electric potential at point A is

$$V = k_e \sum_i \frac{q_i}{r_i} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)$$

$$\times\left(\frac{-15.0\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}} + \frac{27.0\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}}\right)$$

$$= \boxed{5.39\ \text{kV}}$$

(b) The electric potential at point B is

$$V = k_e \sum_i \frac{q_i}{r_i} = \left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)$$

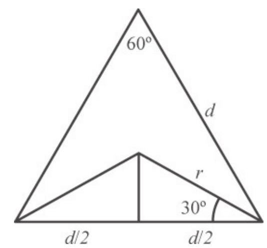
$$\times\left(\frac{-15.0\times10^{-9}\text{ C}}{1.00\times10^{-2}\text{ m}} + \frac{27.0\times10^{-9}\text{ C}}{1.00\times10^{-2}\text{ m}}\right)$$

$$= \boxed{10.8 \text{ kV}}$$

P25.15 By symmetry, a line from the center to each vertex forms a 30° angle with each side of the triangle. The figure shows the relationship between the length d of a side of the equilateral triangle and the distance r from a vertex to the center:

$$r\cos 30.0° = d/2$$

$$\rightarrow r = d/(2\cos 30.0°)$$



ANS. FIG. P25.15

The electric potential at the center is

$$V = k_e \sum_i \frac{q_i}{r_i}$$

$$= k_e\left(\frac{Q}{d/(2\cos 30.0°)} + \frac{Q}{d/(2\cos 30.0°)} + \frac{2Q}{d/(2\cos 30.0°)}\right)$$

$$V = (4)\left(2\cos 30.0°\, k_e \frac{Q}{d}\right) = \boxed{6.93 k_e \frac{Q}{d}}$$

***P25.16** (a) From Equation 25.12, the electric potential due to the two charges is

$$V = k_e \sum_i \frac{q_i}{r_i} = \left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)$$

$$\times\left(\frac{5.00\times10^{-9}\text{ C}}{0.175\text{ m}} + \frac{-3.00\times10^{-9}\text{ C}}{0.175\text{ m}}\right) = \boxed{103 \text{ V}}$$

(b) The potential energy of the pair of charges is

$$U = \frac{k_e q_1 q_2}{r_{12}} = \left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)$$

$$\times\frac{\left(5.00\times10^{-9}\text{ C}\right)\left(-3.00\times10^{-9}\text{ C}\right)}{0.350\text{ m}}$$

$$= \boxed{-3.85\times10^{-7}\text{ J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges by an infinite distance (that is, to bring them to a state of zero potential energy).

***P25.17** (a) In an empty universe, the 20.0-nC charge can be placed at its location with no energy investment. At a distance of 4.00 cm, it creates a potential

$$V_1 = \frac{k_e q_1}{r} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(20.0\times10^{-9}\ \text{C})}{0.040\ 0\ \text{m}} = 4.50\ \text{kV}$$

To place the 10.0-nC charge there we must put in energy

$$U_{12} = q_2V_1 = (10.0\times10^{-9}\ \text{C})(4.50\times10^3\ \text{V}) = 4.50\times10^{-5}\ \text{J}$$

Next, to bring up the –20.0-nC charge requires energy

$$\begin{aligned} U_{23}+U_{13} &= q_3V_2+q_3V_1 = q_3(V_2+V_1) \\ &= (-20.0\times10^{-9}\ \text{C})(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \\ &\qquad\times\left(\frac{10.0\times10^{-9}\ \text{C}}{0.040\ 0\ \text{m}}+\frac{20.0\times10^{-9}\ \text{C}}{0.080\ 0\ \text{m}}\right) \\ &= -4.50\times10^{-5}\ \text{J} - 4.50\times10^{-5}\ \text{J} \end{aligned}$$

The total energy of the three charges is

$$U_{12}+U_{23}+U_{13} = \boxed{-4.50\times10^{-5}\ \text{J}}$$

(b) The three fixed charges create this potential at the location where the fourth is released:

$$\begin{aligned} V &= V_1+V_2+V_3 \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \\ &\quad\times\left(\frac{20.0\times10^{-9}\ \text{C}}{\sqrt{(0.040\ 0\ \text{m})^2+(0.030\ 0\ \text{m})^2}}\right. \\ &\qquad\left.+\frac{10.0\times10^{-9}\ \text{C}}{0.030\ 0\ \text{m}}-\frac{20.0\times10^{-9}\ \text{C}}{\sqrt{(0.040\ 0\ \text{m})^2+(0.030\ 0\ \text{m})^2}}\right) \\ V &= 3.00\times10^3\ \text{V} \end{aligned}$$

Energy of the system of four charged objects is conserved as the fourth charge flies away:

$$\left(\frac{1}{2}mv^2+qV\right)_i = \left(\frac{1}{2}mv^2+qV\right)_f$$

$$0+(40.0\times10^{-9}\ \text{C})(3.00\times10^3\ \text{V}) = \frac{1}{2}(2.00\times10^{-13}\ \text{kg})v^2+0$$

$$v = \sqrt{\frac{2(1.20\times10^{-4}\ \text{J})}{2\times10^{-13}\ \text{kg}}} = \boxed{3.46\times10^4\ \text{m/s}}$$

P25.18 (a) $V_A = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{Q}{d} + \frac{2Q}{d\sqrt{2}}\right) = k_e \frac{Q}{d}\left(1+\sqrt{2}\right)$

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{5.00\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}}\right)\left(1+\sqrt{2}\right) = \boxed{5.43\ \text{kV}}$$

(b) $V_B = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{Q}{d\sqrt{2}} + \frac{2Q}{d}\right) = k_e \frac{Q}{d}\left(\frac{1}{\sqrt{2}}+2\right)$

$$V_B = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{5.00\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}}\right)\left(\frac{1}{\sqrt{2}}+2\right) = \boxed{6.08\ \text{kV}}$$

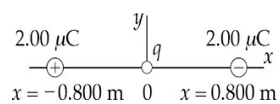
(c) $V_B - V_A = k_e \frac{Q}{d}\left(\frac{1}{\sqrt{2}}+2\right) - k_e\frac{Q}{d}\left(1+\sqrt{2}\right) = k_e\frac{Q}{d}\left(\frac{1}{\sqrt{2}}+1-\sqrt{2}\right)$

$$V_B - V_A = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{5.00\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}}\right)\left(\frac{1}{\sqrt{2}}+1-\sqrt{2}\right)$$

$$= \boxed{658\ \text{V}}$$

P25.19 (a) Since the charges are equal and placed symmetrically, $\boxed{F = 0}$.

(b) Since $F = qE = 0$, $\boxed{E = 0}$.



ANS. FIG. P25.19

(c) $V = 2k_e \frac{q}{r}$

$$= 2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{2.00\times10^{-6}\ \text{C}}{0.800\ \text{m}}\right)$$

$$V = 4.50\times10^4\ \text{V} = \boxed{45.0\ \text{kV}}$$

P25.20 At a distance r from a charged particle, the voltage is $V = \frac{k_e Q}{r}$ and the field magnitude is $E = \frac{k_e|Q|}{r^2}$.

(a) $r = \frac{|V|}{|E|} = \frac{3.00\times10^3\ \text{V}}{5.00\times10^2\ \text{V/m}} = \boxed{6.00\ \text{m}}$

(b) $V = -3\,000\ \text{V} = \frac{Q}{4\pi\epsilon_0\,(6.00\ \text{m})}$

Then,

$$Q = (6.00\ \text{m})(-3\,000\ \text{V})(4\pi\epsilon_0) = \boxed{-2.00\ \mu\text{C}}$$

P25.21 (a) Each charge is a distance $\sqrt{a^2+a^2}/2 = a/\sqrt{2}$ from the center.

$$V = k_e \sum_i \frac{q_i}{r_i} = 4k_e\left(\frac{Q}{a/\sqrt{2}}\right) = \boxed{4\sqrt{2}k_e\frac{Q}{a}}$$

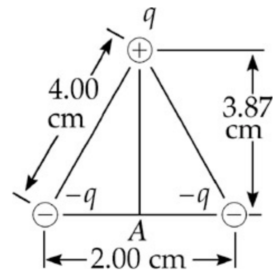
(b) The potential at infinity is zero. The work done by an external agent is

$$W = q\Delta V = q\left(V_f - V_i\right) = q\left(4\sqrt{2}k_e\frac{Q}{a} - 0\right) = \boxed{4\sqrt{2}k_e\frac{qQ}{a}}$$

P25.22 The charges at the base vertices are $d/2 = 0.010\,0$ m from point A, and the charge at the top vertex is

$$\sqrt{(2d)^2 - \left(\frac{d}{2}\right)^2} = \frac{\sqrt{15}}{2}d$$

from point A.



ANS. FIG. P25.22

$$V = \sum_i k_e \frac{q_i}{r_i}$$

$$= k_e\left(\frac{-q}{d/2} + \frac{-q}{d/2} + \frac{q}{d\sqrt{15}/2}\right) = k_e\frac{q}{d}\left(-4 + \frac{2}{\sqrt{15}}\right)$$

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{7.00\times10^{-6}\ \text{C}}{0.020\,0\ \text{m}}\right)\left(-4+\frac{2}{\sqrt{15}}\right)$$

$$= \boxed{-1.10\times10^7\ \text{V}}$$

P25.23 (a) $E_x = \dfrac{k_e q_1}{x^2} + \dfrac{k_e q_2}{(x-2.00)^2} = 0$ becomes $E_x = k_e\left(\dfrac{+q}{x^2} + \dfrac{-2q}{(x-2.00)^2}\right) = 0$

Dividing by k_e, $\quad 2qx^2 = q(x-2.00)^2$

or $\quad x^2 + 4.00x - 4.00 = 0.$

Therefore $E = 0$ when $x = \dfrac{-4.00 \pm \sqrt{16.0+16.0}}{2} = -\boxed{-4.83\ \text{m}}$.

