27

Current and Resistance

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ27.1 Answer (d). One ampere–hour is (1 C/s)(3 600 s) = 3 600 coulombs. The ampere–hour rating is the quantity of charge that the battery can lift though its nominal potential difference.

OQ27.2 (**i**) Answer (e). We require $\rho L/A_A = 3\rho L/A_B$. Then $A_A/A_B = 1/3$.

(**ii**) Answer (d). $\pi r_A{}^2/\pi r_B{}^2 = 1/3$ gives $r_A/r_B = 1/\sqrt{3}$.

OQ27.3 The ranking is c > a > b > d > e. Because

(a) $I = \Delta V / R,$ so the current becomes 3 times larger.

(b) $P = I\Delta V = I^2R,$ so the current is $\sqrt{3}$ times larger.

(c) R is 1/4 as large, so the current is 4 times larger.

(d) R is 2 times larger, so the current is 1/2 as large.

(e) R increases by a small percentage, so the current has a small decrease.

OQ27.4 (**i**) Answer (a). The cross-sectional area decreases, so the current density increases, thus the drift speed must increase.

(ii) Answer (a). The cross-sectional area decreases, so the resistance per unit length, $R/L = \rho/A$, increases.

OQ27.5 Answer (c). $I = \Delta V / R = 1.00\text{ V} / 10.0\ \Omega = 0.100\text{ A} = 0.100\text{ C/s}$. Because current is constant, $I = dq/dt = \Delta q/\Delta t$, and we find that

$$\Delta q = I\Delta t = (0.100\text{ C/s})(20.0\text{ s}) = 2.00\text{ C}$$

OQ27.6 Answer (c). The resistances are: $R_1 = \rho L/A = \rho L/\pi r^2$, $R_2 = \rho L/\pi(2r)^2 = (1/4)\rho L/\pi r^2$, $R_3 = \rho(2L)/\pi(3r)^2 = (2/9)\rho L/\pi r^2$.

OQ27.7 Answer (a). The new cross-sectional area is three times the original. Originally, $R = \dfrac{\rho L}{A}$. Finally, $R_f = \dfrac{\rho(L/3)}{3A} = \dfrac{\rho L}{9A} = \dfrac{R}{9}$.

OQ27.8 Answer (b). Using $R_0 = 10.0\ \Omega$ at $T = 20.0\ °\text{C}$, we have $R = R_0(1+\alpha\Delta T)$ or

$$\alpha = \frac{R/R_0 - 1}{\Delta T} = \frac{10.6/10.0 - 1}{(90.0°\text{C} - 20.0°\text{C})} = 8.57\times 10^{-4}\ °\text{C}^{-1}$$

At $T = -20.0°\text{C}$, we have

$$\begin{aligned} R &= R_0(1+\alpha\Delta T) \\ &= (10.0\ \Omega)\left[1 + 8.57\times 10^{-4}\ °\text{C}^{-1}(-20.0°\text{C} - 20.0°\text{C})\right] = 9.66\ \Omega \end{aligned}$$

OQ27.9 Answer (a). $R = V/I = 2\text{ V}/2\text{ A} = 1\ \Omega$.

OQ27.10 Answer (c). Compare resistances:

$$\frac{R_A}{R_B} = \frac{\rho L_A/\pi(d_A/2)^2}{\rho L_B/\pi(d_B/2)^2} = \frac{L_A}{L_B}\frac{d_B{}^2}{d_A{}^2} = \frac{(2L_B)}{L_B}\frac{d_B{}^2}{(2d_B)^2} = \frac{2}{4} = \frac{1}{2}$$

Compare powers: $\dfrac{P_A}{P_B} = \dfrac{\Delta V^2/R_A}{\Delta V^2/R_B} = \dfrac{R_B}{R_A} = 2$.

OQ27.11 Answer (e). $R_A = \dfrac{\rho_A L}{A} = \dfrac{(2\rho_B)L}{A} = 2R_B$. Therefore,

$$\frac{P_A}{P_B} = \frac{\Delta V^2/R_A}{\Delta V^2/R_B} = \frac{R_B}{R_A} = \frac{1}{2}$$

OQ27.12 **(i)** Answer (a). $P = \Delta V^2/R$, and ΔV is the same for both bulbs, so the 25 W bulb must have higher resistance so that it will have lower power.

(ii) Answer (b). ΔV is the same for both bulbs, so the 100 W bulb must have lower resistance so that it will have more current.

OQ27.13 Answer (d). Because wire B has twice the radius, it has four times the cross-sectional area of wire A. For wire A, $R_A = R = \rho L/A$. For wire B, $R_B = \rho(2L)/(4A) = (1/2)\rho L/A = R/2$.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ27.1 Choose the voltage of the power supply you will use to drive the heater. Next calculate the required resistance R as $\frac{\Delta V^2}{P}$. Knowing the resistivity ρ of the material, choose a combination of wire length and cross-sectional area to make $\left(\frac{\ell}{A}\right)=\left(\frac{R}{\rho}\right)$. You will have to pay for less material if you make both ℓ and A smaller, but if you go too far the wire will have too little surface area to radiate away the energy; then the resistor will melt.

CQ27.2 Geometry and resistivity. In turn, the resistivity of the material depends on the temperature.

CQ27.3 The conductor does not follow Ohm's law, and must have a resistivity that is current-dependent, or more likely temperature-dependent.

CQ27.4 In a normal metal, suppose that we could proceed to a limit of zero resistance by lengthening the average time between collisions. The classical model of conduction then suggests that a constant applied voltage would cause constant acceleration of the free electrons. The drift speed and the current would increase steadily in time.

It is not the situation envisioned in the question, but we can actually switch to zero resistance by substituting a superconducting wire for the normal metal. In this case, the drift velocity of electrons is established by vibrations of atoms in the crystal lattice; the maximum current is limited; and it becomes impossible to establish a potential difference across the superconductor.

CQ27.5 The resistance of copper *increases* with temperature, while the resistance of silicon *decreases* with increasing temperature. The conduction electrons are scattered more by vibrating atoms when copper heats up. Silicon's charge carrier density increases as temperature increases and more atomic electrons are promoted to become conduction electrons.

CQ27.6 The amplitude of atomic vibrations increases with temperature. Atoms can then scatter electrons more efficiently.

CQ27.7 Because there are so many electrons in a conductor (approximately 10^{28} electrons/m^3) the average velocity of charges is very slow. When you connect a wire to a potential difference, you establish an electric field everywhere in the wire nearly instantaneously, to make electrons start drifting everywhere all at once.

CQ27.8 Voltage is a measure of potential difference, not of current. "Surge" implies a flow—and only charge, in coulombs, can flow through a system. It would also be correct to say that the victim carried a certain current, in amperes.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 21.1 Electric Current

***P27.1** The drift speed of electrons in the line is

$$v_d = \frac{I}{nqa} = \frac{I}{n|e|\left(\pi d^2/4\right)}$$

The time to travel the 200-km length of the line is then

$$\Delta t = \frac{L}{v_d} = \frac{Ln|e|\left(\pi d^2\right)}{4I}$$

Substituting numerical values,

$$\Delta t = \frac{\left(200\times10^3\text{ m}\right)\left(8.50\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\pi(0.02\text{ m})^2}{4(1\ 000\text{ A})}$$

$$= \left(8.55\times10^8\text{ s}\right)\left(\frac{1\text{ yr}}{3.156\times10^7\text{ s}}\right) = \boxed{27.1\text{ yr}}$$

***P27.2** The period of revolution for the sphere is $T = \dfrac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I = \dfrac{q}{T} = \boxed{\dfrac{q\omega}{2\pi}}$.

P27.3 We use $I = nqAv_d$, where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$m = \frac{27.0\text{ g}}{N_A} = \frac{27.0\text{ g}}{6.02\times10^{23}} = 4.49\times10^{-23}\text{ g/atom}$$

Thus,

$$n = \frac{\rho}{m} = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70 \text{ g/cm}^3}{4.49 \times 10^{-23} \text{ g/atom}}$$

$$n = 6.02 \times 10^{22} \text{ atoms/cm}^3 = 6.02 \times 10^{28} \text{ atoms/m}^3$$

Therefore,

$$v_d = \frac{I}{nqA} = \frac{5.00 \text{ A}}{\left(6.02 \times 10^{28} \text{ m}^{-3}\right)\left(1.60 \times 10^{-19} \text{ C}\right)\left(4.00 \times 10^{-6} \text{ m}^2\right)}$$

$$= 1.30 \times 10^{-4} \text{ m/s}$$

or, $v_d = \boxed{0.130 \text{ mm/s}}$.

P27.4 The period of the electron in its orbit is $T = 2\pi r/v$, and the current represented by the orbiting electron is

$$I = \frac{\Delta Q}{\Delta t} = \frac{|e|}{T} = \frac{v|e|}{2\pi r}$$

$$= \frac{\left(2.19 \times 10^6 \text{ m/s}\right)\left(1.60 \times 10^{-19} \text{ C}\right)}{2\pi\left(5.29 \times 10^{-11} \text{ m}\right)}$$

$$= 1.05 \times 10^{-3} \text{ C/s} = \boxed{1.05 \text{ mA}}$$

P27.5 If N is the number of protons, each with charge e, that hit the target in time Δt, the average current in the beam is $I = \Delta Q / \Delta t = Ne / \Delta t$, giving

$$N = \frac{I(\Delta t)}{e} = \frac{\left(125 \times 10^{-6} \text{ C/s}\right)(23.0 \text{ s})}{1.60 \times 10^{-19} \text{ C/proton}} = \boxed{1.80 \times 10^{16} \text{ protons}}$$

P27.6 (a) From Example 27.1 in the textbook, the density of charge carriers (electrons) in a copper wire is $n = 8.46 \times 10^{28}$ electrons/m^3. With $A = \pi r^2$ and $|q| = e$, the drift speed of electrons in this wire is

$$v_d = \frac{I}{n|q|A} = \frac{I}{ne\left(\pi r^2\right)}$$

$$= \frac{3.70 \text{ C/s}}{\left(8.46 \times 10^{28} \text{ m}^{-3}\right)\left(1.60 \times 10^{-19} \text{ C}\right)\pi\left(1.25 \times 10^{-3} \text{ m}\right)^2}$$

$$= \boxed{5.57 \times 10^{-5} \text{ m/s}}$$

(b) The drift speed is smaller because more electrons are being conducted. To create the same current, therefore, the drift speed need not be as great.

P27.7 From $I = \frac{dQ}{dt}$, we have $dQ = I\,dt$.

From this, we derive the general integral: $Q = \int dQ = \int I\ dt$

In all three cases, define an end-time, T: $Q = \int_0^T I_0 e^{-t/\tau} dt$

Integrating from time $t = 0$ to time $t = T$: $Q = \int_0^T (-I_0\tau) e^{-t/\tau}\left(-\frac{dt}{\tau}\right)$

We perform the integral and set $Q = 0$ at $t = 0$ to obtain

$$Q = -I_0\tau\left(e^{-T/\tau} - e^0\right) = I_0\tau\left(1 - e^{-T/\tau}\right)$$

(a) If $T = \tau$: $Q(\tau) = I_0\tau\left(1 - e^{-1}\right) = \boxed{(0.632)I_0\tau}$

(b) If $T = 10\tau$: $Q(10\tau) = I_0\tau\left(1 - e^{-10}\right) = \boxed{(0.999\,95)I_0\tau}$

(c) If $T = \infty$: $Q(\infty) = I_0\tau\left(1 - e^{-\infty}\right) = \boxed{I_0\tau}$

P27.8 (a) $J = \frac{I}{A} = \frac{5.00\ \text{A}}{\pi\left(4.00 \times 10^{-3}\ \text{m}\right)^2} = \boxed{99.5\ \text{kA/m}^2}$

(b) $\boxed{\text{Current is the same.}}$

(c) The cross-sectional area is greater; therefore $\boxed{\text{the current density is smaller.}}$

(d) $A_2 = 4A_1$ or $\pi r_2^2 = 4\pi r_1^2$ so $r_2 = 2r_1 = \boxed{0.800\ \text{cm}}$.

