26

Capacitance and Dielectrics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ26.1 (i) Answer (a). Because $C = \kappa \epsilon_0 A/d$ and the dielectric constant κ increases.

(ii) Answer (a). Because ΔV is constant, and C increases, so $Q = C\Delta V$ increases.

(iii) Answer (c).

(iv) Answer (a). Because ΔV is constant, and C increases, $U_E = \frac{1}{2}C(\Delta V)^2$ increases.

OQ26.2 Answer (b). The capacitance of a metal sphere is proportional to its radius ($C = Q/V = R/k_e$), and its volume is proportional to radius cubed; therefore, the capacitance of a metal sphere is proportional to the cube root of the volume: $3^{1/3}$.

OQ26.3 Answer (a).

$$C = \frac{\kappa \in_0 A}{d} = \frac{(1.00\times10^{2})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.00\times10^{-4}\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 8.85\times10^{-11}\ \text{F} \quad \text{or} \quad 88.5\ \text{pF}$$

OQ26.4 Answer (c). The voltage remains constant, but C decreases by a factor of 2 because $C = \kappa \in_0 A/d \to \kappa \in_0 A/(2d) = C/2$. Therefore,

$$U_E = \frac{1}{2}C(\Delta V)^2 \quad \to \quad \left(\frac{1}{2}\right)\left(\frac{1}{2}C\right)(\Delta V)^2 = \frac{1}{2}U_E$$

OQ26.5 Answer (b). Choice (a) is not true because $1/C_{\text{eq}}$ is always larger than $1/C_1 + 1/C_2 + 1/C_3$. Choice (c) is not true because capacitors in series carry the same charge Q, and the voltage across capacitance C_i *is* $\Delta V_i = Q/C_i$. Choices (d) and (e) are not true because capacitors in series carry the same charge.

OQ26.6 Answer (b). Let C = the capacitance of an individual capacitor, and C_S represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$Q = C\Delta V_{\text{charge}} = (5.00\times10^{-4}\ \text{F})(800\ \text{V}) = 0.400\ \text{C}$$

While being discharged in series,

$$\Delta V_{\text{discharge}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\ \text{C}}{5.00\times10^{-5}\ \text{F}} = 8.00\ \text{kV}$$

(or 10 times the original voltage).

OQ26.7 (i) Answer (b), because $Q = C\Delta V$.

(ii) Answer (a), because $U_E = \frac{1}{2}C(\Delta V)^2$.

OQ26.8 Answer (d). Let C_2 be the capacitance of the large capacitor and C_1 that of the small one. The equivalent capacitance is

$$C_{\text{eq}} = \frac{1}{1/C_1 + 1/C_2} = \frac{1}{\left(\frac{C_1 + C_2}{C_1 C_2}\right)} = C_1\left(\frac{C_2}{C_2 + C_1}\right)$$

This is slightly less than C_1.

OQ26.9 Answer (a). Charge Q remains fixed, but the capacitance doubles: $C = \kappa \in_0 A/d \to (2\kappa) \in_0 A/d = 2C$. Therefore, $\Delta V = Q/C \to Q/(2C) = \Delta V/2$.

OQ26.10 **(i)** Answer (c). For capacitors in parallel, choices (a), (b), (d), and (e) are not true because the potential difference ΔV is the same, and the charge across capacitance C_i is $Q_i = C_i \Delta V$.

(ii) Answer (e). Although the charges on capacitors in series are the same, the equivalent capacitance is less than the capacitance of any of the capacitors in the group, because $1/C_{eq}$ is always larger than $1/C_1 + 1/C_2 + 1/C_3$; therefore, choices (a) and (c) are not true. Choices (b), (c), and (d) are not true because the charge Q is the same, and choice (c) is also not true because the potential difference across capacitance C_i is $\Delta V_i = Q/C_i$.

OQ26.11 Answer (b). The charge stays constant but C decreases by a factor of 2 because $C = \kappa \in_0 A/d \to \kappa \in_0 A/(2d) = C/2$. Therefore,

$$U_E = \frac{Q^2}{2C} \quad \to \quad \frac{Q^2}{2\left(\frac{1}{2}C\right)} = 2U_E$$

OQ26.12 We find the capacitance, voltage, charge, and energy for each capacitor.

(a) $C = 20\ \mu\text{F}$ $\Delta V = 4\ \text{V}$ $Q = C\Delta V = 80\ \mu\text{C}$
$U_E = \frac{1}{2}Q\Delta V = 160\ \mu\text{J}$

(b) $C = 30\ \mu\text{F}$ $\Delta V = Q/C = 3\ \text{V}$ $Q = 90\ \mu\text{C}$
$U_E = 135\ \mu\text{J}$

(c) $C = Q/\Delta V = 40\ \mu\text{F}$ $\Delta V = 2\ \text{V}$ $Q = 80\ \mu\text{C}$
$U_E = 80\ \mu\text{J}$

(d) $C = 10\ \mu\text{F}$ $\Delta V = (2U/C)^{1/2} = 5\ \text{V}$ $Q = 50\ \mu\text{C}$
$U_E = 125\ \mu\text{J}$

(e) $C = 2U_E/(\Delta V)^2 = 5\ \mu\text{F}$ $\Delta V = 10\ \text{V}$ $Q = 50\ \mu\text{C}$
$U_E = 250\ \mu\text{J}$

(i) The ranking by capacitance is c > b > a > d > e.

(ii) The ranking by voltage ΔV is e > d > a > b > c.

(iii) The ranking by charge Q is b > a = c > d = e.

(iv) The ranking by energy U_E is e > a > b > d > c.

OQ26.13 (a) False. (b) True. The equation $C = Q/\Delta V$ implies that as charge Q approaches zero, the voltage ΔV also approaches zero so that their ratio remains constant.

OQ26.14 (i) Answer (b). Because $C = \kappa \in_0 A/d$ and the plate separation d increases.

(ii) Answer (c).

(iii) Answer (c). Because $E = Q/\kappa \in_0 A$ remains the same.

(iv) Answer (a). Because $\Delta V = Ed$ and d increases.

ANSWERS TO CONCEPTUAL QUESTIONS

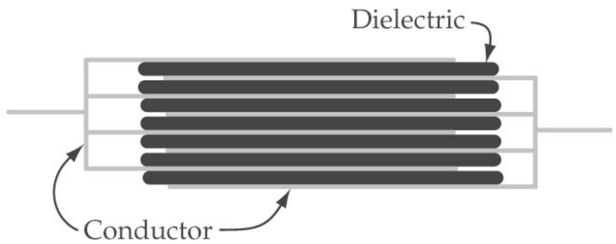
CQ26.1 (a) The capacitor may be charged!

(b) Discharge the capacitor by connecting its terminals together.

CQ26.2 Put a material with higher dielectric strength between the plates, or evacuate the space between the plates. At very high voltages, you may want to cool off the plates or choose to make them of a different chemically stable material, because atoms in the plates themselves can ionize, showing *thermionic emission* under high electric fields.

CQ26.3 The primary choice would be the dielectric. You would want to choose a dielectric that has a large dielectric constant and dielectric strength, such as strontium titanate, where $\kappa \approx 233$ (Table 26.1). A convenient choice could be thick plastic or Mylar. Secondly, geometry would be a factor. To maximize capacitance, one would want the individual plates as close as possible, since the capacitance is proportional to the inverse of the plate separation—hence the need for a dielectric with a high dielectric strength. Also, one would want to build, instead of a single parallel plate capacitor, several capacitors in parallel. This could be achieved through "stacking" the plates of the capacitor. For example, you can alternately lay down sheets of a conducting material, such as aluminum foil, sandwiched between sheets of insulating dielectric. Making sure that none of the conducting sheets are in contact with their immediate neighbors, connect every other plate together. ANS. FIG. CQ26.3 illustrates this idea.

This technique is often used when "home-brewing" signal capacitors for radio applications, as they can withstand huge potential differences without flashover (without either discharge between plates around the dielectric or dielectric breakdown). One variation on this technique is to sandwich together flexible materials such as aluminum roof flashing and thick plastic, so the whole product can be rolled up into a "capacitor burrito" and placed in an insulating tube, such as a PVC pipe, and then filled with motor oil (again to prevent flashover).



ANS. FIG. CQ26.3

CQ26.4 The dielectric decreases the electric field between the plates, causing the potential difference to decrease for the same amount of charge. More charge may be placed on the capacitor before the capacitor experiences dielectric breakdown (resulting in charge jumping from one plate to the other, and in a path being burned through the dielectric) because the electric forces between charges on opposite plates are smaller. The capacitor can have a higher maximum operating voltage, allowing it to hold more charge.

CQ26.5 The work done, $W = Q\Delta V$, is the work done by an external agent, like a battery, to move a charge through a potential difference, ΔV. To determine the energy in a charged capacitor, we must add the work done to move bits of charge from one plate to the other. Initially, there is no potential difference between the plates of an uncharged capacitor. As more charge is transferred from one plate to the other, the potential difference increases, meaning that more work is needed to transfer each additional bit of charge. The total work is given by $W = \frac{1}{2}Q\Delta V$. Another explanation is that the charge Q is moved through an average potential difference $\frac{1}{2}\Delta V$, requiring total work $W = \frac{1}{2}Q\Delta V$.

***CQ26.6** The potential difference must decrease. Since there is no external power supply, the charge on the capacitor, Q, will remain constant—that is, assuming that the resistance of the meter is sufficiently large. Adding a dielectric *increases* the capacitance, which must therefore *decrease* the potential difference between the plates.

CQ26.7 A capacitor stores energy in the electric field between the plates. This is most easily seen when using a "dissectible" capacitor. If the capacitor is charged, carefully pull it apart into its component pieces. One will find that very little residual charge remains on each plate. When reassembled, the capacitor is suddenly "recharged"—by induction—due to the electric field set up and "stored" in the dielectric. This proves to be an instructive classroom demonstration, especially when you ask a student to reconstruct the capacitor without supplying him/her with any rubber gloves or other insulating material. (Of course, this is *after* they sign a liability waiver.)

CQ26.8 The work you do to pull the plates apart becomes additional electric potential energy stored in the capacitor. The charge is constant and the capacitance decreases but the potential difference increases to drive up the potential energy $\frac{1}{2}Q\Delta V$. The electric field between the plates is constant in strength but fills more volume as you pull the plates apart.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 20.1 Definition of Capacitance

P26.1 (a) From Equation 26.1 for the definition of capacitance, $C = \frac{Q}{\Delta V}$, we have

$$\Delta V = \frac{Q}{C} = \frac{27.0\ \mu C}{3.00\ \mu\text{F}} = \boxed{9.00\ \text{V}}$$

(b) Similarly,

$$\Delta V = \frac{Q}{C} = \frac{36.0\ \mu C}{3.00\ \mu\text{F}} = \boxed{12.0\ \text{V}}$$

P26.2 (a) $C = \frac{Q}{\Delta V} = \frac{10.0 \times 10^{-6}\ \text{C}}{10.0\ \text{V}} = 1.00 \times 10^{-6}\ \text{F} = \boxed{1.00\ \mu\text{F}}$

(b) $\Delta V = \frac{Q}{C} = \frac{100 \times 10^{-6}\ \text{C}}{1.00 \times 10^{-6}\ \text{F}} = \boxed{100\ \text{V}}$

P26.3 (a) $Q = C\Delta V = (4.00 \times 10^{-6}\text{ F})(12.0\text{ V}) = 4.80 \times 10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C\Delta V = (4.00 \times 10^{-6}\text{ F})(1.50\text{ V}) = 6.00 \times 10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

Section 26.2 Calculating Capacitance

P26.4 (a) For a spherical capacitor with inner radius a and outer radius b,

$$C = \frac{ab}{k_e(b-a)} = \frac{(0.070\,0\text{ m})(0.140\text{ m})}{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(0.140\text{ m} - 0.070\,0\text{ m})}$$
$$= \boxed{15.6\text{ pF}}$$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{4.00 \times 10^{-6}\text{ C}}{1.56 \times 10^{-11}\text{ F}} = 2.57 \times 10^5\text{ V} = \boxed{257\text{ kV}}$

P26.5 (a) The capacitance of a cylindrical capacitor is

$$C = \frac{\ell}{2k_e \ln(b/a)}$$
$$= \frac{50.0\text{ m}}{2(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\ln(7.27\text{ mm}/2.58\text{ mm})}$$
$$= \boxed{2.68\text{ nF}}$$

(b) Method 1: $\Delta V = 2k_e\lambda \ln\left(\dfrac{b}{a}\right)$

$$\lambda = \frac{Q}{\ell} = \frac{8.10 \times 10^{-6}\text{ C}}{50.0\text{ m}} = 1.62 \times 10^{-7}\text{ C/m}$$

$$\Delta V = 2(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.62 \times 10^{-7}\text{ C/m})\ln\left(\frac{7.27\text{ mm}}{2.58\text{ mm}}\right)$$
$$= \boxed{3.02\text{ kV}}$$

Method 2: $\Delta V = \dfrac{Q}{C} = \dfrac{8.10 \times 10^{-6}\text{ C}}{2.68 \times 10^{-9}\text{ F}} = \boxed{3.02\text{ kV}}$

P26.6 (a) $C = \dfrac{\kappa \epsilon_0 A}{d} = \dfrac{(1.00)(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1.00 \times 10^3\text{ m})^2}{800\text{ m}}$

$= \boxed{11.1\text{ nF}}$

(b) The potential between ground and cloud is

$$\Delta V = Ed = \left(3.00\times10^{6}\ \text{N/C}\right)(800\ \text{m}) = 2.40\times10^{9}\ \text{V}$$

$$Q = C(\Delta V) = \left(11.1\times10^{-9}\ \text{C / V}\right)\left(2.40\times10^{9}\ \text{V}\right) = \boxed{26.6\ \text{C}}$$