(Note that the positive root does not correspond to a physically valid situation.)

(b) Assuming $0 < x < 2.00$ m, the potential is zero when

$$V = \frac{k_e q_1}{x} + \frac{k_e q_2}{2.00 - x} = 0 \quad \text{or} \quad \frac{V}{k_e} = \left[\frac{(q)}{x} + \frac{(-2q)}{2.00-x}\right] = 0$$

giving $\quad q(2.00-x)=2qx \quad$ or $\quad x=\frac{2.00}{3}=\boxed{0.667\text{ m}}$

For $x > 2.00$ m , the potential is zero when

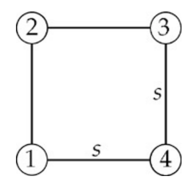
$$\frac{V}{k_e}=\left[\frac{(q)}{x}+\frac{(-2q)}{x-2.00}\right]=0 \quad \text{or} \quad q(x-2.00)=2qx$$

This has no positive x solution. Physically, the total potential cannot be zero for any point where $x > 2.00$ m because that point is closer to charge $-2q$, so its potential is always more negative than the potential from charge q is positive. For $x < 0$, the potential is zero when $\frac{V}{k_e}=\left[\frac{(q)}{|x|}+\frac{(-2q)}{|2.00+x|}\right]=0$, giving

$$\frac{q}{|x|}<\frac{2q}{2.00+|x|} \quad \text{or} \quad q(2.00+|x|)=2q|x|$$

which has the solution $|x|$ = 2.00 correspond to $x=\boxed{-2.00\text{ m}}$.

P25.24 The work required equals the sum of the potential energies for all pairs of charges. No energy is involved in placing q_4 at a given position in empty space. When q_3 is brought from far away and placed close to q_4, the system potential energy can be expressed as q_3V_4, where V_4 is the potential at the position of q_3 established by charge q_4. When q_2 is brought into the system, it interacts with two other charges, so we have two additional terms q_2V_3 and q_2V_4 in the total potential energy. Finally, when we bring the fourth charge q_1 into the system, it interacts with three other charges, giving us three more energy terms. Thus, the complete expression for the energy is:



ANS. FIG. P25.24

$$U=U_1+U_2+U_3+U_4$$

$$U=0+U_{12}+(U_{13}+U_{23})+(U_{14}+U_{24}+U_{34})$$

$$U=0+\frac{k_eQ^2}{s}+\frac{k_eQ^2}{s}\left(\frac{1}{\sqrt{2}}+1\right)+\frac{k_eQ^2}{s}\left(1+\frac{1}{\sqrt{2}}+1\right)$$

$$U=\frac{k_eQ^2}{s}\left(4+\frac{2}{\sqrt{2}}\right)=\boxed{5.41\frac{k_eQ^2}{s}}$$

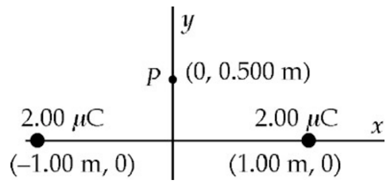
We can visualize the term $\left(4+\frac{2}{\sqrt{2}}\right)$ as arising directly from the 4 side pairs and 2 face diagonal pairs.

P25.25 (a) The electric potential at $y = 0.500$ m on the y axis is given by

$$V = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} = 2\left(\frac{k_e q}{r}\right)$$

$$V = 2\left(\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\ \text{C})}{\sqrt{(1.00\ \text{m})^2 + (0.500\ \text{m})^2}}\right)$$

$$V = 3.22\times10^4\ \text{V} = \boxed{32.2\ \text{kV}}$$



ANS. FIG. P25.25

(b) The change in potential energy of the system when a third charge is brought to this point is

$$U = qV = (-3.00\times10^{-6}\ \text{C})(3.22\times10^4\ \text{J/C}) = \boxed{-9.65\times10^{-2}\ \text{J}}$$

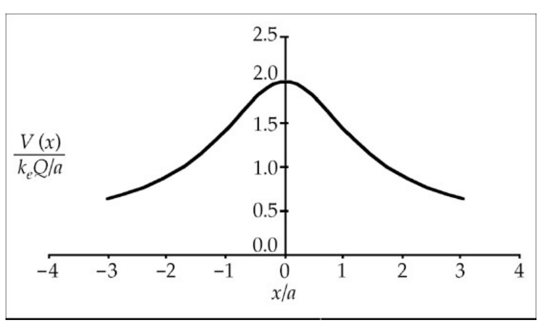
P25.26 (a) The potential due to the two charges along the x axis is

$$V(x) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{\sqrt{x^2+a^2}} + \frac{k_e(+Q)}{\sqrt{x^2+(-a)^2}}$$

$$V(x) = \frac{2k_e Q}{\sqrt{x^2+a^2}} = \frac{k_e Q}{a}\left(\frac{2}{\sqrt{(x/a)^2+1}}\right)$$

$$\frac{V(x)}{(k_e Q/a)} = \boxed{\frac{2}{\sqrt{(x/a)^2+1}}}$$

ANS. FIG. P25.26(a) shows the plot of this function for $|x/a| < 3$.



ANS. FIG. P25.26(a)

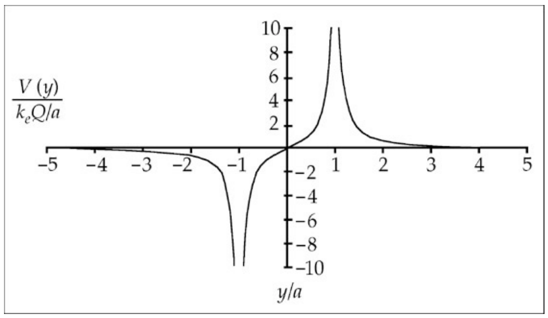
(b) The potential due to the two charges along the y axis is

$$V(y) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{|y-a|} + \frac{k_e(-Q)}{|y+a|}$$

$$V(y) = \frac{k_e Q}{a}\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_e Q/a)} = \boxed{\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)}$$

ANS. FIG. P25.26(b) shows the plot of this function for $|y/a| < 4$.



ANS. FIG. P25.26(b)

P25.27 The total change in potential energy is the sum of the change in potential energy of the $q_1 - q_4$, $q_2 - q_4$, and $q_3 - q_4$ particle systems:

$$U_e = q_4 V_1 + q_4 V_2 + q_4 V_3 = q_4 k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$U_e = (10.0 \times 10^{-6}\text{ C})^2 (8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)$$

$$\times\left(\frac{1}{0.600\text{ m}} + \frac{1}{0.150\text{ m}} + \frac{1}{\sqrt{(0.600\text{ m})^2 + (0.150\text{ m})^2}}\right)$$

$$U_e = \boxed{8.95\text{ J}}$$

P25.28 (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$, located at a finite distance from the charges, at which this total potential is zero.

(b) $V = \frac{k_e q}{a} + \frac{k_e q}{a} = \boxed{\frac{2k_e q}{a}}$

P25.29 Each charge creates equal potential at the center. The total potential is

$$V = 5\left[\frac{k_e(-q)}{R}\right] = \boxed{-\frac{5k_e q}{R}}$$

P25.30 The original electrical potential energy is

$$U_e = qV = q\frac{k_e q}{d}$$

In the final configuration we have mechanical equilibrium. The spring and electrostatic forces on each charge are

$$F_{\text{spring}} + F_{\text{charge}} = -k(2d) + q\frac{k_e q}{(3d)^2} = 0$$

Then $k = \dfrac{k_e q^2}{18d^3}$

In the final configuration the total potential energy is

$$\frac{1}{2}kx^2 + qV = \frac{1}{2}\frac{k_e q^2}{18d^3}(2d)^2 + q\frac{k_e q}{3d} = \frac{4}{9}\frac{k_e q^2}{d}$$

The missing energy must have become internal energy, as the system is isolated:

$$\Delta U + \Delta E_{\text{int}} = 0$$

$$\frac{4k_e q^2}{9d} - \frac{k_e q^2}{d} + \Delta E_{\text{int}} = 0$$

The increase in internal energy of the system is then

$$\Delta E_{\text{int}} = \boxed{\frac{5k_e q^2}{9d}}$$

P25.31 Consider the two spheres as a system.

(a) Conservation of momentum:

$$0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2\left(-\hat{\mathbf{i}}\right) \quad \text{or} \quad v_2 = \frac{m_1 v_1}{m_2}$$

By conservation of energy,

$$0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$$

and $\dfrac{k_e q_1 q_2}{r_1 + r_2} - \dfrac{k_e q_1 q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}\dfrac{m_1^2 v_1^2}{m_2}$, which yields

$$v_1 = \sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$$

suppressing units,

$$v_1 = \sqrt{\frac{2(0.700)(8.99\times10^9)(2\times10^{-6})(3\times10^{-6})}{(0.100)(0.800)}\left(\frac{1}{8\times10^{-3}} - \frac{1}{1.00}\right)}$$

$$= \boxed{10.8\ \text{m/s}}$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{(0.100\ \text{kg})(10.8\ \text{m/s})}{0.700\ \text{kg}} = \boxed{1.55\ \text{m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P25.32 Consider the two spheres as a system.