(e) $\boxed{I = 5.00\ \text{A}}$

(f) $J_2 = \frac{1}{4}J_1 = \frac{1}{4}\left(9.95 \times 10^4\ \text{A/m}^2\right) = \boxed{2.49 \times 10^4\ \text{A/m}^2}$

P27.9 We are given $q = 4t^3 + 5t + 6$. The area is

$$A = \left(2.00\ \text{cm}^2\right)\left(\frac{1.00\ \text{m}}{100\ \text{cm}}\right)^2 = 2.00 \times 10^{-4}\ \text{m}^2$$

(a) $I(1.00\ \text{s}) = \left.\frac{dq}{dt}\right|_{t=1.00\ \text{s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\ \text{s}} = \boxed{17.0\ \text{A}}$

(b) $J = \frac{I}{A} = \frac{17.0\ \text{A}}{2.00 \times 10^{-4}\ \text{m}^2} = \boxed{85.0\ \text{kA/m}^2}$

P27.10 (a) We obtain the speed of each deuteron from $K = \frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2.00\times10^6)(1.60\times10^{-19}\text{ J})}{2(1.67\times10^{-27}\text{ kg})}} = 1.38\times10^7\text{ m/s}$$

The time between deuterons passing a stationary point is t in $I = \frac{q}{t}$, so

$$t = \frac{q}{I} = \frac{1.60\times10^{-19}\text{ C}}{10.0\times10^{-6}\text{ C/s}} = 1.60\times10^{-14}\text{ s}$$

So the distance between individual deuterons is

$$vt = (1.38\times10^7\text{ m/s})(1.60\times10^{-14}\text{ s}) = \boxed{2.21\times10^{-7}\text{ m}}$$

(b) One nucleus will put its nearest neighbor at potential

$$V = \frac{k_e q}{r} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{2.21\times10^{-7}\text{ m}}$$
$$= 6.49\times10^{-3}\text{ V}$$

This is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

P27.11 (a) $J = \frac{I}{A} = \frac{8.00\times10^{-6}\text{ A}}{\pi(1.00\times10^{-3}\text{ m})^2} = \boxed{2.55\text{ A/m}^2}$

(b) From $J = nev_d$, we have

$$n = \frac{J}{ev_d} = \frac{2.55\text{ A/m}^2}{(1.60\times10^{-19}\text{ C})(3.00\times10^8\text{ m/s})} = \boxed{5.31\times10^{10}\text{ m}^{-3}}$$

(c) From $I = \frac{\Delta Q}{\Delta t}$, we have

$$\Delta t = \frac{\Delta Q}{I} = \frac{N_A e}{I} = \frac{(6.02\times10^{23})(1.60\times10^{-19}\text{ C})}{8.00\times10^{-6}\text{ A}}$$
$$= \boxed{1.20\times10^{10}\text{ s}}$$

(This is about 382 years!)

P27.12 To find the total charge passing a point in a given amount of time, we use $I = \dfrac{dq}{dt}$, from which we can write

$$q = \int dq = \int I\,dt = \int_0^{1/240\text{ s}} (100\text{ A})\sin\left(\frac{120\pi t}{\text{s}}\right)dt$$

$$q = \frac{-100\text{ C}}{120\pi}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right] = \frac{+100\text{ C}}{120\pi} = \boxed{0.265\text{ C}}$$

P27.13 The molar mass of silver = 107.9 g/mole and the volume V is

$$V = (\text{area})(\text{thickness}) = \left(700\times10^{-4}\text{ m}^2\right)\left(0.133\times10^{-3}\text{ m}\right)$$
$$= 9.31\times10^{-6}\text{ m}^3$$

The mass of silver deposited is

$$m_{\text{Ag}} = \rho V = \left(10.5\times10^3\text{ kg/m}^3\right)\left(9.31\times10^{-6}\text{ m}^3\right)$$
$$= 9.78\times10^{-2}\text{ kg}$$

And the number of silver atoms deposited is

$$N = \left(9.78\times10^{-2}\text{ kg}\right)\left(\frac{6.02\times10^{23}\text{ atoms}}{107.9\text{ g}}\right)\left(\frac{1\,000\text{ g}}{1\text{ kg}}\right)$$
$$= 5.45\times10^{23}\text{ atoms}$$

The current is then

$$I = \frac{\Delta V}{R} = \frac{12.0\text{ V}}{1.80\ \Omega} = 6.67\text{ A} = 6.67\text{ C/s}$$

The time interval required for the silver coating is

$$\Delta t = \frac{\Delta Q}{I} = \frac{Ne}{I} = \frac{\left(5.45\times10^{23}\right)\left(1.60\times10^{-19}\text{ C}\right)}{6.67\text{ C/s}}$$
$$= 1.31\times10^4\text{ s} = \boxed{3.64\text{ h}}$$

Section 27.2 Resistance

P27.14 From Equation 27.7, we obtain

$$I = \frac{\Delta V}{R} = \frac{120\text{ V}}{240\ \Omega} = 0.500\text{ A} = \boxed{500\text{ mA}}$$

***P27.15** From Ohm's law, $R = \Delta V / I$, and from Equation 27.10,

$$R = \rho\ell / A = \rho\ell / \left(\pi d^2 / 4\right)$$

Solving for the resistivity gives

$$\rho = \left(\frac{\pi d^2}{4\ell}\right)R = \left(\frac{\pi d^2}{4\ell}\right)\left(\frac{\Delta V}{I}\right) = \left[\frac{\pi\left(2.00\times10^{-3}\text{ m}\right)^2}{4(50.0\text{ m})}\right]\left(\frac{9.11\text{ V}}{36.0\text{ A}}\right)$$

$$= 1.59\times10^{-8}\ \Omega\cdot\text{m}$$

Then, from Table 27.2, we see that the wire is made of $\boxed{\text{silver}}$.

P27.16 $\Delta V = IR$ and $R = \dfrac{\rho\ell}{A}$. The area is

$$A = \left(0.600\text{ mm}^2\right)\left(\frac{1.00\text{ m}}{1\,000\text{ mm}}\right)^2 = 6.00\times10^{-7}\text{ m}^2$$

From the potential difference, we can solve for the current, which gives

$$\Delta V = \frac{I\rho\ell}{A} \quad\to\quad I = \frac{\Delta VA}{\rho\ell} = \frac{(0.900\text{ V})\left(6.00\times10^{-7}\text{ m}^2\right)}{\left(5.60\times10^{-8}\ \Omega\cdot\text{m}\right)(1.50\text{ m})}$$

$$I = \boxed{6.43\text{ A}}$$

P27.17 From the definition of resistance,

$$R = \frac{\Delta V}{I} = \frac{120\text{ V}}{13.5\text{ A}} = \boxed{8.89\ \Omega}$$

P27.18 Using $R = \dfrac{\rho L}{A}$ and data from Table 27.2, we have

$$\rho_{\text{Cu}}\frac{L_{\text{Cu}}}{\pi r_{\text{Cu}}^2} = \rho_{\text{Al}}\frac{L_{\text{Al}}}{\pi r_{\text{Al}}^2} \to \frac{r_{\text{Al}}^2}{r_{\text{Cu}}^2} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}$$

which yields

$$\frac{r_{\text{Al}}}{r_{\text{Cu}}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} = \sqrt{\frac{2.82\times10^{-8}\ \Omega\cdot\text{m}}{1.70\times10^{-8}\ \Omega\cdot\text{m}}} = \boxed{1.29}$$

P27.19 (a) Given total mass $m = \rho_m V = \rho_m A\ell \quad\to\quad A = \dfrac{m}{\rho_m\ell}$, where $\rho_m \equiv$ mass density.

Taking $\rho \equiv$ resistivity, $R = \dfrac{\rho\ell}{A} = \dfrac{\rho\ell}{m/\rho_m\ell} = \dfrac{\rho\rho_m\ell^2}{m}$.

Thus,

$$\ell = \sqrt{\frac{mR}{\rho\rho_m}} = \sqrt{\frac{\left(1.00\times10^{-3}\ \text{kg}\right)(0.500\ \Omega)}{\left(1.70\times10^{-8}\ \Omega\cdot\text{m}\right)\left(8.92\times10^{3}\ \text{kg/m}^3\right)}}$$

$$= \boxed{1.82\ \text{m}}$$

(b) $V = \dfrac{m}{\rho_m}$, or $\pi r^2 \ell = \dfrac{m}{\rho_m}$

Thus,

$$r = \sqrt{\frac{m}{\pi\rho_m \ell}} = \sqrt{\frac{1.00\times10^{-3}\ \text{kg}}{\pi\left(8.92\times10^{3}\ \text{kg/m}^3\right)(1.82\ \text{m})}} = 1.40\times10^{-4}\ \text{m}$$

The diameter is twice this distance: diameter = $\boxed{280\ \mu\text{m}}$

P27.20 (a) Given total mass $m = \rho_m V = \rho_m A\ell \quad\rightarrow\quad A = \dfrac{m}{\rho_m \ell}$, where

$\rho_m \equiv$ mass density.

Taking $\rho \equiv$ resistivity, $R = \dfrac{\rho\ell}{A} = \dfrac{\rho\ell}{m/\rho_m\ell} = \dfrac{\rho\rho_m\ell^2}{m}$.

Thus, $\ell = \boxed{\sqrt{\dfrac{mR}{\rho\rho_m}}}$.

(b) Volume $V = \dfrac{m}{\rho_m}$, or

$$\frac{1}{4}\pi d^2 \ell = \frac{m}{\rho_m}$$

$$d = \sqrt{\frac{4}{\pi}}\left(\frac{m}{\rho_m}\frac{1}{\ell}\right)^{1/2} = \sqrt{\frac{4}{\pi}}\left(\frac{m}{\rho_m}\sqrt{\frac{\rho\rho_m}{mR}}\right)^{1/2} = \sqrt{\frac{4}{\pi}}\left(\sqrt{\frac{m^2}{\rho_m^2}\frac{\rho\rho_m}{mR}}\right)^{1/2}$$

$$= \boxed{\sqrt{\frac{4}{\pi}}\left(\frac{\rho m}{\rho_m R}\right)^{1/4}}$$

P27.21 (a) From the definition of resistance,

$$R = \frac{\Delta V}{I} = \frac{120\ \text{V}}{9.25\ \text{A}} = \boxed{13.0\ \Omega}$$

(b) The resistivity of Nichrome (from Table 27.2) is $1.50 \times 10^{-6}\ \Omega \cdot \text{m}$.

We find the length of wire from

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi r^2}$$

solving for the length ℓ gives

$$\ell = \frac{R\pi r^2}{\rho} = \frac{(13.0\ \Omega)\pi\left(2.50 \times 10^{-3}\ \text{m}\right)^2}{\left(1.50 \times 10^{-6}\ \Omega \cdot \text{m}\right)} = \boxed{170\ \text{m}}$$

Section 27.3 A Model for Electrical Conduction

***P27.22** (a) n is $\boxed{\text{unaffected}}$.

(b) $|J| = \frac{I}{A} \propto I$ so it $\boxed{\text{doubles}}$.