P26.7 We have $Q = C\Delta V$ and $C = \epsilon_0 A / d$. Thus, $Q = \epsilon_0 A\Delta V/d$

The surface charge density on each plate has the same magnitude, given by

$$\sigma = \frac{Q}{A} = \frac{\epsilon_0 \Delta V}{d}$$

Thus,

$$d = \frac{\epsilon_0 \Delta V}{Q/A} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(150\ \text{V})}{\left(30.0\times10^{-9}\ \text{C/cm}^2\right)}$$

$$d = \left(4.43\times10^{-2}\,\frac{\text{V}\cdot\text{C}\cdot\text{cm}^2}{\text{N}\cdot\text{m}^2}\right)\frac{\left(1\ \text{m}^2\right)}{\left(10^4\ \text{cm}^2\right)}\frac{\text{J}}{\text{V}\cdot\text{C}}\frac{\text{N}\cdot\text{m}}{\text{J}} = \boxed{4.43\ \mu\text{m}}$$

P26.8 (a) $C = \dfrac{\kappa\,\epsilon_0 A}{d} = \dfrac{(1.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.30\times10^{-4}\ \text{m}^2\right)}{1.50\times10^{-3}\ \text{m}}$

$= 1.36\times10^{-12}\ \text{F} = \boxed{1.36\ \text{pF}}$

(b) $Q = C\Delta V = \left(1.36\ \text{pF}\right)\left(12.0\ \text{V}\right) = \boxed{16.3\ \text{pC}}$

(c) $E = \dfrac{\Delta V}{d} = \dfrac{12.0\ \text{V}}{1.50\times10^{-3}\ \text{m}} = \boxed{8.00\times10^{3}\ \text{V/m}}$

P26.9 (a) The potential difference between two points in a uniform electric field is $\Delta V = Ed$, so

$$E = \frac{\Delta V}{d} = \frac{20.0\ \text{V}}{1.80\times10^{-3}\ \text{m}} = \boxed{1.11\times10^{4}\ \text{V/m}}$$

(b) The electric field between capacitor plates is $E = \dfrac{\sigma}{\epsilon_0}$, so $\sigma = \epsilon_0 E$:

$$\sigma = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.11\times10^{4}\ \text{V/m}\right) = 9.83\times10^{-8}\ \text{C/m}^2$$

$$= \boxed{98.3\ \text{nC/m}^2}$$

(c) For a parallel-plate capacitor, $C = \dfrac{\epsilon_0 A}{d}$:

$$C = \frac{\left(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}\right)\left(7.60 \times 10^{-4}\ \mathrm{m^2}\right)}{1.80 \times 10^{-3}\ \mathrm{m}}$$

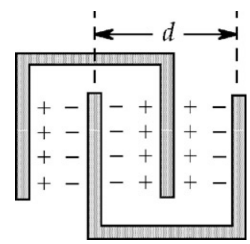
$$= 3.74 \times 10^{-12}\ \mathrm{F} = \boxed{3.74\ \mathrm{pF}}$$

(d) The charge on each plate is $Q = C\Delta V$:

$$Q = \left(3.74 \times 10^{-12}\ \mathrm{F}\right)(20.0\ \mathrm{V}) = \boxed{74.7\ \mathrm{pC}}$$

P26.10 With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With $\theta = 0$, the overlap area is that of a semi-circle, $\dfrac{\pi R^2}{2}$. By proportion, the effective area of a single sheet of charge is

$$\frac{(\pi - \theta)R^2}{2}$$



ANS. FIG. P26.10

When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is $2N - 1$ and the total capacitance is

$$C = (2N-1)\frac{\epsilon_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1)\epsilon_0(\pi - \theta)R^2/2}{d/2}$$

$$= \boxed{\frac{(2N-1)\epsilon_0(\pi - \theta)R^2}{d}}$$

P26.11 (a) The electric field outside a spherical charge distribution of radius R is $E = k_e q/r^2$. Therefore,

$$q = \frac{Er^2}{k_e} = \frac{\left(4.90 \times 10^4\ \mathrm{N/C}\right)(0.210\ \mathrm{m})^2}{8.99 \times 10^9\ \mathrm{N \cdot m^2 / C^2}} = 0.240\ \mu\mathrm{C}$$

Then

$$\sigma = \frac{q}{A} = \frac{0.240 \times 10^{-6}\ \mathrm{C}}{4\pi(0.120\ \mathrm{m})^2} = \boxed{1.33\ \mu\mathrm{C/m^2}}$$

(b) For an isolated charged sphere of radius R,

$$C = 4\pi\epsilon_0 r = 4\pi\left(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}\right)(0.120\ \mathrm{m}) = \boxed{13.3\ \mathrm{pF}}$$

P26.12 $\sum F_y = 0: \quad T\cos\theta - mg = 0$

$\sum F_x = 0: \quad T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$,

so $E = \dfrac{mg}{q}\tan\theta$

and $\Delta V = Ed = \boxed{\dfrac{mgd\tan\theta}{q}}$.

Section 26.3 Combinations of Capacitors

P26.13 (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{\text{eq}} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = \boxed{9.00\ \text{V}}$$

(c) $Q_5 = C\Delta V = (5.00\ \mu\text{F})(9.00\ \text{V}) = \boxed{45.0\ \mu\text{C}}$

$Q_{12} = C\Delta V = (12.0\ \mu\text{F})(9.00\ \text{V}) = \boxed{108\ \mu\text{C}}$

P26.14 (a) In series capacitors add as

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{12.0\ \mu\text{F}}$$

$$C_{\text{eq}} = \boxed{3.53\ \mu\text{F}}$$

(c) We must answer part (c) first before we can answer part (b). The charge on the equivalent capacitor is

$$Q_{\text{eq}} = C_{\text{eq}}\Delta V = (3.53\ \mu\text{F})(9.00\ \text{V}) = 31.8\ \mu\text{C}$$

Each of the series capacitors has this same charge on it.

So $Q_1 = Q_2 = \boxed{31.8\ \mu\text{C}}$.

(b) The potential difference across each is

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{6.35\ \text{V}}$$

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}} = \boxed{2.65\ \text{V}}$$

P26.15 (a) When connected in series, the equivalent capacitance is $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$, or

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.20\ \mu\text{F}} + \frac{1}{8.50\ \mu\text{F}} \rightarrow C_{\text{eq}} = \boxed{2.81\ \mu\text{F}}$$

(b) When connected in parallel, the equivalent capacitance is

$$C_{\text{eq}} = C_1 + C_2 = 4.20\ \mu\text{F} + 8.50\ \mu\text{F} = \boxed{12.70\ \mu\text{F}}$$

P26.16 (a) When connected in series, the equivalent capacitance is $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$, or

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.50\ \mu\text{F}} + \frac{1}{6.25\ \mu\text{F}} \rightarrow C_{\text{eq}} = 1.79\ \mu\text{F}$$

Capacitors in series carry the same charge as their equivalent capacitance:

$$Q = C_{\text{eq}}(\Delta V) = (1.79\ \mu\text{F})(6.00\ \text{V}) = \boxed{10.7\ \mu\text{C}} \text{ on each capacitor}$$

(b) When connected in parallel, each capacitor has the same potential difference across it. The charge stored on each capacitor is then

For $C_1 = 2.50\ \mu\text{F}$: $Q_1 = C_1(\Delta V) = (2.50\ \mu\text{F})(6.00\ \text{V}) = \boxed{15.0\ \mu\text{C}}$

For $C_2 = 6.25\ \mu\text{F}$: $Q_2 = C_2(\Delta V) = (6.25\ \mu\text{F})(6.00\ \text{V}) = \boxed{37.5\ \mu\text{C}}$

P26.17 (a) In $\boxed{\text{series}}$, to reduce the effective capacitance:

$$\frac{1}{32.0\ \mu\text{F}} = \frac{1}{34.8\ \mu\text{F}} + \frac{1}{C_s} \rightarrow \frac{1}{C_s} = \frac{1}{32.0\ \mu\text{F}} - \frac{1}{34.8\ \mu\text{F}}$$

$$\rightarrow C_s = \boxed{398\ \mu\text{F}}$$

(b) In $\boxed{\text{parallel}}$, to increase the total capacitance:

$$29.8\ \mu\text{F} + C_p = 32.0\ \mu\text{F}$$

$$C_p = \boxed{2.20\ \mu\text{F}}$$

P26.18 The capacitance of the combination of extra capacitors must be $\frac{7}{3}C - C = \frac{4}{3}C$. The possible combinations are: one capacitor: C; two capacitors: 2C or $\frac{1}{2}C$; three capacitors: $3C$, $\frac{1}{3}C$, $\frac{2}{3}C$ or $\frac{3}{2}C$. None of these is $\frac{4}{3}C$, so the desired capacitance cannot be achieved.

P26.19 (a) The equivalent capacitance of the series combination in the upper branch is

$$\frac{1}{C_{\text{upper}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3.00\ \mu\text{F}} + \frac{1}{6.00\ \mu\text{F}} \rightarrow C_{\text{upper}} = 2.00\ \mu\text{F}$$

Likewise, the equivalent capacitance of the series combination in the lower branch is

$$\frac{1}{C_{\text{lower}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} \rightarrow C_{\text{lower}} = 1.33\ \mu\text{F}$$

These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is

$$C_{\text{eq}} = C_{\text{upper}} + C_{\text{lower}} = 2.00\ \mu\text{F} + 1.33\ \mu\text{F} = \boxed{3.33\ \mu\text{F}}$$

(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. Each of the capacitors in series combination holds the same charge as that on the equivalent capacitor. For the upper branch:

$$Q_3 = Q_6 = Q_{\text{upper}} = C_{\text{upper}}(\Delta V) = (2.00\ \mu\text{F})(90.0\ \text{V}) = 180\ \mu\text{C}\ \text{s}$$

so, $\boxed{180\ \mu\text{C on the 3.00-}\mu\text{F and the 6.00-}\mu\text{F capacitors}}$

For the lower branch:

$$Q_2 = Q_4 = Q_{\text{lower}} = C_{\text{lower}}(\Delta V) = (1.33\ \mu\text{F})(90.0\ \text{V}) = 120\ \mu\text{C}$$

so, $\boxed{120\ \mu\text{C on the 2.00-}\mu\text{F and 4.00-}\mu\text{F capacitors}}$

(c) The potential difference across each of the capacitors in the circuit is:

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$$\boxed{60.0\ \text{V across the 3.00-}\mu\text{F and the 2.00-}\mu\text{F capacitors}}$$

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$$\boxed{30.0\ \text{V across the 6.00-}\mu\text{F and the 4.00-}\mu\text{F capacitors}}$$

P26.20 (a) Capacitors 2 and 3 are in parallel and present equivalent capacitance 6*C*. This is in series with capacitor 1, so the battery sees capacitance $\left[\frac{1}{3C} + \frac{1}{6C}\right]^{-1} = \boxed{2C}$.

(b) If they were initially uncharged, C_1 stores the same charge as C_2 and C_3 together. With greater capacitance, C_3 stores more charge than C_2. Then $\boxed{Q_1 > Q_3 > Q_2}$.

(c) The $(C_2 \parallel C_3)$ equivalent capacitor stores the same charge as C_1. Since it has greater capacitance, $\Delta V = \frac{Q}{C}$ implies that it has smaller potential difference across it than C_1. In parallel with each other, C_2 and C_3 have equal voltages: $\boxed{\Delta V_1 > \Delta V_2 = \Delta V_3}$.

(d) If C_3 is increased, the overall equivalent capacitance increases. More charge moves through the battery and Q increases. As ΔV_1 increases, ΔV_2 must decrease so Q_2 decreases. Then Q_3 must increase even more: $\boxed{Q_3 \text{ and } Q_1 \text{ increase; } Q_2 \text{ decreases}}$.