(a) Conservation of momentum:

$$0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \left(-\hat{\mathbf{i}}\right)$$

or $$v_2 = \frac{m_1 v_1}{m_2}.$$

By conservation of energy,

$$0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$$

and $$\frac{k_e q_1 q_2}{r_1 + r_2} - \frac{k_e q_1 q_2}{d} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}\frac{m_1^2 v_1^2}{m_2}.$$

$$v_1 = \boxed{\sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

$$v_2 = \left(\frac{m_1}{m_2}\right)v_1 = \boxed{\sqrt{\frac{2m_1 k_e q_1 q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P25.33 A cube has 12 edges and 6 faces. Consequently, there are 12 edge pairs separated by s, 2 × 6 = 12 face diagonal pairs separated by $\sqrt{2}s$, and 4 interior diagonal pairs separated by $\sqrt{3}s$.

$$U = \frac{k_e q^2}{s}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right] = \boxed{22.8\frac{k_e q^2}{s}}$$

P25.34 Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$0 + \frac{4k_e q^2}{L} + \frac{2k_e q^2}{\sqrt{2}L} = 4\left(\frac{1}{2}mv^2\right) + \frac{4k_e q^2}{2L} + \frac{2k_e q^2}{2\sqrt{2}L}$$

$$\left(2 + \frac{1}{\sqrt{2}}\right)\frac{k_e q^2}{L} = 2mv^2$$

Solving for the speed gives

$$v = \boxed{\sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{k_e q^2}{mL}}}$$

P25.35 Using conservation of energy for the alpha particle-nucleus system, we have $K_f + U_f = K_i + U_i$.

But $U_i = \frac{k_e q_\alpha q_{\text{gold}}}{r_i}$ and $r_i \approx \infty$. Thus, $U_i = 0$.

Also, $K_f = 0$ ($v_f = 0$ at turning point),

so $U_f = K_i$

or $\frac{k_e q_\alpha q_{\text{gold}}}{r_{\text{min}}} = \frac{1}{2}m_\alpha v_\alpha^2$

$$r_{\text{min}} = \frac{2k_e q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2}$$

$$= \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.00\times10^7\ \text{m/s}\right)^2}$$

$$= 2.74\times10^{-14}\ \text{m} = \boxed{27.4\ \text{fm}}$$

Section 25.4 Obtaining the Value of the Electric Field from the Electric Potential

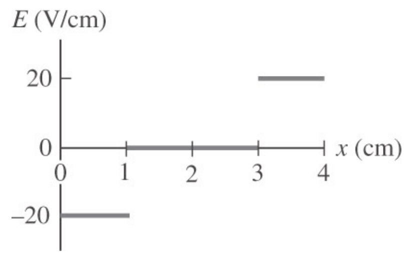
P25.36 $E_x = -\dfrac{\partial V}{\partial x} \quad \rightarrow \quad E_x = -\dfrac{\Delta V}{\Delta x} = -\text{ (slope of line)}$

The sign indicates the direction of the x component of the field.

$x = 0$ to 1 cm: $\quad E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{20\text{ V} - 0}{1\text{ cm}} = -20\text{ V/cm}$

$x = 1$ to 3 cm: $\quad E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{0}{2\text{ cm}} = 0\text{ V/m}$

$x = 3$ to 4 cm: $\quad E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{0 - 20\text{ V}}{1\text{ cm}} = +20\text{ V/cm}$



ANS. FIG. P25.36

P25.37 $V = a + bx = 10.0\text{ V} + (-7.00\text{ V/m})x$

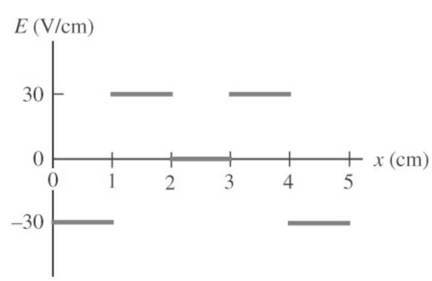
(a) At $x = 0$, $\quad V = \boxed{10.0\text{ V}}$

At $x = 3.00$ m, $\quad V = \boxed{-11.0\text{ V}}$

At $x = 6.00$ m, $\quad V = \boxed{-32.0\text{ V}}$

(b) $E = -\dfrac{dV}{dx} = -b = -(-7.00\text{ V/m}) = \boxed{7.00\text{ N/C in the } +x \text{ direction}}$

P25.38 $E_x = -\dfrac{\partial V}{\partial x} \quad \rightarrow \quad E_x = -\dfrac{\Delta V}{\Delta x} = -\text{ (slope of line)}$



ANS. FIG. P25.38

The sign indicates the direction of the x component of the field.

$x = 0$ to 1 cm: $E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{30\text{ V} - 0}{1\text{ cm}} = -30\text{ V/cm}$

$x = 1$ to 2 cm: $E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{0 - 30\text{ V}}{2\text{ cm}} = 30\text{ V/m}$

$x = 2$ to 3 cm: $E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{0}{1\text{ cm}} = 0\text{ V/cm}$

$x = 3$ to 4 cm: $E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{-30\text{ V} - 0}{1\text{ cm}} = +30\text{ V/cm}$

$x = 4$ to 5 cm: $E_x = -\dfrac{\Delta V}{\Delta x} = -\dfrac{0 - (-30\text{ V})}{1\text{ cm}} = -30\text{ V/cm}$

P25.39 (a) $V = 5x - 3x^2y + 2yz^2$, where x, y and z are in meters and V is in volts.

$$E_x = -\frac{\partial V}{\partial x} = -5 + 6xy$$
$$E_y = -\frac{\partial V}{\partial y} = +3x^2 - 2z^2$$
$$E_z = -\frac{\partial V}{\partial z} = -4yz$$

which gives

$$\boxed{\vec{\mathbf{E}} = (-5 + 6xy)\hat{\mathbf{i}} + (3x^2 - 2z^2)\hat{\mathbf{j}} - 4yz\hat{\mathbf{k}}}$$

(b) Evaluate E at (1.00, 0, −2.00) m, suppressing units,

$$E_x = -5 + 6(1.00)(0) = -5.00$$
$$E_y = 3(1.00)^2 - 2(-2.00)^2 = -5.00$$
$$E_z = -4(0)(-2.00) = 0$$

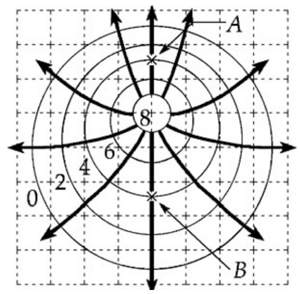
which gives

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(-5.00)^2 + (-5.00)^2 + 0^2} = \boxed{7.07\text{ N/C}}$$

P25.40 (a) $\boxed{E_A > E_B \text{ since } E = \dfrac{\Delta V}{\Delta s}}$

(b) $E_B = -\dfrac{\Delta V}{\Delta s} = -\dfrac{(6-2)\text{ V}}{2\text{ cm}} = \boxed{200\text{ N/C}}$ down

(c) ANS. FIG. P25.40 shows a sketch of the field lines.



ANS. FIG. P25.40

P25.41 (a) For $r < R$, $V = \dfrac{k_e Q}{R}$

$$E_r = -\frac{dV}{dr} = \boxed{0}$$

(b) For $r \geq R$, $V = \dfrac{k_e Q}{r}$

$$E_r = -\frac{dV}{dr} = -\left(-\frac{k_e Q}{r^2}\right) = \boxed{\frac{k_e Q}{r^2}}$$

P25.42 For a general expression for the potential on the y-axis, replace the a with y. The y component of the electric field is

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_e Q}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + y^2}}{y}\right)\right]$$

$$E_y = \frac{k_e Q}{\ell y}\left[1 - \frac{y^2}{\ell^2 + y^2 + \ell\sqrt{\ell^2 + y^2}}\right] = \boxed{\frac{k_e Q}{y\sqrt{\ell^2 + y^2}}}$$

Section 25.5 Electric Potential Due to Continuous Charge Distributions

P25.43 The potential difference between the two points is

$$\Delta V = V_{2R} - V_0 = \frac{k_e Q}{\sqrt{R^2 + (2R)^2}} - \frac{k_e Q}{R} = \frac{k_e Q}{R}\left(\frac{1}{\sqrt{5}} - 1\right)$$

$$= \boxed{-0.553\frac{k_e Q}{R}}$$

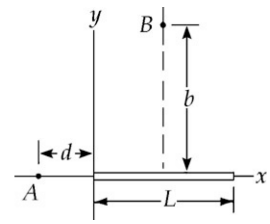
P25.44 $V = \int dV = \frac{1}{4\pi \in_0} \int \frac{dq}{r}$

All bits of charge are at the same distance from O. So

$$V = \frac{1}{4\pi \in_0}\left(\frac{Q}{R}\right) = \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-7.50\times10^{-6}\ \text{C}}{0.140\ \text{m}/\pi}\right)$$
$$= \boxed{-1.51\ \text{MV}}$$

P25.45 (a) As a linear charge density, λ has units of C/m. So $\alpha = \lambda/x$ must have units of C/m^2:

$$[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\text{C}}{\text{m}}\cdot\left(\frac{1}{\text{m}}\right) = \boxed{\frac{\text{C}}{\text{m}^2}}$$



ANS. FIG. P25.45

(b) Consider a small segment of the rod at location x and of length dx. The amount of charge on it is $\lambda\,dx = (\alpha x)\,dx$. Its distance from A is $d + x$, so its contribution to the electric potential at A is

$$dV = k_e \frac{dq}{r} = k_e \frac{\alpha x\,dx}{d+x}$$

Relative to $V = 0$ infinitely far away, to find the potential at A we must integrate these contributions for the whole rod, from $x = 0$ to $x = L$. Then $V = \int_{\text{all } q} dV = \int_0^L \frac{k_e \alpha x}{d+x}\,dx.$