(c) $J = nev_d$ so v_d $\boxed{\text{doubles}}$.

(d) $\tau = \frac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to a temperature change in the conductor.

***P27.23** $J = \sigma E$ so $\sigma = \frac{J}{E} = \frac{6.00 \times 10^{-13}\ \text{A/m}^2}{100\ \text{V/m}} = \boxed{6.00 \times 10^{-15}\ (\Omega \cdot \text{m})^{-1}}$.

P27.24 (a) From Appendix C, the molar mass of iron is

$$M_{\text{Fe}} = 55.85\ \text{g/mol} = (55.85\ \text{g/mol})\left(1\ \text{kg}/10^3\ \text{g}\right)$$
$$= \boxed{5.58 \times 10^{-2}\ \text{kg/mol}}$$

(b) From Table 14.1, the density of iron is $\rho_{\text{Fe}} = 7.86 \times 10^3\ \text{kg/m}^3$, so the molar density is

$$(\text{molar density})_{\text{Fe}} = \frac{\rho_{\text{Fe}}}{M_{\text{Fe}}} = \frac{7.86 \times 10^3\ \text{kg/m}^3}{5.58 \times 10^{-2}\ \text{kg/mol}}$$
$$= \boxed{1.41 \times 10^5\ \text{mol/m}^3}$$

(c) The density of iron atoms is

$$\text{density of atoms} = N_A(\text{molar density})$$
$$= \left(6.02 \times 10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(1.41 \times 10^5\ \frac{\text{mol}}{\text{m}^3}\right)$$
$$= \boxed{8.49 \times 10^{28}\ \frac{\text{atoms}}{\text{m}^3}}$$

(d) With two conduction electrons per iron atom, the density of charge carriers is

$$n = (\text{charge carriers/atom})(\text{density of atoms})$$
$$= \left(2\ \frac{\text{electrons}}{\text{atom}}\right)\left(8.49\times10^{28}\ \frac{\text{atoms}}{\text{m}^3}\right)$$
$$= \boxed{1.70\times10^{29}\ \text{electrons/m}^3}$$

(e) With a current of $I = 30.0$ A and cross-sectional area $A = 5.00 \times 10^{-6}$ m^2, the drift speed of the conduction electrons in this wire is

$$v_d = \frac{I}{nqA} = \frac{30.0\ \text{C/s}}{(1.70\times10^{29}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(5.00\times10^{-6}\ \text{m}^3)}$$
$$= \boxed{2.21\times10^{-4}\ \text{m/s}}$$

P27.25 From Equations 27.16 and 27.13, the resistivity and drift velocity can be related to the electric field within the copper wire:

$$\rho = \frac{m}{ne^2\tau} \to \tau = \frac{m}{\rho ne^2}$$

and

$$v_d = \frac{eE}{m}\tau = \frac{eE}{m}\frac{m}{\rho ne^2} = \frac{E}{\rho ne} \qquad \to \qquad E = \rho nev_d$$

where n is the electron density. From Example 27.1,

$$n = \frac{N_A\rho_{Cu}}{M} = \frac{(6.02\times10^{23}\ \text{mol}^{-1})(8\ 920\ \text{kg/m}^3)}{0.063\ 5\ \text{kg/mol}} = 8.46\times10^{28}\ \text{m}^{-3}$$

The electric field is then

$$E = \rho nev_d$$
$$E = (1.7\times10^{-8}\ \Omega\cdot\text{m})(8.46\times10^{28}\ \text{m}^{-3})$$
$$\times(1.60\times10^{-19}\ \text{C})(7.84\times10^{-4}\ \text{m/s})$$
$$= \boxed{0.18\ \text{V/m}}$$

Section 27.4 Resistance and Temperature

P27.26 $R = R_0[1+\alpha(\Delta T)]$ gives

$$140\ \Omega = (19.0\ \Omega)\left[1+\left(4.50\times10^{-3}/°\text{C}\right)\Delta T\right]$$

Solving,

$$\Delta T = 1.42\times10^{3}\ °\text{C} = T - 20.0°\text{C}$$

And the final temperature is $\boxed{T = 1.44\times10^{3}\ °\text{C}}$

P27.27 If we ignore thermal expansion, the change in the material's resistivity with temperature $\rho = \rho_0[1+\alpha\Delta T]$ implies that the change in resistance is $R - R_0 = R_0\alpha\Delta T$. The fractional change in resistance is defined by $f = (R - R_0)/R_0$. Therefore,

$$f = \frac{R_0\alpha\Delta T}{R_0} = \alpha\Delta T = \left(5.00\times10^{-3}\ °\text{C}^{-1}\right)(50.0°\text{C} - 25.0°\text{C}) = \boxed{0.12}$$

***P27.28** At the low temperature T_C we write

$$R_C = \frac{\Delta V}{I_C} = R_0[1+\alpha(T_C - T_0)]$$

where $T_0 = 20.0°\text{C}$. At the high temperature T_h,

$$R_h = \frac{\Delta V}{I_h} = \frac{\Delta V}{1\ \text{A}} = R_0[1+\alpha(T_h - T_0)]$$

Then,

$$\frac{(\Delta V)/(1.00\ \text{A})}{(\Delta V)/I_C} = \frac{1+\left(3.90\times10^{-3}\ (°\text{C})^{-1}\right)(58.0°\text{C} - 20.0°\text{C})}{1+\left(3.90\times10^{-3}\ (°\text{C})^{-1}\right)(-88.0°\text{C} - 20.0°\text{C})}$$

and $$I_C = (1.00\ \text{A})\left(\frac{1.15}{0.579}\right) = \boxed{1.98\ \text{A}}.$$

P27.29 We use Equation 27.20 and refer to Table 27.2:

$$\begin{aligned} R &= R_0\left[1+\alpha\left(T - T_0\right)\right] \\ &= (6.00\ \Omega)\left[1+\left(3.8\times10^{-3}\ (°\text{C})^{-1}\right)(34.0°\text{C} - 20.0°\text{C})\right] \\ &= \boxed{6.32\ \Omega} \end{aligned}$$

P27.30 (a) From $R = \rho L/A$, the initial resistance of the mercury is

$$R_i = \frac{\rho L_i}{A_i} = \frac{\rho L_i}{\pi d_i^2/4} = \frac{\left(9.58\times10^{-7}\ \Omega\cdot\text{m}\right)(1.000\,0\ \text{m})}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^2/4} = \boxed{1.22\ \Omega}$$

(b) Since the volume of mercury is constant, $V = A_f \cdot L_f = A_i \cdot L_i$ gives the final cross-sectional area as $A_f = A_i \cdot \left(L_i/L_f\right)$. Thus, the final resistance is given by $R_f = \dfrac{\rho L_f}{A_f} = \dfrac{\rho L_f^2}{A_i \cdot L_i}$. The fractional change in the resistance is then

$$\frac{\Delta R}{R} = \frac{R_f - R_i}{R_i} = \frac{R_f}{R_i} - 1 = \frac{\rho L_f^2/\left(A_i \cdot L_i\right)}{\rho L_i/A_i} - 1 = \left(\frac{L_f}{L_i}\right)^2 - 1$$

$$\frac{\Delta R}{R} = \left(\frac{100.040\ 0\,\text{cm}}{100.000\ 0\,\text{cm}}\right)^2 - 1 = \boxed{8.00 \times 10^{-4}\ \text{increase}}$$

***P27.31** (a) The resistance at 20.0°C is

$$R_0 = \frac{\rho \ell}{A} = \frac{\left(1.7 \times 10^{-8}\ \Omega \cdot \text{m}\right)(34.5\ \text{m})}{\pi\left(0.25 \times 10^{-3}\ \text{m}\right)^2} = 2.99\ \Omega$$

and the current is

$$I = \frac{\Delta V}{R_0} = \frac{9.00\ \text{V}}{3.00\ \Omega} = \boxed{3.01\ \text{A}}$$

(b) At 30.0°C, from Equation 27.20,

$$R = R_0\left[1 + \alpha(\Delta T)\right]$$
$$= (2.99\ \Omega)\left[1 + \left(3.9 \times 10^{-3}\ (°\text{C})^{-1}\right)(30.0°\text{C} - 20.0°\text{C})\right] = 3.10\ \Omega$$

The current is then

$$I = \frac{\Delta V}{R_0} = \frac{9.00\ \text{V}}{3.10\ \Omega} = \boxed{2.90\ \text{A}}$$

P27.32 (a) We require two conditions:

$$R = \frac{\rho_1 \ell_1}{\pi r^2} + \frac{\rho_2 \ell_2}{\pi r^2} \qquad \textbf{[1]}$$

where carbon = 1 and Nichrome = 2, and for any ΔT

$$R = \frac{\rho_1 \ell_1}{\pi r^2}\left(1 + \alpha_1 \Delta T\right) + \frac{\rho_2 \ell_2}{\pi r^2}\left(1 + \alpha_2 \Delta T\right) \qquad \textbf{[2]}$$

Setting equations [1] and [2] equal to each other, we have

$$\frac{\rho_1 \ell_1}{\pi r^2} + \frac{\rho_2 \ell_2}{\pi r^2} = \frac{\rho_1 \ell_1}{\pi r^2}\left(1 + \alpha_1 \Delta T\right) + \frac{\rho_2 \ell_2}{\pi r^2}\left(1 + \alpha_2 \Delta T\right)$$

simplifying,

$$\frac{\rho_1\ell_1}{\pi r^2}+\frac{\rho_2\ell_2}{\pi r^2}=\frac{\rho_1\ell_1}{\pi r^2}+\frac{\rho_1\ell_1}{\pi r^2}\alpha_1\Delta T+\frac{\rho_2\ell_2}{\pi r^2}+\frac{\rho_2\ell_2}{\pi r^2}\alpha_2\Delta T$$

or $\frac{\rho_2\ell_2}{\pi r^2}\alpha_2\Delta T=-\frac{\rho_1\ell_1}{\pi r^2}\alpha_1\Delta T$, which gives

$$\rho_2\ell_2\alpha_2=-\rho_1\ell_1\alpha_1 \qquad \textbf{[3]}$$

The two equations [1] and [3] are just sufficient to determine ℓ_1 and ℓ_2. $\boxed{\text{The design goal can be met.}}$

(b) From Table 27.2, $\alpha_1=-0.5\times10^{-3}\,(^\circ\text{C})^{-1}$ and $\alpha_2=0.4\times10^{-3}\,(^\circ\text{C})^{-1}$. Use equation [3] to solve for ℓ_2 in terms of ℓ_1:

$$\ell_2=-\frac{\rho_1}{\rho_2}\frac{\alpha_1}{\alpha_2}\ell_1$$

then substitute this into equation [1]:

$$R=\frac{\rho_1\ell_1}{\pi r^2}+\frac{\rho_2}{\pi r^2}\left(-\frac{\rho_1}{\rho_2}\frac{\alpha_1}{\alpha_2}\ell_1\right)=\frac{\rho_1}{\pi r^2}\left(1-\frac{\alpha_1}{\alpha_2}\right)\ell_1$$

$$10.0\ \Omega=\frac{\left(3.5\times10^{-5}\Omega\cdot\text{m}\right)}{\pi(1.50\times10^{-3}\text{m})^2}\left(1-\frac{-0.5\times10^{-3}}{0.4\times10^{-3}}\right)\ell_1$$

$$\rightarrow \ell_1=0.898\ \text{m}$$

and so

$$\ell_2=-\frac{\rho_1}{\rho_2}\frac{\alpha_1}{\alpha_2}\ell_1=-\frac{\left(3.5\times10^{-5}\Omega\cdot\text{m}\right)}{\left(1.50\times10^{-6}\Omega\cdot\text{m}\right)}\left(\frac{-0.5\times10^{-3}}{0.4\times10^{-3}}\right)\ell_1=26.2\ \text{m}$$