P26.21 Call C the capacitance of one capacitor and n the number of capacitors. The equivalent capacitance for n capacitors in parallel is

$$C_p = C_1 + C_2 + \cdots + C_n = nC$$

The relationship for n capacitors in series is

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} = \frac{n}{C}$$

Therefore,

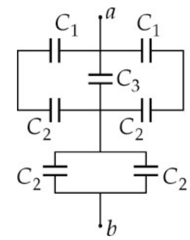
$$\frac{C_p}{C_s} = \frac{nC}{C/n} = n^2 \quad \text{or} \quad n = \sqrt{C_p/C_s} = \sqrt{100} = \boxed{10}$$

P26.22 (a) In the upper section, each C_1-C_2 pair, on either side of C_3, are in series:

$$C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu\text{F}$$

and both C_1-C_2 pairs are in parallel to C_3:

$$C_{\text{upper}} = 2(3.33) + 2.00 = 8.67\ \mu\text{F}$$



ANS. FIG. P26.22

In the lower section, the C_2-C_2 pair are in parallel:

$$C_{\text{lower}} = 2(10.0) = 20.0\ \mu\text{F}$$

The upper section is in series with the lower section:

$$C_{\text{eq}} = \left(\frac{1}{8.67} + \frac{1}{20.0}\right)^{-1} = \boxed{6.05\ \mu\text{F}}$$

(b) Capacitors in series carry the same charge as their equivalent capacitor; therefore, the upper section, equivalent to a 8.67-μF capacitor, and the lower section, equivalent to a 20.0-μF capacitor, carry the same charge as a 6.05-μF capacitor:

$$Q_{\text{upper}} = Q_{\text{eq}} = C_{\text{eq}}\Delta V = (6.05\ \mu F)(60.0\ \text{V}) \simeq 363\ \mu C$$

The upper section is equivalent to capacitor C_3 and two 3.33-μF capacitors in parallel, and the voltage across each is the same as that across a 8.67-μF capacitor:

$$\Delta V_{\text{upper}} = \frac{Q_{\text{eq}}}{C_{\text{eq}}} = \frac{363\ \mu C}{8.67\ \mu F} \simeq 41.9\ \text{V}$$

Therefore, the charge on C_3 is

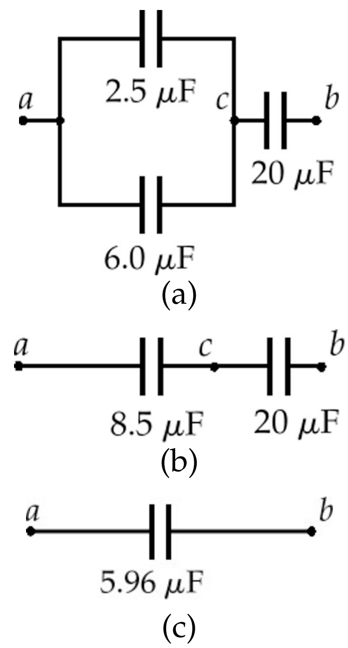
$$Q_3 = C_3 \Delta V_3 \simeq (2.00\ \mu F)(41.9\ \text{V}) = \boxed{83.7\ \mu C}$$

P26.23 (a) We simplify the circuit of Figure P26.23 in three steps as shown in ANS. FIG. P26.23 panels (a), (b), and (c). First, the 15.0-μF and 3.00-μF capacitors in series are equivalent to

$$\frac{1}{(1/15.0\ \mu\text{F})+(1/3.00\ \mu\text{F})} = 2.50\ \mu\text{F}$$

Next, the 2.50-μF capacitor combines in parallel with the 6.00-μF capacitor, creating an equivalent capacitance of 8.50 μF. At last, this 8.50-μF equivalent capacitor and the 20.0-μF capacitor are in series, equivalent to

$$\frac{1}{(1/8.50\ \mu\text{F})+(1/20.00\ \mu\text{F})} = \boxed{5.96\ \mu\text{F}}$$



ANS. FIG. P26.23

(b) We find the charge on each capacitor and the voltage across each by working backwards through solution figures (c)–(a), alternately applying $Q = C\Delta V$ and $\Delta V = Q/C$ to every capacitor, real or equivalent. For the 5.96-μF capacitor, we have

$$Q = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V}) = \boxed{89.5\ \mu\text{C}}$$

Thus, if *a* is higher in potential than *b*, just 89.5 μC flows between the wires and the plates to charge the capacitors in each picture. In (b) we have, for the 8.5-μF capacitor,

$$\Delta V_{ac} = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{8.50\ \mu\text{F}} = 10.5\ \text{V}$$

and for the 20.0-μF capacitor in (b), (a), and the original circuit, we have Q_{20} = 89.5 μC. Then

$$\Delta V_{cb} = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$$

Next, the circuit in diagram (a) is equivalent to that in (b), so ΔV_{cb} = 4.47 V and ΔV_{ac} = 10.5 V.

For the 2.50-μF capacitor, ΔV = 10.5 V and

$$Q = C\Delta V = (2.50\ \mu\text{F})(10.5\ \text{V}) = \boxed{26.3\ \mu\text{C}}$$

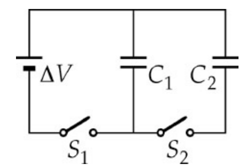
For the 6.00-μF capacitor, $\Delta V = 10.5$ V and

$$Q_6 = C\Delta V = (6.00\ \mu\text{F})(10.5\ \text{V}) = \boxed{63.2\ \mu\text{C}}$$

Now, 26.3 μC having flowed in the upper parallel branch in (a), back in the original circuit we have $Q_{15} = 26.3\ \mu$C and $Q_3 = 26.3\ \mu$C.

P26.24 (a) $C = \dfrac{Q}{\Delta V}$. When S_1 is closed, the charge on C_1 will be

$$Q = C\Delta V = (6.00\ \mu\text{F})(20.0\ \text{V}) = \boxed{120\ \mu\text{C}}$$



ANS. FIG. P26.24

(b) When S_1 is opened and S_2 is closed, the total charge will remain constant and be shared by the two capacitors. We let primed symbols represent the new charges on the capacitors, in $Q_1' = 120\,\mu\text{C} - Q_2'$. The potential differences across the two capacitors will be equal.

$$\Delta V' = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \quad \text{or} \quad \frac{120\ \mu\text{C} - Q_2'}{6.00\ \mu\text{F}} = \frac{Q_2'}{3.00\ \mu\text{F}}$$

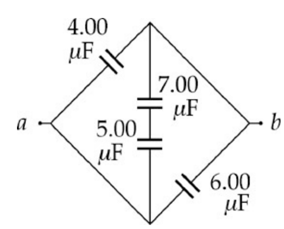
Then we do the algebra to find

$$Q_2' = \frac{360}{9.00}\ \mu\text{C} = \boxed{40.0\ \mu\text{C}}$$

and $Q_1' = 120\ \mu\text{C} - 40.0\ \mu\text{C} = \boxed{80.0\ \mu\text{C}}$.

P26.25 $C_s = \left(\dfrac{1}{5.00} + \dfrac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$

$C_p = 2.92 + 4.00 + 6.00 = \boxed{12.9\ \mu\text{F}}$



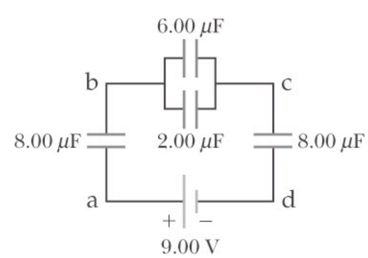
ANS. FIG. P26.25

P26.26 (a) First, we replace the parallel combination between points b and c by its equivalent capacitance,

$C_{\text{bc}} = 2.00\ \mu\text{F} + 6.00\ \mu\text{F} = 8.00\ \mu\text{F}$.

Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_{\text{ab}}} + \frac{1}{C_{\text{bc}}} + \frac{1}{C_{\text{cd}}} = \frac{3}{8.00\ \mu\text{F}}$$



ANS. FIG. P26.26

giving

$$C_{eq} = \frac{8.00\ \mu\text{F}}{3} = \boxed{2.67\ \mu\text{F}}$$

(b) The charge on each capacitor in the series is the same as the charge on the equivalent capacitor:

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq}(\Delta V_{ad}) = (2.67\ \mu\text{F})(9.00\ \text{V}) = 24.0\ \mu\text{C}$$

Then, note that $\Delta V_{bc} = \frac{Q_{bc}}{C_{bc}} = \frac{24.0\ \mu\text{C}}{8.00\ \mu\text{F}} = 3.00\ \text{V}$. The charge on each capacitor in the original circuit is:

On the 8.00 μF between a and b:

$$Q_8 = Q_{ab} = \boxed{24.0\ \mu\text{C}}$$

On the 8.00 μF between c and d:

$$Q_8 = Q_{cd} = \boxed{24.0\ \mu\text{C}}$$

On the 2.00 μF between b and c:

$$Q_2 = C_2(\Delta V_{bc}) = (2.00\ \mu\text{F})(3.00\ \text{V}) = \boxed{6.00\ \mu\text{C}}$$

On the 6.00 μF between b and c:

$$Q_6 = C_6(\Delta V_{bc}) = (6.00\ \mu\text{F})(3.00\ \text{V}) = \boxed{18.0\ \mu\text{C}}$$

(c) We earlier found that $\Delta V_{bc} = 3.00\ \text{V}$. The two 8.00 μF capacitors have the same voltage: $\Delta V_8 = \Delta V_8 = \frac{Q}{C} = \frac{24.0\ \mu\text{C}}{8.00\ \mu\text{F}} = 3.00\ \text{V}$, so we conclude that the potential difference across each capacitor is the same: $\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00\ \text{V}}$.

P26.27 $C_p = C_1 + C_2$ and $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$. Substitute $C_2 = C_p - C_1$:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$$

Simplifying,

$$C_1^2 - C_1C_p + C_pC_s = 0$$

$$C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_pC_s}}{2} = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$$

We choose arbitrarily the + sign. (This choice can be arbitrary, since with the case of the minus sign, we would get the same two answers with their names interchanged.)

$$C_1 = \frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$$

$$= \frac{1}{2}(9.00 \text{ pF}) + \sqrt{\frac{1}{4}(9.00 \text{ pF})^2 - (9.00 \text{ pF})(2.00 \text{ pF})}$$

$$= \boxed{6.00 \text{ pF}}$$

$$C_2 = C_p - C_1 = \frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$$

$$= \frac{1}{2}(9.00 \text{ pF}) - 1.50 \text{ pF} = \boxed{3.00 \text{ pF}}$$

P26.28 $C_p = C_1 + C_2$ and $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$.

Substitute

$$C_2 = C_p - C_1: \quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1\left(C_p - C_1\right)}$$

Simplifying,

$$C_1^2 - C_1C_p + C_pC_s = 0$$

and $$C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_pC_s}}{2} = \boxed{\frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed). Then, from $C_2 = C_p - C_1$, we obtain

$$C_2 = \boxed{\frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$$

P26.29 For C_1 connected by itself, $C_1\Delta V = 30.8\ \mu\text{C}$ where ΔV is the battery voltage: $\Delta V = \frac{30.8\ \mu\text{C}}{C_1}$.

For C_1 and C_2 in series:

$$\left(\frac{1}{1/C_1 + 1/C_2}\right)\Delta V = 23.1\ \mu\text{C}$$

substituting, $\dfrac{30.8\ \mu\text{C}}{C_1} = \dfrac{23.1\ \mu\text{C}}{C_1} + \dfrac{23.1\ \mu\text{C}}{C_2}$ which gives $C_1 = 0.333C_2$

For C_1 and C_3 in series:

$$\left(\frac{1}{1/C_1 + 1/C_3}\right)\Delta V = 25.2\ \mu\text{C}$$

$$\frac{30.8\ \mu\text{C}}{C_1} = \frac{25.2\ \mu\text{C}}{C_1} + \frac{25.2\ \mu\text{C}}{C_3} \quad \text{which gives} \quad C_1 = 0.222C_3$$

For all three:

$$Q = \left(\frac{1}{1/C_1 + 1/C_2 + 1/C_3}\right)\Delta V = \frac{C_1\Delta V}{1 + C_1/C_2 + C_1/C_3}$$

$$= \frac{30.8\ \mu\text{C}}{1 + 0.333 + 0.222} = \boxed{19.8\ \mu\text{C}}$$

This is the charge on each one of the three.

Section 26.4 Energy Stored in a Charged Capacitor

P26.30 From $U_E = \frac{1}{2}C\Delta V^2$, we have

$$\Delta V = \sqrt{\frac{2U_E}{C}} = \sqrt{\frac{2(300\ \text{J})}{30.0 \times 10^{-6}\ \text{F}}} = \boxed{4.47 \times 10^3\ \text{V}}$$

P26.31 The energy stored in the capacitor is given by

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}(54.0 \times 10^{-6}\ \text{C})(12.0\ \text{V}) = \boxed{3.24 \times 10^{-4}\ \text{J}}$$

P26.32 (a) $U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(12.0\ \text{V})^2 = \boxed{216\ \mu\text{J}}$

(b) $U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{54.0\ \mu\text{J}}$

P26.33 (a) $Q = C\Delta V = (150 \times 10^{-12}\ \text{F})(10 \times 10^3\ \text{V}) = 1.50 \times 10^{-6}\ \text{C} = \boxed{1.50\ \mu\text{C}}$

(b) From $U_E = \frac{1}{2}C(\Delta V)^2$,

$$\Delta V = \sqrt{\frac{2U_E}{C}} = \sqrt{\frac{2(250 \times 10^{-6}\ \text{J})}{150 \times 10^{-12}\ \text{F}}} = 1.83 \times 10^3\ \text{V} = \boxed{1.83\ \text{kV}}$$

P26.34 (a) The equivalent capacitance of a series combination of C_1 and C_2 is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{18.0\ \mu\text{F}} + \frac{1}{36.0\ \mu\text{F}} \rightarrow C_{\text{eq}} = \boxed{12.0\ \mu\text{F}}$$

(b) This series combination is connected to a 12.0-V battery, the total stored energy is

$$U_{E,\text{ eq}} = \frac{1}{2}C_{\text{eq}}(\Delta V)^2 = \frac{1}{2}\left(12.0\times10^{-6}\ \text{F}\right)(12.0\ \text{V})^2 = \boxed{8.64\times10^{-4}\ \text{J}}$$

(c) Capacitors in series carry the same charge as their equivalent capacitor. The charge stored on each of the two capacitors in the series combination is

$$Q_1 = Q_2 = Q_{\text{total}} = C_{\text{eq}}(\Delta V) = (12.0\ \mu\text{F})(12.0\ \text{V})$$
$$= 144\ \mu\text{C} = 1.44\times10^{-4}\ \text{C}$$

and the energy stored in each of the individual capacitors is:

18.0 μF capacitor:

$$U_{E1} = \frac{Q_1^2}{2C_1} = \frac{\left(1.44\times10^{-4}\ \text{C}\right)^2}{2\left(18.0\times10^{-6}\ \text{F}\right)} = \boxed{5.76\times10^{-4}\ \text{J}}$$

36.0 μF capacitor:

$$U_{E2} = \frac{Q_2^2}{2C_2} = \frac{\left(1.44\times10^{-4}\ \text{C}\right)^2}{2\left(36.0\times10^{-6}\ \text{F}\right)} = \boxed{2.88\times10^{-4}\ \text{J}}$$

(d) $U_{E1} + U_{E2} = 5.76\times10^{-4}\ \text{J} + 2.88\times10^{-4}\ \text{J} = 8.64\times10^{-4}\ \text{J} = U_{E,\text{ eq}}$, which is one reason why the 12.0 μF capacitor is considered to be equivalent to the two capacitors.

