28

Direct-Current Circuits

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ28.1 Answer (a). When the breaker trips to off, current does not go through the device.

OQ28.2 (**i**) Answer (d). The terminal potential difference is $\Delta V = \mathcal{E} - Ir$, where current I within the battery is considered positive when it flows from the negative to the positive terminal. When $I = 0$, $\Delta V = \mathcal{E}$

(**ii**) Answer (b). When the battery is absorbing electrical energy, the current within the battery flows from the positive to the negative terminal; in this case, I is considered negative, making $\Delta V = \mathcal{E} - Ir = \mathcal{E} + |I|r > \mathcal{E}$.

OQ28.3 Answer (c). In a series connection, the same current exists in each element. The potential difference across a resistor in this series connection is directly proportional to the resistance of that resistor, $\Delta V = IR$, and independent of its location within the series connection.

OQ28.4 Answer (b). because the appliances are connected in parallel, the total power used is proportion to the total current:

$$\sum P_i = \sum I_i \Delta V = \Delta V \sum I_i \rightarrow \sum I_i = \frac{\sum P_i}{\Delta V}$$

or

$$\sum I_i = \frac{P_{\text{heater}} + P_{\text{toaster}} + P_{\text{oven}}}{\Delta V}$$
$$= \frac{\left(1.30\times 10^3 + 1.00\times 10^3 + 1.54\times 10^3\right)\text{ W}}{120\text{ V}} = \boxed{32.0\text{ A}}$$

OQ28.5 Answer (b). When the two identical resistors are in series, the current supplied by the battery is $I = \Delta V/2R$, and the total power delivered is $P_s = (\Delta V)I = (\Delta V)^2/2R$. With the resistors connected in parallel, the potential difference across each resistor is ΔV and the power delivered to each resistor is $P_1 = (\Delta V)^2/R$. Thus, the total power delivered in this case is

$$P_p = 2P_1 = 2\frac{(\Delta V)^2}{R} = 4\left[\frac{(\Delta V)^2}{2R}\right] = 4P_s = 4(8.0\text{ W}) = 32\text{ W}$$

OQ28.6 Answer (a), (d). According to the relationship for resistors in series,

$$R_{\text{eq}} = R_1 + R_2 + \cdots$$

the sum R_{eq} is always larger than any of the resistances R_1, R_2, etc.

OQ28.7 Answer (d). The equivalent resistance for the series combination of five identical resistors is $R_{\text{eq}} = 5R$, and the equivalent capacitance of five identical capacitors in parallel is $C_{\text{eq}} = 5C$. The time constant for the circuit is therefore $\tau = R_{\text{eq}}C_{\text{eq}} = (5R)(5C) = 25RC$.

OQ28.8 Answers (b) and (d). The current is the same in each series resistor, as described by Kirchhoff's junction rule. The potential difference in each resistor is different because $\Delta V = IR$ and each R is different.

OQ28.9 Answer (a). The potential is the same across each parallel resistor, but the current and power in each resistor is different because $I = \Delta V/R$ and $P = I\Delta V$ and each R is different.

OQ28.10 Answer (b) and (c). The same potential difference exists across all elements connected in parallel with each other, while the current through each element is inversely proportional to the resistance of that element $(I = \Delta V/R)$.

OQ28.11 Answer (b). Each headlight's terminals are connected to the positive and negative terminals of the battery so that each headlight can operate if the other is burned out.

OQ28.12 **(i)** The ranking of potentials are: $a > d > b = c > e$. For both batteries to be delivering electric energy, currents are in the direction a to b, and d to c, and so current flows downward through e. Point e is at zero potential. Points b and c are at the same higher potential, d (equal to 9 V) is still higher, and a (equal to 12 V) is highest of all.

(ii) The ranking of magnitudes of current are: $e > a = b > c = d$. The current through e must be the sum of the other two currents. The change in potential from a to b is greater than the change in potential from d to c, so the current from a to b must be greater.

OQ28.13 Answer (b). According to the relationship for resistors in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

the larger the sum on the right-hand side of the equation, $1/R_1 + 1/R_2 + \cdots$, the smaller the equivalent resistance R_{eq}; therefore, R_{eq} is always smaller than any of the resistances R_1, R_2, etc.

OQ28.14 Answers: **(i)** (b) **(ii)** (a) **(iii)** (a) **(iv)** (b) **(v)** (a) **(vi)** (a). Closing the switch lights lamp C. The action increases the battery current so it decreases the terminal voltage of the battery. Lamps A and B are in series, so they carry the same current, but when the terminal voltage of the battery drops, the total voltage drops across lamps A and B combined, thus reducing the potential difference across each. Total power delivered to the lamps increases because the current through the battery increases.

OQ28.15 Answers: (**i**) (a) (**ii**) (d) (**iii**) (a) (**iv**) (a) (**v**) d (**vi**) (a). Closing the switch removes lamp C from the circuit, decreasing the resistance seen by the battery, and so increasing the current in the battery. Lamps A and B are in series, so the potential difference across each is proportional to the current. Total power delivered to the lamps increases because the current through the battery increases.

ANSWERS TO CONCEPTUAL QUESTIONS

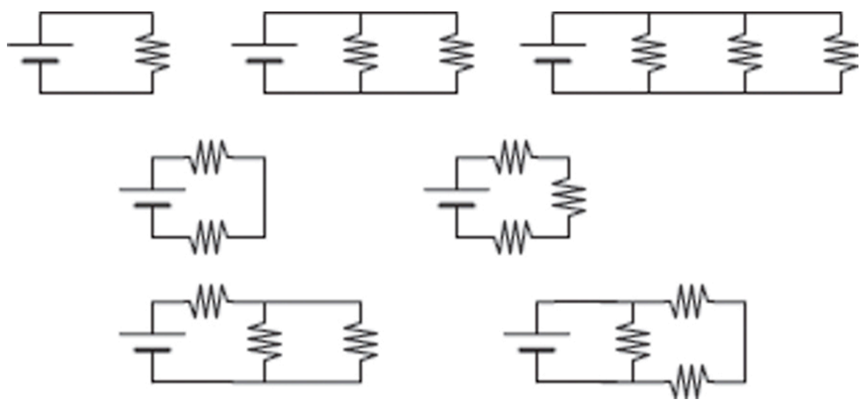
CQ28.1 (a) No. As is the case with the bird in CQ28.3, the resistance of a small length of wire is small, so the potential change along that length is small.

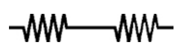
(b) No! When she eventually touches the ground, she will act as a connection to ground, resulting in perhaps several thousand volts across her.

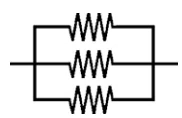
CQ28.2 Answer their question with a challenge. If the student is just looking at a diagram, provide the materials to build the circuit. If you are looking at a circuit where the second bulb really is fainter, get the student to unscrew them both and interchange them. But check that the student's understanding of potential has not been impaired: if you add wires to bypass and short out the first bulb, the second gets brighter.

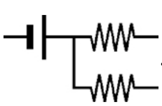
CQ28.3 Because the resistance of a short length of wire is small, the change in potential along that length is small; therefore, there is essentially zero *difference* in potential between the bird's feet. Then negligible current goes through the bird. The resistance through the bird's body between its feet is much larger than the resistance through the wire between the same two points.

CQ28.4



CQ28.5 Two runs in series:  = one run down one slope followed by a second run down a second slope.

Three runs in parallel:  = parallel runs down the same hill so that the change in elevation is the same for each.

Junction of one lift and two runs: .

Gustav Robert Kirchhoff, Professor of Physics at Heidelberg and Berlin, was master of the obvious. A junction rule: The number of skiers coming into any junction must be equal to the number of skiers leaving. A loop rule: the total change in altitude must be zero for any skier completing a closed path.

CQ28.6 The bulb will light up for a while immediately after the switch is closed. As the capacitor charges, the bulb gets progressively dimmer. When the capacitor is fully charged the current in the circuit is zero and the bulb does not glow at all. If the value of RC is small, this whole process might occupy a very short time interval.

CQ28.7 (a) The hospital maintenance worker is right. A hospital room is full of electrical grounds, including the bed frame. If your grandmother touched the faulty knob and the bed frame at the same time, she could receive quite a jolt, as there would be a potential difference of 120 V across her. If the 120 V is DC, the shock could send her into ventricular fibrillation, and the hospital staff could use the defibrillator you read about in Chapter 26. If the 120 V is AC, which is most likely, the current could produce external and internal burns along the path of conduction.

(b) Likely no one got a shock from the radio back at home because her bedroom contained no electrical grounds—no conductors connected to zero volts. Just like the bird in CQ28.3, granny could touch the "hot" knob without getting a shock so long as there was no path to ground to supply a potential difference across her. A new appliance in the bedroom or a flood could make the radio lethal. Repair it or discard it. Enjoy the news from Lake Wobegon on the new plastic radio.

CQ28.8 (a) Both 120-V and 240-V lines can deliver injurious or lethal shocks, but there is a somewhat better safety factor with the lower voltage. To say it a different way, the insulation on a 120-V line can be thinner.

(b) On the other hand, a 240-V device carries less current to operate a device with the same power, so the conductor itself can be thinner. Finally, the last step-down transformer can also be somewhat smaller if it has to go down only to 240 volts from the high voltage of the main power line.

CQ28.9 No. If there is one battery in a circuit, the current inside it will be from its negative terminal to its positive terminal. Whenever a battery is delivering energy to a circuit, it will carry current in this direction. On the other hand, when another source of emf is charging the battery in question, it will have a current pushed through it from its positive terminal to its negative terminal.

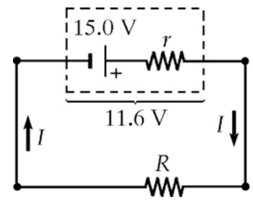
CQ28.10 In Figure 20.13, temperature is similar to electric potential, and temperature difference $\Delta T = T_h - T_c$ is similar to voltage ΔV. Energy transfer is similar to electric current. The upper picture is similar to a series circuit, where the resistors (rods) carry the same current (energy transfer by conduction), and the sum of the voltages (temperature differences) across the rods equals the total voltage (total temperature difference) across both resistors (rods). The lower picture is similar to a parallel circuit, where the resistors (rods) have the same voltage (temperature difference) but carry different currents (energy transfer by conduction).

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 28.1 Electromotive Force

P28.1 (a) Combining Joule's law, $P = I\Delta V$, and the definition of resistance, $\Delta V = IR$, gives

$$R = \frac{(\Delta V)^2}{P} = \frac{(11.6\text{ V})^2}{20.0\text{ W}} = \boxed{6.73\ \Omega}$$



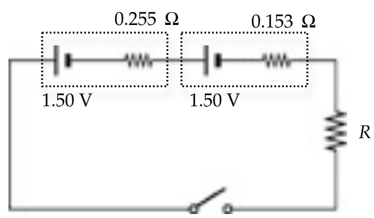
ANS. FIG. P28.1

(b) The electromotive force of the battery must equal the voltage drops across the resistances: $\mathcal{E} = IR + Ir$, where $I = \Delta V / R$.

$$r = \frac{(\mathcal{E} - IR)}{I} = \frac{(\mathcal{E} - \Delta V)R}{\Delta V}$$

$$= \frac{(15.0\text{ V} - 11.6\text{ V})(6.73\ \Omega)}{11.6\text{ V}} = \boxed{1.97\ \Omega}$$

P28.2 The total resistance is $R = \dfrac{3.00\text{ V}}{0.600\text{ A}} = 5.00\ \Omega.$



ANS. FIG. P28.2

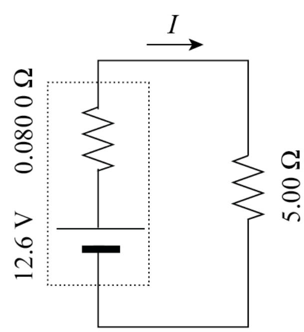
(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\dfrac{P_{\text{batteries}}}{P_{\text{total}}} = \dfrac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.081\ 6 = \boxed{8.16\%}$

P28.3 (a) Here $\mathcal{E} = I(R + r)$,

so

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.6\text{ V}}{(5.00\ \Omega + 0.080\ 0\ \Omega)}$$
$$= 2.48\text{ A}.$$



ANS. FIG. P28.3(a)

Then,

$$\Delta V = IR = (2.48\text{ A})(5.00\ \Omega)$$
$$= \boxed{12.4\text{ V}}$$

(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively. Then,

$$I_1 = I_2 + 35.0 \text{ A}$$

and $\mathcal{E} - I_1R - I_2R = 0$

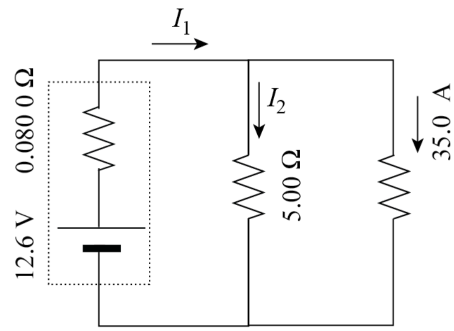
so

$$\mathcal{E} = (I_2 + 35.0 \text{ A})(0.080\,0\ \Omega) + I_2(5.00\ \Omega) = 12.6 \text{ V}$$

giving $I_2 = 1.93$ A.

Thus,

$$\Delta V_2 = (1.93 \text{ A})(5.00\ \Omega) = \boxed{9.65 \text{ V}}$$



ANS. FIG. P28.3(b)

P28.4 (a) At maximum power transfer, $r = R$. Equal powers are delivered to r and R. The efficiency is $\boxed{50.0\%}$.

(b) For maximum fractional energy transfer to R, we want zero energy absorbed by r, so we want $r = \boxed{0}$.

(c) $\boxed{\text{High efficiency.}}$ The electric company's economic interest is to minimize internal energy production in its power lines, so that it can sell a large fraction of the energy output of its generators to the customers.

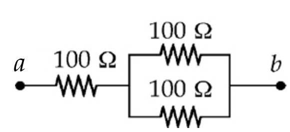
(d) $\boxed{\text{High power transfer.}}$ Energy by electric transmission is so cheap compared to the sound system that she does not spend extra money to buy an efficient amplifier.

Section 28.2 Resistors in Series and Parallel

P28.5 (a) Since all the current in the circuit must pass through the series 100-Ω resistor,

$$P_{\max} = I_{\max}^2 R$$

so $$I_{\max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{25.0 \text{ W}}{100\ \Omega}} = 0.500 \text{ A}.$$



ANS. FIG. P28.5

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1} \Omega = 150\ \Omega$$

$$\Delta V_{\max} = R_{eq} I_{\max} = \boxed{75.0 \text{ V}}$$

(b) From a to b in the circuit, the power delivered is

$$P_{\text{series}} = \boxed{25.0 \text{ W}} \text{ for the first resistor, and}$$

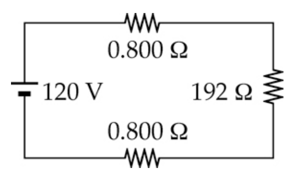
$$P_{\text{parallel}} = I^2 R = (0.250 \text{ A})^2 (100 \text{ }\Omega) = \boxed{6.25 \text{ W}}$$

for each of the two parallel resistors.

(c) $P = I\Delta V = (0.500 \text{ A})(75.0 \text{ V}) = \boxed{37.5 \text{ W}}$

P28.6 (a) The 120-V potential difference is applied across the series combination of the two conductors in the extension cord and the lightbulb. The potential difference across the lightbulb is less than 120 V, and its power is less than 75 W.

(b) See the circuit diagram in ANS. FIG. P28.6; the 192-Ω resistor is the lightbulb (see below).



ANS. FIG. P28.6

(c) First, find the operating resistance of the lightbulb:

$$P = \frac{(\Delta V)^2}{R}$$

or $$R = \frac{(\Delta V)^2}{P} = \frac{(120 \text{ V})^2}{75.0 \text{ W}} = 192 \text{ }\Omega$$

From the circuit, the total resistance is 193.6 Ω. The current is

$$I = \frac{120 \text{ V}}{193.6 \text{ }\Omega} = 0.620 \text{ A}$$

so the power delivered to the lightbulb is

$$P = I^2 \Delta R = (0.620 \text{ A})^2 (192 \text{ }\Omega) = \boxed{73.8 \text{ W}}$$

P28.7 The equivalent resistance of the parallel combination of three identical resistors is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{3}{R} \quad \text{or} \quad R_p = \frac{R}{3}$$

The total resistance of the series combination between points a and b is then

$$R_{ab} = R + R_p + R = 2R + \frac{R}{3} = \boxed{\frac{7}{3}R}$$

P28.8 (a) By Ohm's law, the current in A is $\boxed{I_A = \mathcal{E}/R}$. The equivalent resistance of the series combination of bulbs B and C is $2R$. Thus, the current in each of these bulbs is $\boxed{I_B = I_C = \mathcal{E}/2R}$.

(b) B and C have the same brightness because they carry the same current.

(c) A is brighter than B or C because it carries twice as much current.

P28.9 If we turn the given diagram on its side and change the lengths of the wires, we find that it is the same as ANS. FIG. P28.9(a). The 20.0-Ω and 5.00-Ω resistors are in series, so the first reduction is shown in ANS. FIG. P28.9(b). In addition, since the 10.0-Ω, 5.00-Ω, and 25.0-Ω resistors are then in parallel, we can solve for their equivalent resistance as:

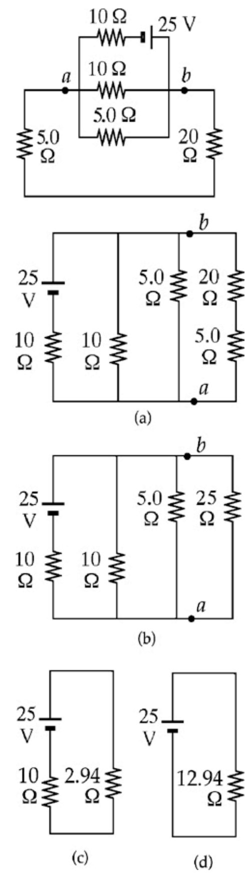
$$\frac{1}{R_{\text{eq}}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega} \to R_{\text{eq}} = 2.94\ \Omega$$

This is shown in ANS. FIG. P28.9(c), which in turn reduces to the circuit shown in ANS. FIG. P28.9(d), from which we see that the total resistance of the circuit is 12.94 Ω.

Next, we work backwards through the diagrams applying $I = \frac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The total 12.94-Ω resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In ANS. FIG. P28.9(c), this 1.93 A goes through the 2.94-Ω equivalent resistor to give a potential difference of:



ANS. FIG. P28.9

$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

From ANS. FIG. P28.9(b), we see that this potential difference is the same as the potential difference ΔV_{ab} across the 10-Ω resistor and the 5.00-Ω resistor.