Therefore, $\boxed{\ell_1=0.898\ \text{m and } \ell_2=26.2\ \text{m.}}$

P27.33 (a) The resistivity is computed from $\rho=\rho_0\left[1+\alpha\left(T-T_0\right)\right]$:

$$\rho=\left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)\left[1+\left(3.90\times10^{-3}\ ^\circ\text{C}^{-1}\right)\left(30.0^\circ\text{C}\right)\right]$$

$$=\boxed{3.15\times10^{-8}\ \Omega\cdot\text{m}}$$

(b) The current density is

$$J=\sigma E=\frac{E}{\rho}=\left(\frac{0.200\ \text{V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}}\right)\left(\frac{1\ \Omega\cdot\text{A}}{\text{V}}\right)=\boxed{6.35\times10^{6}\ \text{A/m}^2}$$

(c) The current density is related to the current by $J=\frac{I}{A}=\frac{I}{\pi r^2}$.

$$I=J\left(\pi r^2\right)=\left(6.35\times10^{6}\ \text{A/m}^2\right)\left[\pi\left(5.00\times10^{-5}\ \text{m}\right)^2\right]=\boxed{49.9\ \text{mA}}$$

(d) The mass density gives the number-density of free electrons; we assume that each atom donates one conduction electron:

$$n=\left(\frac{2.70\times 10^{3}\,\text{kg}}{\text{m}^{3}}\right)\left(\frac{1\text{ mol}}{26.98\text{ g}}\right)\left(\frac{10^{3}\,\text{g}}{\text{kg}}\right)\left(\frac{6.02\times 10^{23}\text{ free e}^{-}}{1\text{ mol}}\right)$$
$$=6.02\times 10^{28}\ \text{e}^{-}/\text{m}^{3}$$

Now $J = nqv_d$ gives the drift speed as

$$v_d=\frac{J}{nq}=\frac{6.35\times 10^{6}\ \text{A/m}^{2}}{\left(6.02\times 10^{28}\ \text{e}^{-}/\text{m}^{3}\right)\left(-1.60\times 10^{-19}\ \text{C/e}^{-}\right)}$$
$$=\boxed{-6.59\times 10^{-4}\ \text{m/s}}$$

The sign indicates that the electrons drift opposite to the field and current.

(e) The applied voltage is $\Delta V = E\ell = (0.200\ \text{V/m})(2.00\ \text{m}) = \boxed{0.400\ \text{V}}$.

P27.34 For aluminum,

$$\alpha_E = 3.90\times 10^{-3}\ {}^{\circ}\text{C}^{-1}\quad \text{(Table 27.2)}$$

and $$\alpha = 24.0\times 10^{-6}\ {}^{\circ}\text{C}^{-1}\quad \text{(Table 19.1)}$$

The resistance is then

$$R=\frac{\rho\ell}{A}=\frac{\rho_0(1+\alpha_E\Delta T)\ell(1+\alpha\Delta T)}{A(1+\alpha\Delta T)^2}=R_0\frac{(1+\alpha_E\Delta T)}{(1+\alpha\Delta T)}$$
$$=(1.23\ \Omega)\left[\frac{1+\left(3.90\times 10^{-3}\ ({}^{\circ}\text{C})^{-1}\right)(120^{\circ}\text{C}-20.0^{\circ}\text{C})}{1+\left(24.0\times 10^{-6}\ ({}^{\circ}\text{C})^{-1}\right)(120^{\circ}\text{C}-20.0^{\circ}\text{C})}\right]$$
$$=\boxed{1.71\ \Omega}$$

P27.35 Room temperature is $T_0 = 20.0°$. From Equation 27.19,

$$\rho_{\text{Al}}=(\rho_0)_{\text{Al}}\left[1+\alpha_{\text{Al}}(T-T_0)\right]=3(\rho_0)_{\text{Cu}}$$

Then, substituting numerical values from Table 27.2 gives

$$T-T_0=\frac{1}{\alpha_{\text{Al}}}\left[\frac{3(\rho_0)_{\text{Cu}}}{(\rho_0)_{\text{Al}}}-1\right]$$
$$=\frac{1}{3.9\times 10^{-3}\ ({}^{\circ}\text{C})^{-1}}\left[\frac{3\left(1.7\times 10^{-8}\ \Omega\cdot\text{m}\right)}{2.82\times 10^{-8}\ \Omega\cdot\text{m}}-1\right]$$

and solving for the temperature gives

$$T - 20.0\ °\text{C} = 207°\text{C}$$

$$T = \boxed{227°\text{C}}$$

where we have assumed three significant figures throughout.

Section 27.6 Electrical Power

***P27.36** (a) $P = (\Delta V)I = \left(300\times10^3\ \text{J/C}\right)\left(1.00\times10^3\ \text{C/s}\right) = \boxed{3.00\times10^8\ \text{W}}$

A large electric generating station, fed by a trainload of coal each day, converts energy faster.

(b) $I = \dfrac{P}{A} = \dfrac{P}{\pi r^2}$

$$P = I\left(\pi r^2\right) = \left(1\ 370\ \text{W/m}^2\right)\left[\pi(6.37\times10^6\ \text{m})^2\right] = \boxed{1.75\times10^{17}\ \text{W}}$$

Terrestrial solar power is immense compared to lightning and compared to all human energy conversions.

***P27.37** $P = 0.800(1\,500\ \text{hp})(746\ \text{W/hp}) = 8.95\times10^5\ \text{W}$

Then, from $P = I\Delta V$,

$$I = \frac{P}{\Delta V} = \frac{8.95\times10^5\ \text{W}}{2\ 000\ \text{V}} = \boxed{448\ \text{A}}$$

P27.38 From Equation 27.21,

$$P = I\Delta V = 500\times10^{-6}\ \text{A}\left(15\times10^3\ \text{V}\right) = \boxed{7.50\ \text{W}}$$

P27.39 (a) From Equation 27.21,

$$P = I\Delta V \to I = P/\Delta V = \left(1.00\times10^3\ \text{W}\right)/(120\ \text{V}) = \boxed{8.33\ \text{A}}$$

(b) From Equation 27.23,

$$P = \Delta V^2/R \to R = \Delta V^2/P = (120\ \text{V})^2/\left(1.00\times10^3\ \text{W}\right) = \boxed{14.4\ \Omega}$$

P27.40 From Equation 27.21,

$$P = I\Delta V = \left(0.200\times10^{-3}\ \text{A}\right)\left(75.0\times10^{-3}\ \text{V}\right)$$

$$= 15.0\times10^{-6}\ \text{W} = \boxed{15.0\ \mu\text{W}}$$

P27.41 From Equation 27.21,

$$P = I\Delta V = \left(350 \times 10^{-3}\ \text{A}\right)(6.00\ \text{V}) = \boxed{2.10\ \text{W}}$$

P27.42 If the tank has good insulation, essentially all of the energy electrically transmitted to the heating element becomes internal energy in the water: $\Delta E_{(\text{internal})} = E_{(\text{electrical})}$. Our symbol $E_{(\text{electrical})}$ represents the same thing as the textbook's T_{ET}, namely electrically transmitted energy.

Since $\Delta E_{(\text{internal})} = mc\Delta T$ and $E_{(\text{electrical})} = P\Delta t = (\Delta V)^2\,\Delta t / R$

where $c = 4\,186\ \text{J/kg}\cdot{}^\circ\text{C}$

the resistance is

$$R = \frac{(\Delta V)^2\,\Delta t}{cm\Delta T} = \frac{(240\ \text{V})^2(1\,500\ \text{s})}{\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(109\ \text{kg}\right)(29.0^\circ\text{C})} = \boxed{6.53\ \Omega}$$

P27.43 From $P = (\Delta V)^2/R$, we find that

$$R = \frac{\left(\Delta V_i\right)^2}{P} = \frac{(120\ \text{V})^2}{100\ \text{W}} = 144\ \Omega$$

The final current is

$$I_f = \frac{\Delta V_f}{R} = \frac{140\ \text{V}}{144\ \Omega} = 0.972\ \text{A}$$

The power during the surge is

$$P = \frac{\left(\Delta V_f\right)^2}{R} = \frac{(140\ \text{V})^2}{144\ \Omega} = 136\ \text{V}$$

So the percentage increase is

$$\frac{136\ \text{W} - 100\ \text{W}}{100\ \text{W}} = 0.361 = \boxed{36.1\%}$$

P27.44 You pay the electric company for energy transferred in the amount $E = P\Delta t$.

(a) $$P\,\Delta t = (40\ \text{W})(2\ \text{weeks})\left(\frac{7\ \text{d}}{1\ \text{week}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.110\ \$}{\text{kWh}}\right)$$

$$= \boxed{\$1.48}$$

(b) $$P\,\Delta t = (970\ \text{W})(3\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.110\ \$}{\text{kWh}}\right) = \boxed{\$0.005\ 34}$$

(c) $P\Delta t = (5\ 200\ \text{W})(40\ \text{min})\left(\dfrac{1\ \text{h}}{60\ \text{min}}\right)\left(\dfrac{\text{k}}{1\ 000}\right)\left(\dfrac{0.110\ \$}{\text{kWh}}\right) = \boxed{\$0.381}$

P27.45 (a) The total energy stored in the battery is

$$\Delta U_E = q(\Delta V) = It(\Delta V)$$
$$= (55.0\ \text{A}\cdot\text{h})(12.0\ \text{V})\left(\frac{1\ \text{C}}{1\ \text{A}\cdot\text{s}}\right)\left(\frac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right)\left(\frac{1\ \text{W}\cdot\text{s}}{1\ \text{J}}\right)$$
$$= 660\ \text{W}\cdot\text{h} = \boxed{0.660\ \text{kWh}}$$

(b) The value of the electricity is

$$\text{Cost} = (0.660\ \text{kWh})\left(\frac{\$0.110}{1\ \text{kWh}}\right) = \boxed{\$0.072\ 6}$$

P27.46 (a) The resistance of 1.00 m of 12-gauge copper wire is

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi(d/2)^2} = \frac{4\rho\ell}{\pi d^2} = \frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(1.00\ \text{m})}{\pi(0.205\times10^{-2}\ \text{m})^2}$$
$$= 5.2\times10^{-3}\ \Omega$$

The rate of internal energy production is

$$P = I\Delta V = I^2R = (20.0\ \text{A})^2(5.2\times10^{-3}\ \Omega) = \boxed{2.1\ \text{W}}$$

(b) $R = \dfrac{4\rho\ell}{\pi d^2} = \dfrac{4(2.82\times10^{-8}\ \Omega\cdot\text{m})(1.00\ \text{m})}{\pi(0.205\times10^{-2}\ \text{m})^2} = 8.54\times10^{-3}\ \Omega$

$$P = I\Delta V = I^2R = (20.0\ \text{A})^2(8.54\times10^{-3}\ \Omega) = \boxed{3.42\ \text{W}}$$

(c) It would not be as safe. If surrounded by thermal insulation, it would get much hotter than a copper wire.