(e) The total energy of the equivalent capacitance will always equal the sum of the energies stored in the individual capacitors.

(f) If C_1 and C_2 were connected in parallel rather than in series, the equivalent capacitance would be $C_{\text{eq}} = C_1 + C_2 = 18.0\ \mu\text{F} + 36.0\ \mu\text{F} = 54.0\ \mu\text{F}$. If the total energy stored in this parallel combination is to be the same as stored in the original series combination, it is necessary that

$$\frac{1}{2}C_{\text{eq}}(\Delta V)^2 = U_{E,\text{ eq}}$$

From which we obtain

$$\Delta V = \sqrt{\frac{2U_{E,\text{ eq}}}{C_{\text{eq}}}} = \sqrt{\frac{2\left(8.64\times10^{-4}\text{ J}\right)}{54.0\times10^{-6}\text{ F}}} = \boxed{5.66\text{ V}}$$

(g) Because the potential difference is the same across the two capacitors when connected in parallel, and $U_E = \frac{1}{2}C(\Delta V)^2$, $\boxed{\text{the larger capacitor } C_2 \text{ stores more energy.}}$

P26.35 (a) Because the capacitors are connected in parallel, their voltage remains the same:

$$\begin{aligned} U_E &= \frac{1}{2}C(\Delta V)^2 + \frac{1}{2}C(\Delta V)^2 = C(\Delta V)^2 \\ &= \left(10.0\times10^{-6}\ \mu\text{F}\right)(50.0\text{ V})^2 \\ &= \boxed{2.50\times10^{-2}\text{ J}} \end{aligned}$$

(b) Because $C = \frac{\kappa\epsilon_0 A}{d}$ and $d \to 2d$, the altered capacitor has new capacitance to $C' = \frac{C}{2}$. The total charge is the same as before:

$$Q_{\text{initial}} = Q_{\text{final}}$$

$$C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V')$$

$$2C(\Delta V) = \frac{3}{2}C(\Delta V') \quad \to \quad \Delta V' = \frac{4}{3}\Delta V = \frac{4}{3}(50.0\text{ V}) = \boxed{66.7\text{ V}}$$

(c) New $U_E' = \frac{1}{2}C(\Delta V')^2 + \frac{1}{2}\left(\frac{1}{2}C\right)(\Delta V')^2 = \frac{3}{4}C(\Delta V')^2 = \frac{3}{4}C\left(\frac{4\Delta V}{3}\right)^2$

$$U_E' = \frac{4}{3}C(\Delta V)^2 = \frac{4}{3}U_E = \frac{4}{3}\left(2.50\times10^{-2}\text{ J}\right) = \boxed{3.30\times10^{-2}\text{ J}}$$

(d) $\boxed{\text{Positive work is done by the agent pulling the plates apart.}}$

P26.36 Before the capacitors are connected, each has voltage ΔV and charge Q.

(a) Connecting plates of like sign places the capacitors in parallel, so the voltage on each capacitor remains the same.

$$U_{E,\text{ total}} = \frac{1}{2}C(\Delta V)^2 + \frac{1}{2}C(\Delta V)^2 = \boxed{C(\Delta V)^2}$$

(b) Because $C = \frac{\epsilon_0 A}{d}$, the altered capacitor has new capacitance $C' = \frac{\epsilon_0 A}{2d} = \frac{C}{2}$, and this change in capacitance results in a new potential difference $\Delta V'$ across the parallel capacitors. We can solve for the new potential difference because the total charge remains the same:

$$2Q = C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V') \quad \rightarrow \quad \boxed{\Delta V' = \frac{4\Delta V}{3}}$$

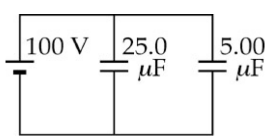
(c) Each capacitor has potential difference $\Delta V'$:

$$U'_{E,\,\text{total}} = \frac{1}{2}C(\Delta V')^2 + \frac{1}{2}C'(\Delta V')^2 = \frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 + \frac{1}{2}\left(\frac{C}{2}\right)\left(\frac{4\Delta V}{3}\right)^2$$

$$= \frac{12}{9}C(\Delta V)^2 = \boxed{4C\frac{(\Delta V)^2}{3}}$$

(d) $\boxed{\text{Positive work is done by the agent pulling the plates apart.}}$

P26.37 (a) The circuit diagram for capacitors connected in parallel is shown in ANS. FIG. P26.37(a).



ANS. FIG. P26.37(a)

(b) $U_E = \frac{1}{2}C(\Delta V)^2$, and

$$C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F} = 30.0\ \mu\text{F}$$

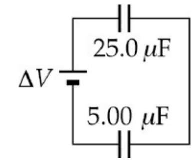
$$U_E = \frac{1}{2}(30.0\times10^{-6})(100)^2 = \boxed{0.150\ \text{J}}$$

(c) $$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$$

$$U_E = \frac{1}{2}C(\Delta V)^2$$

$$\Delta V = \sqrt{\frac{2U_E}{C}} = \sqrt{\frac{2(0.150\ \text{J})}{4.17\times10^{-6}\ \text{F}}} = \boxed{268\ \text{V}}$$

(d) The circuit diagram for capacitors connected in series is shown in ANS. FIG. P26.37(d).



ANS. FIG. P26.37(d)

P26.38 To prove this, we follow the hint, and calculate the work done in separating the plates, which equals the potential energy stored in the charged capacitor:

$$U_E = \frac{1}{2}\frac{Q^2}{C} = \int F dx$$

Now from the fundamental theorem of calculus, $dU_E = Fdx$

and $$F = \frac{d}{dx}U_E = \frac{d}{dx}\left(\frac{Q^2}{2C}\right) = \frac{1}{2}\frac{d}{dx}\left(\frac{Q^2}{A\in_0/x}\right).$$

Performing the differentiation,

$$F = \frac{1}{2}\frac{d}{dx}\left(\frac{Q^2 x}{A\in_0}\right) = \boxed{\frac{Q^2}{2\in_0 A}}$$

P26.39 The energy transferred is

$$T_{\text{ET}} = \frac{1}{2}Q\Delta V = \frac{1}{2}(50.0\text{ C})\left(1.00\times10^8\text{ V}\right) = 2.50\times10^9\text{ J}$$

and 1% of this (or $\Delta E_{\text{int}} = 2.50\times10^7$ J) is absorbed by the tree. If m is the amount of water boiled away, then

$$\Delta E_{\text{int}} = m\left(4\,186\text{ J/kg}\cdot°\text{C}\right)(100°\text{C} - 30.0°\text{C}) + m\left(2.26\times10^6\text{ J/kg}\right)$$

$$= 2.50\times10^7\text{ J}$$

giving $m = \boxed{9.79\text{ kg}}$.

P26.40 (a) According to Equation 26.2, we may think of a sphere of radius R that holds charge Q as having a capacitance $C = \dfrac{R}{k_e}$. The energy stored is

$$U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(\frac{R}{k_e}\right)\left(\frac{k_e Q}{R}\right)^2 = \boxed{\frac{k_e Q^2}{2R}}$$

(b) The total energy is

$$U_E = U_{E1} + U_{E2} = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2} = \frac{1}{2}\frac{q_1^2}{R_1/k_e} + \frac{1}{2}\frac{(Q-q_1)^2}{R_2/k_e}$$

$$= \boxed{\frac{k_e q_1^2}{2R_1} + \frac{k_e(Q-q_1)^2}{2R_2}}$$

(c) For a minimum we set $\dfrac{dU_E}{dq_1} = 0$:

$$\frac{2k_e q_1}{2R_1} + \frac{2k_e(Q - q_1)}{2R_2}(-1) = 0$$

which gives

$$R_2 q_1 = R_1 Q - R_1 q_1 \quad \rightarrow \quad q_1 = \boxed{\frac{R_1 Q}{R_1 + R_2}}$$

(d) $q_2 = Q - q_1 = \boxed{\dfrac{R_2 Q}{R_1 + R_2}}$

(e) $V_1 = \dfrac{k_e q_1}{R_1} = \dfrac{k_e R_1 Q}{R_1(R_1 + R_2)} \rightarrow \boxed{V_1 = \dfrac{k_e Q}{R_1 + R_2}}$, and

$$V_2 = \frac{k_e q_2}{R_2} = \frac{k_e R_2 Q}{R_2(R_1 + R_2)} \rightarrow \boxed{V_2 = \frac{k_e Q}{R_1 + R_2}}$$

(f) $V_1 - V_2 = \boxed{0}$

P26.41 Originally, the capacitance of each pair of plates is $C = \dfrac{\epsilon_0 A}{d}$, but after the switch is closed and the distance d is changed to $d' = 0.500d$, the plates have new capacitance

$$C' = \frac{\epsilon_0 A}{d'} = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d} = 2C$$

The capacitors are identical and in series, so each has half the total voltage $(\Delta V) = 100$ V.

(a) The plates are in series, so each collects the same charge:

$$Q = C'(\Delta V) = 2C(\Delta V) = 2(2.00\ \mu\text{C})(100\ \text{V}) = \boxed{400\ \mu\text{C}}$$

(b) Each plate contributes half of the total electric field between the plates, $\dfrac{E}{2} = \dfrac{\sigma}{2\epsilon_0} = \dfrac{Q}{2\epsilon_0 A}$, where $Q = 2C(\Delta V)$ is the magnitude of the charge on a plate, from (a) above. The electric force that each plate exerts on the charge of its neighboring plate is

$$F = Q\frac{E}{2} = \frac{Q^2}{2\epsilon_0 A} = \frac{[2C(\Delta V)]^2}{2\epsilon_0 A} = \frac{2C^2(\Delta V)^2}{(\epsilon_0 A/d)d} = \frac{2C(\Delta V)^2}{d}$$

and this force is balanced by the spring force $F = kx$ on each plate.

Each spring stretches by distance $x = \frac{d}{4}$, so we obtain

$$\frac{2C(\Delta V)^2}{d} = k\frac{d}{4}$$

and solving for the force constant gives

$$k = \frac{8C(\Delta V)^2}{d^2} = \frac{8(2.00\times10^{-6}\ \text{F})(100\ \text{V})^2}{(8.00\times10^{-3}\ \text{m})^2} = \boxed{2.50\ \text{kN/m}}$$

Section 26.5 Capacitors with Dielectrics

P26.42 (a) Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them.

(b) Suppose the plastic has $\kappa \approx 3$, $E_{\text{max}} \sim 10^7\ \text{V/m}$, and thickness $1\ \text{mil} = \frac{2.54\ \text{cm}}{1\ 000}$.

Then, $C = \frac{\kappa \epsilon_0 A}{d} \sim \frac{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.400\ \text{m}^2)}{2.54\times10^{-5}\ \text{m}} \sim \boxed{10^{-6}\ \text{F}}$

(c) $\Delta V_{\text{max}} = E_{\text{max}} d \sim (10^7\ \text{V/m})(2.54\times10^{-5}\ \text{m}) \sim \boxed{10^2\ \text{V}}$

P26.43 $Q_{\text{max}} = C\Delta V_{\text{max}}$, but $\Delta V_{\text{max}} = E_{\text{max}} d$.

Also, $C = \frac{\kappa \epsilon_0 A}{d}$.

Thus, $Q_{\text{max}} = \frac{\kappa \epsilon_0 A}{d}(E_{\text{max}} d) = \kappa \epsilon_0 A E_{\text{max}}$.

(a) With air between the plates, from Table 26.1, the dielectric constant is $\kappa = 1.00$, and the dielectric strength is $E_{\text{max}} = 3.00\times10^6\ \text{V/m}$. Therefore,

$$\begin{aligned} Q_{\text{max}} &= \kappa \epsilon_0 A E_{\text{max}} \\ &= (8.85\times10^{-12}\ \text{F/m})(5.00\times10^{-4}\ \text{m}^2)(3.00\times10^6\ \text{V/m}) \\ &= \boxed{13.3\ \text{nC}} \end{aligned}$$

(b) With polystyrene between the plates, from Table 26.1, $\kappa = 2.56$ and $E_{\text{max}} = 24.0 \times 10^6$ V/m.

$$\begin{aligned} Q_{\text{max}} &= \kappa \in_0 AE_{\text{max}} \\ &= 2.56\left(8.85\times10^{-12}\text{ F/m}\right)\left(5.00\times10^{-4}\text{ m}^2\right) \\ &\qquad\times\left(24.0\times10^6\text{ V/m}\right) \\ &= \boxed{272\text{ nC}} \end{aligned}$$

P26.44 (a) Note that the charge on the plates remains constant at the original value, Q_0, as the dielectric is inserted. Thus, the change in the potential difference, $\Delta V = Q/C$, is due to a change in capacitance alone. The ratio of the final and initial capacitances is

$$\frac{C_f}{C_i} = \frac{\kappa \in_0 A/d}{\in_0 A/d} = \kappa$$

and $$\frac{C_f}{C_i} = \frac{Q_0/(\Delta V)_f}{Q_0/(\Delta V)_i} = \frac{(\Delta V)_i}{(\Delta V)_f} = \frac{85.0\text{ V}}{25.0\text{ V}} = 3.40$$

Thus, the dielectric constant of the inserted material is $\boxed{\kappa = 3.40}$.

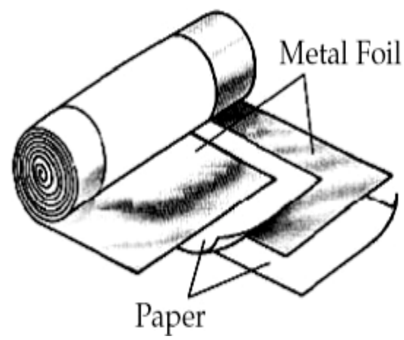
(b) The material is probably $\boxed{\text{nylon}}$ (see Table 26.1).