To perform the integral, make a change of variables to

$u = d + x$, $du = dx$, $u(\text{at } x = 0) = d$, and $u(\text{at } x = L) = d + L$:

$$V = \int_d^{d+L} \frac{k_e \alpha (u-d)}{u}\,du = k_e\alpha \int_d^{d+L} du - k_e\alpha d \int_d^{d+L}\left(\frac{1}{u}\right)du$$

$$V = k_e\alpha u\Big|_d^{d+L} - k_e\alpha d \ln u\Big|_d^{d+L}$$
$$= k_e\alpha(d + L - d) - k_e\alpha d\left[\ln(d+L) - \ln d\right]$$

$$V = \boxed{k_e\alpha\left[L - d\ln\left(1 + \frac{L}{d}\right)\right]}$$

P25.46 $V = \int \frac{k_e dq}{r} = k_e \int \frac{\alpha x dx}{\sqrt{b^2 + (L/2 - x)^2}}$

Let $z = \frac{L}{2} - x$. Then $x = \frac{L}{2} - z$, and $dx = -dz$.

$$V = k_e \alpha \int \frac{(L/2 - z)(-dz)}{\sqrt{b^2 + z^2}} = -\frac{k_e \alpha L}{2} \int \frac{dz}{\sqrt{b^2 + z^2}} + k_e \alpha \int \frac{z dz}{\sqrt{b^2 + z^2}}$$

$$= -\frac{k_e \alpha L}{2} \ln\left(z + \sqrt{z^2 + b^2}\right) + k_e \alpha \sqrt{z^2 + b^2}$$

$$V = -\frac{k_e \alpha L}{2} \ln\left[\left(\frac{L}{2} - x\right) + \sqrt{\left(\frac{L}{2} - x\right)^2 + b^2}\right]_0^L + k_e \alpha \sqrt{\left(\frac{L}{2} - x\right)^2 + b^2}\Bigg|_0^L$$

$$V = -\frac{k_e \alpha L}{2} \ln\left[\frac{L/2 - L + \sqrt{(L/2)^2 + b^2}}{L/2 + \sqrt{(L/2)^2 + b^2}}\right]$$

$$+ k_e \alpha \left[\sqrt{\left(\frac{L}{2} - L\right)^2 + b^2} - \sqrt{\left(\frac{L}{2}\right)^2 + b^2}\right]$$

$$V = \boxed{-\frac{k_e \alpha L}{2} \ln\left[\frac{\sqrt{b^2 + (L^2/4)} - L/2}{\sqrt{b^2 + (L^2/4)} + L/2}\right]}$$

P25.47 $V = k_e \int\limits_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int\limits_{\text{semicircle}} \frac{\lambda ds}{R} + k_e \int_{R}^{3R} \frac{\lambda dx}{x}$

$$V = -k_e \lambda \ln(-x)\Big|_{-3R}^{-R} + \frac{k_e \lambda}{R} \pi R + k_e \lambda \ln x\Big|_R^{3R}$$

$$V = k_e \lambda \ln \frac{3R}{R} + k_e \lambda \pi + k_e \lambda \ln 3 = \boxed{k_e \lambda (\pi + 2\ln 3)}$$

Section 25.6 Electric Potential Due to a Charged Conductor

P25.48 No. A conductor of any shape forms an equipotential surface. If the conductor is a sphere of radius R, and if it holds charge Q, the electric field at its surface is $E = k_e Q/R^2$ and the –potential of the surface is $V = k_e Q/R$; thus, if we know E and R, we can find V. However, if the surface varies in shape, there is no clear way to relate electric field at a point on the surface to the potential of the surface.

P25.49 Substituting given values into $V = \frac{k_e Q}{r}$, with $Q = Nq$:

$$7.50 \times 10^3 \text{ V} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) Q}{0.300 \text{ m}}$$

Substituting $q = 2.50 \times 10^{-7}$ C,

$$N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C}/e^-} = \boxed{1.56 \times 10^{12} \text{ electrons}}$$

P25.50 For points on the surface and outside, the sphere of charge behaves like a charged particle at its center, both for creating field and potential.

(a) Inside a conductor when charges are not moving, the electric field is zero and the potential is uniform, the same as on the surface, and $E = \boxed{0}$.

$$V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(26.0 \times 10^{-6} \text{ C})}{0.140 \text{ m}} = \boxed{1.67 \text{ MV}}$$

(b) $$E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(26.0 \times 10^{-6} \text{ C})}{(0.200 \text{ m})^2}$$

$= \boxed{5.84 \text{ MN/C}}$ away

$$V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(26.0 \times 10^{-6} \text{ C})}{0.200 \text{ m}} = \boxed{1.17 \text{ MV}}$$

(c) $$E = \frac{k_e q}{R^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(26.0 \times 10^{-6} \text{ C})}{(0.140 \text{ m})^2}$$

$= \boxed{11.9 \text{ MN/C}}$ away

$$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$$

P25.51 (a) Both spheres must be at the same potential according to $\frac{k_e q_1}{r_1} = \frac{k_e q_2}{r_2}$, where also $q_1 + q_2 = 1.20 \times 10^{-6}$ C.

Then $q_1 = \frac{q_2 r_1}{r_2}$ and

$$\frac{q_2 r_1}{r_2} + q_2 = 1.20 \times 10^{-6} \text{ C}$$

$$q_2 = \frac{1.20\times10^{-6}\ \text{C}}{1+6.00\ \text{cm}/2.00\ \text{cm}} = 0.300\times10^{-6}\ \text{C}$$

on the smaller sphere.

$$q_1 = 1.20\times10^{-6}\ \text{C} - 0.300\times10^{-6}\ \text{C} = 0.900\times10^{-6}\ \text{C}$$

$$V = \frac{k_e q_1}{r_1} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(0.900\times10^{-6}\ \text{C}\right)}{6.00\times10^{-2}\ \text{m}}$$

$$= \boxed{1.35\times10^5\ \text{V}}$$

(b) Outside the larger sphere,

$$\vec{\mathbf{E}}_1 = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}} = \frac{V_1}{r_1}\hat{\mathbf{r}} = \frac{1.35\times10^5\ \text{V}}{0.060\ 0\ \text{m}}\hat{\mathbf{r}} = \boxed{2.25\times10^6\ \text{V/m away}}$$

Outside the smaller sphere,

$$\vec{\mathbf{E}}_2 = \frac{1.35\times10^5\ \text{V}}{0.020\ 0\ \text{m}}\hat{\mathbf{r}} = \boxed{6.74\times10^6\ \text{V/m away}}$$

The smaller sphere carries less charge but creates a much stronger electric field than the larger sphere.

Section 25.8 Applications of Electrostatics

P25.52 From the maximum allowed electric field, we can find the charge and potential that would create this situation. Since we are only given the diameter of the dome, we will assume that the conductor is spherical, which allows us to use the electric field and potential equations for a spherical conductor.

(a) $E_{\text{max}} = 3.00\times10^6\ \text{V/m} = \dfrac{k_e Q}{r^2} = \dfrac{k_e Q}{r}\left(\dfrac{1}{r}\right) = V_{\text{max}}\left(\dfrac{1}{r}\right)$

$$V_{\text{max}} = E_{\text{max}} r = \left(3.00\times10^6\ \text{V/m}\right)\left(0.150\ \text{m}\right) = \boxed{450\ \text{kV}}$$

(b) $\dfrac{k_e Q_{\text{max}}}{r^2} = E_{\text{max}} \quad \left\{\text{or } \dfrac{k_e Q_{\text{max}}}{r} = V_{\text{max}}\right\}$

$$Q_{\text{max}} = \frac{E_{\text{max}} r^2}{k_e} = \frac{\left(3.00\times10^6\ \text{V/m}\right)\left(0.150\ \text{m}\right)^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{7.51\ \mu\text{C}}$$

Additional Problems

P25.53 From Equation 25.13, solve for the separation distance of the electron and proton:

$$U = k_e \frac{q_1 q_2}{r_{12}} \quad \rightarrow \quad r_{12} = k_e \frac{q_1 q_2}{U} = -k_e \frac{e^2}{U}$$

The separation distance r_{12} between the electron and proton is the same as the radius r of the orbit of the electron. Substitute numerical values:

$$r = -\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \frac{\left(1.6 \times 10^{-19} \text{ C}\right)^2}{-13.6 \text{ eV}} \left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}\right)$$
$$= 1.06 \times 10^{-10} \text{ m}$$

Set this equal to $r = n^2(0.052\,9 \text{ nm})$ and solve for n:

$$r = n^2(0.052\ 9 \text{ nm}) = 1.06 \times 10^{-10} \text{ m} = 0.106 \text{ nm}$$

Which gives $n = 1.42$. Because n is not an integer, this is not possible. Therefore, the energy given cannot be possible for an allowed state of the atom.