P27.47 The power of the lamp is $P = I\Delta V = U/\Delta t$, where U is the energy transformed. Then the energy you buy, in standard units, is

$$U = \Delta V I \Delta t$$
$$= (110\ \text{V})(1.70\ \text{A})(1\ \text{day})\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{3\ 600\ \text{s}}{\text{h}}\right)\left(\frac{1\ \text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{1\ \text{C}}{\text{A}\cdot\text{s}}\right)$$
$$= 16.2\ \text{MJ}$$

In kilowatt hours, the energy is

$$U = \Delta V I \Delta t$$
$$= (110\ \text{V})(1.70\ \text{A})(1\ \text{day})\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{1\ \text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{1\ \text{C}}{\text{A}\cdot\text{s}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)$$
$$= 4.49\ \text{kWh}$$

So operating the lamp costs (4.49 kWh)($0.110/kWh) = $\boxed{\$0.494\ /\text{day}}$.

P27.48 The energy taken in by electric transmission for the fluorescent bulb is

$$P\Delta t = 11\ \text{J/s}(100\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 3.96\times10^{6}\ \text{J}$$

$$\text{cost} = 3.96\times10^{6}\ \text{J}\left(\frac{\$0.110}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\ \text{s}}\right) = \$0.121$$

For the incandescent bulb,

$$P\Delta t = 40\ \text{W}(100\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 1.44\times10^{7}\ \text{J}$$

$$\text{cost} = 1.44\times10^{7}\ \text{J}\left(\frac{\$0.110}{3.6\times10^{6}\ \text{J}}\right) = \$0.440$$

$$\text{savings} = \$0.440 - \$0.121 = \boxed{\$0.319}$$

P27.49 First, we compute the resistance of the wire:

$$R = \frac{\rho\ell}{A} = \frac{(1.50\times10^{-6}\ \Omega\cdot\text{m})25.0\ \text{m}}{\pi(0.200\times10^{-3}\ \text{m})^2} = 298\ \Omega$$

The potential drop across the wire is then

$$\Delta V = IR = (0.500\ \text{A})(298\ \Omega) = 149\ \text{V}$$

(a) The magnitude of the electric field in the wire is

$$E = \frac{\Delta V}{\ell} = \frac{149\ \text{V}}{25.0\ \text{m}} = \boxed{5.97\ \text{V/m}}$$

(b) The power delivered to the wire is

$$P = (\Delta V)I = (149\ \text{V})(0.500\ \text{A}) = \boxed{74.6\ \text{W}}$$

(c) We use Equation 27.20 and Table 27.2:

$$R = R_0\left[1+\alpha(T-T_0)\right] = (298\ \Omega)\left[1+(0.400\times10^{-3}/^\circ\text{C})320^\circ\text{C}\right]$$
$$= 337\ \Omega$$

To find the power delivered, we first compute the current flowing through the wire:

$$I = \frac{\Delta V}{R} = \frac{149\ \text{V}}{337\ \Omega} = 0.443\ \text{A}$$

then,

$$P = (\Delta V) I = (149\ \text{V})(0.443\ \text{A}) = \boxed{66.1\ \text{W}}$$

P27.50 The battery takes in energy by electric transmission:

$$P\Delta t = (\Delta V) I (\Delta t) = (2.3\ \text{J/C})(13.5 \times 10^{-3}\ \text{C/s})(4.2\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right)$$
$$= 469\ \text{J}$$

It puts out energy by electric transmission:

$$(\Delta V) I (\Delta t) = (1.6\ \text{J/C})(18 \times 10^{-3}\ \text{C/s})(2.4\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 249\ \text{J}$$

(a) efficiency $= \dfrac{\text{useful output}}{\text{total input}} = \dfrac{249\ \text{J}}{469\ \text{J}} = \boxed{0.530}$

(b) The only place for the missing energy to go is into internal energy:

$$469\ \text{J} = 249\ \text{J} + \Delta E_{\text{int}}$$
$$\Delta E_{\text{int}} = \boxed{221\ \text{J}}$$

(c) We imagine toasting the battery over a fire with 221 J of heat input:

$$Q = mc\Delta T$$
$$\Delta T = \frac{Q}{mc} = \frac{221\ \text{J}}{(0.015\ \text{kg})(975\ \text{J/kg}\cdot{}^\circ\text{C})} = \boxed{15.1^\circ\text{C}}$$

P27.51 We compute the resistance of the wire from

$$P = \frac{(\Delta V)^2}{R} \rightarrow R = \frac{(\Delta V)^2}{P} = \frac{(110\ \text{V})^2}{500\ \text{W}} = 24.2\ \Omega$$

(a) Then, Equation 27.10, $R = \dfrac{\rho}{A}\ell$, gives us the length of wire used:

$$\ell = \frac{RA}{\rho} = \frac{(24.2\ \Omega)\pi(2.50 \times 10^{-4}\ \text{m})^2}{1.50 \times 10^{-6}\ \Omega\cdot\text{m}} = \boxed{3.17\ \text{m}}$$

(b) From Equation 27.20, the resistance of the wire at this temperature is

$$R = R_0[1+\alpha\Delta T] = 24.2\ \Omega\left[1+\left(0.400\times10^{-3}\right)\left(1\,200-20\right)\right]$$
$$= 35.6\ \Omega$$

The power delivered to the coil is then

$$P = \frac{(\Delta V)^2}{R} = \frac{(110\ \text{V})^2}{35.6\ \Omega} = \boxed{340\ \text{W}}$$

P27.52 We find the energy transferred into a number N of these clocks in one year:

$$\begin{aligned} T_{\text{ET}} &= P_{\text{total}}\Delta t = NP_{\text{one clock}}\Delta t \\ &= \left(270\times10^6\ \text{clocks}\right)(2.50\ \text{W/clock}) \\ &\qquad \times\left(365\ \text{d/yr}\right)(24\ \text{h/d})(1\ \text{kW}/1000\ \text{W}) \\ &= 5.91\times10^9\ \text{kWh} \end{aligned}$$

Divide this energy into the total cost claimed by the politician to find the cost of the electricity:

$$\text{cost} = \frac{\$100\times10^6}{5.91\times10^9\ \text{kWh}} = \$0.017\ /\ \text{kWh}$$

This is significantly lower than the average cost of electricity in the United States. While the situation is not actually impossible, the politician would have a better argument by using the actual average cost of electricity in the United States, which would raise his estimate of the total cost to operate the clocks to about $650 million every year.

P27.53 At operating temperature,

(a) $P = I\Delta V = (1.53\ \text{A})(120\ \text{V}) = \boxed{184\ \text{W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$$R = R_0(1+\alpha\Delta T)$$

$$\frac{120\ \text{V}}{1.53\ \text{A}} = \left(\frac{120\ \text{V}}{1.80\ \text{A}}\right)\left[1+\left(0.400\times10^{-3}\ (^\circ\text{C})^{-1}\right)\Delta T\right]$$

which gives

$$\Delta T = 441^\circ\text{C}$$

and $T = 20.0^\circ\text{C} + 441^\circ\text{C} = \boxed{461^\circ\text{C}}$

P27.54 Consider a 400-W blow dryer used for ten minutes daily for a year. The energy transferred to the dryer is

$$P\,\Delta t = (400\ \text{J/s})(600\ \text{s/d})(365\ \text{d})$$
$$\approx 9\times 10^{7}\ \text{J}\left(\frac{1\ \text{kWh}}{3.6\times 10^{6}\ \text{J}}\right)\approx 20\ \text{kWh}$$

We suppose that electrically transmitted energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$$\text{Cost} \approx (20\ \text{kWh})(\$0.10/\text{kWh}) = \$2\ \boxed{\sim \$1}$$

P27.55 We first compute the power delivered to the resistor:

$$P = I\Delta V = (2.00\ \text{A})(120\ \text{V}) = 240\ \text{W}$$

The change in internal energy of the water as it is heated from 23.0°C to 100°C is

$$\Delta E_{\text{int}} = (0.500\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^{\circ}\text{C})(77.0^{\circ}\text{C}) = 161\ \text{kJ}$$

The time interval required to heat the water is then

$$\Delta t = \frac{\Delta E_{\text{int}}}{P} = \frac{1.61\times 10^{5}\ \text{J}}{240\ \text{W}} = \boxed{672\ \text{s}}$$

P27.56 (a) We know that

$$\text{efficiency} = \frac{\text{mechanical power output}}{\text{total power input}}$$
$$= 0.900 = \frac{(2.50\ \text{hp})(746\ \text{W}/1\ \text{hp})}{(120\ \text{V})\,I}$$

from which, we calculate the current as

$$I = \frac{1\,860\ \text{J/s}}{0.9(120\ \text{V})} = \frac{2\,070\ \text{J/s}}{120\ \text{V}} = \boxed{17.3\ \text{A}}$$

(b) The energy delivered to the motor in 3.00 h is

$$\text{energy input} = P_{\text{input}}\Delta t = (2\,070\ \text{J/s})[3.00(3\,600\ \text{s})]$$
$$= 2.24\times 10^{7}\ \text{J} = \boxed{22.4\ \text{MJ}}$$

(c) At \$0.110/kWh, the cost of running the motor for 3.00 h is

$$\text{cost} = (2.24\times 10^{7}\ \text{J})\left(\frac{\$0.110}{1\ \text{kWh}}\right)\left(\frac{\text{k}}{10^{3}}\frac{\text{J}}{\text{W s}}\frac{\text{h}}{3\,600\ \text{s}}\right) = \boxed{\$0.684}$$

Additional Problems

***P27.57** From Equation 27.22, $P = \frac{(\Delta V)^2}{R}$, we find that the total resistance needed in the wire is

$$R = \frac{(\Delta V)^2}{P} = \frac{(20\ \text{V})^2}{48\ \text{W}} = 8.3\ \Omega$$

We then solve for the length of the wire from Equation 27.10:

$$\ell = \frac{RA}{\rho} = \frac{(8.3\ \Omega)\left(4.0\times 10^{-6}\ \text{m}^2\right)}{3.0\times 10^{-8}\ \Omega\cdot\text{m}} = 1.1\times 10^3\ \text{m} = \boxed{1.1\ \text{km}}$$

P27.58 At T_0 = 20.0°, $R = R_0$. Then, from Equation 27.20,

$$R = R_0\left[1+\alpha\left(T - T_0\right)\right] = 2R_0$$

Solving for the change in temperature gives

$$T - T_0 = \frac{1}{\alpha} = \frac{1}{3.9\times 10^{-3}\ (°\text{C})^{-1}}$$

$$T - 20.0°\text{C} = 256°\text{C} \quad\rightarrow\quad T = \boxed{276°\text{C}}$$

P27.59 We find the amount of current each headlight draws:

$$P = I\Delta V \rightarrow I = \frac{P}{\Delta V} = \frac{36.0\ \text{W}}{12.0\ \text{V}} = 3.00\ \text{A}$$

For two headlights, the total current from battery is 6.00 A. The battery rating is the total amount of charge the battery can deliver, without being recharged, over a time interval Δt at a rate (current) I:

$$\Delta Q = I\Delta t = 90.0\ \text{A}\cdot\text{h}$$

The total time interval to discharge the battery is then

$$\Delta t = \frac{\Delta Q}{I} = \frac{90.0\ \text{A}\cdot\text{h}}{6.00\ \text{A}} = \boxed{15.0\ \text{h}}$$

P27.60 (a) $P = I\Delta V = \frac{(\Delta V)^2}{R} \rightarrow R = \frac{(\Delta V)^2}{P}$

Lightbulb A: $R = \frac{(\Delta V)^2}{P} = \frac{(120\ \text{V})^2}{25.0\ \text{W}} = \boxed{576\ \Omega}$

Lightbulb B: $R = \frac{(\Delta V)^2}{P} = \frac{(120\ \text{V})^2}{100\ \text{W}} = \boxed{144\ \Omega}$

(b) $I = \frac{Q}{\Delta t} = \frac{P}{\Delta V} \rightarrow \Delta t = \frac{Q\Delta V}{P} = \frac{(1.00\ \text{C})(120\ \text{V})}{25.0\ \text{W}} = \boxed{4.80\ \text{s}}$

(c) The charge is the same. It is at a location that is lower in potential.