Thus we have first found the answer to part (b), which is

$$\Delta V_{ab} = \boxed{5.68\ \text{V}}$$

Since the current through the 20.0-Ω resistor is also the current through the 25.0-Ω line *ab*,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$

P28.10 (a) Connect two 50-Ω resistors in parallel to get 25 Ω. Then connect that parallel combination in series with a 20-Ω for a total resistance of 45 Ω.

(b) Connect two 50-Ω resistors in parallel to get 25 Ω. Also, connect two 20-Ω resistors in parallel to get 10 Ω. Then, connect these two combinations in series with each other to obtain 35 Ω.

P28.11 When *S* is open, R_1, R_2, and R_3 are in series with the battery. Thus,

$$R_1 + R_2 + R_3 = \frac{6\ \text{V}}{10^{-3}\ \text{A}} = 6\ \text{k}\Omega \qquad \textbf{[1]}$$

When *S* is closed in position *a*, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. Thus,

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\ \text{V}}{1.2\times 10^{-3}\ \text{A}} = 5\ \text{k}\Omega \qquad \textbf{[2]}$$

When *S* is closed in position *b*, R_1 and R_2 are in series with the battery and R_3 is shorted. Thus,

$$R_1 + R_2 = \frac{6\ \text{V}}{2\times 10^{-3}\ \text{A}} = 3\ \text{k}\Omega \qquad \textbf{[3]}$$

Subtracting [3] from [1] gives $R_3 = 3\ \text{k}\Omega$.

Subtracting [2] from [1] gives $R_2 = 2\ \text{k}\Omega$.

Then, from [3], $R_1 = 1\ \text{k}\Omega$.

Answers: (a) $\boxed{R_1 = 1.00\ \text{k}\Omega}$ (b) $\boxed{R_2 = 2.00\ \text{k}\Omega}$ (c) $\boxed{R_3 = 3.00\ \text{k}\Omega}$

P28.12 When *S* is open, R_1, R_2, and R_3 are in series with the battery. Thus,

$$R_1 + R_2 + R_3 = \frac{\varepsilon}{I_0} \qquad \textbf{[1]}$$

When S is closed in position a, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. Thus,

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{\mathcal{E}}{I_a} \qquad \textbf{[2]}$$

When S is closed in position b, R_1 and R_2 are in series with the battery. R_3 is shorted. Thus:

$$R_1 + R_2 = \frac{\mathcal{E}}{I_b} \qquad \textbf{[3]}$$

Subtracting [3] from [1] gives

$$(R_1 + R_2 + R_3) - (R_1 + R_2) = \frac{\mathcal{E}}{I_0} - \frac{\mathcal{E}}{I_b}$$

$$R_3 = \mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_b}\right)$$

Subtracting [2] from [1] gives

$$(R_1 + R_2 + R_3) - \left(R_1 + \frac{1}{2}R_2 + R_3\right) = \frac{\mathcal{E}}{I_0} - \frac{\mathcal{E}}{I_a}$$

$$\frac{1}{2}R_2 = \mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right)$$

$$R_2 = 2\mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right)$$

Then, from [3],

$$R_1 + R_2 = \frac{\mathcal{E}}{I_b}$$

$$R_1 = \frac{\mathcal{E}}{I_b} - R_2$$

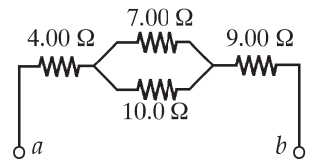
$$R_1 = \frac{\mathcal{E}}{I_b} - 2\mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right)$$

$$R_1 = \mathcal{E}\left(-\frac{2}{I_0} + \frac{2}{I_a} + \frac{1}{I_b}\right)$$

Answers: (a) $\boxed{R_1 = \mathcal{E}\left(-\frac{2}{I_0} + \frac{2}{I_a} + \frac{1}{I_b}\right)}$ (b) $\boxed{R_2 = 2\mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right)}$

(c) $\boxed{R_3 = \mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_b}\right)}$

***P28.13** (a) The equivalent resistance of the two parallel resistors is



ANS. FIG. P28.13

$$R_p = \frac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$$

Thus,

$$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$$

(b) $\Delta V = IR$

$34.0\ \text{V} = I(17.1\ \Omega)$

$I = \boxed{1.99\ \text{A}}$ for the $4.00\text{-}\Omega$ and $9.00\text{-}\Omega$ resistors.

Applying $\Delta V = IR$,

$(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$

$8.18\ \text{V} = I(7.00\ \Omega)$

so $I = \boxed{1.17\ \text{A}}$ for the $7.00\text{-}\Omega$ resistor. Finally,

$8.18\ \text{V} = I(10.0\ \Omega)$

so $I = \boxed{0.818\ \text{A}}$ for the $10.0\text{-}\Omega$ resistor.

P28.14 (a) The resistance between a and b $\boxed{\text{decreases.}}$ The original resistance is

$$R + \frac{1}{\frac{1}{90+10}+\frac{1}{10+90}} = R + 50\ \Omega$$

Closing the switch changes the resistance to

$$R + \frac{1}{\frac{1}{90}+\frac{1}{10}} + \frac{1}{\frac{1}{10}+\frac{1}{90}} = R + 18\ \Omega$$

(b) We require $R + 18\ \Omega = 0.50(R + 50\ \Omega)$, so $R = \boxed{14.0\ \Omega}$.

P28.15 Denoting the two resistors as x and y, and suppressing units,

$$x + y = 690,\ \text{and}\ \frac{1}{150} = \frac{1}{x} + \frac{1}{y}$$

$$\frac{1}{150} = \frac{1}{x} + \frac{1}{690-x} = \frac{(690-x)+x}{x(690-x)}$$

$$x^2 - 690x + 103\,500 = 0$$

$$x = \frac{690 \pm \sqrt{(690)^2 - 414\,000}}{2}$$

$$x = \boxed{470\ \Omega} \quad y = \boxed{220\ \Omega}$$

P28.16 (a) The resistors 2, 3, and 4 can be combined to a single $2R$ resistor. This is in series with resistor 1, with resistance R, so the equivalent resistance of the whole circuit is $3R$. In series, potential difference is shared in proportion to the resistance, so resistor 1 gets $\frac{1}{3}$ of the battery voltage and the 2-3-4 parallel combination gets $\frac{2}{3}$ of the battery voltage. This is the potential difference across resistor 4, but resistors 2 and 3 must share this voltage. $\frac{1}{3}$ goes to 2 and $\frac{2}{3}$ to 3. The ranking by potential difference is

$$\Delta V_4 > \Delta V_3 > \Delta V_1 > \Delta V_2$$

Based on the reasoning above the potential differences are

$$\boxed{\Delta V_1 = \frac{\mathcal{E}}{3},\ \Delta V_2 = \frac{2\mathcal{E}}{9},\ \Delta V_3 = \frac{4\mathcal{E}}{9},\ \Delta V_4 = \frac{2\mathcal{E}}{3}}$$

(b) All the current goes through resistor 1, so it gets the most. The current then splits at the parallel combination. Resistor 4 gets more than half, because the resistance in that branch is less than in the other branch. Resistors 2 and 3 have equal currents because they are in series. The ranking by current is

$$I_1 > I_4 > I_2 = I_3$$

Resistor 1 has a current of I. Because the resistance of 2 and 3 in series is twice that of resistor 4, twice as much current goes through 4 as through 2 and 3. The current through the resistors are

$$\boxed{I_1 = I,\ I_2 = I_3 = \frac{I}{3},\ I_4 = \frac{2I}{3}}$$

(c) Increasing resistor 3 increases the equivalent resistance of the entire circuit. The current in the circuit, which is the current through resistor 1, decreases. This decreases the potential difference across resistor 1, increasing the potential difference across the parallel combination. With a larger potential difference the current through resistor 4 is increased. With more current through 4, and less in the circuit to start with, the current through resistors 2 and 3 must decrease. To summarize,

$$\boxed{I_4 \text{ increases and } I_1,\ I_2,\ \text{and } I_3 \text{ decrease}}$$

(d) If resistor 3 has an infinite resistance it blocks any current from passing through that branch, and the circuit effectively is just resistor 1 and resistor 4 in series with the battery. The circuit now has an equivalent resistance of $4R$. The current in the circuit drops to $\frac{3}{4}$ of the original current because the resistance has increased by $\frac{4}{3}$. All this current passes through resistors 1 and 4, and none passes through 2 or 3. Therefore,

$$\boxed{I_1 = \frac{3I}{4},\ I_2 = I_3 = 0,\ I_4 = \frac{3I}{4}}$$

P28.17 (a) The parallel combination of the 6.0 Ω and 12 Ω resistors has an equivalent resistance of

$$\frac{1}{R_{p1}} = \frac{1}{6.0\ \Omega} + \frac{1}{12\ \Omega} = \frac{2+1}{12\ \Omega}$$

or $$R_{p1} = \frac{12\ \Omega}{3} = 4.0\ \Omega$$

Similarly, the equivalent resistance of the 4.0 Ω and 8.0 Ω parallel combination is

$$\frac{1}{R_{p2}} = \frac{1}{4.0\ \Omega} + \frac{1}{8.0\ \Omega} = \frac{2+1}{8.0\ \Omega}$$

or $$R_{p2} = \frac{8\ \Omega}{3}$$

The total resistance of the series combination between points *a* and *b* is then

$$R_{ab} = R_{p1} + 5.0\ \Omega + R_{p2} = 4.0\ \Omega + 5.0\ \Omega + \frac{8.0}{3}\ \Omega$$

$$= \frac{35}{3}\ \Omega = \boxed{11.7\ \Omega}$$

(b) If $\Delta V_{ab} = 35$ V, the total current from a to b is $I_{ab} = \Delta V_{ab}/R_{ab} =$ $35\text{ V}/(35\ \Omega/3) = 3.0\text{ A}$ and the potential differences across the two parallel combinations are

$$\Delta V_{p1} = I_{ab}R_{p1} = (3.0\text{ A})(4.0\ \Omega) = 12\text{ V}$$

and $\Delta V_{p2} = I_{ab}R_{p2} = (3.0\text{ A})\left(\frac{8.0}{3}\ \Omega\right) = 8.0\text{ V}$

so the individual currents through the various resistors are:

$$I_{12} = \Delta V_{p1}/12\ \Omega = \boxed{1.0\text{ A}}$$

$$I_6 = \Delta V_{p1}/6.0\ \Omega = \boxed{2.0\text{ A}}$$

$$I_5 = I_{ab} = \boxed{3.0\text{ A}}$$

$$I_8 = \Delta V_{p2}/8.0\ \Omega = \boxed{1.0\text{ A}}$$

and $I_4 = \Delta V_{p2}/4.0\ \Omega = \boxed{2.0\text{ A}}$

P28.18 We assume that the metal wand makes low-resistance contact with the person's hand and that the resistance through the person's body is negligible compared to the resistance R_{shoes} of the shoe soles. The equivalent resistance seen by the power supply is 1.00 MΩ + R_{shoes}. The current through both resistors is $\frac{50.0\text{ V}}{1.00\text{ M}\Omega + R_{\text{shoes}}}$. The voltmeter displays

$$\Delta V = I(1.00\text{ M}\Omega)$$

$$\Delta V = \frac{50.0\text{ V}}{1.00\text{ M}\Omega + R_{\text{shoes}}} = 1.00\text{ M}\Omega$$

(a) We solve to obtain

$$50.0\text{ V}(1.00\text{ M}\Omega) = \Delta V(1.00\text{ M}\Omega) + \Delta V(R_{\text{shoes}})$$

$$R_{\text{shoes}} = \frac{(1.00\text{ M}\Omega)(50.0 - \Delta V)}{\Delta V}$$

or

$$R_{\text{shoes}} = \frac{50.0 - \Delta V}{\Delta V}$$

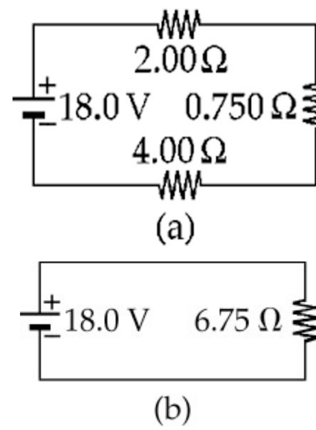
where resistance is measured in MΩ.

(b) With $R_{\text{shoes}} \to 0$, the current through the person's body is

$$\frac{50.0 \text{ V}}{1.00 \text{ M}\Omega} = 50.0 \ \mu\text{A} \quad \boxed{\text{The current will never exceed 50 } \mu\text{A.}}$$

P28.19 To find the current in each resistor, we find the resistance seen by the battery. The given circuit reduces as shown in ANS. FIG. P28.19(a), since

$$\frac{1}{(1/1.00 \ \Omega) + (1/3.00 \ \Omega)} = 0.750 \ \Omega$$



ANS. FIG. P28.19

In ANS. FIG. P28.19(b),

$$I = 18.0 \text{ V}/6.75 \ \Omega = 2.67 \text{ A}$$

This is also the current in ANS. FIG. P28.19(a), so the 2.00-Ω and 4.00-Ω resistors convert powers

$$P_2 = I\Delta V = I^2 R = (2.67 \text{ A})^2 (2.00 \ \Omega) = \boxed{14.2 \text{ W}}$$

and $P_4 = I^2 R = (2.67 \text{ A})^2 (4.00 \ \Omega) = \boxed{28.4 \text{ W}}$

The voltage across the 0.750-Ω resistor in ANS. FIG. P28.19(a), and across both the 3.00-Ω and the 1.00-Ω resistors in Figure P28.19, is

$$\Delta V = IR = (2.67 \text{ A})(0.750 \ \Omega) = \boxed{2.00 \text{ V}}$$

Then for the 3.00-Ω resistor,

$$I = \frac{\Delta V}{R} = \frac{2.00 \text{ V}}{3.00 \ \Omega}$$

and the power is

$$P_3 = I\Delta V = \left(\frac{2.00 \text{ V}}{3.00 \ \Omega}\right)(2.00 \text{ V}) = \boxed{1.33 \text{ W}}$$

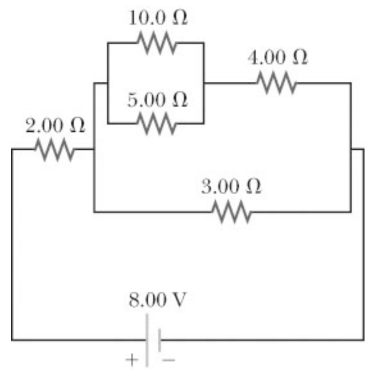
For the 1.00-Ω resistor,

$$I = \frac{2.00 \text{ V}}{1.00 \ \Omega} \quad \text{and} \quad P_1 = \left(\frac{2.00 \text{ V}}{1.00 \ \Omega}\right)(2.00 \text{ V}) = \boxed{4.00 \text{ W}}$$

P28.20 The resistance of the combination of extra resistors must be $\frac{7}{3}R - R = \frac{4}{3}R$. The possible combinations are: one resistor: R; two resistors: $2R$, $\frac{1}{2}R$; three resistors: $3R$, $\frac{1}{3}R$, $\frac{2}{3}R$, $\frac{3}{2}R$. None of these is $\frac{4}{3}R$, so the desired resistance cannot be achieved.

P28.21 (a) The equivalent resistance of this first parallel combination is

$$\frac{1}{R_{p1}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} \quad \text{or} \quad R_{p1} = 3.33\ \Omega$$



ANS. FIG. P28.21

For this series combination,

$$R_{\text{upper}} = R_{p1} + 4.00\ \Omega = 7.33\ \Omega$$

For the second parallel combination,

$$\frac{1}{R_{p2}} = \frac{1}{R_{\text{upper}}} + \frac{1}{3.00\ \Omega} = \frac{1}{7.33\ \Omega} + \frac{1}{3.00\ \Omega} \quad \text{or} \quad R_{p2} = 2.13\ \Omega$$

For the second series combination (and hence the entire resistor network)

$$R_{\text{total}} = 2.00\ \Omega + R_{p2} = 2.00\ \Omega + 2.13\ \Omega = 4.13\ \Omega$$

The total current supplied by the battery is

$$I_{\text{total}} = \frac{\Delta V}{R_{\text{total}}} = \frac{8.00\ \text{V}}{4.13\ \Omega} = 1.94\ \text{A}$$

The potential drop across the 2.00 Ω resistor is

$$\Delta V_2 = R_2 I_{\text{total}} = (2.00\ \Omega)(1.94\ \text{A}) = 3.88\ \text{V}$$

The potential drop across the second parallel combination must be

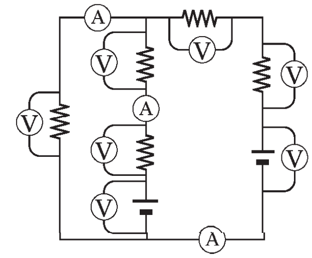
$$\Delta V_{p2} = \Delta V - \Delta V_2 = 8.00\ \text{V} - 3.88\ \text{V} = \boxed{4.12\ \text{V}}$$

(b) So the current through the 3.00 Ω resistor is

$$I_{\text{total}} = \frac{\Delta V_{p2}}{R_3} = \frac{4.12\ \text{V}}{3.00\ \Omega} = \boxed{1.38\ \text{A}}$$

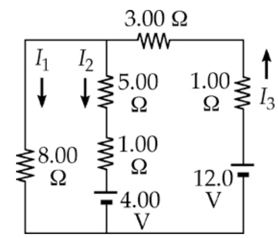
Section 28.3 Kirchhoff 's Rules

***P28.22** We need one voltmeter across each resistor and each battery. These are shown with (V) in ANS. FIG. P28.22. From Kirchhoff's junction rule, we need one ammeter in each segment of the circuit. Ammeters are shown with (A) in ANS. FIG. P28.22. ANS. FIG. P28.22 is the complete answer to this problem.



ANS. FIG. P28.22

P28.23 We name currents I_1, I_2, and I_3 as shown in ANS. FIG. P28.23. From Kirchhoff's current rule, $I_3 = I_1 + I_2$.