(c) The presence of a dielectric weakens the field between plates, and the weaker field, for the same charge on the plates, results in a smaller potential difference. If the dielectric only partially filled the space between the plates, the field is weakened only within the dielectric and not in the remaining air-filled space, so the potential difference would not be as small. $\boxed{\text{The voltage would lie somewhere between 25.0 V and 85.0 V.}}$

P26.45 (a) $$C = \frac{\kappa \in_0 A}{d} = \frac{2.10\left(8.85\times10^{-12}\text{ F/m}\right)\left(1.75\times10^{-4}\text{ m}^2\right)}{4.00\times10^{-5}\text{ m}} = 8.13\times10^{-11}\text{F}$$
$$= \boxed{81.3\text{ pF}}$$

(b) $$\Delta V_{\text{max}} = E_{\text{max}}d = \left(60.0\times10^6\text{ V/m}\right)\left(4.00\times10^{-5}\text{ m}\right) = \boxed{2.40\text{ kV}}$$

P26.46 ANS. FIG. P26.46 exaggerates how the strips can be offset to avoid contact between the two foils. It shows how a second paper strip can be used to roll the capacitor into a convenient cylindrical shape with electrical contacts at the two ends. We suppose that the overlapping width of the two metallic strips is still $w = 7.00$ cm. Then for the area of the plates we have $A = \ell w$ in $C = \kappa\epsilon_0 A/d = \kappa\epsilon_0 \ell w/d$. Solving the equation gives



ANS. FIG. P26.46

$$\ell = \frac{Cd}{\kappa\epsilon_0 w} = \frac{(9.50\times10^{-8}\ \text{F})(2.50\times10^{-5}\ \text{m})}{3.70(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.070\ 0\ \text{m})} = \boxed{1.04\ \text{m}}$$

P26.47 Originally, $C_i = \frac{\epsilon_0 A}{d} = \frac{Q}{(\Delta V)_i}$.

(a) The charge is the same before and after immersion, with value

$$Q = C_i(\Delta V)_i = \frac{\epsilon_0 A(\Delta V)_i}{d}$$

$$Q = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(25.0\times10^{-4}\ \text{m}^2)(250\ \text{V})}{1.50\times10^{-2}\ \text{m}}$$

$$= \boxed{369\ \text{pC}}$$

(b) Finally,

$$C_f = \frac{\kappa\epsilon_0 A}{d} = \frac{Q}{(\Delta V)_f}:$$

$$C_f = \frac{(80)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(25.0\times10^{-4}\ \text{m}^2)}{1.50\times10^{-2}\ \text{m}}$$

$$= \boxed{1.20\times10^{-10}\ \text{F}}$$

and $(\Delta V)_f = \frac{Q}{C_f} = \frac{C_i(\Delta V)_i}{C_f} = \frac{(\epsilon_0 A/d)}{(\kappa\epsilon_0 A/d)}(\Delta V)_i = \frac{(\Delta V)_i}{\kappa} = \frac{250\ \text{V}}{80}$

$$= \boxed{3.10\ \text{V}}$$

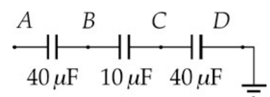
(c) Originally, $U_i = \frac{1}{2}C_i(\Delta V)_i^2 = \frac{\epsilon_0 A(\Delta V)_i^2}{2d}$.

Finally, $U_f = \frac{1}{2}C_f(\Delta V)_f^2 = \frac{1}{2}\left(\frac{\kappa\epsilon_0 A}{d}\right)\left(\frac{(\Delta V)_i}{\kappa}\right)^2 = \frac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa}$, where, from Table 26.1, $\kappa = 80$ for distilled water. So,

$$\Delta U = U_f - U_i$$
$$= \frac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa} - \frac{\epsilon_0 A(\Delta V)_i^2}{2d}$$
$$= \frac{\epsilon_0 A(\Delta V)_i^2}{2d}\left(\frac{1}{\kappa} - 1\right) = \frac{\epsilon_0 A(\Delta V)_i^2 (1-\kappa)}{2d\kappa}$$

$$\Delta U = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0\times10^{-4}\ \text{m}^2\right)(250\ \text{V})^2(1-80)}{2\left(1.50\times10^{-2}\ \text{m}\right)(80)}$$
$$= -4.55\times10^{-8}\ \text{J} = \boxed{-45.5\ \text{nJ}}$$

P26.48 The given combination of capacitors is equivalent to the circuit diagram shown in ANS. FIG. P26.48.



ANS. FIG. P26.48

Put charge Q on point A. Then,

$$Q = (40.0\ \mu\text{F})\Delta V_{AB} = (10.0\ \mu\text{F})\Delta V_{BC} = (40.0\ \mu\text{F})\Delta V_{CD}$$

So, $\Delta V_{BC} = 4\Delta V_{AB} = 4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC} = 15.0$ V. When this occurs,

$$\Delta V_{AB} = \Delta V_{CD} = \frac{1}{4}(\Delta V_{BC}) = 3.75\ \text{V}$$

and $V_{AD} = V_{AB} + V_{BC} + V_{CD} = 3.75\ \text{V} + 15.0\ \text{V} + 3.75\ \text{V} = \boxed{22.5\ \text{V}}$.

P26.49 (a) We use the equation $U_E = Q^2/2C$ to find the potential energy of the capacitor. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are $U_{E,i} = \dfrac{Q^2}{2C_i}$ and $U_{E,f} = \dfrac{Q^2}{2C_f}$. But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_{E,f} = \kappa\dfrac{Q^2}{2C_i} = \kappa U_{E,i}$. Since the work done by the external force in removing the dielectric equals the change in potential energy, we have

$$W = U_f - U_i = \kappa U_i - U_i = (\kappa - 1)U_i = (\kappa - 1)\frac{Q^2}{2C_i}$$

To express this relation in terms of potential difference ΔV_i, we substitute $Q = C_i(\Delta V_i)$, and evaluate:

$$W = \frac{1}{2}C_i(\Delta V_i)^2(\kappa - 1) = \frac{1}{2}(2.00\times10^{-9}\text{ F})(100\text{ V})^2(5.00 - 1.00)$$
$$= 4.00\times10^{-5}\text{ J} = \boxed{40.0\ \mu\text{J}}$$

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \dfrac{Q}{C_f}$.

Substituting $C_f = \dfrac{C_i}{\kappa}$ and $Q = C_i(\Delta V_i)$ gives

$$\Delta V_f = k\Delta V_i = 5.00(100\text{ V}) = \boxed{500\text{ V}}$$

Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

Section 26.6 Electric Dipole in an Electric Field

P26.50 (a) The displacement from negative to positive charge is

$$2\vec{\mathbf{a}} = (-1.20\hat{\mathbf{i}} + 1.10\hat{\mathbf{j}})\text{ mm} - (1.40\hat{\mathbf{i}} - 1.30\hat{\mathbf{j}})\text{ mm}$$
$$= (-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}})\times10^{-3}\text{ m}$$

The electric dipole moment is $\vec{\mathbf{p}} = 2\vec{\mathbf{a}}q$

$$\vec{\mathbf{p}} = (3.50\times10^{-9}\text{ C})(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}})\times10^{-3}\text{ m}$$
$$= \boxed{(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}})\times10^{-12}\text{ C}\cdot\text{m}}$$

(b) The torque exerted by the field on the dipole is

$$\vec{\tau} = \vec{\mathbf{p}}\times\vec{\mathbf{E}}$$
$$= \left[(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}})\times10^{-12}\text{ C}\cdot\text{m}\right]\times\left[(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}})\times10^{3}\text{ N/C}\right]$$
$$= (+44.6\hat{\mathbf{k}} - 65.5\hat{\mathbf{k}})\times10^{-9}\text{ N}\cdot\text{m} = \boxed{-2.09\times10^{-8}\,\hat{\mathbf{k}}\text{ N}\cdot\text{m}}$$

(c) Relative to zero energy when it is perpendicular to the field, the dipole has potential energy

$$U = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}$$
$$= -\left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times 10^{-12}\ \text{C}\cdot\text{m}\right] \cdot\left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right)\times 10^{3}\ \text{N/C}\right]$$
$$= (71.0 + 41.2)\times 10^{-9}\ \text{J} = \boxed{112\ \text{nJ}}$$

(d) For convenience we compute the magnitudes

$$\left|\vec{\mathbf{p}}\right| = \sqrt{(9.10)^2 + (8.40)^2}\times 10^{-12}\ \text{C}\cdot\text{m} = 12.4\times 10^{-12}\ \text{C}\cdot\text{m}$$

and $\left|\vec{\mathbf{E}}\right| = \sqrt{(7.80)^2 + (4.90)^2}\times 10^{3}\ \text{N/C} = 9.21\times 10^{3}\ \text{N/C}$

The maximum potential energy occurs when the dipole moment is opposite in direction to the field, and is

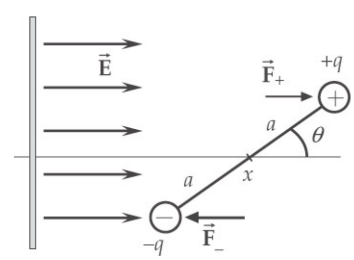
$$U_{\text{max}} = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}} = -\left|\vec{\mathbf{p}}\right|\left|\vec{\mathbf{E}}\right|(-1) = \left|\vec{\mathbf{p}}\right|\left|\vec{\mathbf{E}}\right| = 114\ \text{nJ}$$

The minimum potential energy configuration is the stable equilibrium position with the dipole aligned with the field. The value is $U_{\text{min}} = -114\ \text{nJ}$

Then the difference, representing the range of potential energies available to the dipole, is $U_{\text{max}} - U_{\text{min}} = \boxed{228\ \text{nJ}}$.

P26.51 (a) The electric field produced by the line of charge has radial symmetry about the y axis. According to Equation 24.7 in Example 24.4, the electric field to the right of the y axis is

$$\vec{\mathbf{E}} = E(r)\hat{\mathbf{i}} = 2k_e\frac{\lambda}{r}\hat{\mathbf{i}}$$



ANS. FIG. P26.51

Let x = 25.0 cm represent the coordinate of the center of the dipole charge, and let $2a$ = 2.00 cm represent the distance between the charges. Then $r_- = x - a\cos\theta$ is the coordinate of the negative charge and $r_+ = x + a\cos\theta$ is the coordinate of the positive charge.

The force on the positive charge is

$$\vec{\mathbf{F}}_+ = qE(r_+)\hat{\mathbf{i}} = q\left(2k_e\frac{\lambda}{r_+}\hat{\mathbf{i}}\right) = 2k_e\frac{q\lambda}{x + a\cos\theta}\hat{\mathbf{i}}$$

and the force on the negative charge is

$$\vec{\mathbf{F}}_- = -qE(r_-)\hat{\mathbf{i}} = -q\left(2k_e\frac{\lambda}{r_-}\hat{\mathbf{i}}\right) = -2k_e\frac{q\lambda}{x - a\cos\theta}\hat{\mathbf{i}}$$

The force on the dipole is

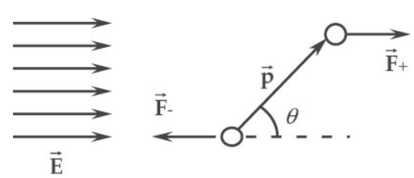
$$\begin{aligned}\vec{\mathbf{F}} = \vec{\mathbf{F}}_+ + \vec{\mathbf{F}}_- &= \left(2k_e\frac{q\lambda}{x + a\cos\theta} - 2k_e\frac{q\lambda}{x - a\cos\theta}\right)\hat{\mathbf{i}}\\ &= 2k_e q\lambda\left(\frac{1}{x + a\cos\theta} - \frac{1}{x - a\cos\theta}\right)\hat{\mathbf{i}}\\ &= 2k_e q\lambda\left[\frac{(x - a\cos\theta) - (x + a\cos\theta)}{x^2 + (a\cos\theta)^2}\right]\hat{\mathbf{i}}\\ &= -\left[\frac{4k_e aq\lambda\cos\theta}{x^2 + (a\cos\theta)^2}\right]\hat{\mathbf{i}}\end{aligned}$$

Substituting numerical values and suppressing units,

$$\begin{aligned}\vec{\mathbf{F}} &= -\frac{4(8.99\times10^9)(0.010\ 0)(10.0\times10^{-6})(2.00\times10^{-6})\cos 35.0^\circ}{(0.250)^2 + [(0.010\ 0)(\cos 35.0^\circ)]^2}\hat{\mathbf{i}}\\ &= \boxed{-9.42\times10^{-2}\,\hat{\mathbf{i}}\ \text{N}}\end{aligned}$$

P26.52 Let x represent the coordinate of the negative charge. Then $x + 2a\cos\theta$ is the coordinate of the positive charge. The force on the negative charge is $\vec{\mathbf{F}}_- = -qE(x)\hat{\mathbf{i}}$. The force on the positive charge is

$$\vec{\mathbf{F}}_+ = +qE(x + 2a\cos\theta)\hat{\mathbf{i}} \approx q\left[E(x) + \left(\frac{dE}{dx}\right)(2a\cos\theta)\right]\hat{\mathbf{i}}$$



ANS. FIG. P26.52

The force on the dipole is altogether

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_- + \vec{\mathbf{F}}_+ = q\frac{dE}{dx}(2a\cos\theta)\hat{\mathbf{i}} = \boxed{p\frac{dE}{dx}\cos\theta\,\hat{\mathbf{i}}}$$

Section 26.7 An Atomic Description of Dielectrics

P26.53 (a) Consider a gaussian surface in the form of a cylindrical pillbox with ends of area $A' << A$ parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss's law becomes

$$EA' + EA' = \frac{Q}{\in A}A', \text{ so } \boxed{E = \frac{Q}{2\in A}} \text{ directed away from the}$$

positive sheet.