(d) $P = \dfrac{\Delta U}{\Delta t} \rightarrow \Delta t = \dfrac{\Delta U}{P} = \dfrac{1.00\text{ J}}{25.0\text{ W}} = \boxed{0.040\ 0\text{ s}}$

(e) Because of energy conservation, the energy entering and leaving the lightbulb is the same. Energy enters the lightbulb by electric transmission and leaves by heat and electromagnetic radiation.

(f) $\Delta U = P\Delta t = (25.0\text{ J/s})(86\ 400\text{ s/d})(30.0\text{ d}) = 64.8 \times 10^6\text{ J}$

$$\text{Cost} = (64.8 \times 10^6\text{ J})\left(\frac{\$0.110\ 0}{\text{kWh}}\right)\left(\frac{\text{k}}{1\ 000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\ 600\text{ s}}\right) = \boxed{\$1.98}$$

P27.61 The resistance of one wire is $\left(\dfrac{0.500\ \Omega}{\text{mi}}\right)(100\text{ mi}) = 50.0\ \Omega$.

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$$IR = (1\ 000\text{ A})(50.0\ \Omega) = 50.0\text{ kV}$$

Then it radiates as heat power

$$P = I\Delta V = (1\ 000\text{ A})(50.0 \times 10^3\text{ V}) = \boxed{50.0\text{ MW}}$$

P27.62 (a) From $\rho = \dfrac{RA}{\ell} = \dfrac{(\Delta V)}{I}\dfrac{A}{\ell}$ we compute

ℓ (m)	R (Ω)	ρ ($\Omega\cdot$m)
0.540	7.25	9.80×10^{-7}
1.028	14.1	9.98×10^{-7}
1.543	21.1	1.00×10^{-6}

(b) $\bar{\rho} = \boxed{9.93 \times 10^{-7}\ \Omega\cdot\text{m}}$

(c) The average value is within 1% of the tabulated value of $1.00 \times 10^{-6}\ \Omega\cdot\text{m}$ given in Table 27.2.

P27.63 The original stored energy is $U_{E,i} = \frac{1}{2}Q\Delta V_i = \frac{1}{2}\frac{Q^2}{C}$.

(a) When the switch is closed, charge Q distributes itself over the plates of C and $3C$ in parallel, presenting equivalent capacitance $4C$. Then the final potential difference is $\boxed{\Delta V_f = \frac{Q}{4C}}$ for both.

(b) The smaller capacitor then carries charge $C\Delta V_f = \frac{Q}{4C}C = \boxed{\frac{Q}{4}}$.

The larger capacitor carries charge $3C\frac{Q}{4C} = \boxed{\frac{3Q}{4}}$.

(c) The smaller capacitor stores final energy $\frac{1}{2}C(\Delta V_f)^2 = \frac{1}{2}C\left(\frac{Q}{4C}\right)^2 = \boxed{\frac{Q^2}{32C}}$. The larger capacitor possesses energy $\frac{1}{2}3C\left(\frac{Q}{4C}\right)^2 = \boxed{\frac{3Q^2}{32C}}$.

(d) The total final energy is $\frac{Q^2}{32C} + \frac{3Q^2}{32C} = \frac{Q^2}{8C}$. The loss of potential energy is the energy appearing as internal energy in the resistor:

$$\frac{Q^2}{2C} = \frac{Q^2}{8C} + \Delta E_{\text{int}} \quad \text{so} \quad \Delta E_{\text{int}} = \boxed{\frac{3Q^2}{8C}}$$

P27.64 (a) The heater should put out constant power

$$P = \frac{Q}{\Delta t} = \frac{mc(T_f - T_i)}{\Delta t}$$
$$= \frac{(0.250\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 20^\circ\text{C})}{(4\text{ min})}\left(\frac{1\text{ min}}{60\text{ s}}\right)$$
$$= 349\text{ J/s}$$

Then its resistance should be described by

$$P = \frac{(\Delta V)^2}{R} \rightarrow R = \frac{(\Delta V)^2}{P} = \frac{(120\text{ J/C})^2}{349\text{ J/s}} = 41.3\ \Omega$$

Its resistivity at 100 °C is given by

$$\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right] = \left(1.50\times10^{-6}\ \Omega\cdot\text{m}\right)\left[1+0.4\times10^{-3}(80)\right]$$
$$= 1.55\times10^{-6}\ \Omega\cdot\text{m}$$

Then for a wire of circular cross section, from Equation 27.10,

$$R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2} = \rho\frac{\ell 4}{\pi d^2}$$

$$41.3\ \Omega = \left(1.55\times10^{-6}\ \Omega\cdot\text{m}\right)\frac{4\ell}{\pi d^2}$$

$$\frac{\ell}{d^2} = 2.09\times10^{+7}/\text{m} \quad \text{or} \quad d^2 = \left(4.77\times10^{-8}\ \text{m}\right)\ell$$

One possible choice is $\ell = 0.900$ m and $d = 2.07\times10^{-4}$ m. If ℓ and d are made too small, the surface area will be inadequate to transfer heat into the water fast enough to prevent overheating of the filament. To make the volume less than 0.5 cm^3, we want ℓ and d less than those described by $\frac{\pi d^2}{4}\ell = 0.5\times10^{-6}\ \text{m}^3$.

Substituting $d^2 = \left(4.77\times10^{-8}\ \text{m}\right)\ell$ gives

$\frac{\pi}{4}\left(4.77\times10^{-8}\ \text{m}\right)\ell^2 = 0.5\times10^{-6}\ \text{m}^3$, $\ell = 3.65$ m and $d = 4.18\times10^{-4}$ m. Thus our answer is:

Any diameter d and length ℓ related by $d^2 = (4.77\times10^{-8})\ell$, where d and ℓ are in meters.

(b) Yes; for $V = 0.500\ \text{cm}^3$ of Nichrome, $\ell = 3.65$ m and $d = 0.418$ mm.

***P27.65** The power the beam delivers to the target is

$$P = I\Delta V = \left(25.0\times10^{-3}\ \text{A}\right)\left(4.00\times10^{6}\ \text{V}\right) = 1.00\times10^{5}\ \text{W}$$

The mass of cooling water that must flow through the tube each second if the rise in the water temperature is not to exceed 50°C is found from

$$Q = P\Delta t = (\Delta m)c\Delta T$$

Therefore,

$$\frac{\Delta m}{\Delta t} = \frac{P}{c\Delta T} = \frac{1.00\times10^{5}\ \text{J/s}}{\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(50.0^\circ\text{C}\right)} = \boxed{0.478\ \text{kg/s}}$$

P27.66 (a) Since $P = I\Delta V$, we have

$$I = \frac{P}{\Delta V} = \frac{8.00 \times 10^3\ \text{W}}{12.0\ \text{V}} = \boxed{667\ \text{A}}$$

(b) From $P = U / \Delta t$, the time the car runs is

$$\Delta t = \frac{\Delta U}{P} = \frac{2.00 \times 10^7\ \text{J}}{8.00 \times 10^3\ \text{W}} = 2.50 \times 10^3\ \text{s}$$

So it moves a distance of

$$\Delta x = v\Delta t = (20.0\ \text{m/s})(2.50 \times 10^3\ \text{s}) = \boxed{50.0\ \text{km}}$$

P27.67 (a) Assuming the change in V is uniform:

$$E_x = -\frac{dV(x)}{dx} \to E_x = -\frac{\Delta V}{\Delta x} = -\frac{(0 - 4.00\ \text{V})}{(0.500\ \text{m} - 0)} = +8.00\ \text{V/m}$$

or $\boxed{8.00\ \text{V/m in the positive } x \text{ direction.}}$

(b) From Equation 27.10, we have

$$R = \frac{\rho\ell}{A} = \frac{(4.00 \times 10^{-8}\ \Omega \cdot \text{m})(0.500\ \text{m})}{\pi(1.00 \times 10^{-4}\ \text{m})^2} = \boxed{0.637\ \Omega}$$

(c) From Equation 27.7,

$$I = \frac{\Delta V}{R} = \frac{4.00\ \text{V}}{0.637\ \Omega} = \boxed{6.28\ \text{A}}$$

(d) From Equation 27.5, the current density is given by

$$J = \frac{I}{A} = \frac{6.28\ \text{A}}{\pi(1.00 \times 10^{-4}\ \text{m})^2} = 2.00 \times 10^8\ \text{A/m}^2 = \boxed{200\ \text{MA/m}^2}$$

The field and the current are both in the x direction.

(e) We intend to derive the equivalent of Equation 27.6. We start with the definition of current density, $J = I/A$, and, using Equations 27.7 and 27.10, note that the current is given by

$$I = \frac{\Delta V}{R} = \frac{E\ell}{R} = \frac{EA}{\rho}$$

Then,

$$J = \frac{I}{A} = \frac{EA/\rho}{A} = \frac{E}{\rho}$$

so

$$E = \rho J = (4.00\times10^{-8}\ \Omega\cdot\text{m})(2.00\times10^{8}\ \text{A/m}^2) = \boxed{8.00\ \text{V/m}}$$

P27.68 (a) Assuming the change in V is uniform:

$$E_x = -\frac{dV(x)}{dx} \to E_x = -\frac{\Delta V}{\Delta x} = -\frac{0-V}{L-0} = +\frac{V}{L}$$

Therefore, the electric field is $\boxed{V/L \text{ in the positive } x \text{ direction.}}$

(b) From Equation 27.10, we have

$$R = \frac{\rho\ell}{A} = \frac{\rho L}{\pi d^2/4} = \boxed{4\rho L/\pi d^2}$$

(c) From Equation 27.7,

$$I = \Delta V/R = \boxed{V\pi d^2/4\rho L}$$

(d) From Equation 27.5, the current density is given by

$$J = \frac{I}{A} = \frac{V\pi d^2/4\rho L}{\pi d^2/4} = \boxed{V/\rho L \text{ in the positive } x \text{ direction}}$$

The field and the current both have the same direction.