ANS. FIG. P28.23

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$$12.0 \text{ V} - (4.00 \ \Omega)I_3 - (6.00 \ \Omega)I_2 - 4.00 \text{ V} = 0$$

$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00 \ \Omega)I_2 - 4.00 \text{ V} + (8.00 \ \Omega)I_1 = 0$$

or $$(8.00 \ \Omega)I_1 = 4.00 + (6.00 \ \Omega)I_2$$

Solving the above linear system (by substituting $I_1 + I_2$ for I_3), we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = \dfrac{4}{3}I_1 - \dfrac{2}{3} \end{cases}$$

and to the single equation

$$8 = 4I_1 + 10\left(\frac{4}{3}I_1 - \frac{2}{3}\right) = \frac{52}{3}I_1 - \frac{20}{3}$$

which gives

$$I_1 = \frac{3}{52}\left(8 + \frac{20}{3}\right) = 0.846 \text{ A}$$

Then $$I_2 = I_2 = \frac{4}{3}(0.846) - \frac{2}{3} = 0.462$$

and $I_3 = I_1 + I_2 = 1.31\text{ A}$

give $\boxed{I_1 = 846\text{ mA}, I_2 = 462\text{ mA}, I_3 = 1.31\text{ A}}$

(a) The results are: 0.846 A down in the 8.00-Ω resistor; 0.462 A down in the middle branch; 1.31 A up in the right-hand branch.

(b) For 4.00-V battery:

$$\Delta U = P\Delta t = (\Delta V)I\Delta t = (4.00\text{ V})(-0.462\text{ A})(120\text{ s}) = -222\text{ J}$$

For 12.0-V battery:

$$\Delta U = (12.0\text{ V})(1.31\text{ A})(120\text{ s}) = 1.88\text{ kJ}$$

The results are: –222 J by the 4.00-V battery and 1.88 kJ by the 12.0-V battery.

(c) To the 8.00-Ω resistor:

$$\Delta U = I^2R\Delta t = (0.846\text{ A})^2(8.00\ \Omega)(120\text{ s}) = \boxed{687\text{ J}}$$

To the 5.00-Ω resistor:

$$\Delta U = (0.462\text{ A})^2(5.00\ \Omega)(120\text{ s}) = \boxed{128\text{ J}}$$

To the 1.00-Ω resistor in the center branch:

$$(0.462\text{ A})^2(1.00\ \Omega)(120\text{ s}) = \boxed{25.6\text{ J}}$$

To the 3.00-Ω resistor:

$$(1.31\text{ A})^2(3.00\ \Omega)(120\text{ s}) = \boxed{616\text{ J}}$$

To the 1.00-Ω resistor in the right-hand branch:

$$(1.31\text{ A})^2(1.00\ \Omega)(120\text{ s}) = \boxed{205\text{ J}}$$

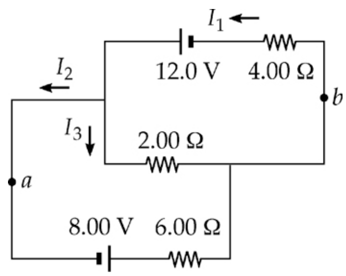
(d) Chemical energy in the 12.0-V battery is transformed into internal energy in the resistors. The 4.00-V battery is being charged, so its chemical potential energy is increasing at the expense of some of the chemical potential energy in the 12.0-V battery.

(e) Either sum the results in part (b): –222 J + 1.88 kJ = 1.66 kJ,

or in part (c): 687 J + 128 J + 25.6 J + 616 J +205 J = 1.66 kJ

The total amount of energy transformed is 1.66 kJ.

P28.24 We name the currents I_1, I_2, and I_3 and arbitrarily choose current directions as labeled in ANS. FIG. P28.24.



ANS FIG. P28.24

(a) From the point rule for the junction below point *b*,

$$-I_1 + I_2 + I_3 = 0 \quad \textbf{[1]}$$

Traversing the top loop counterclockwise gives the voltage loop equation

$$+12.0 \text{ V} - (2.00\ ) I_3 - (4.00\ ) I_1 = 0 \quad \textbf{[2]}$$

Traversing the bottom loop CCW,

$$+8.00 \text{ V} - (6.00\ ) I_2 + (2.00\ ) I_3 = 0 \quad \textbf{[3]}$$

Solving for I_1 from equation [2],

$$I_1 = \frac{12.0 \text{ V} - (2.00\ \Omega) I_3}{4.00\ \Omega}$$

Solving for I_2 from equation [3],

$$I_2 = \frac{8.00 \text{ V} + (2.00\ \Omega) I_3}{6.00\ \Omega}$$

Substituting both of these values into equation [1], we find

$$-(3.00 \text{ V} - 0.500 I_3) + 1.33 \text{ V} + 0.333 I_3 + I_3 = 0$$

so $\quad -1.67 \text{ V} + 1.833 I_3 = 0$

and the current in the 2.00-Ω resistor is $\boxed{I_3 = 909 \text{ mA}}$

(b) Through the center wire,

$$V_a - (0.909 \text{ A})(2.00\ \Omega) = V_b$$

Therefore,

$$V_b - V_a = \boxed{-1.82 \text{ V}}, \text{ with } V_a > V_b$$

P28.25 (a) Let I_6 represent the current in the ammeter and the top 6-Ω resistor. The bottom 6-Ω resistor has the same potential difference across it, so it carries an equal current.

We assume both I_6 in the upper branch and I_6 in the lower branch flow to the right. We assume current I_{10} flows to the left through the 10-Ω resistor. For the top loop we have

$$6.00 - 10.0I_{10} - 6.00I_6 = 0 \quad \rightarrow \quad I_{10} = 0.6 - 0.6\,I_6 \qquad \textbf{[1]}$$

We assume current I_5 flows to the left through the 5-Ω resistor. For the bottom loop,

$$4.50 - 5.00I_5 - 6.00I_6 = 0 \quad \rightarrow \quad I_5 = 0.9 - 1.2\,I_6 \qquad \textbf{[2]}$$

For the junctions on the left side, taken together,

$$+I_{10} + I_5 - I_6 - I_6 = 0 \qquad \textbf{[3]}$$

Substituting I_{10} and I_5 into [3], we have

$$(0.6 - 0.6\,I_6) + (0.9 - 1.2\,I_6) - 2\,I_6 = 0 \rightarrow I_6 = 1.5/3.8 = \boxed{0.395\text{ A}}$$

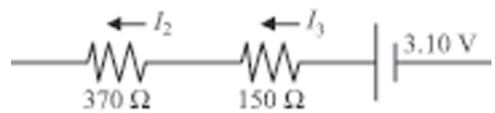
(b) The loop theorem for the little loop containing the voltmeter gives

$$+\,6.00\text{ V} - \Delta V - 4.50\text{ V} = 0 \quad \rightarrow \quad \Delta V = \boxed{1.50\ V}$$

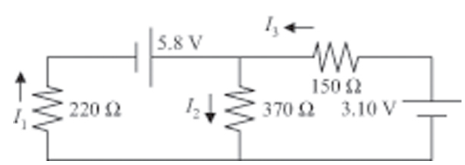
P28.26 (a) The first equation represents Kirchhoff's loop theorem. We choose to think of it as describing a clockwise trip around the left-hand loop in a circuit; see ANS. FIG. P28.26(a).

ANS. FIG. P28.26(a)

For the right-hand loop see ANS. FIG. P28.26(b). The junctions must be between the 5.80-V emf and the 370-Ω resistor and between the 370-Ω resistor and the 150-Ω resistor. Then we have ANS. FIG. P28.26(c). This is consistent with the third equation,



ANS. FIG. P28.26(b)



ANS. FIG. P28.26(c)

$$I_1 + I_3 - I_2 = 0$$
$$I_2 = I_1 + I_3$$

(b) Suppressing units, we substitute:

$$-220I_1 + 5.80 - 370I_1 - 370I_3 = 0$$
$$+370I_1 + 370I_3 + 150I_3 - 3.10 = 0$$

Next, $I_3 = \dfrac{5.80 - 590I_1}{370}$

$$370I_1 + \frac{520}{370}(5.80 - 590I_1) - 3.1 = 0$$
$$370I_1 + 8.15 - 829I_1 - 3.10 = 0$$

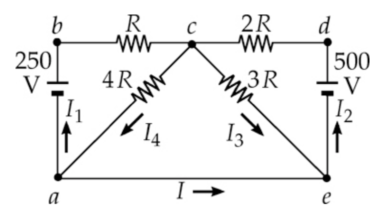
$$I_1 = \frac{5.05\ \text{V}}{459\ \Omega} = \boxed{\text{11.0 mA in the 220-}\Omega\text{ resistor and out of the positive pole of the 5.80-V battery}}$$

$$I_3 = \frac{5.80 - 590(0.011\,0)}{370} = -1.87\ \text{mA}$$

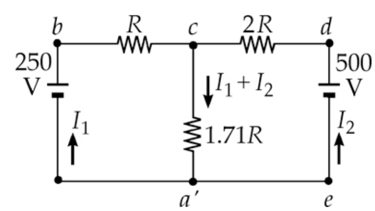
$\boxed{\text{The current is 1.87 mA in the 150-}\Omega\text{ resistor and out of the negative pole of the 3.10-V battery.}}$

$$I_2 = 11.0 - 1.87 = \boxed{9.13\ \text{mA in the 370-}\Omega\text{ resistor}}$$

P28.27 Label the currents in the branches as shown in ANS. FIG. P28.27(a). Reduce the circuit by combining the two parallel resistors as shown in ANS. FIG. P28.27(b).



ANS. FIG. P28.27(a)



ANS. FIG. P28.27(b)

Apply Kirchhoff's loop rule to both loops in ANS. FIG. P28.27(b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250\ \text{V}$$

$$(1.71R)I_1 + (3.71R)I_2 = 500\ \text{V}$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0\ \text{mA}$$

$$I_2 = 130.0\ \text{mA}$$

From ANS. FIG. P28.27(b), $V_c - V_a = (I_1 + I_2)(1.71R) = 240$ V.

Thus, from ANS. FIG. P28.27(a), $I_4 = \dfrac{V_c - V_a}{4R} = \dfrac{240\ \text{V}}{4\,000\ \Omega} = 60.0$ mA.

Finally, applying Kirchhoff's point rule at point *a* in ANS. FIG. P28.27(a) gives:

$$I = I_4 - I_1 = 60.0\ \text{mA} - 10.0\ \text{mA} = +50.0\ \text{mA}$$

or $I = \boxed{50.0 \text{ mA from point } a \text{ to point } e}$.

P28.28 Using Kirchhoff's rules and suppressing units,

$$12.0 - (0.01)I_1 - (0.06)I_3 = 0 \quad \textbf{[1]}$$

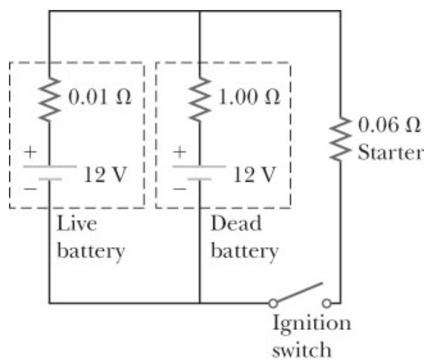
$$12.0 + (1.00)I_2 - (0.06)I_3 = 0 \quad \textbf{[2]}$$

and $I_1 = I_2 + I_3$. **[3]**

Substitute [3] into [1]:

$$12.0 - (0.01)(I_2 + I_3) - (0.06)I_3 = 0$$

$$12.0 - (0.01)I_2 - (0.07)I_3 = 0 \quad \textbf{[4]}$$



ANS. FIG. P28.28

Solving [4] and [2] simultaneously gives

(a) $I_3 = 172 \text{ A} = \boxed{172 \text{ A downward}}$ in the starter.

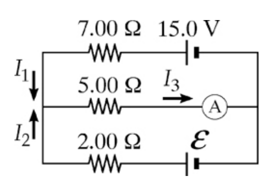
(b) $I_2 = -1.70 \text{ A} = \boxed{1.70 \text{ A upward}}$ in the dead battery.

(c) No, the current in the dead battery is upward in Figure P28.28, so it is not being charged. The dead battery is providing a small amount of power to operate the starter, so it is not really "dead."

P28.29 (a) For the upper loop:

$$+15.0 \text{ V} - (7.00 \text{ }\Omega)I_1 - (2.00 \text{ A})(5.00 \text{ }\Omega) = 0$$

$5.00 = 7.00I_1$ so $\boxed{I_1 = 0.714 \text{ A}}$



ANS. FIG. P28.29

(b) For the center-left junction:

$$I_3 = I_1 + I_2 = 2.00 \text{ A}$$

where I_3 is the current through the ammeter (assumed to travel to the right):

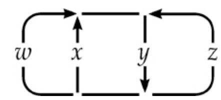
$$0.714 + I_2 = 2.00 \quad \text{so} \quad \boxed{I_2 = 1.29 \text{ A}}$$

(c) For the lower loop:

$$+\mathcal{E} - (2.00\ \Omega)(1.29\text{ A}) - (5.00\ \Omega)(2.00\text{ A}) = 0 \rightarrow \boxed{\mathcal{E} = 12.6\text{ V}}$$

P28.30 Name the currents as shown in ANS. FIG. P28.30. Then

$$y = w + x + z$$



ANS. FIG. P28.30

The loop equations are (suppressing units):

$$\left.\begin{array}{l} -200w - 40.0 + 80.0x = 0 \\ -80.0x + 40.0 + 360 - 20.0y = 0 \\ +360 - 20.0y - 70.0z + 80.0 = 0 \end{array}\right\} \rightarrow \left\{\begin{array}{l} x = 2.50w + 0.500 \\ 20.0 = 4.00x + 1.00y \\ 22.0 = 1.00y + 3.50z \end{array}\right.$$

Use $y = w + x + z$ to eliminate y by substitution:

$$\left.\begin{array}{l} x = 2.50w + 0.500 \\ 20.0 = 4.00x + 1.00y \rightarrow 20.0 = 4.00x + 1.00(w + x + z) \\ 22.0 = 1.00y + 3.50z \rightarrow 22.0 = 1.00(w + x + z) + 3.50z \end{array}\right\}$$

$$\rightarrow \left\{\begin{array}{l} x = 2.50w + 0.500 \\ 20.0 = 5.00x + 1.00w + 1.00z \\ 22.0 = 1.00w + 1.00x + 4.50z \end{array}\right.$$

Eliminate x:

$$\left.\begin{array}{l} 20.0 = 5.00(2.50w + 0.500) + 1.00w + 1.00z \\ 22.0 = 1.00w + 1.00(2.50w + 0.500) + 4.50z \end{array}\right\}$$

$$\rightarrow \left\{\begin{array}{l} 17.5 = 13.5w + 1.00z \\ 21.5 = 3.50w + 4.50z \end{array}\right.$$

Eliminate $z = 17.5 - 13.5w$ to obtain

$$21.5 = 3.50w + 4.50(17.5 - 13.5w)$$
$$21.5 = 3.50w + 4.50(17.5) - 4.50(13.5w)$$
$$\rightarrow 57.25 = 57.25w \rightarrow w = 1.00$$

(a) $w = \boxed{1.00 \text{ A upward in the 200-}\Omega\text{ resistor}}$

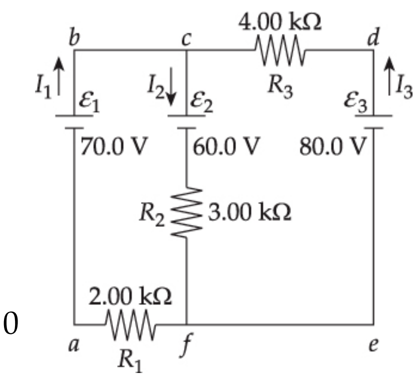
$$z = 17.5 - 13.5w = 17.5 - 13.5(1.00)$$
$$= \boxed{4.00 \text{ A upward in the } 70.0\text{-}\Omega \text{ resisor}}$$

$$x = 2.50w + 0.500 = 2.50(1.00) + 0.500$$
$$= \boxed{3.00 \text{ A upward in the } 80.0\text{-}\Omega \text{ resistor}}$$

$$y = w + x + z = 1.00 + 3.00 + 4.00$$
$$= \boxed{8.00 \text{ A downward in the } 20.0\text{-}\Omega \text{ resistor}}$$

(b) For the 200-Ω resistor, $\Delta V = IR = (1.00 \text{ A})(200\ \Omega) = \boxed{200 \text{ V}}$

***P28.31** (a) We name the currents I_1, I_2, and I_3 as shown in ANS. FIG. P28.31. Applying Kirchhoff's loop rule to loop *abcfa* gives



ANS. FIG. P28.31

$$+\mathcal{E}_1 - \mathcal{E}_2 - R_2 I_2 - R_1 I_1 = 0$$

or,

$$70.0 \text{ V} - 60.0 \text{ V} - (3.00 \text{ k}\Omega) I_2 - (2.00 \text{ k}\Omega) I_1 = 0$$

which gives

$$3I_2 + 2I_1 = 10.0 \text{ mA}$$

or $\quad I_1 = 5.00 \text{ mA} - 1.50 I_2$ **[1]**

Applying the loop rule to loop *edcfe* yields

$$+\mathcal{E}_3 - R_3 I_3 - \mathcal{E}_2 - R_2 I_2 = 0$$

which gives

$$80.0 \text{ V} - (4.00 \text{ k}\Omega) I_3 - 60.0 \text{ V} - (3.00 \text{ k}\Omega) I_2 = 0$$

or $\quad 3I_2 + 4I_3 = 20.0 \text{ mA}$

and $\quad I_3 = 5.00 \text{ mA} - 0.750 I_2$ **[2]**

Finally, applying Kirchhoff's junction rule at junction *c* gives

$I_2 = I_1 + I_3$ **[3]**

Substituting equations [1] and [2] into [3] yields

$$I_2 = 5.00 \text{ mA} - 1.50 I_2 + 5.00 \text{ mA} - 0.750 I_2$$

or $\quad 3.25 I_2 = 10.0 \text{ mA}$

This gives $I_2 = \frac{10.0 \text{ mA}}{3.25} = \boxed{3.08 \text{ mA}}$. Then, equation [1] yields

$$I_1 = 5.00 \text{ mA} - 1.50 I_2 = 5.00 \text{ mA} - 1.50(3.08 \text{ mA}) = \boxed{0.385 \text{ mA}}$$

and from equation [2],

$$I_3 = 5.00 \text{ mA} - 0.750 I_2 = 5.00 \text{ mA} - 0.750(3.08 \text{ mA})$$
$$= \boxed{2.69 \text{ mA}}$$

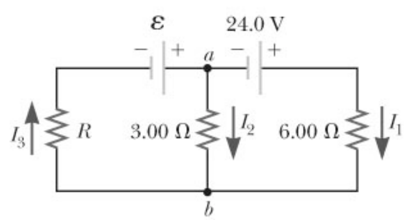
(b) Start at point *c* and go to point *f*, recording changes in potential to obtain

$$V_f - V_c = -\mathcal{E}_2 - R_2 I_2$$
$$= -60.0 \text{ V} - \left(3.00 \times 10^3 \ \Omega\right)\left(3.08 \times 10^{-3} \text{ A}\right) = -69.2 \text{ V}$$

or $|\Delta V|_{cf} = \boxed{69.2 \text{ V and point c is at the higher potential}}$.