P25.54 (a) The field within the conducting Earth is zero. The field is downward, so the Earth is negatively charged. Treat the surface of Earth at this location as a charged conducting plane: thus, use

$$E = \sigma / \epsilon_0$$

which gives

$$\sigma = E\epsilon_0 = (120 \text{ N/C})\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)$$
$$= \boxed{1.06 \text{ nC/m}^2\text{, negative}}$$

(b) $$Q = \sigma A = \sigma 4\pi r^2 = (-1.06 \times 10^{-9} \text{ C/m}^2)(4\pi)(6.37 \times 10^6 \text{ m})^2$$
$$= \boxed{-542 \text{ kC}}$$

(c) The Earth acts like a conducting sphere:

$$V = \frac{k_e Q}{R} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)(-5.42 \times 10^5 \text{ C})}{6.37 \times 10^6 \text{ m}} = \boxed{-764 \text{ MV}}$$

(d) Electric potential decreases in the direction of the electric field; therefore, the potential is greater at greater heights:

$$V_{head} - V_{feet} = Ed = (120 \text{ N/C})(1.75 \text{ m}) = 210 \text{ V}.$$

$\rightarrow$ $\boxed{\text{The person's head is higher in potential by 210 V.}}$

(e) Like charges repel:

$$F_E = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(5.42\times10^5\ \text{C})^2(0.273)}{(3.84\times10^8\ \text{m})^2}$$

$$F_E = 4.88\times10^3\ \text{N} = \boxed{4.88\times10^3\ \text{N}\ \ \text{away from Earth}}$$

(f) The gravitational force is

$$F_G = \frac{GM_E M_M}{r^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(5.98\times10^{24}\ \text{kg})(7.36\times10^{22}\ \text{kg})}{(3.84\times10^8\ \text{m})^2}$$

$$F_G = 1.99\times10^{20}\ \text{N}$$

Comparing the two forces,

$$\frac{F_G}{F_E} = \frac{1.99\times10^{20}\ \text{N}}{4.88\times10^3\ \text{N}} = 4.08\times10^{16}$$

> The gravitational force is in the opposite direction and 4.08×10^{16} times larger. Electrical forces are negligible in accounting for planetary motion.

P25.55 Assume the particles move along the x direction.

(a) Momentum is constant within the isolated system throughout the process. We equate it at the large-separation initial point and the point c of closest approach.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1c} + m_2\vec{\mathbf{v}}_{2c}$$

$$m_1 v\hat{\mathbf{i}} + 0 = m_1\vec{\mathbf{v}}_c + m_2\vec{\mathbf{v}}_c$$

$$\vec{\mathbf{v}}_c = \frac{m_1 v}{m_1+m_2}\hat{\mathbf{i}} = \frac{\left(2.00\times10^{-3}\ \text{kg}\right)(21.0\ \text{m/s})}{7.00\times10^{-3}\ \text{kg}}\hat{\mathbf{i}} = \boxed{6.00\hat{\mathbf{i}}\ \text{m/s}}$$

(b) Energy is conserved within the isolated system. Compare energy terms between the large-separation initial point and the point of closest approach:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 + 0 = \frac{1}{2}\left(m_1+m_2\right)v_c^2 + \frac{k_e q_1 q_2}{r_c}$$

$$\frac{1}{2}m_1 v^2 + 0 = \frac{1}{2}\left(m_1+m_2\right)\left(\frac{m_1 v}{m_1+m_2}\right)^2 + \frac{k_e q_1 q_2}{r_c}$$

$$\rightarrow m_1v^2 + 0 = \frac{m_1^2v^2}{m_1+m_2} + 2\frac{k_eq_1q_2}{r_c}$$

$$\rightarrow (m_1+m_2)m_1v^2 - m_1^2v^2 = 2\frac{k_eq_1q_2(m_1+m_2)}{r_c}$$

$$m_1m_2v^2 = 2\frac{k_eq_1q_2(m_1+m_2)}{r_c}$$

$$r_c = \frac{2k_eq_1q_2(m_1+m_2)}{m_1m_2v^2}$$

$$= \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(15.0\times10^{-6}\ \text{C})(8.50\times10^{-6}\ \text{C})(7.00\times10^{-3}\ \text{kg})}{(2.00\times10^{-3}\ \text{kg})(5.00\times10^{-3}\ \text{kg})(21.0\ \text{m/s})^2}$$

$$= \boxed{3.64\ \text{m}}$$

(c) The overall elastic collision is described by conservation of momentum:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$m_1v\hat{\mathbf{i}} + 0 = m_1v_{1f}\hat{\mathbf{i}} + m_2v_{2f}\hat{\mathbf{i}}$$

and by the relative velocity equation:

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

$$v - 0 = v_{2f} - v_{1f} \rightarrow v_{2f} = v + v_{1f}$$

We substitute the expression for v_{2f} into the momentum equation:

$$m_1v = m_1v_{1f} + m_2v_{2f}$$

$$m_1v = m_1v_{1f} + m_2(v + v_{1f})$$

$$m_1v = m_1v_{1f} + m_2v + m_2v_{1f}$$

$$m_1v - m_2v = m_1v_{1f} + m_2v_{1f}$$

$$(m_1 - m_2)v = (m_1 + m_2)v_{1f}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v = \left(\frac{2.00\,\text{g} - 5.00\,\text{g}}{2.00\,\text{g} + 5.00\,\text{g}}\right)(21.0\ \text{m/s})$$

$$= -9.00\ \text{m/s}$$

Therefore, the velocity of the particle of mass m_1 is $\boxed{-9.00\ \hat{\mathbf{i}}\ \text{m/s}}$.

(d) Substitute the expression for v_{1f} back into $v_{2f} = v + v_{1f}$:

$$v_{2f} = v + v_{1f} = (21.0\ \text{m/s}) + (-9.00\ \text{m/s}) = 12.0\ \text{m/s}$$

Therefore, the velocity of the particle of mass m_2 is $\boxed{12.0\ \hat{\mathbf{i}}\ \text{m/s}}$.

P25.56 Assume the particles move along the x direction.

(a) Momentum is constant within the isolated system throughout the process. We equate it at the large-separation initial point and the point c of closest approach.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$m_1 v\hat{\mathbf{i}} + 0 = m_1 v_c\hat{\mathbf{i}} + m_2 v_c\hat{\mathbf{i}} \quad \rightarrow \quad v_c = \boxed{\frac{m_1 v}{m_1 + m_2}}$$

(b) Energy is conserved within the isolated system. Compare energy terms between the large-separation initial point and the point of closest approach:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 + 0 = \frac{1}{2}(m_1 + m_2)v_c^2 + \frac{k_e q_1 q_2}{r_c}$$

$$\frac{1}{2}m_1 v^2 + 0 = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 v}{m_1 + m_2}\right)^2 + \frac{k_e q_1 q_2}{r_c}$$

$$\rightarrow m_1 v^2 + 0 = \frac{{m_1}^2 v^2}{m_1 + m_2} + 2\frac{k_e q_1 q_2}{r_c}$$

$$\rightarrow (m_1 + m_2)m_1 v^2 - {m_1}^2 v^2 = 2\frac{k_e q_1 q_2 (m_1 + m_2)}{r_c}$$

$$m_1 m_2 v^2 = 2\frac{k_e q_1 q_2 (m_1 + m_2)}{r_c} \rightarrow r_c = \boxed{\frac{2k_e q_1 q_2 (m_1 + m_2)}{m_1 m_2 v^2}}$$

(c) The overall elastic collision is described by conservation of momentum:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$m_1 v\hat{\mathbf{i}} + 0 = m_1 v_{1f}\hat{\mathbf{i}} + m_2 v_{2f}\hat{\mathbf{i}} \quad \rightarrow \quad m_1 v = m_1 v_{1f} + m_2 v_{2f}$$

and by the relative velocity equation:

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

$$v - 0 = v_{2f} - v_{1f} \quad \rightarrow \quad v_{2f} = v + v_{1f}$$

We substitute the expression for v_{2f} into the momentum equation:

$$m_1 v = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v = m_1 v_{1f} + m_2\left(v + v_{1f}\right)$$

$$m_1 v = m_1 v_{1f} + m_2 v + m_2 v_{1f}$$

$$m_1 v - m_2 v = m_1 v_{1f} + m_2 v_{1f}$$

$$(m_1 - m_2)v = (m_1 + m_2)v_{1f} \rightarrow v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v$$

Therefore, the velocity of the particle of mass m_1 is $\boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)v\,\hat{\mathbf{i}}}$.

(d) Substitute the expression for v_{1f} back into $v_{2f} = v + v_{1f}$:

$$v_{2f} = v + v_{1f} = v + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v$$

$$= \left[\frac{(m_1 + m_2) + (m_1 - m_2)}{m_1 + m_2}\right]v = \left(\frac{2m_1}{m_1 + m_2}\right)v$$

Therefore, the velocity of the particle of mass m_2 is $\boxed{\left(\frac{2m_1}{m_1 + m_2}\right)v\,\hat{\mathbf{i}}}$.

P25.57 The two spheres of charge have together electric potential energy

$$U = qV = k_e \frac{q_1 q_2}{r_{12}}$$

$$= \left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(38)(54)\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{(5.50 + 6.20) \times 10^{-15}\ \text{m}}$$

$$= 4.04 \times 10^{-11}\ \text{J} = \boxed{253\ \text{MeV}}$$

P25.58 (a) To make a spark 5 mm long in dry air between flat metal plates requires potential difference

$$\Delta V = Ed = \left(3 \times 10^6\ \text{V/m}\right)\left(5 \times 10^{-3}\ \text{m}\right) = 1.5 \times 10^4\ \text{V}\ \boxed{\sim 10^4\ \text{V}}$$

(b) The area of your skin is perhaps 1.5 m^2, so model your body as a sphere with this surface area. Its radius is given by $1.5\ \text{m}^2 = 4\pi r^2$, $r = 0.35$ m. We require that you are at the potential found in part (a), with $V = \frac{k_e q}{r}$. Then,

$$q = \frac{Vr}{k_e} = \frac{1.5 \times 10^4\ \text{V}(0.35\ \text{m})}{8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$$

$$q = 5.8 \times 10^{-7}\ \text{C}\ \boxed{\sim 10^{-6}\ \text{C}}$$

P25.59 We have $V_1 = k_eQ/R = 200$ V and $V_2 = k_eQ/(R + 10 \text{ cm}) = 150$ V.