(e) We intend to derive the equivalent of Equation 27.6. We start with the definition of current density, $J = I/A$, and, using Equations 27.7 and 27.10, note that the current is given by

$$I = \frac{\Delta V}{R} = \frac{E\ell}{R} = \frac{EA}{\rho}$$

Then,

$$J = \frac{I}{A} = \frac{EA/\rho}{A} = \frac{E}{\rho}$$

so $$E = \rho J = \rho\left(\frac{V}{\rho L}\right) = \frac{V}{L}$$

P27.69 Since there are 2 wires, the total length is $\ell = 100$ m. The resistance of the wires is

$$R = \left(\frac{0.108\ \Omega}{300\ \text{m}}\right)(100\ \text{m}) = 0.036\,0\ \Omega$$

(a) We find the potential difference at the customer's house from

$$(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120\ \text{V} - (110\ \text{A})(0.036\,0\ \Omega) = \boxed{116\ \text{V}}$$

(b) The power delivered to the customer is

$$P = I(\Delta V) = (110\ \text{A})(116\ \text{V}) = \boxed{12.8\ \text{kW}}$$

(c) The power dissipated in the wires, or the energy produced in the wires, is

$$P_{\text{wires}} = I^2R = (110\ \text{A})^2(0.036\,0\ \Omega) = \boxed{436\ \text{W}}$$

P27.70 The original resistance is $R_i = \rho L_i/A_i$. The new length is

$$L = L_i + \delta L_i = L_i(1+\delta)$$

(a) Constancy of volume implies $AL = A_iL_i$ so

$$A = \frac{A_iL_i}{L} = \frac{A_iL_i}{L_i(1+\delta)} = \frac{A_i}{(1+\delta)}$$

The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho L_i(1+\delta)}{A_i/(1+\delta)} = R_i(1+\delta)^2 = R_i(1+2\delta+\delta^2)$$

(b) The result is exact if the assumptions are precisely true. Our derivation contains no approximation steps where delta is assumed to be small.

P27.71 (a) A thin cylindrical shell of radius r, thickness dr, and length L contributes resistance

$$dR = \frac{\rho\, d\ell}{A} = \frac{\rho\, dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

$$R = \frac{\rho}{2\pi L}\int_{r_c}^{r_b}\frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)}$$

(b) In this equation $\dfrac{\Delta V}{I} = \dfrac{\rho}{2\pi L}\ln\left(\dfrac{r_b}{r_a}\right)$.

Solving, we get $\boxed{\rho = \dfrac{2\pi L\Delta V}{I\ln(r_b/r_a)}}$.

P27.72 The value of 11.4 A is what results from substituting the given voltage and resistance into Equation 27.7. However, the resistance measured for a lightbulb with an ohmmeter is not the resistance at which it operates because of the change in resistivity with temperature. The higher resistance of the filament at the operating temperature brings the current down significantly.

P27.73 Let α be the temperature coefficient at 20.0°C, and α' be the temperature coefficient at 0°C. Then $\rho = \rho_0\left[1+\alpha(T-20.0°\text{C})\right]$ and $\rho = \rho'\left[1+\alpha'(T-0°\text{C})\right]$ must both give the correct resistivity at any temperature T. That is, we must have:

$$\rho_0\left[1+\alpha(T-20.0°\text{C})\right] = \rho'\left[1+\alpha'(T-0°\text{C})\right] \qquad \textbf{[1]}$$

Setting $T = 0$ in equation [1] yields:

$$\rho' = \rho_0\left[1-\alpha(20.0°\text{C})\right]$$

and setting $T = 20.0°\text{C}$ in equation [1] gives:

$$\rho_0 = \rho'\left[1+\alpha'(20.0°\text{C})\right]$$

Substitute ρ' from the first of these results into the second to obtain:

$$\rho_0 = \rho_0\left[1-\alpha(20.0°\text{C})\right]\left[1+\alpha'(20.0°\text{C})\right]$$

Therefore,

$$1+\alpha'(20.0°\text{C}) = \frac{1}{1-\alpha(20.0°\text{C})}$$

which simplifies:

$$\alpha'(20.0°\text{C}) = \frac{1}{1-\alpha(20.0°\text{C})} - 1 = \frac{1-\left[1-\alpha(20.0°\text{C})\right]}{1-\alpha(20.0°\text{C})}$$

$$\alpha'(20.0°\text{C}) = \frac{\alpha(20.0°\text{C})}{1-\alpha(20.0°\text{C})} \rightarrow \alpha' = \frac{\alpha}{1-\alpha(20.0°\text{C})}$$

Therefore,

$$\alpha' = \frac{\alpha}{\left[1-\alpha(20.0°\text{C})\right]} = \frac{3.8\times10^{-3}\ (°\text{C})^{-1}}{\left[1-\left(3.8\times10^{-3}\ (°\text{C})^{-1}\right)(20.0°\text{C})\right]}$$

$$= \boxed{4.1\times10^{-3}\ (°\text{C})^{-1}}$$

P27.74 (a) We begin from $\Delta V = -E \cdot \ell$ or $dV = -E \cdot dx$. Then,

$$\Delta V = -IR = -E \cdot \ell$$

and the current is

$$I = \frac{dq}{dt} = \frac{E \cdot \ell}{R} = \frac{A}{\rho \ell} E \cdot \ell = \frac{A}{\rho} E = -\sigma A \frac{dV}{dx} = \boxed{\sigma A \left| \frac{dV}{dx} \right|}$$

(b) Current flows in the direction of decreasing voltage. Energy flows by heat in the direction of decreasing temperature.

P27.75 We begin with

$$R = \frac{\rho \ell}{A} = \frac{\rho_0 \left[1 + \alpha (T - T_0)\right] \ell_0 \left[1 + \alpha' (T - T_0)\right]}{A_0 \left[1 + \alpha' (T - T_0)\right]^2}$$

$$= \frac{\rho_0 \ell_0}{A_0} \frac{1 + \alpha (T - T_0)}{1 + \alpha' (T - T_0)}$$

For copper (for $T_0 = 20.0°C$): $\rho_0 = 1.700 \times 10^{-8} \ \Omega \cdot \text{m}$, $\alpha = 3.900 \times 10^{-3} \ °C^{-1}$, and $\alpha' = 17.00 \times 10^{-6} \ °C^{-1}$. Then,

$$R = \frac{\rho_0 \ell_0}{A_0} \frac{1 + \alpha (T - T_0)}{1 + \alpha' (T - T_0)}$$

$$R = \frac{(1.700 \times 10^{-8})(2.000)}{\pi (0.1000 \times 10^{-3})^2} \left[\frac{1 + (3.900 \times 10^{-3} \ °C^{-1})(80.00°C)}{1 + (17.00 \times 10^{-6} \ °C^{-1})(80.00°C)} \right]$$

$$R = \boxed{1.418 \ \Omega}$$

P27.76 The wire has length ℓ, and radius r; its cross-sectional area is A (πr^2, if circular), which is proportional to r^2. Because both ℓ and r change with a temperature variation ΔT according to $L = L_0 (1 + \alpha' \Delta T)$, the cross-sectional area changes according to $A = A_0 (1 + \alpha' \Delta T)^2$.

Calling $R_0 = \frac{\rho_0 \ell_0}{A_0}$ at temperature T_0, we have

$$R_0 = \frac{\rho_0 \ell_0}{A_0} \rightarrow R = \frac{\rho_0 \left[1 + \alpha (T - T_0)\right] \ell_0 \left[1 + \alpha' (T - T_0)\right]}{A_0 \left[1 + \alpha' (T - T_0)\right]^2}$$

$$= \frac{\rho \ell_0}{A_0} \frac{\left[1 + \alpha (T - T_0)\right]}{\left[1 + \alpha' (T - T_0)\right]} x$$

which gives

$$R = \boxed{R_0 \frac{1+\alpha(T-T_0)}{1+\alpha'(T-T_0)}}$$

P27.77 (a) Think of the device as two capacitors in parallel. The one on the left has $\kappa_1 = 1$, $A_1 = \left(\frac{\ell}{2}+x\right)\ell$. The equivalent capacitance is

$$\frac{\kappa_1 \in_0 A_1}{d} + \frac{\kappa_2 \in_0 A_2}{d} = \frac{\in_0 \ell}{d}\left(\frac{\ell}{2}+x\right) + \frac{\kappa \in_0 \ell}{d}\left(\frac{\ell}{2}-x\right)$$

$$= \boxed{\frac{\in_0 \ell}{2d}(\ell + 2x + \kappa\ell - 2\kappa x)}$$

(b) The charge on the capacitor is $Q = C\Delta V$

$$Q = \frac{\in_0 \ell\Delta V}{2d}(\ell + 2x + \kappa\ell - 2\kappa x)$$

The current is

$$I = \frac{dQ}{dt} = \frac{dQ}{dx}\frac{dx}{dt} = \frac{\in_0 \ell\Delta V}{2d}(0+2+0-2\kappa)v = -\frac{\in_0 \ell\Delta V v}{d}(\kappa - 1)$$

The negative value indicates that the current drains charge from the capacitor. Positive current is $\boxed{\text{clockwise } \frac{\in_0 \ell\Delta V v}{d}(\kappa-1)}$.

P27.78 (a) The resistance of the dielectric block is $R = \frac{\rho\ell}{A} = \frac{d}{\sigma A}$.

The capacitance of the capacitor is $C = \frac{\kappa \in_0 A}{d}$.

Then $RC = \frac{d}{\sigma A}\frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0}{\sigma}$ is a characteristic of the material only.

(b) The resistance between the plates of the capacitor is

$$R = \frac{\kappa \in_0}{\sigma C} = \frac{\rho\kappa \in_0}{C}$$

$$= \frac{(75\times 10^{16}\ \Omega\cdot\text{m})(3.78)(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{14.0\times 10^{-9}\ \text{F}}$$

$$= \boxed{1.79\times 10^{15}\ \Omega}$$

P27.79 The volume of the gram of gold is given by $\rho = \dfrac{m}{V}$.

$$V = \frac{m}{\rho} = \frac{10^{-3}\text{ kg}}{19.3\times10^{3}\text{ kg/m}^3} = 5.18\times10^{-8}\text{ m}^3 = A\left(2.40\times10^{3}\text{ m}\right)$$

Then, $A = 2.16\times10^{-11}\text{ m}^2$ and the resistance is

$$R = \frac{\rho\ell}{A} = \frac{\left(2.44\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.4\times10^{3}\text{ m}\right)}{2.16\times10^{-11}\text{ m}^2} = \boxed{2.71\times10^{6}\ \Omega}$$

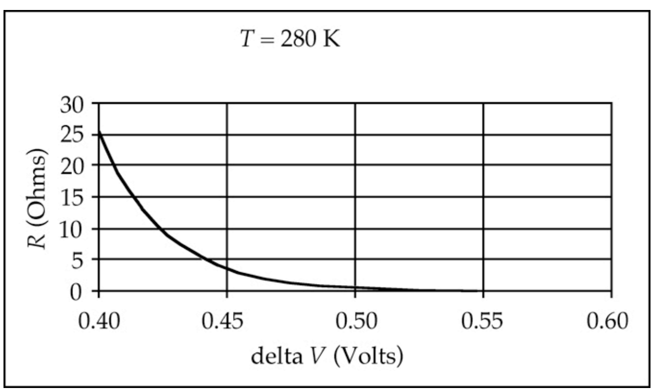
P27.80 Evaluate $I = I_0\left[\exp\left(\dfrac{e\Delta V}{k_B T}\right) - 1\right]$ and $R = \dfrac{\Delta V}{I}$ with

$I_0 = 1.00\times10^{-9}$ A, $e = 1.60\times10^{-19}$ C, and $k_B = 1.38\times10^{-23}$ J/K.

Parts (a) and (b): The following includes a partial table of calculated values and a graph for each of the specified temperatures.