P28.32 Following the path of I_1 from *a* to *b*, and recording changes in potential gives

$$V_b - V_a = +24.0 \text{ V} - (6.00 \ \Omega)(3.00 \text{ A}) = +6.00 \text{ V}$$



ANS. FIG. P28.32

Now, following the path of I_2 from *a* to *b*, and recording changes in potential gives

$$V_b - V_a = -(3.00 \ \Omega) I_2 = +6.00 \text{ V} \quad \rightarrow \quad I_2 = -2.00 \text{ A}$$

which means the initial choice of the direction of I_2 in Figure P28.32 was incorrect. Applying Kirchhoff's junction rule at point *a* gives

$$I_3 = I_1 + I_2 = 3.00 \text{ A} + (-2.00 \text{ A}) = 1.00 \text{ A}$$

The results are:

(a) $\boxed{I_2 \text{ is directed from } b \text{ toward } a \text{ and has a magnitude of 2.00 A.}}$

(b) $\boxed{I_3 = 1.00 \text{ A}}$ and flows in the direction shown in Figure P28.32.

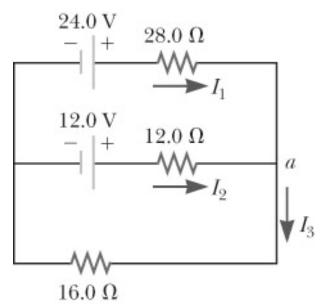
(c) No. Neither of the equations used to find I_2 and I_3 contained $\mathcal{E}$ and R. The third equation that we could generate from Kirchhoff's rules contains both the unknowns. Therefore, we have only one equation with two unknowns.

P28.33 (a) Applying Kirchhoff's junction rule at point *a* gives

$$I_3 = I_1 + I_2 \qquad \textbf{[1]}$$

Using the loop rule on the lower loop yields

$$+12.0 - 12.0I_2 - 16.0I_3 = 0 \quad \text{or} \quad I_2 = 1.00 - \frac{4.00I_3}{3.00} \qquad \textbf{[2]}$$



ANS. FIG. P28.33

Applying the loop rule to loop forming the outer perimeter of the circuit gives

$$+24.0 - 28.0I_1 - 16.0I_3 = 0 \quad \text{or} \quad I_1 = \frac{24.0 - 16.0I_3}{28.0} \qquad \textbf{[3]}$$

Substituting equations [2] and [3] into [1] yields

$$I_3 = \frac{24.0 - 16.0I_3}{28.0} + 1.00 - \frac{4.00I_3}{3.00}$$

and multiplying by 84 to eliminate fractions:

$$84.0I_3 = 72.0 - 48.0I_3 + 84.0 - 112I_3$$

$$244I_3 = 156$$

$$\boxed{I_3 = 0.639\ \text{A}}$$

Then, equation [2] gives $\boxed{I_2 = 0.148\ \text{A}}$ and equation [3] yields $\boxed{I_1 = 0.492\ \text{A}}$.

(b) The power delivered to each of the resistors in this circuit is:

$$P_{28.0\ \Omega} = I_1^2 R_{28.0\ \Omega} = (0.492\ \text{A})^2 (28.0\ \Omega) = \boxed{6.77\ \text{W}}$$

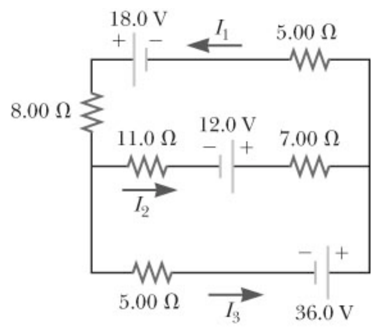
$$P_{12.0\ \Omega} = I_2^2 R_{12.0\ \Omega} = (0.148\ \text{A})^2 (12.0\ \Omega) = \boxed{0.261\ \text{W}}$$

$$P_{16.0\ \Omega} = I_3^2 R_{16.0\ \Omega} = (0.639\ \text{A})^2 (16.0\ \Omega) = \boxed{6.54\ W}$$

P28.34 (a) Going counterclockwise around the upper loop and suppressing units, Kirchhoff's loop rule gives

$$-11.0I_2 + 12.0 - 7.00I_2 - 5.00I_1 + 18.0 - 8.00I_1 = 0$$

or $\boxed{13.0I_1 + 18.0I_2 = 30.0}$. **[1]**



ANS. FIG. P28.34

(b) Going counterclockwise around the lower loop:

$$-5.00I_3 + 36.0 + 7.00I_2 - 12.0 + 11.0I_2 = 0$$

or $\boxed{18.0I_2 - 5.00I_3 = -24.0}$. **[2]**

(c) Applying the junction rule at the node in the left end of the circuit gives $\boxed{I_1 - I_2 - I_3 = 0}$ **[3]**

(d) Solving equation [3] for I_3 yields $\boxed{I_3 = I_1 - I_2}$ **[4]**

(e) Substituting equation [4] into [2] gives

$$5.00(I_1 - I_2) - 18.0I_2 = 24.0$$

or $\boxed{5.00I_1 - 23.0I_2 = 24.0}$. **[5]**

(f) Solving equation [5] for I_1 yields $I_1 = (24.0 + 23.0I_2)/5$. Substituting this into equation [1] gives

$$13.0I_1 + 18.0I_2 = 30.0$$

$$13.0\frac{(24.0 + 23.0I_2)}{5.00} + 18.0I_2 = 30.0$$

$$13.0(24.0 + 23.0I_2) + 5.00(18.0I_2) = 5.00(30.0)$$

$$389I_2 = -162 \quad \rightarrow \quad I_2 = -162/389 \quad \rightarrow \quad \boxed{I_2 = -0.416\ \text{A}}$$

Then, from equation [2], $I_1 = (30 - 18I_2)/13$ which yields

$$\boxed{I_1 = 2.88\ \text{A}}$$

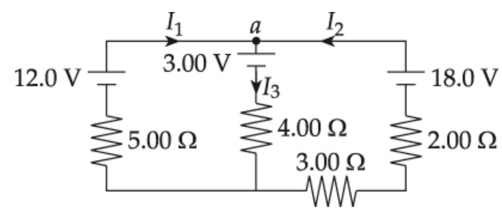
(g) Equation [4] gives

$$I_3 = I_1 - I_2 = 2.88\ \text{A} - (-0.416\ \text{A}) \quad \rightarrow \quad \boxed{I_3 = 3.30\ \text{A}}$$

(h) The negative sign in the answer for I_2 means that this current flows in the opposite direction to that shown in the circuit diagram and assumed during the solution. That is, the actual current in the middle branch of the circuit flows from right to left and has a magnitude of 0.416 A.

***P28.35** Refer to ANS. FIG. P28.35. Applying Kirchhoff's junction rule at junction *a* gives

$$I_3 = I_1 + I_2 \qquad \textbf{[1]}$$



ANS. FIG. P28.35

Using Kirchhoff's loop rule on the leftmost loop yields

$$-3.00\ \text{V} - (4.00\ \Omega)I_3 - (5.00\ \Omega)I_1 + 12.0\ \text{V} = 0$$

so $$I_1 = \frac{9.00\ \text{A} - 4.00I_3}{5.00} = 1.80\ \text{A} - 0.800I_3 \qquad \textbf{[2]}$$

For the rightmost loop,

$$-3.00\ \text{V} - (4.00\ \Omega)I_3 - (3.00\ \Omega + 2.00\ \Omega)I_2 + 18.0\ \text{V} = 0$$

and $$I_2 = \frac{15.0\ \text{A} - 4.00I_3}{5.00} = 3.00\ \text{A} - 0.800I_3 \qquad \textbf{[3]}$$

Substituting equations [2] and [3] into [1] and simplifying gives $2.60I_3 = 4.80$ A and $I_3 = 1.846$ A. Then equations [2] and [3] yield $I_1 = 0.323$ A and $I_2 = 1.523$ A.

Therefore, the potential differences across the resistors are

$$\Delta V_2 = I_2(2.00\ \Omega) = (1.523\ \text{A})(2.00\ \Omega) = \boxed{3.05\ \text{V}}$$

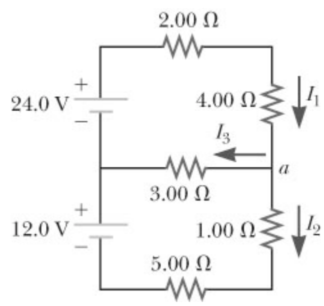
$$\Delta V_3 = I_2(3.00\ \Omega) = (1.523\ \text{A})(3.00\ \Omega) = \boxed{4.57\ \text{V}}$$

$$\Delta V_4 = I_3(4.00\ \Omega) = (1.846\ \text{A})(4.00\ \Omega) = \boxed{7.38\ \text{V}}$$

$$\Delta V_5 = I_1(5.00\ \Omega) = (0.323\ \text{A})(5.00\ \Omega) = \boxed{1.62\ \text{V}}$$

P28.36 (a) [No.] Some simplification could be made by recognizing that the 2.0 Ω and 4.0 Ω resistors are in series, adding to give a total of 6.0 Ω; and the 5.0 Ω and 1.0 Ω resistors form a series combination with a total resistance of 6.0 Ω.

[The circuit cannot be simplified any further, and Kirchhoff's rules must be used to analyze it.]



ANS. FIG. P28.36

(b) Applying Kirchhoff's junction rule at junction *a* gives

$$I_1 = I_2 + I_3 \qquad \textbf{[1]}$$

Using Kirchhoff's loop rule on the upper loop yields

$$+24.0\text{ V} - (2.00 + 4.0)I_1 - (3.00)I_3 = 0$$

or $$I_3 = 8.00\text{ A} - 2.00I_1 \qquad \textbf{[2]}$$

and for the lower loop,

$$+12.0\text{ V} + (3.00)I_3 - (1.00 + 5.00)I_2 = 0$$

Using equation [2], this reduces to

$$I_2 = \frac{12.0\text{ V} + 3.00(8.00\text{ A} - 2.00I_1)}{6.00}$$

giving

$$I_2 = 6.00\text{ A} - 1.00I_1 \qquad \textbf{[3]}$$

Substituting equations [2] and [3] into [1] gives $\boxed{I_1 = 3.50\text{ A}}$

(c) Then, equation [3] gives $\boxed{I_2 = 2.50\text{ A}}$, and

(d) Equation [2] yields $\boxed{I_3 = 1.00\text{ A}}$

Section 28.4 *RC* Circuits

P28.37 (a) The time constant of the circuit is

$$\tau = RC = (100\ \Omega)\left(20.0\times10^{-6}\ \text{F}\right) = 2.00\times10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$$

(b) The maximum charge on the capacitor is given by Equation 28.13:

$$Q_{\text{max}} = C\mathcal{E} = \left(20.0\times10^{-6}\ \text{F}\right)(9.00\ \text{V}) = \boxed{1.80\times10^{-4}\ \text{C}}$$

(c) We use $q(t) = Q_{\text{max}}\left(1-e^{-t/RC}\right)$, when $t = RC$. Then,

$$q(t) = Q_{\text{max}}\left(1-e^{-RC/RC}\right) = Q_{\text{max}}\left(1-e^{-1}\right) = \left(1.80\times10^{-4}\ \text{C}\right)\left(1-e^{-1}\right)$$
$$= \boxed{1.14\times10^{-4}\ \text{C}}$$

P28.38 (a) The time constant is

$$RC = \left(1.00\times10^{6}\ \Omega\right)\left(5.00\times10^{-6}\ \text{F}\right) = \boxed{5.00\ \text{s}}$$

(b) After a long time interval, the capacitor is "charged to thirty volts," separating charges of

$$Q = C\mathcal{E} = \left(5.00\times10^{-6}\ \text{C}\right)(30.0\ \text{V}) = \boxed{150\ \mu\text{C}}$$

(c) $$I(t) = \frac{\mathcal{E}}{R}e^{-t/RC} = \left(\frac{30.0\ \text{V}}{1.00\times10^{6}\ \Omega}\right)\exp\left[\frac{-10.0\ \text{s}}{\left(1.00\times10^{6}\ \Omega\right)\left(5.00\times10^{-6}\ \text{F}\right)}\right]$$
$$= \boxed{4.06\ \mu\text{A}}$$

P28.39 (a) From $I(t) = -I_0 e^{-t/RC}$,

$$I_0 = \frac{Q}{RC} = \frac{5.10\times10^{-6}\ \text{C}}{(1\,300\ \Omega)\left(2.00\times10^{-9}\ \text{F}\right)} = 1.96\ \text{A}$$

$$I(t) = -(1.96\ \text{A})\exp\left[\frac{-9.00\times10^{-6}\ \text{s}}{(1\,300\ \Omega)\left(2.00\times10^{-9}\ \text{F}\right)}\right] = \boxed{-61.6\ \text{mA}}$$

(b) $$q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\frac{-8.00\times10^{-6}\ \text{s}}{(1\,300\ \Omega)\left(2.00\times10^{-9}\ \text{F}\right)}\right]$$
$$= \boxed{0.235\ \mu\text{C}}$$

(c) The magnitude of the maximum current is $I_0 = \boxed{1.96\ \text{A}}$.

P28.40 The potential difference across the capacitor is

$$\Delta V(t) = \Delta V_{\text{max}}\left(1-e^{-t/RC}\right)$$

Using 1 farad = 1 s/Ω,

$$4.00\text{ V} = (10.0\text{ V})\left[1 - e^{-(3.00\text{ s})/\left[R(10.0\times10^{-6}\text{ s}/\Omega)\right]}\right]$$

Therefore,

$$0.400 = 1.00 - e^{-(3.00\times10^{5}\,\Omega)/R}$$

or $$e^{-(3.00\times10^{5}\,\Omega)/R} = 0.600.$$

Taking the natural logarithm of both sides,

$$-\frac{3.00\times10^{5}\ \Omega}{R} = \ln(0.600)$$

and $$R = -\frac{3.00\times10^{5}\ \Omega}{\ln(0.600)} = +5.87\times10^{5}\ \Omega = \boxed{587\ \text{k}\Omega}.$$

P28.41 (a) Before the switch is closed, the two resistors are in series. The time constant is

$$\tau = (R_1 + R_2)C = (1.50\times10^{5}\ \Omega)(10.0\times10^{-6}\ \text{F}) = \boxed{1.50\text{ s}}$$

(b) After the switch is closed, the capacitor discharges through resistor R_2. The time constant is

$$\tau = (1.00\times10^{5}\ \Omega)(10.0\times10^{-6}\ \text{F}) = \boxed{1.00\text{ s}}$$

(c) Before the switch is closed, there is no current in the circuit because the capacitor is fully charged, and the voltage across the capacitor is $\mathcal{E}$. After the switch is closed, current flows clockwise from the battery to resistor R_1 and down through the switch, and current from the capacitor flows counterclockwise and down through the switch to resistor R_2; the result is that the total current through the switch is $I_1 + I_2$.

Going clockwise around the left loop,

$$\mathcal{E} - I_1R_1 = 0 \rightarrow I_1 = \frac{\mathcal{E}}{R_1}$$

so the battery carries current $$I_1 = \frac{10.0\text{ V}}{50.0\times10^{3}\ \Omega} = 200\ \mu\text{A}.$$

Going counterclockwise around the right loop,

$$\frac{q}{C} - I_2R_2 = 0 \rightarrow I_2 = \frac{q}{R_2C} = \frac{\mathcal{E}}{R_2}e^{-t/(R_2C)}$$

so the 100-kΩ resistor carries current of magnitude

$$I_2 = \frac{\mathcal{E}}{R_2}e^{-t/RC} = \left(\frac{10.0 \text{ V}}{100\times 10^3 \ \Omega}\right)e^{-t/1.00 \text{ s}}$$

and the switch carries downward current

$$I_1 + I_2 = \boxed{200 \ \mu\text{A} + (100 \ \mu\text{A})e^{-t/1.00 \text{ s}}}$$

P28.42 (a) Before the switch is closed, the two resistors are in series. The time constant is

$$\tau = \boxed{(R_1 + R_2)C}$$

(b) After the switch is closed, the capacitor discharges through resistor R_2. The time constant is

$$\tau = \boxed{R_2 C}$$

(c) Before the switch is closed, there is no current in the circuit because the capacitor is fully charged, and the voltage across the capacitor is $\mathcal{E}$. After the switch is closed, current flows clockwise from the battery to resistor R_1 and down through the switch, and current from the capacitor flows counterclockwise and down through the switch to resistor R_2; the result is that the total current through the switch is $I_1 + I_2$. Going clockwise around the left loop,

$$\mathcal{E} - I_1 R_1 = 0 \rightarrow I_1 = \frac{\mathcal{E}}{R_1} \text{ is the current in the battery.}$$

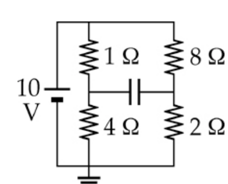
Going counterclockwise around the right loop,

$$\frac{q}{C} - I_2 R_2 = 0 \rightarrow I_2 = \frac{q}{R_2 C} = \frac{\mathcal{E}}{R_2}e^{-t/(R_2 C)}$$

is the magnitude of the current in R_2. The total current through the switch is

$$I_1 + I_2 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2}e^{-t/(R_2 C)} = \boxed{\mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}e^{-t/(R_2 C)}\right)}$$

P28.43 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.



ANS. FIG. P28.43(a)

$$I_L = \frac{10.0 \text{ V}}{5.00 \ \Omega} = 2.00 \text{ A}$$

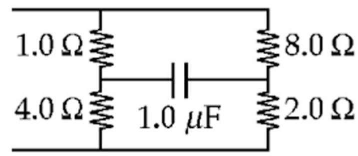
$$V_L = 10.0 \text{ V} - (2.00 \text{ A})(1.00 \ \Omega) = 8.00 \text{ V}$$

$$I_R = \frac{10.0 \text{ V}}{10.0 \ \Omega} = 1.00 \text{ A}$$

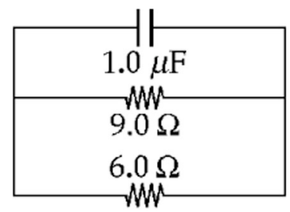
$$V_R = (10.0 \text{ V}) - (8.00 \ \Omega)(1.00 \text{ A}) = 2.00 \text{ V}$$

Therefore, $\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00 \text{ V}}$

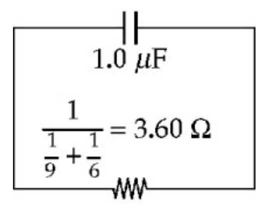
(b) We suppose the battery is pulled out leaving an open circuit. We are left with ANS. FIG. P28.43(b), which can be reduced to the equivalent circuits shown in ANS. FIG. P28.43(c) and ANS. FIG. P28.43(d). From ANS. FIG. P28.43(d), we can see that the capacitor discharges through a 3.60-Ω equivalent resistance.