(a) $$\frac{V_1}{V_2} = \frac{R + 10 \text{ cm}}{R} = \frac{200}{150} \rightarrow 150(R + 10 \text{ cm}) = 200R \rightarrow R = \boxed{30.0 \text{ cm}}$$

(b) From $V_1 = k_e \dfrac{Q}{R}$, we have

$$Q = \frac{V_1 R}{k_e} = \frac{(200 \text{ V})(0.300 \text{ m})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 6.67 \times 10^{-9} \text{ C} = \boxed{6.67 \text{ nC}}$$

(c) We have $V = k_eQ/R = 210$ V and $E = k_eQ/(R + 10 \text{ cm})^2 = 400$ V/m. Therefore,

$$\frac{V}{E} = \frac{(R + 10 \text{ cm})^2}{R} = \frac{210}{400} = \frac{21}{40} \rightarrow 40(R + 0.100)^2 = 21R$$

where R is in meters.

Thus, we have

$$40R^2 + 8R + 0.4 = 21R \rightarrow 40R^2 - 13R + 0.4 = 0$$

There are two possibilities, according to

$$R = \frac{+13 \pm \sqrt{13^2 - 4(40)(0.4)}}{80} = \text{ either } 0.291 \text{ m or } 0.034\,4 \text{ m}$$

$$= \boxed{29.1 \text{ cm or } 3.44 \text{ cm}}$$

(d) If the radius is 29.1 cm,

$$Q = \frac{VR}{k_e} = \frac{(210 \text{ V})(0.291 \text{ m})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 6.79 \times 10^{-9} \text{ C} = \boxed{6.79 \text{ nC}}$$

If the radius is 3.44 cm,

$$Q = \frac{VR}{k_e} = \frac{(210 \text{ V})(0.0344 \text{ m})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 8.04 \times 10^{-10} \text{ C} = \boxed{804 \text{ pC}}$$

(e) $\boxed{\text{No; two answers exist for each part.}}$

P25.60 (a) The exact potential is

$$+\frac{k_e q}{r + a} - \frac{k_e q}{r - a} = +\frac{k_e q}{3a + a} - \frac{k_e q}{3a - a} = \frac{k_e q}{4a} - \frac{2k_e q}{4a} = \boxed{-\frac{k_e q}{4a}}$$

(b) The approximate expression $-2k_eqa/x^2$ gives

$$-2k_eqa/(3a)^2 = -k_eq/4.5a$$

Compare the exact to the approximate solution:

$$\frac{1/4 - 1/4.5}{1/4} = \frac{0.5}{4.5} = 0.111 \cdot$$

The approximate expression $-2k_e qa/x^2$ gives $-k_e q/4.5a$, which is different by only 11.1%.

P25.61 $W = \int_0^Q V dq$, where $V = \dfrac{k_e q}{R}$.

Therefore, $W = \dfrac{k_e Q^2}{2R} = \dfrac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(125 \times 10^{-6} \text{ C})^2}{2(0.100 \text{ m})} = \boxed{702 \text{ J}}$.

P25.62 $W = \int_0^Q V dq$, where $V = \dfrac{k_e q}{R}$. Therefore, $W = \boxed{\dfrac{k_e Q^2}{2R}}$.

P25.63 For a charge at (x = –1 m, y = 0), the radial distance away is given by $\sqrt{(x+1)^2 + y^2}$. So the first term will be the potential it creates if

$$(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)Q_1 = 36 \text{ V·m} \quad \rightarrow \quad Q_1 = 4.00 \text{ nC}$$

The second term is the potential of a charge at (x = 0, y = 2 m) with

$$(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)Q_2 = -45 \text{ V·m} \quad \rightarrow \quad Q_2 = -5.01 \text{ nC}$$

Thus we have $\boxed{4.00 \text{ nC at } (-1.00 \text{ m}, 0) \text{ and } -5.01 \text{ nC at } (0, 2.00 \text{ m}).}$

P25.64 From Example 25.5, the potential along the x axis of a ring of charge of radius R is

$$V = \frac{k_e Q}{\sqrt{R^2 + x^2}}$$

Therefore, the potential at the center of the ring is

$$V = \frac{k_e Q}{\sqrt{R^2 + (0)^2}} = \frac{k_e Q}{R}$$

When we place the point charge Q at the center of the ring, the electric potential energy of the charge–ring system is

$$U = QV = Q\left(\frac{k_e Q}{R}\right) = \frac{k_e Q^2}{R}$$

Now, apply Equation 8.2 to the isolated system of the point charge and the ring with initial configuration being that with the point charge at the center of the ring and the final configuration having the point charge infinitely far away and moving with its highest speed:

$$\Delta K + \Delta U = 0 \quad \rightarrow \quad \left(\frac{1}{2}mv_{\text{max}}^2 - 0\right) + \left(0 - \frac{k_e Q^2}{R}\right) = 0$$

Solve for the maximum speed:

$$v_{\text{max}} = \left(\frac{2k_e Q^2}{mR}\right)^{1/2}$$

Substitute numerical values:

$$v_{\text{max}} = \left(\frac{2\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(50.0 \times 10^{-6} \text{ C}\right)^2}{(0.100 \text{ kg})(0.500 \text{ m})}\right)^{1/2}$$

$$= 30.0 \text{ m/s}$$

Therefore, even if the charge were to accelerate to infinity, it would only achieve a maximum speed of 30.0 m/s, so it cannot strike the wall of your laboratory at 40.0 m/s.

P25.65 In Equation 25.3, $V_2 - V_1 = \Delta V = -\int_1^2 \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$, think about stepping from distance r_1 out to the larger distance r_2 away from the charged line. Then $d\vec{\mathbf{s}} = dr\hat{\mathbf{r}}$, and we can make r the variable of integration:

$$V_2 - V_1 = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi \epsilon_0 r} \hat{\mathbf{r}} \cdot dr\, \hat{\mathbf{r}} \quad \text{with} \quad \hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = 1 \cdot 1\cos 0^\circ = 1$$

The potential difference is

$$V_2 - V_1 = -\frac{\lambda}{2\pi \epsilon_0} \int_{r_1}^{r_2} \frac{dr}{r} = -\frac{\lambda}{2\pi \epsilon_0} \ln r \Big|_{r_1}^{r_2}$$

and

$$V_2 - V_1 = -\frac{\lambda}{2\pi \epsilon_0}\left(\ln r_2 - \ln r_1\right) = \boxed{-\frac{\lambda}{2\pi \epsilon_0} \ln \frac{r_2}{r_1}}$$

P25.66 (a) Modeling the filament as a single charged particle, we obtain

$$V = \frac{k_e Q}{r} = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-9} \text{ C}\right)}{2.00 \text{ m}} = \boxed{7.19 \text{ V}}$$

(b) Modeling the filament as two charged particles, we obtain

$$V = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = k_e\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right)$$

$$= \left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{0.800 \times 10^{-9} \text{ C}}{1.50 \text{ m}} + \frac{0.800 \times 10^{-9} \text{ C}}{2.50 \text{ m}}\right)$$

$$= \boxed{7.67 \text{ V}}$$

(c) Modeling the filament as four charged particles, we obtain

$$V = k_e\left(\frac{Q_1}{r_1}+\frac{Q_2}{r_2}+\frac{Q_3}{r_3}+\frac{Q_4}{r_4}\right)$$
$$= \left(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\times\left(\frac{0.400\times 10^{-9}\ \text{C}}{1.25\ \text{m}}+\frac{0.400\times 10^{-9}\ \text{C}}{1.75\ \text{m}}+\frac{0.400\times 10^{-9}\ \text{C}}{2.25\ \text{m}}+\frac{0.400\times 10^{-9}\ \text{C}}{2.75\ \text{m}}\right)$$
$$= \boxed{7.84\ \text{V}}$$

(d) We represent the exact result as

$$V = \frac{k_e Q}{\ell}\ln\left(\frac{\ell + a}{a}\right)$$
$$= \left[\frac{\left(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-9}\ \text{C}\right)}{2.00\ \text{m}}\right]\ln\left(\frac{3}{1}\right)$$
$$= 7.901\ 2\ \text{V}$$

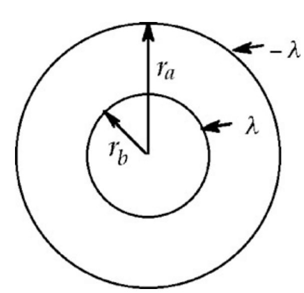
Modeling the line as a set of points works nicely. The exact result, represented as 7.90 V, is approximated to within 0.8% by the four-particle version.

P25.67 We obtain the electric potential at P by integrating:

$$V = k_e\int_a^{a+L}\frac{\lambda dx}{\sqrt{x^2+b^2}} = k_e\lambda\ln\left[x+\sqrt{\left(x^2+b^2\right)}\right]_a^{a+L}$$
$$= \boxed{k_e\lambda\ln\left[\frac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]}$$

P25.68 (a) $V_B - V_A = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ and the field at distance r from a uniformly charged rod (where $r >$ radius of charged rod) is

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k_e\lambda}{r}$$



ANS. FIG. P25.68

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r} dr = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)$$

or $\boxed{\Delta V = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)}$.

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance r from the axis is

$$V = 2k_e\lambda \ln\left(\frac{r_a}{r}\right)$$

The field at r is given by

$$E = -\frac{\partial V}{\partial r} = -2k_e\lambda\left(\frac{r}{r_a}\right)\left(-\frac{r_a}{r^2}\right) = \frac{2k_e\lambda}{r}$$

But, from part (a), $2k_e\lambda = \dfrac{\Delta V}{\ln(r_a/r_b)}$.