(b) In the space between the sheets, each creates field $\dfrac{Q}{2\in A}$ away from the positive and toward the negative sheet. Together, they create a field of

$$\boxed{E = \frac{Q}{\in A}}$$

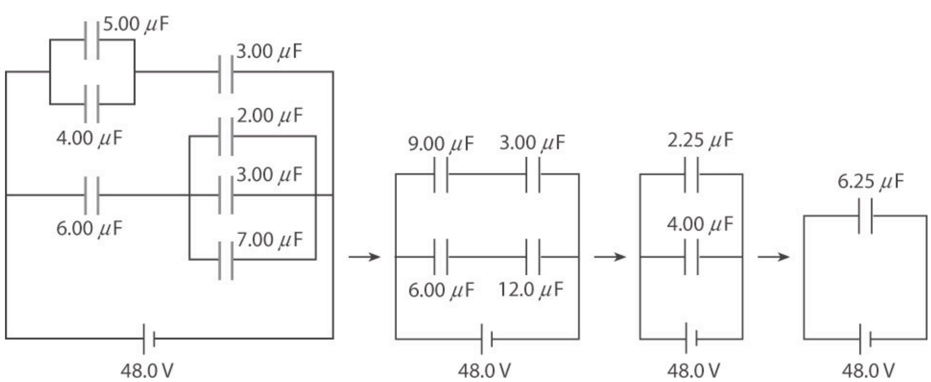
(c) Assume that the field is in the positive x-direction. Then, the potential of the positive plate relative to the negative plate is

$$\Delta V = -\int_{-\text{plate}}^{+\text{plate}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{-\text{plate}}^{+\text{plate}} \frac{Q}{\in A}\hat{\mathbf{i}} \cdot \left(-\hat{\mathbf{i}}dx\right) = \boxed{+\frac{Qd}{\in A}}$$

(d) Capacitance is defined by: $C = \dfrac{Q}{\Delta V} = \dfrac{Q}{Qd/\in A} = \boxed{\dfrac{\in A}{d} = \dfrac{\kappa \in_0 A}{d}}$.

Additional Problems

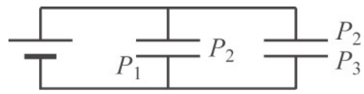
P26.54 The stages for the reduction of this circuit are shown in ANS. FIG. P26.54 below.



ANS. FIG. P26.54

Thus, $C_{\text{eq}} = \boxed{6.25\ \mu\text{F}}$

P26.55 (a) Each face of P_2 carries charge, so the three-plate system is equivalent to what is shown in ANS. FIG. P26.55 below.



ANS. FIG. P26.55

Each capacitor by itself has capacitance

$$C = \frac{\kappa \in_0 A}{d} = \frac{1\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.50\times10^{-4}\ \text{m}^2\right)}{1.19\times10^{-3}\ \text{m}}$$
$$= 5.58\ \text{pF}$$

Then equivalent capacitance $= 5.58\ \text{pF} + 5.58\ \text{pF} = \boxed{11.2\ \text{pF}}$.

(b) $Q = C\Delta V + C\Delta V = \left(11.2\times10^{-12}\ \text{F}\right)(12\ \text{V}) = \boxed{134\ \text{pC}}$

(c) Now P_3 has charge on two surfaces and in effect three capacitors are in parallel:

$$C = 3\left(5.58\ \text{pF}\right) = \boxed{16.7\ \text{pF}}$$

(d) Only one face of P_4 carries charge:

$$Q = C\Delta V = \left(5.58\times10^{-12}\ \text{F}\right)(12\ \text{V}) = \boxed{66.9\ \text{pC}}$$

P26.56 The upper pair of capacitors, 3-μF and 6-μF, are in series. Their equivalent capacitance is

$$\left(\frac{1}{3.00} + \frac{1}{6.00}\right)^{-1} = 2.00\ \mu\text{F}$$

The lower pair of capacitors, 2-μF and 4-μF, are in series. Their equivalent capacitance is

$$\left(\frac{1}{2.00} + \frac{1}{4.00}\right)^{-1} = 1.33\ \mu\text{F}$$

The upper pair are in parallel to the lower pair, so the total capacitance is

$$C_{eq} = 2.00\mu\text{F} + 1.33\mu\text{F} = 3.33\mu\text{F}$$

(a) The total energy stored in the full circuit is then

$$\left(\text{Energy stored}\right)_{\text{total}} = \frac{1}{2}C_{eq}\left(\Delta V\right)^2 = \frac{1}{2}\left(3.33\times10^{-6}\ \text{F}\right)\left(90.0\ \text{V}\right)^2$$
$$= 1.35\times10^{-2}\ \text{J} = 13.5\times10^{-3}\ \text{J} = \boxed{13.5\ \text{mJ}}$$

(b) Refer to P26.19 for the calculation of the charges used below. The energy stored in each individual capacitor is

For 2.00 μF:

$$(\text{Energy stored})_2 = \frac{Q_2^2}{2C_2} = \frac{(120\times10^{-6}\ \text{C})^2}{2(2.00\times10^{-6}\ \text{F})} = 3.60\times10^{-3}\ \text{J}$$

$$= \boxed{3.60\ \text{mJ}}$$

For 3.00 μF:

$$(\text{Energy stored})_3 = \frac{Q_3^2}{2C_3} = \frac{(180\times10^{-6}\ \text{C})^2}{2(3.00\times10^{-6}\ \text{F})} = 5.40\times10^{-3}\ \text{J}$$

$$= \boxed{5.40\ \text{mJ}}$$

For 4.00 μF:

$$(\text{Energy stored})_4 = \frac{Q_4^2}{2C_4} = \frac{(120\times10^{-6}\ \text{C})^2}{2(4.00\times10^{-6}\ \text{F})} = 1.80\times10^{-3}\ \text{J}$$

$$= \boxed{1.80\ \text{mJ}}$$

For 6.00 μF:

$$(\text{Energy stored})_6 = \frac{Q_6^2}{2C_6} = \frac{(180\times10^{-6}\ \text{C})^2}{2(6.00\times10^{-6}\ \text{F})} = 2.70\times10^{-3}\ \text{J}$$

$$= \boxed{2.70\ \text{mJ}}$$

(c) Energy stored $= (3.60 + 5.40 + 1.80 + 2.70)\ \text{mJ} = 13.5\ \text{mJ} = (\text{Energy stored})_{\text{total}}$

$\boxed{\text{The total energy stored by the system equals the sum of the energies stored in the individual capacitors.}}$

***P26.57** From Equation 26.13,

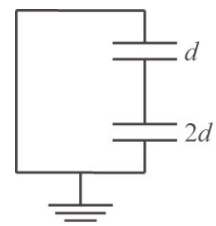
$$u_E = \frac{U_E}{V} = \frac{1}{2}\epsilon_0 E^2$$

Solving for the volume gives

$$V = \frac{U_E}{\frac{1}{2}\epsilon_0 E^2} = \frac{1.00\times10^{-7}\ \text{J}}{\frac{1}{2}(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3\,000\ \text{V/m})^2}$$

$$= \boxed{2.51\times10^{-3}\ \text{m}^3} = (2.51\times10^{-3}\ \text{m}^3)\left(\frac{1\,000\ \text{L}}{\text{m}^3}\right) = \boxed{2.51\ \text{L}}$$

P26.58 Imagine the center plate is split along its midplane and pulled apart. We have two capacitors in parallel, supporting the same ΔV and carrying total charge Q. The upper capacitor has capacitance $C_1 = \dfrac{\epsilon_0 A}{d}$ and the lower $C_2 = \dfrac{\epsilon_0 A}{2d}$. Charge flows from ground onto each of the outside plates so that



ANS. FIG. P26.58

$$Q_1 + Q_2 = Q \quad \text{and} \quad \Delta V_1 = \Delta V_2 = \Delta V.$$

Then $$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_1 d}{\epsilon_0 A} = \frac{Q_2 2d}{\epsilon_0 A} \quad \to \quad Q_1 = 2Q_2 \quad \to \quad 2Q_2 + Q_2 = Q.$$

(a) $Q_2 = \dfrac{Q}{3}$. $\boxed{\text{On the lower plate the charge is } -\dfrac{Q}{3}}$.

$Q_1 = \dfrac{2Q}{3}$. $\boxed{\text{On the upper plate the charge is } -\dfrac{2Q}{3}}$

(b) $\Delta V = \dfrac{Q_1}{C_1} = \boxed{\dfrac{2Qd}{3\epsilon_0 A}}$

P26.59 The dielectric strength is $E_{max} = 2.00 \times 10^8 \text{ V/m} = \dfrac{\Delta V_{max}}{d}$,

so we have for the distance between plates $d = \dfrac{\Delta V_{max}}{E_{max}}$.

Now to also satisfy $C = \dfrac{\kappa \epsilon_0 A}{d} = 0.250 \times 10^{-6}$ F with $\kappa = 3.00$, we combine by substitution to solve for the plate area:

$$A = \frac{Cd}{\kappa \epsilon_0} = \frac{C\Delta V_{max}}{\kappa \epsilon_0 E_{max}} = \frac{(0.250 \times 10^{-6} \text{ F})(4\ 000 \text{ V})}{(3.00)(8.85 \times 10^{-12} \text{ F/m})(2.00 \times 10^8 \text{ V/m})}$$

$$= \boxed{0.188 \text{ m}^2}$$

P26.60 We can use the energy U_C stored in the capacitor to find the potential difference across the plates:

$$U_C = \frac{1}{2}C(\Delta V)^2 \quad \to \quad \Delta V = \sqrt{\frac{2U_C}{C}}$$

When the particle moves between the plates, the change in potential energy of the charge-field system is

$$\Delta U_{\text{system}} = q\Delta V = -q\sqrt{\frac{2U_C}{C}}$$

where we have noted that the potential difference is negative from the positive plate to the negative plate. Apply the isolated system (energy) model to the charge-field system:

$$\Delta K + \Delta U_{\text{system}} = 0 \rightarrow \Delta K = -\Delta U_{\text{system}} = q\sqrt{\frac{2U_C}{C}}$$

Substitute numerical values:

$$\Delta K = \left(-3.00 \times 10^{-6}\ \text{C}\right)\sqrt{\frac{2\left(0.050\ 0\ \text{J}\right)}{10.0 \times 10^{-6}\ \text{F}}} = -3.00 \times 10^{-4}\ \text{J}$$

This decrease in kinetic energy of the particle is more than the energy with which it began. Therefore, the particle does not arrive at the negative plate but rather turns around and moves back to the positive plate.

***P26.61** (a) $V = \dfrac{m}{\rho} = \dfrac{1.00 \times 10^{-12}\ \text{kg}}{1\ 100\ \text{kg/m}^3} = \boxed{9.09 \times 10^{-16}\ \text{m}^3}$

Since $V = \dfrac{4\pi r^3}{3}$, the radius is $r = \left[\dfrac{3V}{4\pi}\right]^{1/3}$, and the surface area is

$$A = 4\pi r^2 = 4\pi\left[\frac{3V}{4\pi}\right]^{2/3} = 4\pi\left[\frac{3\left(9.09 \times 10^{-16}\ \text{m}^3\right)}{4\pi}\right]^{2/3}$$

$$= \boxed{4.54 \times 10^{-10}\ \text{m}^2}$$

(b) $C = \dfrac{\kappa \epsilon_0 A}{d} = \dfrac{(5.00)\left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.54 \times 10^{-10}\ \text{m}^2\right)}{100 \times 10^{-9}\ \text{m}}$

$= \boxed{2.01 \times 10^{-13}\ \text{F}}$

(c) $Q = C(\Delta V) = \left(2.01 \times 10^{-13}\ \text{F}\right)\left(100 \times 10^{-3}\ \text{V}\right) = \boxed{2.01 \times 10^{-14}\ \text{C}}$,

and the number of electronic charges is

$$n = \frac{Q}{e} = \frac{2.01 \times 10^{-14}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = \boxed{1.26 \times 10^5}$$

P26.62 (a) With the liquid filling the space between the plates to height *fd*, the top of the fluid at the air-fluid interface develops an induced dipole layer of charge so that it acts as a thin plate with opposite charge on its upper and lower sides; thus, the partially filled capacitor behaves as two capacitors in series connected at the interface. The upper and lower capacitors have separate capacitances:

$$C_{\text{up}} = \frac{1 \in_0 A}{d(1-f)} \quad \text{and} \quad C_{\text{down}} = \frac{6.5 \in_0 A}{fd}$$

The equivalent series capacitance is

$$C_f = \frac{1}{\dfrac{d(1-f)}{\in_0 A} + \dfrac{fd}{6.5 \in_0 A}} = \frac{6.5 \in_0 A}{6.5d - 6.5df + fd}$$

$$= \left(\frac{\in_0 A}{d}\right)\left(\frac{6.5}{6.5 - 5.5f}\right)$$

$$= \boxed{25.0\ \mu\text{F}(1-0.846f)^{-1}}$$

(b) For $f = 0$, the capacitor is empty so we can expect capacitance $\boxed{25.0\ \mu\text{F}}$. For $f = 0$,

$$C_f = 25.0\ \mu\text{F}(1-0.846f)^{-1} = 25.0\ \mu\text{F}(1-0)^{-1} = 25.0\ \mu\text{F}$$

and $\boxed{\text{the general expression agrees}}$.

(c) For $f = 1$, we expect $6.5(25.0\ \mu\text{F}) = 162\ \mu\text{F}$. For $f = 1$,

$$C_f = 25.0\ \mu\text{F}(1-0.846f)^{-1} = 25.0\ \mu\text{F}(1-0.846)^{-1} = \boxed{162\ \mu\text{F}}$$

and $\boxed{\text{the general expression agrees}}$.