ANS. FIG. P28.43(b)



ANS. FIG. P28.43(c)



ANS. FIG. P28.43(d)

According to $q = Qe^{-t/RC}$,

we calculate that $qC = QCe^{-t/RC}$

and $\Delta V = \Delta V_i e^{-t/RC}$.

We proceed to solve for t:

$$\frac{\Delta V}{\Delta V_i} = e^{-t/RC} \quad \text{or} \quad \frac{\Delta V_i}{\Delta V} = e^{+t/RC}$$

Take natural logarithms of both sides:

$$\ln\left(\frac{\Delta V_i}{\Delta V}\right) = t/RC$$

so $$t = RC\ln\left(\frac{\Delta V_i}{\Delta V}\right)$$

$$= (3.60 \ \Omega)(1.00 \times 10^{-6} \text{ F})\ln\left(\frac{\Delta V_i}{0.100\Delta V_i}\right) = (3.60 \times 10^{-6} \text{ s})\ln 10$$

$$= \boxed{8.29 \ \mu\text{s}}$$

P28.44 We are to calculate

$$\int_0^\infty e^{-2t/RC}\,dt = -\frac{RC}{2}\int_0^\infty e^{-2t/RC}\left(-\frac{2dt}{RC}\right)$$

$$= -\frac{RC}{2}e^{-2t/RC}\Big|_0^\infty = -\frac{RC}{2}\left[e^{-\infty} - e^0\right]$$

$$= -\frac{RC}{2}[0-1] = \boxed{+\frac{RC}{2}}$$

P28.45 (a) The charge remaining on the capacitor after time t is $q = Qe^{-t/\tau}$.

Thus, if $q = 0.750Q$, then

$$0.750Q = Qe^{-t/\tau}$$
$$e^{-t/\tau} = 0.750$$
$$t = -\tau\ln(0.750) = -(1.50\text{ s})\ln(0.750) = \boxed{0.432\text{ s}}$$

(b) $\tau = RC$, so

$$C = \frac{\tau}{R} = \frac{1.50\text{ s}}{250\times10^{3}\ \Omega} = 6.00\times10^{-6}\text{ F} = \boxed{6.00\ \mu\text{F}}$$

Section 28.5 Household Wiring and Electrical Safety

P28.46 (a) $P = I\Delta V$: So for the heater, $I = \dfrac{P}{\Delta V} = \dfrac{1\,500\text{ W}}{120\text{ V}} = \boxed{12.5\text{ A}}$.

For the toaster, $I = \dfrac{750\text{ W}}{120\text{ V}} = \boxed{6.25\text{ A}}$.

And for the grill, $I = \dfrac{1\,000\text{ W}}{120\text{ V}} = \boxed{8.33\text{ A}}$.

(b) The total current drawn is 12.5 A + 6.25 A + 8.33 A = 27.1 A.

The current draw is greater than 25.0 amps, so this circuit will trip the circuit breaker.

***P28.47** From $P = (\Delta V)^2/R$, the resistance of the element is

$$R = \frac{(\Delta V)^2}{P} = \frac{(240\text{ V})^2}{3\ 000\text{ W}} = 19.2\ \Omega$$

When the element is connected to a 120-V source, we find that

(a) $I = \dfrac{\Delta V}{R} = \dfrac{120\text{ V}}{19.2\ \Omega} = \boxed{6.25\text{ A}}$

(b) $P = I\Delta V = (6.25\text{ A})(120\text{ V}) = \boxed{750\text{ W}}$

P28.48 (a) Suppose that the insulation between either of your fingers and the conductor adjacent is a chunk of rubber with contact area 4 mm^2 and thickness 1 mm. Its resistance is

$$R = \frac{\rho\ell}{A} \approx \frac{(10^{13}\ \Omega\cdot\text{m})(10^{-3}\text{ m})}{4\times10^{-6}\text{ m}^2} \approx 2\times10^{15}\ \Omega$$

The current will be driven by 120 V through total resistance (series)

$$2\times10^{15}\ \Omega+10^{4}\ \Omega+2\times10^{15}\ \Omega\approx5\times10^{15}\ \Omega$$

It is: $I=\dfrac{\Delta V}{R}\sim\dfrac{120\text{ V}}{5\times10^{15}\ \Omega}\boxed{\sim10^{-14}\text{ A}}$

(b) The resistors form a voltage divider, with the center of your hand at potential $\dfrac{V_h}{2}$, where V_h is the potential of the "hot" wire. The potential difference between your finger and thumb is

$$\Delta V=IR\sim\left(10^{-14}\text{ A}\right)\left(10^{4}\ \Omega\right)\sim10^{-10}\text{ V}$$

So the points where the rubber meets your fingers are at potentials of

$$\boxed{\sim\frac{V_h}{2}+10^{-10}\text{ V}}\quad\text{and}\quad\boxed{\sim\frac{V_h}{2}-10^{-10}\text{ V}}$$

Additional Problems

P28.49 (a) With the lightbulbs in series, the equivalent resistance is $R_{eq}=3R$, and the current is given by $I=\dfrac{\mathcal{E}}{3R}$. Then,

$$P_{\text{series}}=\mathcal{E}I=\boxed{\frac{\mathcal{E}^2}{3R}}$$

(b) With the lightbulbs in parallel, the equivalent resistance is

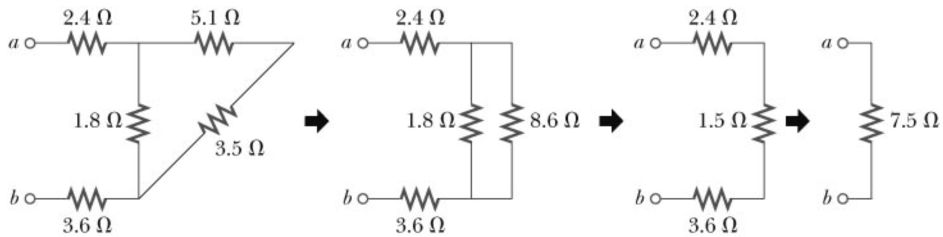
$$R_{eq}=\frac{1}{(1/R)+(1/R)+(1/R)}=\frac{R}{3}$$

the current is given by $I=\dfrac{3\mathcal{E}}{R}$. Then,

$$P_{\text{parallel}}=\mathcal{E}I=\boxed{\frac{3\mathcal{E}^2}{R}}$$

(c) Nine times more power is converted in the $\boxed{\text{parallel}}$ connection.

P28.50 The resistive network between *a* an *b* reduces, in the stages shown in ANS. FIG. P28.50, to an equivalent resistance of $R_{eq} = \boxed{7.49\ \Omega}$.



ANS. FIG. P28.50

P28.51 The set of four batteries boosts the electric potential of each bit of charge that goes through them by 4 × 1.50 V = 6.00 V. The chemical energy they store is

$$\Delta U = q\Delta V = (240\ \text{C})(6.00\ \text{J/C}) = 1\ 440\ \text{J}$$

The radio draws current

$$I = \frac{\Delta V}{R} = \frac{6.00\ \text{V}}{200\ \Omega} = 0.030\ 0\ \text{A}.$$

So, its power is

$$P = I\Delta V = (0.030\ 0\ \text{A})(6.00\ \text{V}) = 0.180\ \text{J/s}$$

Then for the time the energy lasts, we have $P = \dfrac{E}{\Delta t}$:

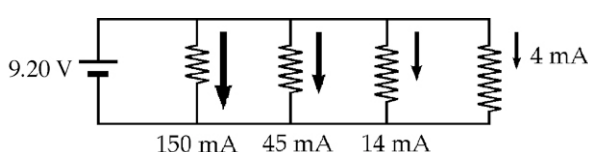
$$\Delta t = \frac{E}{P} = \frac{1\ 440\ \text{J}}{0.180\ \text{J/s}} = 8.00 \times 10^3\ \text{s}$$

We could also compute this from $I = \dfrac{Q}{\Delta t}$:

$$\Delta t = \frac{Q}{I} = \frac{240\ \text{C}}{0.030\ 0\ \text{A}} = 8.00 \times 10^3\ \text{s} = \boxed{2.22\ \text{h}}$$

P28.52 The battery current is

$$(150 + 45.0 + 14.0 + 4.00)\ \text{mA} = 213\ \text{mA}$$



ANS. FIG. P28.52

(a) The resistor with highest resistance is that carrying 4.00 mA. Doubling its resistance will reduce the current it carries to

2.00 mA. Then the total current is (150 + 45 + 14 + 2) mA = 211 mA, nearly the same as before. The ratio is $\frac{211}{213} = \boxed{0.991}$.

(b) The resistor with least resistance carries 150 mA. Doubling its resistance changes this current to 75 mA and changes the total to (75 + 45 + 14 + 4) mA = 138 mA. The ratio is $\frac{138}{213} = \boxed{0.648}$.

(c) The energy flows are precisely analogous to the currents in parts (a) and (b). The ceiling has the smallest R value of the thermal resistors in parallel, so increasing its thermal resistance will produce the biggest reduction in the total energy flow.

P28.53 Several seconds is many time constants, so the capacitor is fully charged and (d) the current in its branch is $\boxed{\text{zero.}}$

For the center loop, Kirchhoff's loop rule gives

$$+8 + (3\ \Omega) I_2 - (5\ \Omega) I_1 = 0$$

or $$I_1 = 1.6 + 0.6 I_2 \qquad \textbf{[1]}$$

For the right-hand loop, Kirchhoff's loop rule gives

$$+4\ \text{V} - (3\ \Omega) I_2 - (5\ \Omega) I_3 = 0$$

or $$I_3 = 0.8 - 0.6 I_2 \qquad \textbf{[2]}$$

For the top junction, Kirchhoff's junction rule gives

$$+ I_1 + I_2 - I_3 = 0 \qquad \textbf{[3]}$$

Now we eliminate I_1 and I_3 by substituting [1] and [2] into [3]. Suppressing units,

$$1.6 + 0.6 I_2 + I_2 - 0.8 + 0.6 I_2 = 0 \rightarrow I_2 = -0.8/2.2 = -0.3636$$

(b) $\boxed{\text{The current in } 3\ \Omega \text{ is } 0.364\text{ A down.}}$

(a) Now, from [2], we find $I_3 = 0.8 - 0.6(-0.364) = \boxed{1.02\text{ A down in } 4\text{ V and in } 5\ \Omega.}$

(c) From [1] we have $I_1 = 1.6 + 0.6(-0.364) = \boxed{1.38\text{ A up in the } 8\text{ V battery}}$.

(e) For the left loop $+3\text{ V} - (Q/6\ \mu\text{F}) + 8\text{ V} = 0$, so $Q = (6\ \mu\text{F})(11\text{ V}) = \boxed{66.0\ \mu\text{C}}$

P28.54 The current in the battery is $\dfrac{15\text{ V}}{10\ \Omega+\dfrac{1}{\frac{1}{5\ \Omega}+\frac{1}{8\ \Omega}}}=1.15\text{ A}.$

The voltage across 5 Ω is 15 V – (10 Ω) (1.15 A) = 3.53 V.

(a) The current in it is 3.53 V/5 Ω = $\boxed{0.706\text{ A.}}$

(b) P = (3.53 V) (0.706 A) = $\boxed{2.49\text{ W.}}$

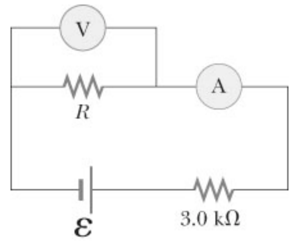
(c) Only the circuit in Figure P28.54(c) requires the use of Kirchhoff's rules for solution. In the other circuits the 5-Ω and 8-Ω resistors are still in parallel with each other.

(d) The power is lowest in Figure P28.54(c). The circuits in Figures P28.54(b) and P28.54(d) have in effect 30-V batteries driving the current. The power is lowest in Figure P28.54(c) because the current in the 10-W resistor is lowest because the battery voltage driving the current is lowest.

P28.55 (a) $R=\dfrac{\Delta V}{I}=\dfrac{6.00\text{ V}}{3.00\times10^{-3}\text{ A}}=2.00\times10^{3}\ \Omega=\boxed{2.00\text{ k}\Omega}$

(b) The resistance in the circuit consists of a series combination with an equivalent resistance of R_{eq} = 5.00 Ω. The emf of the battery is then

$$\varepsilon=IR_{\text{eq}}=\left(3.00\times10^{-3}\text{ A}\right)\left(5.00\times10^{3}\ \Omega\right)=\boxed{15.0\text{ V}}$$



ANS. FIG. P28.55

(c) $\Delta V_3=IR_3=\left(3.00\times10^{-3}\text{ A}\right)\left(3.00\times10^{3}\ \Omega\right)=\boxed{9.00\text{ V}}$

P28.56 The equivalent resistance is $R_{eq} = R + R_p$, where R_p is the total resistance of the three parallel branches;

$$R_p = \left(\frac{1}{120\ \Omega} + \frac{1}{40.0\ \Omega} + \frac{1}{R + 5.00\ \Omega}\right)^{-1}$$

$$= \left(\frac{1}{30.0\ \Omega} + \frac{1}{R + 5.00\ \Omega}\right)^{-1}$$

$$= \frac{(30.0\ \Omega)(R + 5.00\ \Omega)}{R + 35.0\ \Omega}$$

Thus,

$$75.0\ \Omega = R + \frac{(30.0\ \Omega)(R + 5.00\ \Omega)}{R + 35.0\ \Omega} = \frac{R^2 + (65.0\ \Omega)R + 150\ \Omega^2}{R + 35.0\ \Omega}$$

which reduces to

$$R^2 - (10.0\ \Omega)R - 2\,475\ \Omega^2 = 0$$

or $$(R - 55\ \Omega)(R + 45\ \Omega) = 0$$

Only the positive solution is physically acceptable, so $R = \boxed{55.0\ \Omega}$.

P28.57 (a) Using Kirchhoff's loop rule for the closed loop,

$$+12.0 - 2.00I - 4.00I = 0$$

so $I = 2.00$ A

Then,

$$V_b - V_a = +4.00\text{ V} - (2.00\text{ A})(4.00\ \Omega) - (0)(10.0\ \Omega) = -4.00\text{ V}$$

Thus, $|\Delta V_{ab}| = \boxed{4.00\text{ V}}$

(b) $V_b - V_a = -4.00\text{ V} \rightarrow V_a = V_b + 4.00\text{ V}$; thus,

$\boxed{a\text{ is at the higher potential}}$.

P28.58 Find an expression for the power delivered to the load resistance R:

$$P = I^2R = \left(\frac{\mathcal{E}}{r + R}\right)^2 R \quad \rightarrow \quad (r + R)^2 = \frac{\mathcal{E}^2}{P}R = aR$$

where $$a = \frac{\mathcal{E}^2}{P}$$

Carry out the squaring process:

$$r^2 + 2rR + R^2 = aR$$

$$R^2 + (2r - a)R + r^2 = 0$$

$$R^2 + bR + r^2 = 0$$

where $b = 2r - a = 2r - \dfrac{\mathcal{E}^2}{P}$.

Solve the quadratic equation:

$$R = \frac{-b \pm \sqrt{b^2 - 4r^2}}{2}$$

Evaluate b:

$$b = 2(1.20\ \Omega) - \frac{(9.20\ \text{V})^2}{21.2\ \text{W}} = -1.59\ \Omega$$

Substitute numerical values into the expression for R:

$$R = \frac{-(-1.59\ \Omega) \pm \sqrt{(-1.59\ \Omega)^2 - 4(1.20\ \Omega)^2}}{2}$$
$$= \frac{1.59\ \Omega \pm \sqrt{-3.22\ \Omega^2}}{2}$$

There is no real solution to this expression for R. Therefore, no load resistor can extract 21.2 W from this battery.

P28.59 The charging circuit is shown in the left-hand panel of ANS. FIG. P28.59. Kirchhoff's loop rule gives

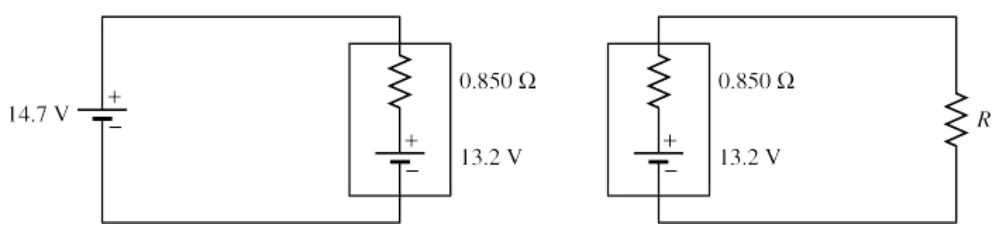
$$+14.7\ \text{V} - 13.2\ \text{V} - I\,(0.850\ \Omega) = 0$$

so the charging current is

$$I = 1.5\ \text{V}/0.850\ \Omega = 1.76\ \text{A}.$$

The charge passing through the battery as it charges is

$$q = I\Delta t = (1.76\ \text{A})(1.80\ \text{h}) = 3.18\ \text{A}\cdot\text{h} = 11.4\ \text{kC}$$



ANS. FIG. P28.59

We can think of this charge as indexing a certain number of chemical reactions, producing a certain quantity of high-energy molecules in the battery. When the battery returns to its original state in discharging, we assume that the same number of reverse reactions uses up all of the high-energy chemical. In our model, the same charge passes through the battery in discharging, in the opposite direction.

The discharge current is then

$$I = \frac{q}{\Delta t} = \frac{3.18\ \text{A}\cdot\text{h}}{7.30\ \text{h}} = 0.435\ \text{A}$$

In the discharge circuit , shown in the right-hand panel of ANS. FIG. P28.59, the loop rule gives

$$13.2\ \text{V} - (0.435\ \text{A})(0.850\ \Omega) - (0.435\ \text{A})R = 0$$

so the load resistance R is 12.8 V/0.435 A = 29.5 Ω. Now we can get around to thinking about energy. The energy output of the 14.7-V power supply is

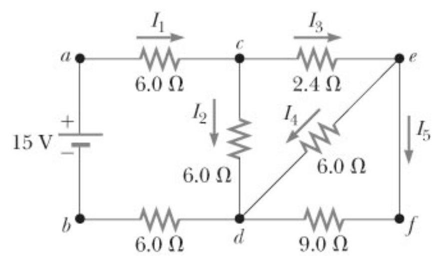
$$q\Delta V = (3.18\ \text{A}\cdot\text{h})(14.7\ \text{V}) = 46.7\ \text{W}\cdot\text{h} = 168\ \text{kJ}$$

The energy delivered to the load during discharge is

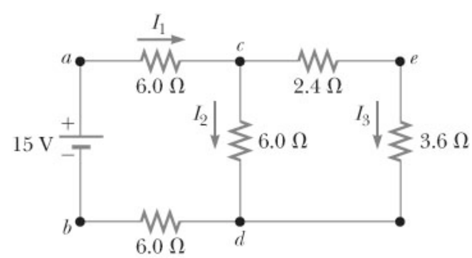
$$q\Delta V = qIR = (3.18\ \text{A}\cdot\text{h})(0.435\ \text{A})(29.5\ \Omega) = 40.8\ \text{W}\cdot\text{h} = 147\ \text{kJ}$$

The storage efficiency is $\dfrac{40.8\ \text{W}\cdot\text{h}}{46.7\ \text{W}\cdot\text{h}} = 0.873 = \boxed{87.3\%}$.