Therefore, $\boxed{E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)}$.

P25.69 (a) The positive plate by itself creates a field

$$E = \frac{\sigma}{2\epsilon_0} = \frac{36.0\times10^{-9}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 2.03\ \text{kN/C}$$

away from the positive plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field $\boxed{4.07\ \text{kN/C}}$ in the space between.

(b) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$\Delta V = V - 0 = -\int_{x_i}^{x_f} E_x dx = -\int_0^{12.0\ \text{cm}} (-4.07\ \text{kN/C})dx$$

The potential difference between the plates is

$$V = \left(4.07\times10^3\ \text{N/C}\right)(0.120\ \text{m}) = \boxed{488\ \text{V}}$$

(c) The positive proton starts from rest and accelerates from higher to lower potential. Taking $V_i = 488$ V and $V_f = 0$, by energy conservation, we find the proton's final kinetic energy.

$$(K+qV)_i = (K+qV)_f \rightarrow K_f = qV_i$$

$$\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$$

$$qV_i = \left(1.60\times10^{-19}\text{ C}\right)(488\text{ V}) = \frac{1}{2}mv_f^2 = \boxed{7.81\times10^{-17}\text{ J}}$$

(d) From the kinetic energy of part (c),

$$K = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(7.81\times10^{-17}\text{ J}\right)}{1.67\times10^{-27}\text{ kg}}} = 3.06\times10^5\text{ m/s} = \boxed{306\text{ km/s}}$$

(e) Using the constant-acceleration equation, $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$,

$$a = \frac{v_f^2 - v_i^2}{2\left(x_f - x_i\right)} = \frac{\left(3.06\times10^5\text{ m/s}\right)^2 - 0}{2(0.120\text{ m})}$$

$$= \boxed{3.90\times10^{11}\text{ m/s}^2} \text{ toward the negative plate}$$

(f) The net force on the proton is given by Newton's second law:

$$\sum F = ma = \left(1.67\times10^{-27}\text{ kg}\right)\left(3.90\times10^{11}\text{ m/s}^2\right)$$

$$= \boxed{6.51\times10^{-16}\text{ N}} \text{ toward the negative plate}$$

(g) The magnitude of the electric field is

$$E = \frac{F}{q} = \frac{6.51\times10^{-16}\text{ N}}{1.60\times10^{-19}\text{ C}} = \boxed{4.07\text{ kN/C}}$$

(h) They are the same.

P25.70 (a) Inside the sphere, $\boxed{E_x = E_y = E_z = 0}$.

(b) Outside,

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-3/2}\right)$$

$$= -\left[0 + 0 + E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2x)\right]$$

$$E_x = \boxed{3E_0 a^3 xz\left(x^2+y^2+z^2\right)^{-5/2}}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-3/2}\right)$$

$$= -E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2} 2y$$

$$E_y = \boxed{3E_0 a^3 yz\left(x^2+y^2+z^2\right)^{-5/2}}$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\left[V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$= E_0 - E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2z) - E_0 a^3\left(x^2+y^2+z^2\right)^{-3/2}$$

$$E_z = \boxed{E_0 + E_0 a^3\left(2z^2 - x^2 - y^2\right)\left(x^2+y^2+z^2\right)^{-5/2}}$$

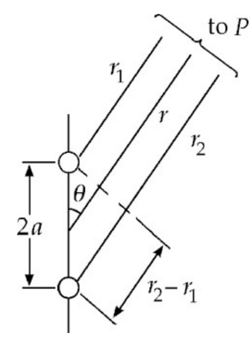
Challenge Problems

P25.71 (a) The total potential is

$$V = \frac{k_e q}{r_1} - \frac{k_e q}{r_2} = \frac{k_e q}{r_1 r_2}\left(r_2 - r_1\right)$$

From the figure, for $r >> a$, $r_2 - r_1 \approx 2a\cos\theta$. Note that r_1 is approximately equal to r_2. Then

$$V \approx \frac{k_e q}{r_1 r_2} 2a\cos\theta \approx \frac{k_e p\cos\theta}{r^2}$$



ANS. FIG. P25.71

(b) $$E_r = -\frac{\partial V}{\partial r} = \boxed{\frac{2k_e p\cos\theta}{r^3}}$$

In spherical coordinates, the θ component of the gradient is $-\frac{1}{r}\left(\frac{\partial}{\partial\theta}\right)$. Therefore,

$$E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) = \boxed{\frac{k_e p\sin\theta}{r^3}}$$

(c) For $r >> a$, $\theta = 90°$: $E_r(90°) = 0$, $E_\theta(90°) = \frac{k_e p}{r^3}$

For $r >> a$, $\theta = 0°$: $E_r(0°) = \frac{2k_e p}{r^3}$, $E_\theta(0°) = 0$

$\boxed{\text{Yes,}}$ these results are reasonable.

(d) $\boxed{\text{No,}}$ because as $r \to 0$, $E \to \infty$. The magnitude of the electric field between the charges of the dipole is not infinite.

(e) Substituting $r_1 \approx r_2 \approx r = (x^2 + y^2)^{1/2}$ and $\cos\theta = \dfrac{y}{\left(x^2+y^2\right)^{1/2}}$ into

$$V = \frac{k_e p\cos\theta}{r^2} \text{ gives } \boxed{V = \frac{k_e p y}{\left(x^2+y^2\right)^{3/2}}}.$$

(f) $\boxed{E_x = -\dfrac{\partial V}{\partial x} = \dfrac{3k_e p x y}{\left(x^2+y^2\right)^{5/2}}}$ and $\boxed{E_y = -\dfrac{\partial V}{\partial y} = \dfrac{k_e p\left(2y^2 - x^2\right)}{\left(x^2+y^2\right)^{5/2}}}$

P25.72 Following the problem's suggestion, we use $dU = Vdq$, where the potential is given by $V = \dfrac{k_e q}{r}$. The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho\left(4\pi r^2 dr\right)$ and the charge q in a sphere of radius r is

$$q = 4\pi\rho\int_0^r r^2 dr = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substituting this into the expression for dU, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e\rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho\left(4\pi r^2 dr\right) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4 dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2\int_0^R r^4 dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

But the *total* charge, $Q = \rho\dfrac{4}{3}\pi R^3$. Therefore, $\boxed{U = \dfrac{3}{5}\dfrac{k_e Q^2}{R}}$.

P25.73 For an element of area which is a ring of radius r and width dr, the incremental potential is given by $dV = \dfrac{k_e dq}{\sqrt{r^2 + x^2}}$, where

$$dq = \sigma dA = Cr\left(2\pi r dr\right)$$

The electric potential is then given by

$$V = C\left(2\pi k_e\right)\int_0^R \frac{r^2 dr}{\sqrt{r^2 + x^2}}$$

From a table of integrals,

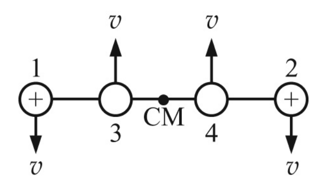
$$\int \frac{r^2 dr}{\sqrt{r^2+x^2}} = \frac{r}{2}\sqrt{r^2+x^2} - \frac{x^2}{2}\ln\left(r+\sqrt{r^2+x^2}\right)$$

The potential then becomes, after substituting and rearranging,

$$V = C(2\pi k_e)\int_0^R \frac{r^2 dr}{\sqrt{r^2+x^2}}$$

$$= \boxed{\pi k_e C\left[R\sqrt{R^2+x^2} + x^2 \ln\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]}$$

P25.74 Take the illustration presented with the problem as an initial picture. No external horizontal forces act on the set of four balls, so its center of mass stays fixed at the location of the center of the square. As the charged balls 1 and 2 swing out and away from each other, balls 3 and 4 move up with equal y-components of velocity. The maximum-kinetic-energy point is illustrated. System energy is conserved because it is isolated:



ANS. FIG. P25.74

$$K_i + U_i = K_f + U_f$$
$$0 + U_i = K_f + U_f$$
$$\rightarrow U_i = K_f + U_f$$

$$\frac{k_e q^2}{a} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{k_e q^2}{3a}$$

$$\frac{2k_e q^2}{3a} = 2mv^2 \quad \rightarrow \quad \boxed{v = \sqrt{\frac{k_e q^2}{3am}}}$$

P25.75 (a) Take the origin at the point where we will find the potential. One ring, of width dx, has charge $\frac{Qdx}{h}$ and, according to Example 25.5, creates potential

$$dV = \frac{k_e Q dx}{h\sqrt{x^2+R^2}}$$

The whole stack of rings creates potential

$$V = \int_{\text{all charge}} dV = \int_d^{d+h} \frac{k_e Q dx}{h\sqrt{x^2+R^2}} = \frac{k_e Q}{h}\ln\left(x+\sqrt{x^2+R^2}\right)\Big|_d^{d+h}$$

$$= \boxed{\frac{k_e Q}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}$$

(b) A disk of thickness dx has charge $\frac{Qdx}{h}$ and charge-per-area $\frac{Qdx}{\pi R^2 h}$. According to Example 25.6, it creates potential

$$dV = 2\pi k_e \frac{Qdx}{\pi R^2 h}\left(\sqrt{x^2 + R^2} - x\right)$$

Integrating,

$$V = \int_d^{d+h} \frac{2k_e Q}{R^2 h}\left(\sqrt{x^2 + R^2}\,dx - x\,dx\right)$$

$$= \frac{2k_e Q}{R^2 h}\left[\frac{1}{2}x\sqrt{x^2 + R^2} + \frac{R^2}{2}\ln\left(x + \sqrt{x^2 + R^2}\right) - \frac{x^2}{2}\right]_d^{d+h}$$

$$\boxed{V = \frac{k_e Q}{R^2 h}\left[(d+h)\sqrt{(d+h)^2 + R^2} - d\sqrt{d^2 + R^2} - 2dh - h^2 + R^2 \ln\left(\frac{d + h + \sqrt{(d+h)^2 + R^2}}{d + \sqrt{d^2 + R^2}}\right)\right]}$$