P26.63 The initial charge on the larger capacitor is

$$Q = C\Delta V = (10.0\ \mu\text{F})(15.0\ \text{V}) = 150\ \mu\text{C}$$

An additional charge q is pushed through the 50.0-V battery, giving the smaller capacitor charge q and the larger charge 150 μC + q.

Then $$50.0\ \text{V} = \frac{q}{5.00\ \mu\text{F}} + \frac{150\ \mu\text{C} + q}{10.0\ \mu\text{F}}.$$

$$500\ \mu\text{C} = 2q + 150\ \mu\text{C} + q$$
$$q = 117\ \mu\text{C}$$

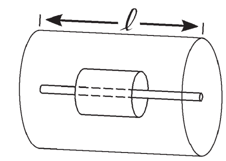
So across the 5.00-μF capacitor,

$$\Delta V = \frac{q}{C} = \frac{117\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{23.3\ \text{V}}$$

Across the 10.0-μF capacitor,

$$\Delta V = \frac{150\ \mu\text{C} + 117\ \mu\text{C}}{10.0\ \mu\text{F}} = \boxed{26.7\ \text{V}}$$

***P26.64** From Gauss's Law, for the electric field inside the cylinder, $2\pi r\ell E = \dfrac{q_{\text{in}}}{\epsilon_0}$.



ANS. FIG. P26.64

so $$E = \frac{\lambda}{2\pi r\,\epsilon_0}.$$

$$\Delta V = -\int_{r_1}^{r_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi r \epsilon_0} dr = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{r_1}{r_2}\right)$$

Recognizing that $\dfrac{\lambda_{\max}}{2\pi\epsilon_0} = E_{\max} r_{\text{inner}}$, we obtain

$$\Delta V = \left(1.20\times10^{6}\ \text{V/m}\right)\left(0.100\times10^{-3}\ \text{m}\right)\ln\left(\frac{25.0\ \text{m}}{0.200\ \text{m}}\right)$$

$$\Delta V_{\max} = \boxed{579\ \text{V}}$$

P26.65 Where the metal block and the plates overlap, the electric field between the plates is zero. The plates do not lose charge in the overlapping region, but opposite charge induced on the surfaces of the inserted portion of the block cancels the field from charge on the plates. The unfilled portion of the capacitor has capacitance

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \ell(\ell - x)}{d}$$

The effective charge on this portion (the charge producing the remaining electric field between the plates) is proportional to the unblocked area:

$$Q = \frac{(\ell - x)Q_0}{\ell}$$

(a) The stored energy is

$$U = \frac{Q^2}{2C} = \frac{\left[(\ell - x)Q_0/\ell\right]^2}{2\epsilon_0 \ell(\ell - x)/d} = \boxed{\frac{Q_0^2 d(\ell - x)}{2\epsilon_0 \ell^3}}$$

(b) $F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{Q_0^2(\ell - x)d}{2\epsilon_0 \ell^3}\right) = +\frac{Q_0^2 d}{2\epsilon_0 \ell^3}$

$\vec{\mathbf{F}} = \boxed{\frac{Q_0^2 d}{2\epsilon_0 \ell^3} \text{ to the right}}$ (into the capacitor: the block is pulled in)

(c) Stress $= \frac{F}{\ell d} = \boxed{\frac{Q_0^2}{2\epsilon_0 \ell^4}}$

(d) The energy density is

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{\sigma}{\epsilon_0}\right)^2 = \frac{1}{2\epsilon_0}\left(\frac{Q}{A}\right)^2 = \frac{1}{2\epsilon_0}\left[\frac{\cancel{(\ell - x)}Q_0/\ell}{\ell\cancel{(\ell - x)}}\right]^2$$

$$= \boxed{\frac{Q_0^2}{2\epsilon_0 \ell^4}}$$

(e) They are precisely the same.

P26.66 (a) Put charge Q on the sphere of radius a and $-Q$ on the other sphere. Relative to $V = 0$ at infinity, because d is larger compared to a and to b.

The potential at the surface of a is approximately $V_a = \frac{k_e Q}{a} - \frac{k_e Q}{d}$

and the potential of b is approximately $V_b = \frac{-k_e Q}{b} + \frac{k_e Q}{d}$.

The difference in potential is $V_a - V_b = \frac{k_e Q}{a} + \frac{k_e Q}{b} - \frac{k_e Q}{d} - \frac{k_e Q}{d}$

and $C = \frac{Q}{V_a - V_b} = \frac{4\pi\epsilon_0}{\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right) - \left(\frac{2}{d}\right)}$

(b) As $d \to \infty$, $\frac{1}{d}$ becomes negligible compared to $\frac{1}{a}$ and $\frac{1}{b}$. Then,

$$C = \frac{4\pi\epsilon_0}{\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right)} \quad \text{and} \quad \frac{1}{C} = \boxed{\frac{1}{4\pi\epsilon_0 a} + \frac{1}{4\pi\epsilon_0 b}}$$

as for two spheres in series.

P26.67 Call the unknown capacitance C_u. The charge remains the same:

$$Q = C_u(\Delta V_i) = (C_u + C)(\Delta V_f)$$

$$\boxed{C_u = \frac{C(\Delta V_f)}{(\Delta V_i) - (\Delta V_f)}} = \frac{(10.0\ \mu\text{F})(30.0\ \text{V})}{(100\ \text{V} - 30.0\ \text{V})} = \boxed{4.29\ \mu\text{F}}$$

P26.68 (a) $C_0 = \frac{\epsilon_0 A}{d} = \frac{Q_0}{\Delta V_0}$ for a capacitor with air or vacuum beteen its plates. When the dielectric is inserted at constant voltage,

$$C = \kappa C_0 = \frac{Q}{\Delta V_0}$$

The original energy is

$$U_{E0} = \frac{C_0(\Delta V_0)^2}{2}$$

and the final energy is

$$U_E = \frac{C(\Delta V_0)^2}{2} = \frac{\kappa C_0(\Delta V_0^2)}{2}$$

therefore,

$$\frac{U_E}{U_{E0}} = \kappa$$

(b) The electric field between the plates polarizes molecules within the dielectric; therefore the field does work on charge within the molecules to create electric dipoles. The extra energy comes from (part of the) electrical work done by the battery in separating that charge.

(c) The charge on the plates increases because the voltage remains the same:

$$Q_0 = C_0 \Delta V_0$$

and $Q = C\Delta V_0 = \kappa C_0 \Delta V_0$

so the charge increases according to $\boxed{\frac{Q}{Q_0} = \kappa}$.

P26.69 Initially (capacitors charged in parallel),

$$q_1 = C_1(\Delta V) = (6.00\ \mu\text{F})(250\ \text{V}) = 1\,500\ \mu\text{C}$$

$$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(250\ \text{V}) = 500\ \mu\text{C}$$

After reconnection (positive plate to negative plate),

$$q'_{\text{total}} = q_1 - q_2 = 1\,000\ \mu\text{C}$$

and $$\Delta V' = \frac{q'_{\text{total}}}{C_{\text{total}}} = \frac{1\,000\ \mu\text{C}}{8.00\ \mu\text{F}} = 125\ \text{V}$$

Therefore,

$$q'_1 = C_1(\Delta V') = (6.00\ \mu\text{F})(125\ \text{V}) = \boxed{750\ \mu\text{C}}$$

$$q'_2 = C_2(\Delta V') = (2.00\ \mu\text{F})(125\ \text{V}) = \boxed{250\ \mu\text{C}}$$

P26.70 The condition that we are testing is that the capacitance increases by less than 10%, or,

$$\frac{C'}{C} < 1.10$$

Substituting the expressions for C and C' from Example 26.1, we have

$$\frac{C'}{C} = \frac{\dfrac{\ell}{2k_e \ln(b/1.10a)}}{\dfrac{\ell}{2k_e \ln(b/a)}} = \frac{\ln(b/a)}{\ln(b/1.10a)} < 1.10$$

This becomes

$$\ln\left(\frac{b}{a}\right) < 1.10\ln\left(\frac{b}{1.10a}\right) = 1.10\ln\left(\frac{b}{a}\right) + 1.10\ln\left(\frac{1}{1.10}\right)$$

$$= 1.10\ln\left(\frac{b}{a}\right) - 1.10\ln(1.10)$$

We can rewrite this as

$$-0.10\ln\left(\frac{b}{a}\right) < -1.10\ln(1.10)$$

$$\ln\left(\frac{b}{a}\right) > 11.0\ln(1.10) = \ln(1.10)^{11.0}$$

where we have reversed the direction of the inequality because we multiplied the whole expression by –1 to remove the negative signs.

Comparing the arguments of the logarithms on both sides of the inequality, we see that

$$\frac{b}{a} > (1.10)^{11.0} = 2.85$$

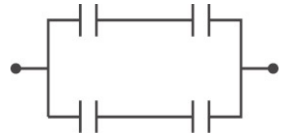
Thus, if $b > 2.85a$, the increase in capacitance is less than 10% and it is more effective to increase ℓ.

P26.71 Placing two identical capacitor is series will split the voltage evenly between them, giving each a voltage of 45 V, but the total capacitance will be half of what is needed. To double the capacitance, another pair of series capacitors must be placed in parallel with the first pair, as shown in ANS. FIG. P26.71A. The equivalent capacitance is

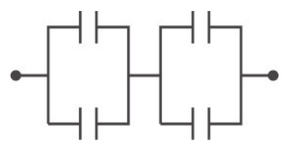
$$\left(\frac{1}{100\ \mu\text{F}} + \frac{1}{100\ \mu\text{F}}\right)^{-1} + \left(\frac{1}{100\ \mu\text{F}} + \frac{1}{100\ \mu\text{F}}\right)^{-1} = 100\ \mu\text{F}$$

Another possibility shown in ANS. FIG. P26.71B: two capacitors in parallel, connected in series to another pair of capacitors in parallel; the voltage across each parallel section is then 45 V. The equivalent capacitance is

$$\frac{1}{(100\ \mu\text{F} + 100\ \mu\text{F})^{-1} + (100\ \mu\text{F} + 100\ \mu\text{F})^{-1}} = 100\ \mu\text{F}$$



ANS. FIG. P26.71A



ANS. FIG. P26.71B

(a) One capacitor cannot be used by itself — it would burn out. She can use two capacitors in series, connected in parallel to another two capacitors in series. Another possibility is two capacitors in parallel, connected in series to another two capacitors in parallel. In either case, one capacitor will be left over.

(b) Each of the four capacitors will be exposed to a maximum voltage of 45 V.

Challenge Problems

P26.72 From Example 26.1, when there is a vacuum between the conductors, the voltage between them is

$$\Delta V = |V_b - V_a| = 2k_e \lambda \ln\left(\frac{b}{a}\right) = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{b}{a}\right)$$

With a dielectric, a factor $1/\kappa$ must be included, and the equation becomes

$$\Delta V = \frac{\lambda}{2\pi\kappa \epsilon_0} \ln\left(\frac{b}{a}\right)$$

The electric field is

$$E = \frac{\lambda}{2\pi\kappa \epsilon_0 r}$$

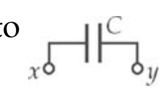
So when $E = E_{\max}$ at $r = a$,

$$\frac{\lambda_{\max}}{2\pi\kappa \epsilon_0} = E_{\max} a \quad \text{and} \quad \Delta V_{\max} = \frac{\lambda_{\max}}{2\pi\kappa \epsilon_0} \ln\left(\frac{b}{a}\right) = E_{\max} a \ln\left(\frac{b}{a}\right)$$

Thus,

$$\Delta V_{\max} = \left(18.0 \times 10^6 \text{ V/m}\right)\left(0.800 \times 10^{-3} \text{ m}\right) \ln\left(\frac{3.00 \text{ mm}}{0.800 \text{ mm}}\right)$$

$$= \boxed{19.0 \text{ kV}}$$

P26.73 According to the suggestion, the combination of capacitors shown is equivalent to 

Then, from ANS. FIG. P26.73,

$$\frac{1}{C} = \frac{1}{C_0} + \frac{1}{C + C_0} + \frac{1}{C_0}$$

$$= \frac{C + C_0 + C_0 + C + C_0}{C_0\left(C + C_0\right)}$$

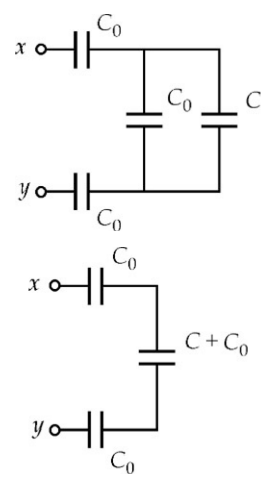
$$C_0 C + C_0^2 = 2C^2 + 3C_0 C$$

$$2C^2 + 2C_0 C - C_0^2 = 0$$

$$C = \frac{-2C_0 \pm \sqrt{4C_0^2 + 4\left(2C_0^2\right)}}{4}$$

Only the positive root is physical:

$$\boxed{C = \frac{C_0}{2}\left(\sqrt{3} - 1\right)}$$



ANS. FIG. P26.73

P26.74 Let charge λ per length be on one wire and $-\lambda$ be on the other. The electric field due to the charge on the positive wire is perpendicular to the wire, radial, and of magnitude

$$E_+ = \frac{\lambda}{2\pi \epsilon_0 r}$$

The potential difference between the surfaces of the wires due to the presence of this charge is

$$\Delta V_1 = -\int_{-\text{wire}}^{+\text{wire}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\frac{\lambda}{2\pi \epsilon_0} \int_{D-r}^{r} \frac{dr}{r} = \frac{\lambda}{2\pi \epsilon_0} \ln\left(\frac{D-r}{r}\right)$$

The presence of the linear charge density $-\lambda$ on the negative wire makes an identical contribution to the potential difference between the wires. Therefore, the total potential difference is

$$\Delta V = 2(\Delta V_1) = \frac{\lambda}{\pi \epsilon_0} \ln\left(\frac{D-r}{r}\right)$$

With D much larger than r we have nearly $\Delta V = \dfrac{\lambda}{\pi \epsilon_0} \ln\left(\dfrac{D}{r}\right)$

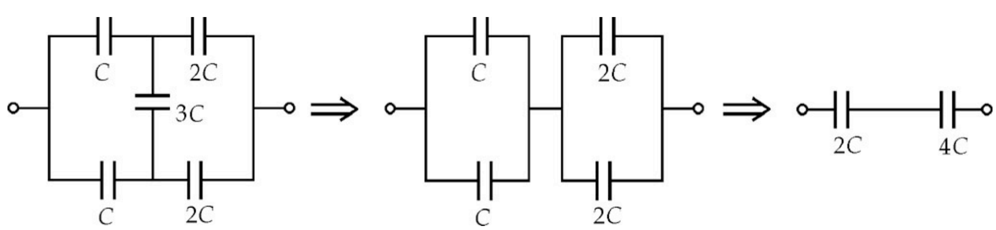
and the capacitance of this system of two wires, each of length ℓ, is

$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\Delta V} = \frac{\lambda \ell}{(\lambda/\pi \epsilon_0)\ln[D/r]} = \frac{\pi \epsilon_0 \ell}{\ln[D/r]}$$

The capacitance per unit length is $\boxed{\dfrac{C}{\ell} = \dfrac{\pi \epsilon_0}{\ln[D/r]}}$.