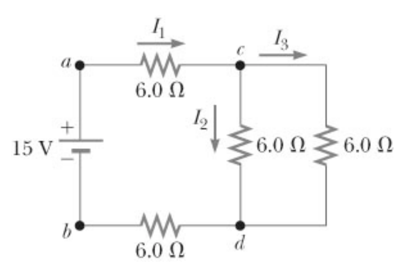
P28.60 (a) The resistors combine to an equivalent resistance of $R_{eq} = \boxed{15.0\ \Omega}$ as shown in ANS. FIGs P28.60(a-e).



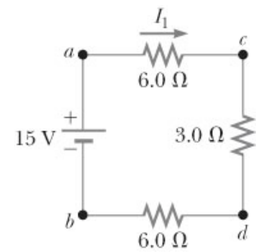
ANS. FIG. P28.60(a)



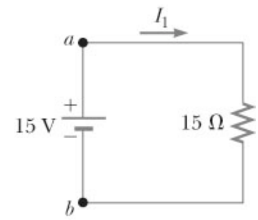
ANS. FIG. P28.60(b)



ANS. FIG. P28.60(c)



ANS. FIG. P28.60(d)



ANS. FIG. P28.60(e)

From ANS. FIG. P28.60(e),

$$I_1 = \frac{\Delta V_{ab}}{R_{eq}} = \frac{15.0\text{ V}}{15.0\ \Omega} = 1.00\text{ A}$$

Then, from ANS. FIG. P28.60(d),

$$\Delta V_{ac} = \Delta V_{db} = I_1(6.00\ \Omega) = 6.00\text{ V}$$

and $\Delta V_{cd} = I_1(3.00\ \Omega) = 3.00\text{ V}$

From ANS. FIG. P28.60(c),

$$I_2 = I_3 = \frac{\Delta V_{cd}}{6.00\ \Omega} = \frac{3.00\text{ V}}{6.00\ \Omega} = 0.500\text{ A}$$

From ANS. FIG. P28.60(b),

$$\Delta V_{ed} = I_3(3.60\ \Omega) = 1.80\text{ V}$$

Then, from ANS. FIG. P28.60(a),

$$I_4 = \frac{\Delta V_{ed}}{6.00\ \Omega} = \frac{1.80\text{ V}}{6.00\ \Omega} = 0.300\text{ A}$$

and $I_5 = \dfrac{\Delta V_{fd}}{9.00\ \Omega} = \dfrac{\Delta V_{ed}}{9.00\ \Omega} = \dfrac{1.80\text{ V}}{9.00\ \Omega} = 0.200\text{ A}$

From ANS. FIG. P28.60(b),

$$\Delta V_{ce} = I_3(2.40\ \Omega) = 1.20\text{ V}.$$

The collected results are:

(b) $\boxed{\Delta V_{ac} = \Delta V_{db} = 6.00\text{ V},\ \Delta V_{ce} = 1.20\text{ V},\ \Delta V_{fd} = \Delta V_{ed} = 1.80\text{ V},\ \Delta V_{cd} = 3.00\text{ V}}$

(c) $\boxed{I_1 = 1.00\text{ A}, I_2 = 0.500\text{ A}, I_3 = 0.500\text{ A}, I_4 = 0.300\text{ A}, I_5 = 0.200\text{ A}}$

(d) The power dissipated in each resistor is found from $P = (\Delta V)^2/R$ with the following results:

$$P_{ac} = \frac{(\Delta V)_{ac}^2}{R_{ac}} = \frac{(6.00\text{ V})^2}{6.00\ \Omega} = \boxed{6.00\text{ W}}$$

$$P_{ce} = \frac{(\Delta V)_{ce}^2}{R_{ce}} = \frac{(1.20\text{ V})^2}{2.40\ \Omega} = \boxed{0.600\text{ W}}$$

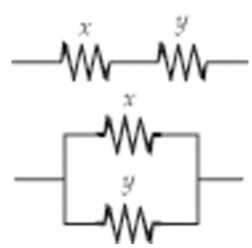
$$P_{ed} = \frac{(\Delta V)_{ed}^2}{R_{ed}} = \frac{(1.80\text{ V})^2}{6.00\ \Omega} = \boxed{0.540\text{ W}}$$

$$P_{fd} = \frac{(\Delta V)_{fd}^2}{R_{fd}} = \frac{(1.80\text{ V})^2}{9.00\ \Omega} = \boxed{0.360\text{ W}}$$

$$P_{cd} = \frac{(\Delta V)_{cd}^2}{R_{cd}} = \frac{(3.00\text{ V})^2}{6.00\ \Omega} = \boxed{1.50\text{ W}}$$

$$P_{db} = \frac{(\Delta V)_{db}^2}{R_{db}} = \frac{(6.00\text{ V})^2}{6.00\ \Omega} = \boxed{6.00\text{ W}}$$

P28.61 Let the two resistances be x and y.



ANS. FIG. P28.61

Then,

$$R_s = x + y = \frac{P_s}{I^2} = \frac{225\text{ W}}{(5.00\text{ A})^2} = 9.00\ \Omega \rightarrow y = 9.00\ \Omega - x$$

and $$R_p = \frac{xy}{x+y} = \frac{P_p}{I^2} = \frac{50.0\text{ W}}{(5.00\text{ A})^2} = 2.00\ \Omega$$

so $$\frac{x(9.00\ \Omega - x)}{x + (9.00\ \Omega - x)} = 2.00\ \Omega$$

$$x^2 - 9.00x + 18.0 = 0$$

Factoring the second equation,

$$(x-6.00)(x-3.00)=0$$

so $x = 6.00\ \Omega$ or $x = 3.00\ \Omega$

Then, $y = 9.00\ \Omega - x$ gives

$$y = 3.00\ \Omega \quad \text{or} \quad y = 6.00\ \Omega$$

There is only one physical answer: The two resistances are $\boxed{6.00\ \Omega}$ and $\boxed{3.00\ \Omega}$.

P28.62 Refer to ANS. FIG. P28.61 above. Let the two resistances be x and y.

Then, $R_s = x + y = \dfrac{P_s}{I^2}$ and $R_p = \dfrac{xy}{x+y} = \dfrac{P_p}{I^2}$.

From the first equation, $y = \dfrac{P_s}{I^2} - x$, and the second

becomes $\dfrac{x\left(P_s/I^2 - x\right)}{x + \left(P_s/I^2 - x\right)} = \dfrac{P_p}{I^2}$ or $x^2 - \left(\dfrac{P_s}{I^2}\right)x + \dfrac{P_s P_p}{I^4} = 0$.

Using the quadratic formula, $x = \dfrac{P_s \pm \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$.

Then, $y = \dfrac{P_s}{I^2} - x$ gives $y = \dfrac{P_s \mp \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$.

The two resistances are $\boxed{\dfrac{P_s + \sqrt{P_s^2 - 4P_sP_p}}{2I^2}}$ and $\boxed{\dfrac{P_s - \sqrt{P_s^2 - 4P_sP_p}}{2I^2}}$.

P28.63 (a) The equivalent capacitance of this parallel combination is

$$C_{\text{eq}} = C_1 + C_2 = 3.00\ \mu\text{F} + 2.00\ \mu\text{F} = 5.00\ \mu\text{F}$$

When fully charged by a 12.0-V battery, the total stored charge before the switch is closed is

$$Q_0 = C_{\text{eq}}(\Delta V) = (5.00\ \mu\text{F})(12.0\ \text{V}) = 60.0\ \mu\text{C}$$

Once the switch is closed, the time constant of the resulting RC circuit is

$$\tau = RC_{\text{eq}} = \left(5.00 \times 10^2\ \Omega\right)(5.00\ \mu\text{F}) = 2.50 \times 10^{-3}\ \text{s} = 2.50\ \text{ms}$$

Thus, at $t = 1.00$ ms after closing the switch, the remaining total stored charge is

$$q = Q_0 e^{-t/\tau} = (60.0\ \mu\text{C})e^{-1.00\ \text{ms}/2.50\ \text{ms}} = (60.0\ \mu\text{C})e^{-0.400} = 40.2\ \mu\text{C}$$

The potential difference across the parallel combination of capacitors is then

$$\Delta V = \frac{q}{C_{eq}} = \frac{40.2\ \mu\text{C}}{5.00\ \mu\text{F}} = 8.04\ \text{V}$$

and the charge remaining on the 3.00 μF capacitor will be

$$q_3 = C_3(\Delta V) = (3.00\ \mu\text{F})(8.04\ \text{V}) = \boxed{24.1\ \mu\text{C}}$$

(b) The charge remaining on the 2.00 μF at this time is

$$q_2 = q - q_3 = 40.2\ \mu\text{C} - 24.1\ \mu\text{C} = \boxed{16.1\ \mu\text{C}}$$

or, alternately,

$$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(8.04\ \text{V}) = \boxed{16.1\ \mu\text{C}}$$

(c) Since the resistor is in parallel with the capacitors, it has the same potential difference across it as do the capacitors at all times. Thus, Ohm's law gives

$$I = \frac{\Delta V}{R} = \frac{8.04\ \text{V}}{5.00\times 10^{2}\ \Omega} = 1.61\times 10^{-2}\ \text{A} = \boxed{16.1\ \text{mA}}$$

P28.64 (a) Around the circuit,

$$\mathcal{E} - I\left(\sum R\right) - (\mathcal{E}_1 + \mathcal{E}_2) = 0$$

Substituting numerical values,

$$40.0\ \text{V} - (4.00\ \text{A})\left[(2.00 + 0.300 + 0.300 + R)\Omega\right] - (6.00 + 6.00)\ \text{V} = 0$$

so $R = \boxed{4.40\ \Omega}$

(b) Inside the supply,

$$P = I^2R = (4.00\ \text{A})^2(2.00\ \Omega) = \boxed{32.0\ \text{W}}$$

(c) Inside both batteries together,

$$P = I^2R = (4.00\ \text{A})^2(0.600\ \Omega) = \boxed{9.60\ \text{W}}$$

(d) For the limiting resistor,

$$P = (4.00\ \text{A})^2(4.40\ \Omega) = \boxed{70.4\ \text{W}}$$

(e) $P = I(\mathcal{E}_1 + \mathcal{E}_2) = (4.00\ \text{A})\left[(6.00 + 6.00)\ \text{V}\right] = \boxed{48.0\ \text{W}}$

P28.65 The total resistance in the circuit is

$$R=\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1}=\left(\frac{1}{2.00\text{ k}\Omega}+\frac{1}{3.00\text{ k}\Omega}\right)^{-1}=1.20\text{ k}\Omega$$

and the total capacitance is

$$C=C_1+C_2=2.00\ \mu\text{F}+3.00\ \mu\text{F}=5.00\ \mu\text{F}$$

Thus, $Q_{\max}=C\mathcal{E}=(5.0\ \mu\text{F})(120\text{ V})=600\ \mu\text{C}$

and $\tau=RC=\left(1.2\times10^{3}\ \Omega\right)\left(5.0\times10^{-6}\text{ F}\right)=6.0\times10^{-3}\text{ s}=\dfrac{6.0\text{ s}}{1\,000}$

The total stored charge at any time t is then

$$q=q_1+q_2=Q_{\max}\left(1-e^{-t/\tau}\right)$$

or $q_1+q_2=(600\ \mu\text{C})\left(1-e^{-1\,000t/6.0\text{ s}}\right)$ **[1]**

Since the capacitors are in parallel with each other, the same potential difference exists across both at any time.

Therefore,

$$(\Delta V)_C=\frac{q_1}{C_1}=\frac{q_2}{C_2}\quad\rightarrow\quad q_2=\left(\frac{C_2}{C_1}\right)q_1=1.5q_1$$ **[2]**

(a) Substituting equation [2] into [1] gives

$$2.5q_1=(600\ \mu\text{C})\left(1-e^{-1\,000t/6.0\text{ s}}\right)$$

$$q_1=\left(\frac{600\ \mu\text{C}}{2.5}\right)\left(1-e^{-t/(6.0\text{ s}/1\,000)}\right)$$

$$q_1=240\ \mu\text{C}\left(1-e^{-t/6\text{ ms}}\right)$$

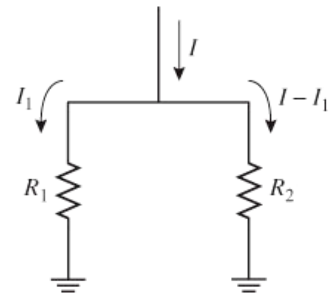
or $\boxed{q=240\left(1-e^{-t/6}\right)}$, $\boxed{\text{where } q \text{ is in microcoulombs and } t \text{ is in milliseconds}}$.

(b) and from equation [2],

$$q_2=1.5q_1=1.5\left[240\ \mu\text{C}\left(1-e^{-t/6\text{ ms}}\right)\right]=360\ \mu\text{C}\left(1-e^{-t/6\text{ ms}}\right)$$

or, $q=\boxed{360\left(1-e^{-t/6}\right)}$, $\boxed{\text{where } q \text{ is in microcoulombs and } t \text{ is in milliseconds}}$.

P28.66 (a) In the diagram we could show the two resistors connected top end to top end and bottom end to bottom end with wires; we represent this connection instead by showing the bottom ends of both resistors connected to ground. The ground represents a conductor that is always at zero volts, and also can carry any current. Think of I, R_1, and R_2 as known quantities. We represent the current in R_1 as the unknown I_1. Then the current in the second resistor must by $I - I_1$. The total potential difference clockwise around the little loop containing both resistors must be zero:



ANS. FIG. P28.66

$$-(I - I_1)R_2 + I_1R_1 + 0$$

We can already solve for I_1 in terms of the total current:

$$-IR_2 + I_1R_2 + I_1R_1 = 0 \quad \rightarrow \quad I_1 = \boxed{IR_2 / (R_1 + R_2)}$$

Then the current in the second resistor is

$$I_2 = I - I_1 = I - IR_2 / (R_1 + R_2) = I(R_1 + R_2 - R_2)/(R_1 + R_2)$$

$$I_2 = \boxed{IR_1 / (R_1 + R_2)}$$

(b) Continue to think of I, R_1, and R_2 as known quantities and I_1 as an unknown. The power being converted by both resistors together is $P = I_1^2R_1 + (I - I_1)^2R_2$. Because the current is squared, the power would be extra large if all of the current went through either one of the resistors with zero current in the other. The minimum power condition must be with a more equitable division of current, and we find it by taking the derivative of P with respect to I_1 and setting the derivative equal to zero:

$$dP/dI_1 = 2\,I_1R_1 + 2(I - I_1)(0 - 1)R_2 = 0$$

Again we can solve directly for the real value of I_1 in

$$I_1R_1 - IR_2 + I_1R_2 = 0 \qquad \text{as} \qquad I_1 = IR_2/(R_1 + R_2)$$

So then again

$$I_2 = I - I_1 = IR_1/(R_1 + R_2)$$

This power-minimizing division of current is the same as the voltage-equalizing division of current that we found in part (a), so the proof is complete.

P28.67 (a) The charge on the capacitor at this instant is

$$q = C\Delta V\left(1 - e^{-t/RC}\right)$$

$$q = \left(1.00\times 10^{-6}\ \text{F}\right)\left(10.0\ \text{V}\right)\left[1 - e^{-10.0\ \text{s}/\left[\left(2.00\times 10^{6}\ \Omega\right)\left(1.00\times 10^{-6}\ \text{F}\right)\right]}\right]$$

$$= \boxed{9.93\ \mu\text{C}}$$

(b) The current in the resistor is given by

$$I = \frac{dq}{dt} = \left(\frac{\Delta V}{R}\right)e^{-t/RC}$$

$$I = \left(\frac{10.0\ \text{V}}{2.00\times 10^{6}\ \Omega}\right)e^{-5.00} = 3.37\times 10^{-8}\ \text{A} = \boxed{33.7\ \text{nA}}$$

(c) Since the energy stored in the capacitor is $U = q^2/2C$, the rate of storing energy is

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right) = \left(\frac{q}{C}\right)\frac{dq}{dt} = \left(\frac{q}{C}\right)I$$

$$\frac{dU}{dt} = \left(\frac{9.93\times 10^{-6}\ \text{C}}{1.00\times 10^{-6}\ \text{C/V}}\right)\left(3.37\times 10^{-8}\ \text{A}\right)$$

$$= 3.34\times 10^{-7}\ \text{W} = \boxed{334\ \text{nW}}$$

(d) $P_{\text{battery}} = I\mathcal{E} = \left(3.37\times 10^{-8}\ \text{A}\right)\left(10.0\ \text{V}\right) = 3.37\times 10^{-7}\ \text{W} = \boxed{337\ \text{nW}}$

The battery power could also be computed as the sum of the instantaneous powers delivered to the resistor and to the capacitor:

$$I^2R + \frac{dU}{dt} = \left(33.7\times 10^{-9}\ \text{A}\right)\left(2.00\times 10^{6}\ \Omega\right) + 334\ \text{nW} = 337\ \text{nW}$$

P28.68 The battery supplies energy at a changing rate

$$\frac{dE}{dt} = P = \mathcal{E}I = \mathcal{E}\left(\frac{\mathcal{E}}{R}e^{-t/RC}\right)$$

Then the total energy put out by the battery is

$$\int dE = \int_{t=0}^{\infty}\frac{\mathcal{E}^2}{R}\exp\left(-\frac{t}{RC}\right)d$$

$$\int dE = \frac{\mathcal{E}^2}{R}(-RC)\int_0^{\infty}\exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right)$$

$$= -\mathcal{E}^2C\exp\left(-\frac{t}{RC}\right)\Bigg|_0^{\infty} = -\mathcal{E}^2C[0-1] = \mathcal{E}^2C$$

The power delivered to the resistor is

$$\frac{dE}{dt} = P = \Delta V_R I = I^2 R = R\frac{\mathcal{E}^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$$

So the total internal energy appearing in the resistor is

$$\int dE = \int_0^\infty \frac{\mathcal{E}^2}{R}\exp\left(-\frac{2t}{RC}\right)dt$$

$$\int dE = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\int_0^\infty \exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right)$$

$$= -\frac{\mathcal{E}^2 C}{2}\exp\left(-\frac{2t}{RC}\right)\Bigg|_0^\infty = -\frac{\mathcal{E}^2 C}{2}[0-1] = \frac{\mathcal{E}^2 C}{2}$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$.