P25.76 The plates create a uniform electric field to the right in the picture, with magnitude

$$\frac{V_0 - (-V_0)}{d} = \frac{2V_0}{d}$$

Assume the ball swings a small distance x to the right so that the thread is at angle θ from the vertical. The ball moves to a place where the voltage created by the plates is lower by

$$-Ex = -\frac{2V_0}{d}x$$

Because its ground connection maintains the ball at $V = 0$, charge q flows from ground onto the ball, so that

$$-\frac{2V_0 x}{d} + \frac{k_e q}{R} = 0 \quad \rightarrow \quad q = \frac{2V_0 x R}{k_e d}$$

Then the ball feels an electric force

$$F = qE = \frac{4V_0^2 x R}{k_e d^2}$$

to the right. For equilibrium, the electric force must be balanced by the horizontal component of string tension according to

$$T\sin\theta = qE = \frac{4V_0^2 xR}{k_e d^2}$$

and the weight of the ball must be balanced by the vertical component of string tension according to $T\cos\theta = mg$. Dividing the expression for the horizontal component by that for the vertical component, we find that

$$\tan\theta = \frac{4V_0^2 xR}{k_e d^2 mg}$$

For very small angles, we can approximate $\tan\theta \sim \sin\theta = \frac{x}{L}$, so the above expression becomes

$$\frac{x}{L} = \frac{4V_0^2 xR}{k_e d^2 mg} \quad \rightarrow \quad V_0 = \left(\frac{k_e d^2 mg}{4RL}\right)^{1/2} \text{ for small } x$$

If V_0 is less than this value, the only equilibrium position of the ball is hanging straight down. If V_0 exceeds this value, the ball will swing over to one plate or the other.

P25.77 For the given charge distribution,

$$V(x,\ y,\ z) = \frac{k_e(q)}{r_1} + \frac{k_e(-2q)}{r_2}$$

where $r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$

and $r_2 = \sqrt{x^2 + y^2 + z^2}$

The surface on which $V(x, y, z) = 0$ is given by

$$k_e q\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0 \qquad \text{or} \qquad 2r_1 = r_2$$

This gives:

$$4(x+R)^2 + 4y^2 + 4z^2 = x^2 + y^2 + z^2$$

which may be written in the form:

$$x^2 + y^2 + z^2 + \left(\frac{8}{3}R\right)x + (0)y + (0)z + \left(\frac{4}{3}R^2\right) = 0 \qquad \textbf{[1]}$$

The general equation for a sphere of radius a centered at (x_0, y_0, z_0) is:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - a^2 = 0$$

or

$$x^2 + y^2 + z^2 + (-2x_0)x + (-2y_0)y + (-2z_0)z + (x_0^2 + y_0^2 + z_0^2 - a^2) = 0 \qquad \textbf{[2]}$$

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V = 0$ is indeed a sphere and that:

$$-2x_0 = \frac{8}{3}R;\ -2y_0 = 0;\ -2z_0 = 0;\ x_0^2 + y_0^2 + z_0^2 - a^2 = \frac{4}{3}R^2$$

Thus, $x_0 = -\frac{4}{3}R$, $y_0 = z_0 = 0$, and $a^2 = \left(\frac{16}{9} - \frac{4}{3}\right)R^2 = \frac{4}{9}R^2$

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\frac{4}{3}R, 0, 0\right)}$, having a radius $\boxed{\frac{2}{3}R}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P25.2 (a) -6.00×10^{-4} J; (b) -50.0 V

P25.4 1.35 MJ

P25.6 See P25.6 for full explanation.

P25.8 (a) -2.31 kV; (b) Because a proton is more massive than an electron, a proton traveling at the same speed as an electron has more initial kinetic energy and requires a greater magnitude stopping potential; (c) $\Delta V_p / \Delta V_e = -m_p / m_e$

P25.10 (a) isolated; (b) electric potential energy and elastic potential energy; (c) $\dfrac{2QE}{k}$; (d) Particle in equilibrium; (e) $\dfrac{QE}{k}$; (f) $\dfrac{d^2x'}{dt^2} = -\dfrac{kx'}{m}$; (g) $2\pi\sqrt{\dfrac{m}{k}}$; (h) The period does not depend on the electric field. The electric field just shifts the equilibrium point for the spring, just like a gravitational field does for an object hanging from a vertical spring.

P25.12 (a) -5.76×10^{-7} V; (b) 3.84×10^{-7} V; (c) Because the charge of the proton has the same magnitude as that of the electron, only the sign of the answer to part (a) would change.

P25.14 (a) 5.39 kV; (b) 10.8 kV

P25.16 (a) 103 V; (b) -3.85×10^{-7} J, positive work must be done

P25.18 (a) 5.43 kV; (b) 6.08 kV; (c) 658 V

P25.20 (a) 6.00 m; (b) -2.00 μC

P25.22 -11.0×10^7 V

P25.24 $5.41\dfrac{k_e Q^2}{s}$

P25.26 (a) $\dfrac{2}{\sqrt{(x/a)^2+1}}$; (b) See ANS. FIG. P25.26(b).

P25.28 (a) no point; (b) $\dfrac{2k_e q}{a}$

P25.30 $\Delta E_{\text{int}} = \dfrac{5k_e q^2}{9d}$

P25.32 (a) $v_1 = \sqrt{\dfrac{2m_2k_eq_1q_2}{m_1(m_1+m_2)}\left(\dfrac{1}{r_1+r_2}-\dfrac{1}{d}\right)}$

and $v_2 = \sqrt{\dfrac{2m_1k_eq_1q_2}{m_2(m_1+m_2)}\left(\dfrac{1}{r_1+r_2}-\dfrac{1}{d}\right)}$; (b) faster than calculated in (a)

P25.34 $v = \sqrt{\left(1+\dfrac{1}{\sqrt{8}}\right)\dfrac{k_eq^2}{mL}}$

P25.36 See ANS. FIG. P25.36.

P25.38 See ANS. FIG. P25.38.

P25.40 (a) $E_A > E_B$ since $E = \dfrac{\Delta V}{\Delta s}$; (b) 200 N/C; (c) See ANS. FIG. P25.40.

P25.42 $E_y = \dfrac{k_eQ}{y\sqrt{\ell^2+y^2}}$

P25.44 –1.51 MV

P25.46 $-\dfrac{k_e\alpha L}{2}\ln\left[\dfrac{\sqrt{b^2+(L^2/4)}-L/2}{\sqrt{b^2+(L^2/4)}+L/2}\right]$

P25.48 No. A conductor of any shape forms an equipotential surface. However, if the surface varies in shape, there is no clear way to relate electric field at a point on the surface to the potential of the surface.

P25.50 (a) 0, 1.67 MV; (b) 5.84 MN/C away, 1.17 MV; (c) 11.9 MN/C away, 1.67 MV

P25.52 (a) 450 kV; 7.51 μC

P25.54 (a) 1.06 nC/m^2, negative; (b) –542 kC; (c) –764 MV; (d) The person's head is higher in potential by 210 V; (e) 4.88×10^3 N away from Earth; (f) The gravitational force is in the opposite direction and 4.08×10^{16} times larger. Electrical forces are negligible in accounting for planetary motion.

P25.56 (a) $\dfrac{m_1v}{m_1+m_2}$; (b) $\dfrac{2k_eq_1q_2(m_1+m_2)}{m_1m_2v^2}$; (c) $\left(\dfrac{m_1-m_2}{m_1+m_2}\right)v\,\hat{\mathbf{i}}$; (d) $\left(\dfrac{2m_1}{m_1+m_2}\right)v\,\hat{\mathbf{i}}$

P25.58 (a) ~10^4 V; (b) ~10^{-6} C

P25.60 (a) $-\dfrac{k_eq}{4a}$; (b) The approximate expression $-2k_eqa/x^2$ gives $-k_eq/4.5$, which is different by only 11.1%.

P25.62 $\dfrac{k_e Q^2}{2R}$

P25.64 Even if the charge were to accelerate to infinity, it would only achieve a maximum speed of 30.0 m/s, so it cannot strike the wall of your laboratory at 40.0 m/s.

P25.66 (a) 7.19 V; (b) 7.67 V; (c) 7.84 V; (d) The exact result, represented as 7.90 V, is approximated to within 0.8% by the four-particle version.

P25.68 (a) $\Delta V = 2k_e\lambda \ln\left(\dfrac{r_a}{r_b}\right)$; (b) $E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)$

P25.70 (a) $E_x = E_y = E_z = 0$; (b) $E_x = 3E_0 a^3 xz\left(x^2+y^2+z^2\right)^{-5/2}$,
$E_y = 3E_0 a^3 yz\left(x^2+y^2+z^2\right)^{-5/2}$, $E_z = E_0 + E_0 a^3\left(2z^2-x^2-y^2\right)\left(x^2+y^2+z^2\right)^{-5/2}$

P25.72 $U = \dfrac{3}{5}\dfrac{k_e Q^2}{R}$

P25.74 $v = \sqrt{\dfrac{k_e q^2}{3am}}$

P25.76 See P25.76 for full explanation.