P26.75 By symmetry, the potential difference across $3C$ is zero, so the circuit reduces to (see ANS. FIG. P26.75):

$$C_{\text{eq}} = \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{8}{6}C = \boxed{\frac{4}{3}C}$$



ANS. FIG. P26.75

P26.76 (a) Consider a strip of width dx and length W at position x from the front left corner. The capacitance of the lower portion of this strip is $\dfrac{\kappa_1 \epsilon_0 W\, dx}{t\, x/L}$. The capacitance of the upper portion is $\dfrac{\kappa_2 \epsilon_0 W\, dx}{t\,(1-x/L)}$.

The series combination of the two elements has capacitance

$$\frac{1}{\dfrac{tx}{\kappa_1 \in_0 WL\,dx}+\dfrac{t(L-x)}{\kappa_2 \in_0 W\,Ldx}}=\frac{\kappa_1\kappa_2 \in_0 W\,Ldx}{\kappa_2 tx+\kappa_1 tL-\kappa_1 tx}$$

The whole capacitance is a combination of elements in parallel:

$$C=\int_0^L \frac{\kappa_1\kappa_2 \in_0 W\,Ldx}{(\kappa_2-\kappa_1)tx+\kappa_1 tL}$$

$$=\frac{1}{(\kappa_2-\kappa_1)t}\int_0^L \frac{\kappa_1\kappa_2 \in_0 W\,L(\kappa_2-\kappa_1)tdx}{(\kappa_2-\kappa_1)tx+\kappa_1 tL}$$

$$=\frac{\kappa_1\kappa_2 \in_0 W\,L}{(\kappa_2-\kappa_1)t}\ln\left[(\kappa_2-\kappa_1)tx+\kappa_1 tL\right]_0^L$$

$$=\frac{\kappa_1\kappa_2 \in_0 WL}{(\kappa_2-\kappa_1)t}\ln\left[\frac{(\kappa_2-\kappa_1)tL+\kappa_1 tL}{0+\kappa_1 tL}\right]$$

$$=\frac{\kappa_1\kappa_2 \in_0 WL}{(\kappa_2-\kappa_1)t}\ln\left[\frac{\kappa_2}{\kappa_1}\right]=\frac{\kappa_1\kappa_2 \in_0 WL}{(-1)(\kappa_2-\kappa_1)t}\ln\left[\left(\frac{\kappa_2}{\kappa_1}\right)^{-1}\right]$$

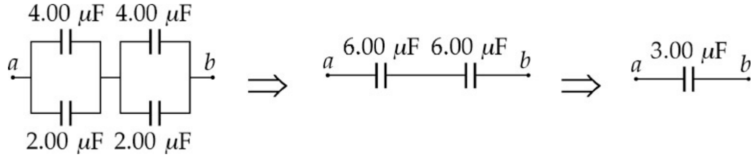
$$=\boxed{\frac{\kappa_1\kappa_2 \in_0 WL}{(\kappa_1-\kappa_2)t}\ln\left[\frac{\kappa_1}{\kappa_2}\right]}$$

(b) The capacitor physically has the same capacitance if it is turned upside down, so the answer should be the same with κ_1 and κ_2 interchanged. We have proven that it has this property in the solution to part (a).

(c) Let $\kappa_1 = \kappa_2\,(1 + x)$. Then $C=\dfrac{\kappa_2(1+x)\kappa_2 \in_0 WL}{\kappa_2 xt}\ln[1+x]$.

As x approaches zero we have $C=\dfrac{\kappa(1+0)\in_0 WL}{xt}x=\dfrac{\kappa \in_0 WL}{t}$ as was to be shown.

P26.77 Assume a potential difference across a and b, and notice that the potential difference across 8.00 μF the capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the circuit shown in ANS. FIG. P26.77, and is $C_{ab}=\boxed{3.00\ \mu\text{F}}$.



ANS. FIG. P26.77

P26.78 (a) The portion of the device containing the dielectric has plate area ℓx and capacitance $C_1 = \frac{\kappa \epsilon_0 \ell x}{d}$. The unfilled part has area $\ell(\ell - x)$ and capacitance $C_2 = \frac{\epsilon_0 \ell(\ell - x)}{d}$. The total capacitance is $C_1 + C_2 = \boxed{\frac{\epsilon_0 \ell}{d}[\ell + x(\kappa - 1)]}$.

(b) The stored energy is $U = \frac{1}{2}\frac{Q^2}{C} = \boxed{\frac{Q^2 d}{2\epsilon_0 \ell[\ell + x(\kappa - 1)]}}$.

(c) $\vec{\mathbf{F}} = -\left(\frac{dU}{dx}\right)\hat{\mathbf{i}} = \boxed{\frac{Q^2 d(\kappa - 1)}{2\epsilon_0 \ell[\ell + x(\kappa - 1)]^2}\hat{\mathbf{i}}}$. When $x = 0$, the original value of the force is $\frac{Q^2 d(\kappa - 1)}{2\epsilon_0 \ell^3}\hat{\mathbf{i}}$. As the dielectric slides in, the charges on the plates redistribute themselves. The force decreases to its final value, when $x = \ell$, of $\frac{Q^2 d(\kappa - 1)}{2\epsilon_0 \ell^3 \kappa^2}\hat{\mathbf{i}}$.

(d) At $x = \frac{\ell}{2}$, $\vec{\mathbf{F}} = \frac{2Q^2 d(\kappa - 1)}{\epsilon_0 \ell^3 (\kappa + 1)^2}\hat{\mathbf{i}}$.

For the constant charge on the capacitor and the initial voltage we have the relationship

$$Q = C_0 \Delta V = \frac{\epsilon_0 \ell^2 \Delta V}{d}$$

Then the force is $\vec{\mathbf{F}} = \frac{2\epsilon_0 \ell(\Delta V)^2(\kappa - 1)}{d(\kappa + 1)^2}\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}} = \frac{2(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(0.050\ 0\ \mathrm{m})(2.00 \times 10^3\ \mathrm{V})^2(4.50 - 1)}{(0.002\ 00\ \mathrm{m})(4.50 + 1)^2}\hat{\mathbf{i}}$$

$$= \boxed{205\hat{\mathbf{i}}\ \mu\mathrm{N}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P26.2 (a) 1.00 μF; (b) 100 V

P26.4 (a) 15.6 pF; (b) 257 kV

P26.6 (a) 11.1 nF; (b) 26.6 C

P26.8 (a) 1.36 pF; (b) 16.3 pC; (c) 8.00×10^3 V/m

P26.10 $\dfrac{(2N-1)\epsilon_0(\pi-\theta)R^2}{d}$

P26.12 $\dfrac{mgd\tan\theta}{q}$

P26.14 (a) 3.53 μF; (b) 6.35 V and 2.65 V; (c) 31.8 μC

P26.16 (a) 10.7 μC; (b) 15.0 μC and 37.5 μC

P26.18 None of the possible combinations of the extra capacitors is $\frac{4}{3}C$, so the desired capacitance cannot be achieved.

P26.20 (a) 2*C*; (b) $Q_1 > Q_3 > Q_2$; (c) $\Delta V_1 > \Delta V_2 > \Delta V_3$; (d) Q_3 and Q_1 increase; Q_2 decreases

P26.22 (a) 6.05 μF; (b) 83.7 μC

P26.24 120 μC; (b) 40.0 μC and 80.0 μC

P26.26 (a) 2.67 μF; (b) 24.0 μC, 24.0 μC, 6.00 μC, 18.0 μC; (c) 3.00 V

P26.28 $C_1 = \frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$ and $C_2 = \frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$

P26.30 4.47×10^3 V

P26.32 (a) 216 μJ; (b) 54.0 μJ

P26.34 (a) 12.0 μF; (b) 8.64×10^{-4} J; (c) $U_1 = 5.76 \times 10^{-4}$ J and $U_2 = 2.88 \times 10^{-4}$ J; (d) $U_1 + U_2 = 5.76\times10^{-4}\,\text{J} + 2.88\times10^{-4}\,\text{J} = 8.64\times10^{-4}\,\text{J} = U_{\text{eq}}$, which is one reason why the 12.0 μF capacitor is considered to be equivalent to the two capacitors; (e) The total energy of the equivalent capacitance will always equal the sum of the energies stored in the individual capacitors; (f) 5.66 V; (g) The larger capacitor C_2 stores more energy.

P26.36 (a) $C(\Delta V)^2$; (b) $\Delta V' = \dfrac{4\Delta V}{3}$; (c) $4C\dfrac{(\Delta V)^2}{3}$; (d) Positive work is done by the agent pulling the plates apart.

P26.38 $\dfrac{Q^2}{2\in_0 A}$

P26.40 (a) $\dfrac{k_e Q^2}{2R}$; (b) $\dfrac{k_e q_1^2}{2R_1}+\dfrac{k_e(Q-q_1)^2}{2R_2}$; (c) $\dfrac{R_1 Q}{R_1+R_2}$; (d) $\dfrac{R_2 Q}{R_1+R_2}$;

(e) $V_1=\dfrac{k_e Q}{R_1+R_2}$ and $V_2=\dfrac{k_e Q}{R_1+R_2}$; (f) 0

P26.42 (a) Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them; (b) 10^{-6} F; (c) 10^2 V

P26.44 (a) $\kappa = 3.40$; (b) nylon; (c) The voltage would lie somewhere between 25.0 V and 85.0 V.

P26.46 1.04 m

P26.48 22.5 V

P26.50 (a) $\left(-9.10\hat{\mathbf{i}}+8.40\hat{\mathbf{j}}\right)\times 10^{-12}\ \text{C}\cdot\text{m}$; (b) $-2.09\times 10^{-8}\,\hat{\mathbf{k}}\ \text{N}\cdot\text{m}$; (c) 112 nJ; (d) 228 nJ

P26.52 $p\dfrac{dE}{dx}\cos\theta\,\hat{\mathbf{i}}$

P26.54 6.25 μF

P26.56 (a) 13.5 mJ; (b) 3.60 mJ, 5.40 mJ, 1.80 mJ, 2.70 mJ; (c) The total energy stored by the system equals the sum of the energies stored in the individual capacitors.

P26.58 (a) On the lower plate the charge is $-\dfrac{Q}{3}$, and on the upper plate the charge is $-\dfrac{2Q}{3}$; (b) $\dfrac{2Qd}{3\in_0 A}$

P26.60 The decrease in kinetic energy of the particle is more than the energy with which it began. Therefore, the particle does not arrive at the negative plate but rather turns around and moves back to the positive plate.

P26.62 (a) $2.50\,\mu\text{F}(1-0.846f)^{-1}$; (b) $25.0\,\mu\text{F}$, the general expression agrees; (c) 162 μF; The general expression agrees.

P26.64 579 V

P26.66 (a) See P26.66(a) for full explanation; (b) $\dfrac{1}{4\pi \epsilon_0 a} + \dfrac{1}{4\pi \epsilon_0 b}$

P26.68 (a) See P26.68(a) for full explanation; (b) The electric field between the plates polarizes molecules within the dielectric; therefore the field does work on charge within the molecules to create electric dipoles. The extra energy comes from (part of the) electrical work done by the battery in separating that charge; (c) $\dfrac{Q}{Q_0} = \kappa$

P26.70 See P26.70 for full mathematical verification.

P26.72 19.0 kV

P26.74 $\dfrac{C}{\ell} = \dfrac{\pi \epsilon_0}{1\text{n}\left[D/r\right]}$

P26.76 (a) $\dfrac{\kappa_1 \kappa_2 \epsilon_0 WL}{(\kappa_1 - \kappa_2)t} \ln\left[\dfrac{\kappa_1}{\kappa_2}\right]$; (b) The capacitor physically has the same capacitance if it is turned upside down, so the answer should be the same with κ_1 and κ_2 interchanged. We have proven that it has this property in the solution to part (a); (c) See P26.76(c) for full explanation.

P26.78 (a) $\dfrac{\epsilon_0 \ell}{d}\left[\ell + x(\kappa - 1)\right]$; (b) $\dfrac{Q^2 d}{2\epsilon_0 \ell\left[\ell + x(\kappa - 1)\right]}$; (c) $\dfrac{Q^2 d(\kappa - 1)}{2\epsilon_0 \ell\left[\ell + x(\kappa - 1)\right]^2}\hat{\mathbf{i}}$;

(d) $205\hat{\mathbf{i}}\ \mu\text{N}$