Thus, energy of the circuit is conserved, $\mathcal{E}^2 C = \frac{1}{2}\mathcal{E}^2 C + \frac{1}{2}\mathcal{E}^2 C$, and resistor and capacitor share equally in the energy from the battery.

P28.69 (a) We find the resistance intrinsic to the vacuum cleaner:

$$P = I\Delta V = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\text{ V})^2}{535\text{ W}} = 26.9\ \Omega$$

with the inexpensive cord, the equivalent resistance is

$$R_{\text{Tot}} = R + 2r$$
$$= 26.9\ \Omega + 2(0.9\ \Omega) = 28.7\ \Omega$$

so the current throughout the circuit is

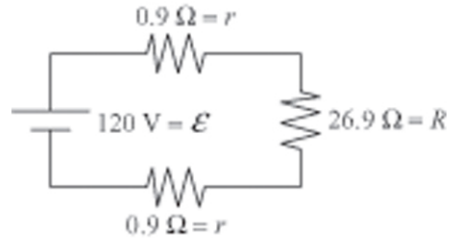
$$I = \frac{\Delta V}{R_{\text{Tot}}} = \frac{120\text{ V}}{28.7\ \Omega} = 4.18\text{ A}$$

and the cleaner power is

$$P_{\text{cleaner}} = I(\Delta V)_{\text{cleaner}} = I^2 R = (4.18\text{ A})^2(26.9\ \Omega) = \boxed{470\text{ W}}$$

In symbols,

$$R_{\text{tot}} = R + 2r,\ I = \frac{\Delta V}{R+2r} \text{ and } P_{\text{cleaner}} = I^2 R = \frac{(\Delta V)^2 R}{(R+2r)^2}$$



ANS. FIG. P28.69

(b) Using $P_{\text{cleaner}} = I^2 R = \dfrac{(\Delta V)^2 R}{(R+2r)^2}$, we find that

$$R + 2r = \left(\frac{(\Delta V)^2 R}{P_{\text{cleaner}}}\right)^{1/2}$$

solving for r gives

$$r = \frac{\Delta V}{2}\left(\frac{R}{P_{\text{cleaner}}}\right)^{1/2} - \frac{R}{2} = \frac{120\text{ V}}{2}\left(\frac{26.9\ \Omega}{525\text{ W}}\right)^{1/2} - \frac{26.9\ \Omega}{2}$$

$$= 0.128\ \Omega = \frac{\rho\ell}{A} = \frac{\rho\ell 4}{\pi d^2}$$

then,

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(15\text{ m})}{\pi(0.128\ \Omega)}\right)^{1/2}$$

$$= \boxed{1.60\text{ mm or more}}$$

(c) To move from 525 W to 532 W will require a lot more copper:

$$r = \frac{\Delta V}{2}\left(\frac{R}{P_{\text{cleaner}}}\right)^{1/2} - \frac{R}{2} = \frac{120\text{ V}}{2}\left(\frac{26.9\ \Omega}{532\text{ W}}\right)^{1/2} - \frac{26.9\ \Omega}{2}$$

$$= 0.037\,9\ \Omega$$

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(15\text{ m})}{\pi\left(0.037\,9\ \Omega\right)}\right)^{1/2}$$

$$= \boxed{2.93\text{ mm or more}}$$

P28.70 (a) When the capacitor is fully charged, no current exists in its branch. The current in the left resistors is $I_L = 5.00\text{ V}/83.0\Omega$. The current in the right resistors is $I_R = 5.00\text{ V}/(2.00\ \Omega + R)$.

Relative to the positive side of the battery, the left capacitor plate is at voltage

$$V_L = 5.00\text{ V} - (3.00\ \Omega)\left(\frac{5.00\text{ V}}{83.0\ \Omega}\right) = (5.00\text{ V})\left(1 - \frac{3.00}{83.0}\right)$$

and the right plate is at voltage

$$V_R = 5.00\text{ V} - \frac{(2.00\ \Omega)(5.00\text{ V})}{2.00\ \Omega + R} = (5.00\text{ V})\left(1 - \frac{2.00}{2.00 + R}\right)$$

where R is in ohms. The voltage across the capacitor is

$$\Delta V = V_L - V_R = (5.00\text{ V})\left(1 - \frac{3.00}{83.0}\right) - (5.00\text{ V})\left(1 - \frac{2.00}{2.00 + R}\right)$$

$$\Delta V = (5.00\text{ V})\left(\frac{2.00}{2.00 + R} - \frac{3.00}{83.0}\right)$$

The charge on the capacitor is

$$q = C\Delta V = (3.00\ \mu\text{C})(5.00\text{ V})\left(\frac{2.00}{2.00 + R} - \frac{3.00}{83.0}\right)$$

$$q = (15.0\ \mu C)\left(\frac{2.00}{2.00 + R} - \frac{3.00}{83.0}\right)$$

$$\boxed{q = \frac{30.0}{2.00 + R} - 0.542, \text{ where } q \text{ is in microcoulombs and } R \text{ is in ohms.}}$$

(b) With $R = 10.0\ \Omega$,

$$q = \frac{30.0}{2.00 + R} - 0.542 = \frac{30.0}{2.00 + 10.0} - 0.542 = \boxed{1.96\ \mu\text{C}}$$

(c) $\boxed{\text{Yes.}}$ Setting $q = 0$, and solving for R,

$$q = (15.0\ \mu C)\left[\frac{2.00}{2.00 + R} - \frac{3.00}{83.0}\right] = 0$$

$$R = \frac{2.00(83.0)}{3.00} - 2.00 = \boxed{53.3\ \Omega}$$

(d) By inspection, the maximum charge occurs for $R = 0$. It is

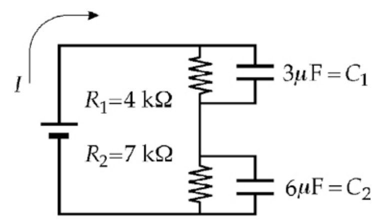
$$q = (15.0\ \mu C)\left[\frac{2.00}{2.00+0} - \frac{3.00}{83.0}\right] = \boxed{14.5\ \mu C}$$

(e) $\boxed{\text{Yes. Taking } R = \infty \text{ corresponds to disconnecting the wire}}$ to remove the branch containing R:

$$|q| = (15.0\ \mu C)\left|\frac{2.00}{2.00+\infty} - \frac{3.00}{83.0}\right| = (15.0\ \mu C)\frac{3.00}{83.0} = \boxed{0.542\ \mu C}$$

P28.71 (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have

$$P = I^2R_2 \quad \text{and} \quad I = \sqrt{\frac{P}{R_2}} = \sqrt{\frac{2.40\ \text{V}\cdot\text{A}}{7\ 000\ \text{V/A}}} = 18.5\ \text{mA}$$



ANS. FIG. P28.71(a)

The potential difference across R_1 and C_1 is

$$\Delta V = IR_1 = (1.85\times10^{-2}\ \text{A})(4\ 000\ \text{V/A}) = 74.1\ \text{V}$$

The charge on C_1 is

$$Q = C_1\Delta V = (3.00\times10^{-6}\ \text{C/V})(74.1\ \text{V}) = \boxed{222\ \mu\text{C}}$$

The potential difference across R_2 and C_2 is

$$\Delta V = IR_2 = (1.85\times10^{-2}\ \text{A})(7\ 000\ \Omega) = 130\ \text{V}$$

The charge on C_2

$$Q = C_2\Delta V = (6.00\times10^{-6}\ \text{C/V})(130\ \text{V}) = 778\ \mu\text{C}$$

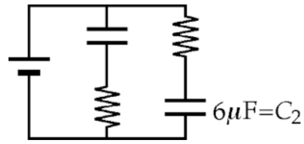
The battery emf is

$$IR_{eq} = I(R_1+R_2) = (1.85\times10^{-2}\ \text{A})(4\ 000\ \Omega + 7\ 000\ \Omega) = 204\ \text{V}$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge on C_2 is

$$Q = C_2\Delta V = (6.00\times10^{-6}\ \text{C/V})(204\ \text{V}) = 1\ 222\ \mu\text{C}$$

for a change of $1\,222\ \mu\text{C} - 778\ \mu\text{C} = \boxed{444\ \mu\text{C}}$.



ANS. FIG. P28.71(b)

P28.72 (a) First determine the resistance of each light bulb. From $P = \dfrac{(\Delta V)^2}{R}$, we have

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\ \text{V})^2}{60.0\ \text{W}} = 240\ \Omega$$

We obtain the equivalent resistance R_{eq} of the network of light bulbs by identifying series and parallel equivalent resistances:

$$R_{\text{eq}} = R_1 + \frac{1}{(1/R_2) + (1/R_3)} = 240\ \Omega + 120\ \Omega = 360\ \Omega$$

The total power dissipated in the 360 Ω is

$$P = \frac{(\Delta V)^2}{R_{\text{eq}}} = \frac{(120\ \text{V})^2}{360\ \Omega} = \boxed{40.0\ \text{W}}$$

(b) The current through the network is given by $\Delta V = IR_{\text{eq}}$:

$$I = \frac{120\ \text{V}}{360\ \Omega} = \frac{1}{3}\ \text{A}$$

The potential difference across R_1 is

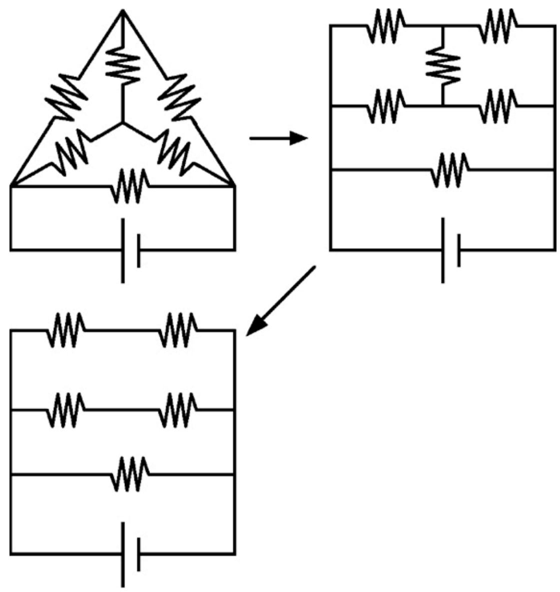
$$\Delta V_1 = IR_1 = \left(\frac{1}{3}\ \text{A}\right)(240\ \Omega) = \boxed{80.0\ \text{V}}$$

The potential difference ΔV_{23} across the parallel combination of R_2 and R_3 is

$$\Delta V_{23} = IR_{23} = \left(\frac{1}{3}\ \text{A}\right)\left(\frac{1}{(1/240\ \Omega) + (1/240\ \Omega)}\right) = \boxed{40.0\ \text{V}}$$

P28.73 (a) First let us flatten the circuit on a 2-D plane as shown in ANS. FIG. P28.73; then reorganize it to a format easier to read. Notice that the two resistors shown in the top horizontal branch carry the same current as the resistors in the horizontal branch second from the top. The center junctions in these two branches are at the same potential. The vertical resistor between these two junctions has no potential difference across it and carries no current. This middle resistor can be removed without affecting the circuit. The remaining resistors over the three parallel branches have equivalent resistance

$$R_{\text{eq}} = \left(\frac{1}{20} + \frac{1}{20} + \frac{1}{10} \right)^{-1} = \boxed{5.00\ \Omega}$$



ANS. FIG. P28.73

(b) So the current through the battery is

$$\frac{\Delta V}{R_{\text{eq}}} = \frac{12.0\text{ V}}{5.00\ \Omega} = \boxed{2.40\text{ A}}$$

P28.74 (a) The emf of the battery is $\boxed{9.30\text{ V}}$.

(b) Its internal resistance is given by

$$\Delta V = 9.30\text{ V} - (3.70\text{ A})r = 0 \quad \rightarrow \quad r = \boxed{2.51\ \Omega}$$

(c) The batteries are in series: Total emf = 2(9.30 V) = $\boxed{18.6\text{ V}}$.

(d) The batteries are in series, so their total internal resistance is $2r = 5.03\ \Omega$. The maximum current is given by

$$I = \frac{\Delta V}{R} = \frac{18.6\text{ V}}{5.03\ \Omega} = \boxed{3.70\text{ A}}$$

(e) For the circuit the total series resistance is

$$R_{\text{eq}} = 2r + 12.0\ \Omega = 17.0\ \Omega$$

and

$$I = \frac{\Delta V}{R} = \frac{18.6 \text{ V}}{17.0 \ \Omega} = \boxed{1.09 \text{ A}}$$

(f) $P = I^2 R = (1.09 \text{ A})^2 (12.0 \ \Omega) = \boxed{14.3 \text{ W}}$

(g) The two 12.0-Ω resistors in parallel are equivalent to one 6.00-Ω, Resistor, and this is in series with the internal resistances of the batteries: R_{eq} = 6.00 Ω + 2r = 11.0 Ω. Therefore, the current in the batteries is

$$I = \frac{\Delta V}{R} = \frac{18.6 \text{ V}}{11.0 \ \Omega} = 1.69 \text{ A}$$

and the terminal voltage across both batteries is

$$\Delta V = \mathcal{E} - I(2r) = 18.6 \text{ V} - (1.69 \text{ A})(5.03 \ \Omega) = 10.1 \text{ V}$$

The power delivered to each resistor is

$$P = \frac{(\Delta V)^2}{R} = \frac{(10.1 \text{ V})^2}{12.0 \ \Omega} = \boxed{8.54 \text{ W}}$$

(h) Because of the internal resistance of the batteries, the terminal voltage of the pair of batteries is not the same in both cases.

P28.75 (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for R_3 : $\boxed{I_{R_3} = 0 \ (\text{steady-state})}$

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For R_1 and R_2:

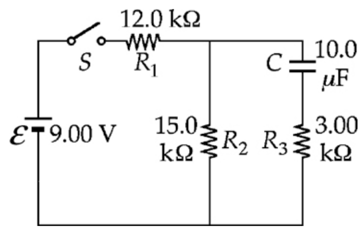
$$I_{(R_1+R_2)} = \frac{\mathcal{E}}{R_1 + R_2} = \frac{9.00 \text{ V}}{(12.0 \text{ k}\Omega + 15.0 \text{ k}\Omega)}$$

$$= \boxed{333 \ \mu\text{A} \ (\text{steady-state})}$$

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0 \ \mu\text{F})(333 \ \mu\text{A})(15.0 \text{ k}\Omega)$$

$$= \boxed{50.0 \ \mu\text{C}}$$



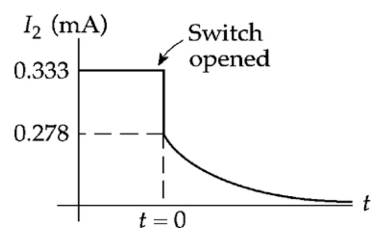
ANS. FIG. P28.75(b)

(c) When the switch is opened, the branch containing R_1 is no longer part of the circuit. The capacitor discharges through $(R_2 + R_3)$ with a time constant of

$$(R_2 + R_3)C = (15.0\text{ k}\Omega + 3.00\text{ k}\Omega)(10.0\ \mu\text{F}) = 0.180\text{ s}$$

The initial current I_i in this discharge circuit is determined by the initial potential difference across the capacitor applied to $(R_2 + R_3)$ in series:

$$I_i = \frac{(\Delta V)_C}{(R_2+R_3)} = \frac{IR_2}{(R_2+R_3)} = \frac{(333\ \mu\text{A})(15.0\text{ k}\Omega)}{(15.0\text{ k}\Omega + 3.00\text{ k}\Omega)} = 278\ \mu\text{A}$$



ANS. FIG. P28.75(c)

Thus, when the switch is opened, the current through R_2 changes instantaneously from 333 μA (downward) to 278 μA (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2} = I_i e^{-t/(R_2+R_3)C} = \boxed{(278\ \mu\text{A})e^{-t/(0.180\text{ s})}\quad (\text{for } t > 0)}$$

(d) The charge q on the capacitor decays from Q_i to $\frac{Q_i}{5}$ according to

$$q = Q_i e^{-t/(R_2+R_3)C}$$

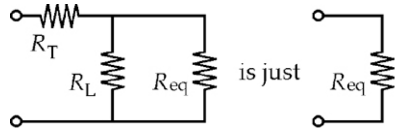
$$\frac{Q_i}{5} = Q_i e^{(-t/0.180\text{ s})}$$

$$5 = e^{t/0.180\text{ s}}$$

$$\ln 5 = \frac{t}{180\text{ ms}}$$

$$t = (0.180\text{ s})(\ln 5) = \boxed{290\text{ ms}}$$

P28.76 From the hint, the equivalent resistance of



That is, $$R_T + \frac{1}{1/R_L + 1/R_{eq}} = R_{eq}$$

$$R_T + \frac{R_L R_{eq}}{R_L + R_{eq}} = R_{eq}$$

$$R_T R_L + R_T R_{eq} + R_L R_{eq} = R_L R_{eq} + R_{eq}^2$$

$$R_{eq}^2 - R_T R_{eq} - R_T R_L = 0$$

$$R_{eq} = \frac{R_T \pm \sqrt{R_T^2 - 4(1)(-R_T R_L)}}{2(1)}$$

Only the + sign is physical:

$$R_{eq} = \frac{1}{2}\left(\sqrt{4R_T R_L + R_T^2} + R_T\right)$$

For example, if $R_T = 1\ \Omega$ and $R_L = 20\ \Omega$, then $R_{eq} = 5\ \Omega$.

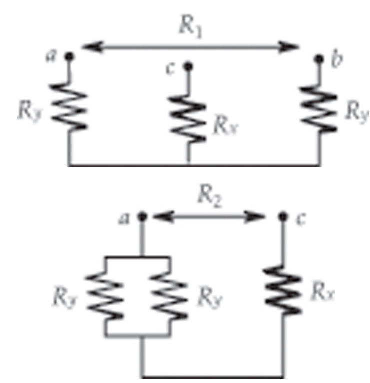
P28.77 (a) For the first measurement, the equivalent circuit is as shown in the top panel of ANS. FIG. P28.77.