(i) For T = 280 K:

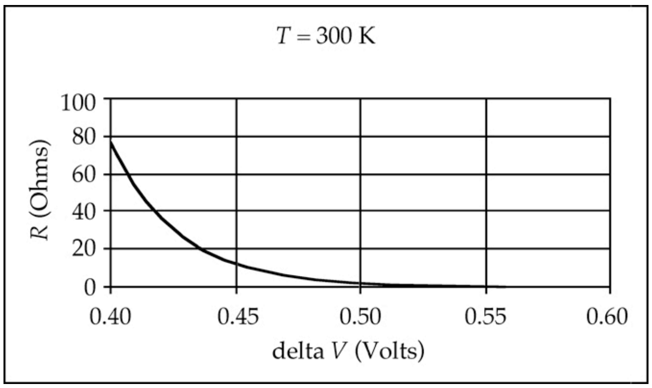
ΔV (V)	I (A)	R (Ω)
0.400	0.015 6	25.6
0.440	0.081 8	5.38
0.480	0.429	1.12
0.520	2.25	0.232
0.560	11.8	0.047 6
0.600	61.6	0.009 7



ANS. FIG. P27.80(i)

(ii) For $T = 300$ K:

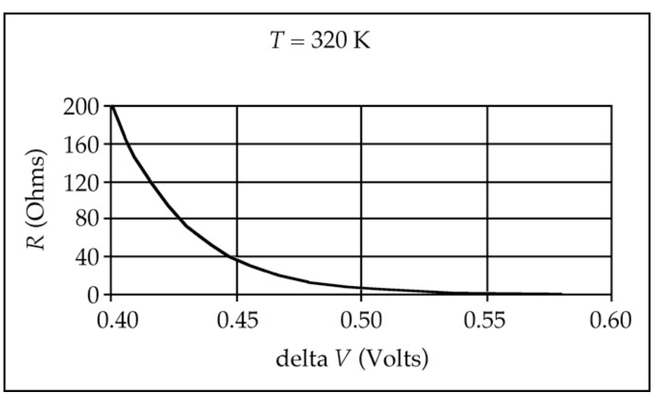
$\Delta V(\text{V})$	$I(\text{A})$	$R(\Omega)$
0.400	0.005	77.3
0.440	0.024	18.1
0.480	0.114	4.22
0.520	0.534	0.973
0.560	2.51	0.223
0.600	11.8	0.051



ANS. FIG. P27.80(ii)

(iii) For $T = 320$ K:

$\Delta V(\text{V})$	$I(\text{A})$	$R(\Omega)$
0.400	0.002 0	203
0.440	0.008 4	52.5
0.480	0.035 7	13.4
0.520	0.152	3.42
0.560	0.648	0.864
0.600	2.76	0.217



ANS. FIG. P27.80(iii)

P27.81 To find the final operating temperature, we begin with

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right]$$

and solve for the temperature T:

$$T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0}-1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I}-1\right]$$

In this case, $I = \frac{I_0}{10}$, so

$$T = T_0 + \frac{1}{\alpha}(9) = 20° + \frac{9}{0.004\,50/°\text{C}} = \boxed{2\,020°\text{C}}$$

Challenge Problems

P27.82 (a) We are given $\alpha = \frac{1}{\rho}\frac{d\rho}{dT}$

Separating variables, $\int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = \int_{T_0}^{T}\alpha\,dT$. We integrate, on both sides, from the physical situation at temperature T_0 to that at temperature T.

Integrating both sides, $\ln(\rho/\rho_0) = \alpha(T - T_0)$

Thus $\boxed{\rho = \rho_0 e^{\alpha(T-T_0)}}$

(b) From the series expansion $e^x \approx 1 + x$, with x much less than 1,

$$\boxed{\rho \approx \rho_0\left[1+\alpha\left(T-T_0\right)\right]}$$

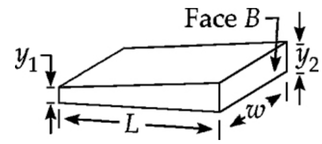
P27.83 A spherical layer within the shell, with radius r and thickness dr, has resistance

$$dR = \frac{\rho dr}{4\pi r^2}$$

The whole resistance is the absolute value of the quantity

$$R = \int_a^b dR = \int_{r_a}^{r_b}\frac{\rho dr}{4\pi r^2} = \frac{\rho}{4\pi}\frac{r^{-1}}{-1}\bigg|_{r_a}^{r_b} = -\frac{\rho}{4\pi}\left(-\frac{1}{r_a}+\frac{1}{r_b}\right) = \frac{\rho}{4\pi}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$

P27.84 Refer to ANS. FIG. P27.84. The current flows generally parallel to L. Consider a slice of the material perpendicular to this current, of thickness dx, and at distance x from face A. Then the other dimensions of the slice are w and y, where by proportion $\frac{y-y_1}{x} = \frac{y_2-y_1}{L}$



ANS. FIG. P27.84

so $y = y_1 + (y_2 - y_1)\frac{x}{L}$. The bit of resistance which this slice contributes is

$$dR = \frac{\rho dx}{A} = \frac{\rho dx}{wy} = \frac{\rho dx}{w\left(y_1 + (y_2 - y_1)(x/L)\right)}$$

The whole resistance is that of all the slices:

$$R = \int_{x=0}^{L} dR = \int_0^L \frac{\rho dx}{w\left(y_1 + (y_2 - y_1)(x/L)\right)}$$

$$= \frac{\rho}{w}\frac{L}{y_2 - y_1}\int_{x=0}^{L}\frac{\left((y_2 - y_1)/L\right)dx}{y_1 + (y_2 - y_1)(x/L)}$$

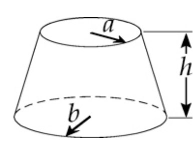
With $u = y_1 + (y_2 - y_1)\frac{x}{L}$ this is of the form $\int \frac{du}{u}$, so

$$R = \frac{\rho L}{w(y_2 - y_1)}\ln\left[y_1 + (y_2 - y_1)(x/L)\right]_{x=0}^{L}$$

$$= \frac{\rho L}{w(y_2 - y_1)}(\ln y_2 - \ln y_1) = \boxed{\frac{\rho L}{w(y_2 - y_1)}\ln\frac{y_2}{y_1}}$$

P27.85 From the geometry of the longitudinal section of the resistor shown in ANS. FIG. P27.85, we see that

$$\frac{(b-r)}{y} = \frac{(b-a)}{h}$$



ANS. FIG. P27.85

From this, the radius at a distance y from the base is $r = (a-b)\frac{y}{h} + b$. For a disk-shaped element of volume $dR = \frac{\rho dy}{\pi r^2}$:

$$R = \frac{\rho}{\pi}\int_0^h \frac{dy}{\left[(a-b)(y/h) + b\right]^2}$$

Using the integral formula $\int \frac{du}{(au+b)^2} = -\frac{1}{a(au+b)}$, $\boxed{R = \frac{\rho}{\pi}\frac{h}{ab}}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P27.2 $\frac{q\omega}{2\pi}$

P27.4 1.05 mA

P27.6 (a) 5.57×10^{-5} m/s; (b) The drift speed is smaller because more electrons are being conducted.

P27.8 (a) 99.5 kA/m^2; (b) The current is the same; (c) The current density is smaller; (d) 0.800 cm; (e) $I = 5.00$ A; (f) 2.49×10^4 A/m^2

P27.10 (a) 2.21×10^{-7} m; (b) The potential of the nearest neighbor is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

P27.12 0.256 C

P27.14 500 mA

P27.16 6.43 A

P27.18 1.29

P27.20 (a) $\sqrt{\frac{mR}{\rho\rho_m}}$; (b) $\sqrt{\frac{4}{\pi}}\left(\frac{\rho m}{\rho_m R}\right)^{1/4}$

P27.22 (a) unaffected; (b) doubles; (c) doubles; (d) unchanged

P27.24 (a) 5.58×10^{-2} kg/mol; (b) 1.41×10^5 mol/m^3; (c) 8.49×10^{28} $\frac{\text{atoms}}{\text{m}^3}$; (d) 1.70×10^{29} electrons/m^3; (e) 2.21×10^{-4} m/s

P27.26 $T = 1.44 \times 10^3$ °C

P27.28 1.98 A

P27.30 (a) 1.22 Ω; (b) 8.00×10^{-4} increase

P27.32 (a) The design goal can be met; (b) $\ell_1 = 0.898$ m and $\ell_2 = 26.2$ m

P27.34 1.71 Ω

P27.36 (a) 3.00×10^8 W; (b) 1.75×10^{17} W

P27.38 7.50 W

P27.40 15.0 μW

P27.42 6.53 Ω

P27.44 (a) \$1.48; (b) \$0.005 34; (c) \$0.381

P27.46 (a) 2.1 W; (b) 3.42 W; (c) It would not be as safe. If surrounded by thermal insulation, it would get much hotter than a copper wire.

P27.48 \$0.319

P27.50 (a) 0.530; (b) 221 J; (c) 15.1°C

P27.52 See P27.52 for full explanation.

P27.54 ~ \$1

P27.56 (a) 17.3 A; (b) 22.4 MJ; (c) \$0.684

P27.58 276°C

P27.60 (a) Lightbulb A = 576 Ω and Lightbulb B = 144 Ω; (b) 4.80 s; (c) The charge is the same. It is at a location that is lower in potential; (d) 0.040 0 s; (e) The energy is the same. Energy enters the lightbulb by electric transmission and leaves by heat and electromagnetic radiation; (f) \$1.98

P27.62 (a) See the table in P27.62(a); (b) $9.93\times10^{-7}\ \Omega\cdot\text{m}$; (c) The average value is within 1% of the tabulated value of $1.00\times10^{-6}\ \Omega\cdot\text{m}$ given in Table 27.2.

P27.64 (a) Any diameter d and length ℓ related by $d^2=\left(4.77\times10^{-8}\right)\ell$, where d and ℓ are in meters; (b) Yes; for V = 0.500 cm^3 of Nichrome, $\ell = 3.65$ m and d = 0.418 mm.

P27.66 (a) 667 A; (b) 50.0 km

P27.68 (a) V/L in the positive x direction; (b) $4\ \rho L/\pi d^2$; (c) $V\pi d^2\ /\ 4\rho L$;

(d) $V/\rho L$ in the positive x direction; (e) $\rho J=\rho\left(\dfrac{V}{\rho L}\right)=\dfrac{V}{L}=E$

P27.70 See P27.70(a) for the full explanation; (b) The result is exact if the assumptions are precisely true. Our derivation contains no approximation steps where delta is assumed to be small.

P27.72 The value of 11.4 A is what results from substituting the given voltage and resistance into Equation 27.7. However, the resistance measured for a lightbulb with an ohmmeter is not the resistance at which it operates because of the change in resistivity with temperature. The higher resistance of the filament at the operating temperature brings the current down significantly.

P27.74 (a) $\sigma A\left|\dfrac{dV}{dx}\right|$; (b) Current flows in the direction of decreasing voltage.

Energy flows by heat in the direction of decreasing temperature.

P27.76 $R = R_0 \dfrac{1+\alpha(T-T_0)}{1+\alpha'(T-T_0)}$

P27.78 (a) See P27.78 for full explanation; (b) $1.79 \times 10^{15}\ \Omega$

P27.80 (a) See Table P27.80 (i), (ii), and (iii); (b) See ANS. FIG. P27.80 (i), (ii), and (iii).

P27.82 (a) $\rho = \rho_0 e^{\alpha(T-T_0)}$; (b) $\rho \approx \rho_0\left[1+\alpha(T-T_0)\right]$

P27.84 $\dfrac{\rho L}{w(y_2-y_1)}\ln\left(\dfrac{y_2}{y_1}\right)$