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

so $$R_y = \frac{1}{2}R_1. \qquad \textbf{[1]}$$

For the second measurement, the equivalent circuit is shown in the bottom panel of ANS. FIG. P28.77. Thus,

$$R_{ac} = R_2 = \frac{1}{2}R_y + R_x \qquad \textbf{[2]}$$



ANS. FIG. P28.77

Substitute [1] into [2] to obtain:

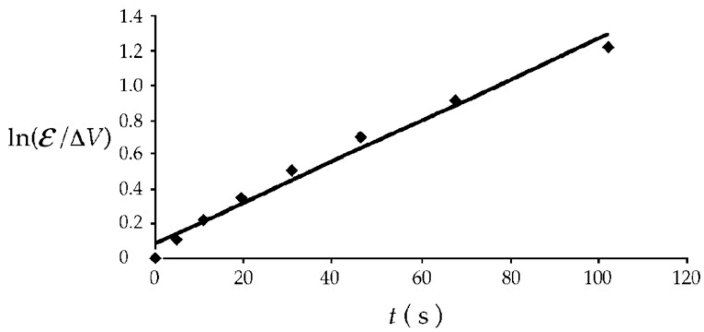
$$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x, \quad \text{or} \quad \boxed{R_x = R_2 - \frac{1}{4}R_1}$$

(b) If $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$, then $\boxed{R_x = 2.75\ \Omega}$.

[The antenna is inadequately grounded] since this exceeds the limit of $2.00\ \Omega$.

P28.78 $\Delta V = \mathcal{E} e^{-t/RC}$ so $\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = \left(\frac{1}{RC}\right)t.$

A plot of $\ln\left(\frac{\mathcal{E}}{\Delta V}\right)$ versus t should be a straight line with slope equal to $\frac{1}{RC}$, as shown in ANS. FIG. P28.78.



ANS. FIG. P28.78

Using the given data values:

(a) A least-square fit to this data yields the graph shown in ANS. FIG. P28.78.

$$\sum x_i = 282, \quad \sum x_i^2 = 1.86 \times 10^4,$$

$$\sum x_i y_i = 244, \ \sum y_i = 4.03, \quad N = 8$$

$$\text{Slope} = \frac{N\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.011\,8$$

t (s)	ΔV (V)	$\ln(\mathcal{E}/\Delta V)$
0	6.19	0
4.87	5.55	0.109
11.1	4.93	0.228
19.4	4.34	0.355
30.8	3.72	0.509
46.6	3.09	0.695
67.3	2.47	0.919
102.2	1.83	1.219

$$\text{Intercept} = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.088\,2$$

The equation of the best fit line is: $\boxed{\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right) = (0.011\,8)t + 0.088\,2}$

(b) Thus, the time constant is

$$\tau = RC = \frac{1}{\text{slope}} = \frac{1}{0.011\,8} = \boxed{84.7\ \text{s}}$$

and the capacitance is

$$C = \frac{\tau}{R} = \frac{84.7\ \text{s}}{10.0 \times 10^6\ \Omega} = \boxed{8.47\ \mu\text{F}}$$

P28.79 A certain quantity of energy $\Delta E_{\text{int}} = P\Delta t$ is required to raise the temperature of the water to 100°C in time interval Δt. For the power delivered to the heaters we have $P = I\Delta V = \dfrac{(\Delta V)^2}{R}$ where ΔV is a constant. Thus, comparing coils 1 and 2, we have for the energy $\Delta E_{\text{int}} = \dfrac{(\Delta V)^2 \Delta t}{R_1} = \dfrac{(\Delta V)^2 2\Delta t}{R_2}$. Therefore, $R_2 = 2R_1$.

(a) When connected in parallel, the coils present equivalent resistance

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_1} + \frac{1}{2R_1} = \frac{3}{2R_1} \quad \rightarrow \quad R_p = \frac{2}{3}R_1.$$

Now,

$$\Delta E_{\text{int}} = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 \Delta t_p}{\frac{2}{3}R_1} \quad \rightarrow \quad \Delta t_p = \boxed{\frac{2}{3}\Delta t}$$

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1 = 3R_1$ and

$$\Delta E_{\text{int}} = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 \Delta t_s}{3R_1} \quad \rightarrow \quad \Delta t_s = \boxed{3\Delta t}$$

P28.80 When connected in series, the equivalent resistance is $R_{\text{eq}} = R_1 + R_2 + \cdots + R_n = nR$. Thus, the current is $I_s = (\Delta V)/R_{\text{eq}}$, and the power consumed by the series configuration is

$$P_s = I_s \Delta V = \frac{(\Delta V)^2}{R_{\text{eq}}} = \frac{(\Delta V)^2}{nR}$$

For the parallel connection, the power consumed by each individual resistor is $P_1 = \frac{(\Delta V)^2}{R}$, and the total power consumption is

$$P_p = nP_1 = \frac{n(\Delta V)^2}{R}$$

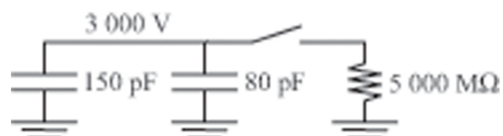
Therefore, $\frac{P_s}{P_p} = \frac{(\Delta V)^2}{nR} \cdot \frac{R}{n(\Delta V)^2} = \frac{1}{n^2}$ or $\boxed{P_s = \frac{1}{n^2} P_p}$

P28.81 We model the person's body and street shoes as shown in ANS. FIG. P28.81. For the discharge to reach 100 V,

$$q(t) = Qe^{-t/RC} = C\Delta V(t) = C\Delta V_0 e^{-t/RC}$$

$$\frac{\Delta V}{\Delta V_0} = e^{-t/RC} \quad \rightarrow \quad \frac{\Delta V_0}{\Delta V} = e^{+t/RC}$$

$$\rightarrow \quad \frac{t}{RC} = \ln\left(\frac{\Delta V_0}{\Delta V}\right)$$



ANS. FIG. P28.81

The equivalent capacitance for parallel capacitors is

150 pF + 80.0 pF = 230 pF.

(a) For $R = 5.00$ MΩ, a change from 3 000 V to 100 V requires that

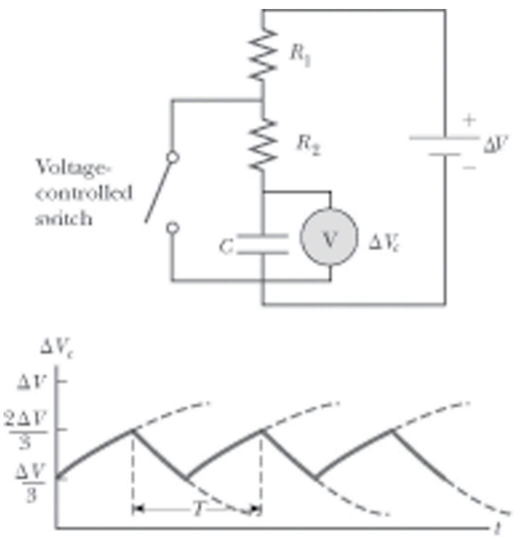
$$t = RC\ln\left(\frac{\Delta V_0}{\Delta V}\right) = \left(5\ 000 \times 10^6\ \Omega\right)\left(230 \times 10^{-12}\ \text{F}\right)\ln\left(\frac{3\ 000\ \text{V}}{100\ \text{V}}\right)$$
$$= \boxed{3.91\ \text{s}}$$

(b) For $R = 1.00$ MΩ, the same change requires that

$$t = RC\ln\left(\frac{\Delta V_0}{\Delta V}\right) = \left(1.00 \times 10^6\ \Omega\right)\left(230 \times 10^{-12}\ \text{F}\right)\ln\left(\frac{3\ 000\ \text{V}}{100\ \text{V}}\right)$$
$$= 7.82 \times 10^{-4}\ \text{s} = \boxed{782\ \mu\text{s}}$$

Challenge Problems

P28.82 Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant R_2C.



ANS. FIG. P28.82

$$\Delta V_C(t) = \left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$$

We want to know when $\Delta V_C(t)$ will reach $\frac{1}{3}\Delta V$.

Therefore, $\frac{1}{3}\Delta V = \left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$

$$e^{-t/R_2C} = \frac{1}{2}$$

$$t_1 = R_2C\ln 2$$

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_1 + R_2)C$:

$$\Delta V_C(t) = \Delta V - \left[\frac{2}{3}\Delta V\right]e^{-t/(R_1+R_2)C}$$

When $\Delta V_C(t) = \frac{2}{3}\Delta V$,

$$\frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-t/(R_1+R_2)C} \quad \text{or} \quad e^{-t/(R_1+R_2)C} = \frac{1}{2}$$

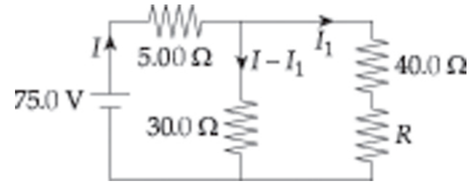
so $t_2 = (R_1 + R_2)C\ln 2$ and $T = t_1 + t_2 = \boxed{(R_1 + 2R_2)C\ln 2}$

P28.83 Assume a set of currents as shown in the circuit diagram in ANS. FIG. P28.83. Applying Kirchhoff's loop rule to the leftmost loop and suppressing units gives

$$+75.0 - (5.00)I - (30.0)(I - I_1) = 0$$

$$75.0 - 35.0I + 30.0I_1 = 0$$

or $7\,I - 6\,I_1 = 15$ **[1]**



ANS. FIG. P28.83

For the rightmost loop, the loop rule gives, suppressing units,

$$-(40.0+R)I_1+(30.0)(I-I_1)=0$$
$$-(70.0+R)I_1+30.0I=0$$

or $$I=\left(\frac{7}{3}+\frac{R}{30}\right)I_1 \qquad \textbf{[2]}$$

Substituting equation [2] into [1] and simplifying gives

$$(310 + 7R)\,I_1 = 450 \qquad \textbf{[3]}$$

Also, it is known that $P_R = I_1^2 R = 20.0\text{ W}$,

so $$R=\frac{20.0\text{ W}}{I_1^2} \qquad \textbf{[4]}$$

Substituting equation [4] into [3] yields

$$310I_1+\frac{140}{I_1}=450$$

or $$310I_1^2-450I_1+140=0$$

Using the quadratic formula,

$$I_1=\frac{-(-450)\pm\sqrt{(-450)^2-4(310)(140)}}{2(310)}=\frac{450\pm 170}{620}$$

yielding $I_1 = 1.00$ A and $I_1 = 0.452$ A. Then, from $R=\dfrac{20.0\text{ W}}{I_1^2}$, we find two possible values for the resistance R. These are:

$$\boxed{R = 20.0\ \Omega \quad \text{or} \quad R = 98.1\ \Omega}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P28.2 (a) 4.59 Ω; (b) 8.16%

P28.4 (a) 50%; (b) 0; (c) High efficiency; (d) High power transfer

P28.6 (a) The 120-V potential difference is applied across the series combination of the two conductors in the extension cord and the lightbulb. The potential difference across the lightbulb is less than 120 V, and its power is less than 75 W; (b) See ANS. FIG. P28.6; (c) 73.8 W

P28.8 (a) $I_A = \mathcal{E}/R,\ I_B = I_C = \mathcal{E}/2R;$ (b) B and C have the same brightness because they carry the same current; (c) A is brighter than B or C because it carries twice as much current.

P28.10 (a) Connect two 50-Ω resistors in parallel to get 25 Ω. Then connect that parallel combination in series with a 20 Ω for a total resistance of 45 Ω; (b) Connect two 50-Ω resistors in parallel to get 2 Ω. Also, connect two 20 Ω resistors in parallel to get 10 Ω. Then connect these two combinations in a series with each other to obtain 35 Ω.

P28.12 (a) $R_1 = \mathcal{E}\left(-\dfrac{2}{I_0} + \dfrac{2}{I_a} + \dfrac{1}{I_b}\right)$; (b) $R_2 = 2\mathcal{E}\left(\dfrac{1}{I_0} - \dfrac{1}{I_a}\right)$; (c) $R_3 = \mathcal{E}\left(\dfrac{1}{I_0} - \dfrac{1}{I_b}\right)$

P28.14 (a) decreases; (b) 14 Ω

P28.16 (a) $\Delta V_1 = \dfrac{\mathcal{E}}{3},\ \Delta V_2 = \dfrac{2\mathcal{E}}{9},\ \Delta V_3 = \dfrac{4\mathcal{E}}{9},\ \Delta V_4 = \dfrac{2\mathcal{E}}{3};$

(b) $I_1 = I,\ I_2 = I_3 = \dfrac{I}{3},\ I_4 = \dfrac{2I}{3}$; (c) I_4 increases and I_1, I_2, and I_3 decrease;

(d) $I_1 = \dfrac{3I}{4},\ I_2 = I_3 = 0,\ I_4 = \dfrac{3I}{4}$

P28.18 (a) See P28.18(a) for the full solution; (b) The current never exceeds 50 μA.

P28.20 None of these is $\dfrac{4}{3}R,$ so the desired resistance cannot be achieved.

P28.22 (a) See ANS. FIG. P28.22

P28.24 (a) I_3 = 909 mA; (b) –1.82 V

P28.26 (a) See ANS. FIG. P28.26; (b) 11.0 mA in the 220-Ω resistor and out of the positive pole of the 5.80-V battery; The current is 1.87 mA in the 150-Ω resistor and out of the negative pole of the 3.10-V battery; 9.13 mA in the 370-Ω resistor

P28.28 (a) 172 A downward; (b) 1.70 A downward; (c) No, the current in the dead battery is upward in Figure P28.28, so it is not being charged. The dead battery is providing a small amount of power to operate the starter, so it is not really "dead."

P28.30 (a) w = 1.00 A upward in 200 Ω; z = 4.00 A upward in 70.0 Ω; x = 3.00 A upward in 80.0 Ω; y = 8.00 A downward in 20.0 Ω; (b) 200 V

P28.32 (a) I_2 is directed from b toward a and has a magnitude of 2.00 A; (b) I_3 = 1.00 A; (c) No. Neither of the equations used to find I_2 and I_3 contained $\mathcal{E}$ and R. The third equation that we could generate from Kirchhoff's rules contains both the unknowns. Therefore, we have only one equation with two unknowns.

P28.34 (a) $13.0I_1 + 18.0I_2 = 30.0$; (b) $18.0I_2 - 5.00I_3 = -24.0$; (c) $I_1 - I_2 - I_3 = 0$; (d) $I_3 = I_1 - I_2$; (e) $5.00I_1 - 23.0I_2 = 24.0$; (f) $I_2 = -0.416$ A and I_1 = 2.88 A; (g) I_3 = 3.30 A; (h) The negative sign in the answer for I_2 means that this current flows in the opposite direction to that shown in the circuit diagram and assumed during the solution. That is, the actual current in the middle branch of the circuit flows from right to left and has a magnitude of 0.416 A.

P28.36 (a) No. The circuit cannot be simplified further, and Kirchhoff's rules must be used to analyze it; (b) I_1 = 3.50 A; (c) I_2 = 2.50 A; (d) I_3 = 1.00 A

P28.38 (a) 5.00 s; (b) 150 μC; (c) 4.06 μA

P28.40 587 kΩ

P28.42 (a) $(R_1 + R_2)C$; (b) R_2C; (c) $\mathcal{E}\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}e^{-t/(R_2C)}\right)$

P28.44 $+\dfrac{RC}{2}$

P28.46 (a) For the heater, 12.5 A; For the toaster, 6.25 A; For the grill, 8.33 A; (b) The current draw is greater than 25.0 amps, so this circuit will trip the circuit breaker.

P28.48 (a) ~10^{-14}; (b) $\sim\dfrac{V_h}{2} + 10^{-10}$ V and $\sim\dfrac{V_h}{2} - 10^{-10}$ V

P28.50 7.49 Ω

P28.52 (a) 0.991; (b) 0.648; (c) The energy flows are precisely analogous to the currents in parts (a) and (b). The ceiling has the smallest R value of the thermal resistors in parallel, so increasing its thermal resistance will produce the biggest reduction in the total energy flow.

P28.54 (a) 0.706 A; (b) 2.49 W; (c) Only the circuit in Figure P28.54(c) requires the use of Kirchhoff's rules for solution. In the other circuits, the 5-Ω and 8-Ω resistors are still in parallel with each other; (c) The power is lowest in Figure P28.54(c). The circuits in Figures P28.54(b) and P28.54(d) have in effect 30-V batteries driving the current.

P28.56 55.0 Ω

P28.58 See P28.58 for full explanation.

P28.60 (a) 15.0 Ω; (b) $\Delta V_{ac} = \Delta V_{db} = 6.00$ V, $\Delta V_{ce} = 1.20$ V, $\Delta V_{fd} = \Delta V_{ed} = 1.80$ V, $\Delta V_{cd} = 3.00$ V; (c) $I_1 = 1.00$ A, $I_2 = 0.500$ A, $I_3 = 0.500$ A, $I_4 = 0.300$ A, $I_5 = 0.200$ A; (d) $P_{ac} = 6.00$ W, $P_{ce} = 0.600$ W, $P_{ed} = 0.540$ W, $P_{fd} = 0.360$ W, $P_{cd} = 1.50$ W, $P_{db} = 6.00$ W

P28.62 $\dfrac{P_s + \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$ and $\dfrac{P_s - \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

P28.64 (a) 4.40 Ω; (b) 32.0 W; (c) 9.60 W; (d) 70.4 W; (e) 48.0 W

P28.66 (a) $I_1 = \dfrac{IR_2}{R_1 + R_2}$ and $\dfrac{IR_1}{R_1 + R_2} = I_2$; (b) See P28.66(b) for full proof.

P28.68 See P28.68 for full explanation.

P28.70 (a) $q = \dfrac{30.0}{2.00 + R} - 0.542$, where q is in microcoulombs and R is in ohms; (b) 1.96 μC; (c) Yes; 53.3 Ω; (d) 14.5 μC; (e) Yes. Taking $R = \infty$ corresponds to disconnecting the wire; 0.542 μC

P28.72 (a) 40.0 W; (b) 80.0 V and 40.0 V

P28.74 (a) 9.30 V; (b) 2.51 Ω; (c) 18.6 V; (d) 3.70 A; (e) 1.09 A; (f) 14.3 W; (g) 8.54 W; (h) Because of the internal resistance of the batteries, the terminal voltage of the pair of batteries is not the same in both cases.

P28.76 See P28.76 for full explanation.

P28.78 (a) $\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right) = (0.011\,8)t + 0.088\,2$; (b) The time constant is 84.7 s and the capacitance is 8.47 μF.

P28.80 $P_s = \dfrac{1}{n^2} P_p$

P28.82 $(R_1 + 2R_2)C \ln 2$