30

Sources of the Magnetic Field

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ30.1 **(i)** Answer (b). The field is proportional to the current. **(ii)** Answer (d). The field is inversely proportional to the length of the solenoid. **(iii)** Answer (b). The field is proportional to the number of turns. **(iv)** Answer (c). The field does not depend on the radius of the solenoid. All the questions can be answered by referring to Equation 30.17, $B = \frac{\mu_0 NI}{\ell}$.

OQ30.2 Answer (c). Newton's third law describes the relationship.

OQ30.3 (a) No. At least two would be of like sign, so they would repel. (b) Yes, if all are alike in sign. (c) Yes, if all carry current in the same direction. (d) No. If one current-carrying wire repelled the other two, those two would attract each other.

OQ30.4 Answer (a). The contribution made to the magnetic field at point *P* by the lower wire is directed out of the page, while the contribution due to the upper wire is directed into the page. Since point *P* is equidistant from the two wires, and the wires carry the same magnitude currents, these two oppositely directed contributions to the magnetic field have equal magnitudes and cancel each other.

OQ30.5 Answer (a) and (c). The magnetic field due to the current in the vertical wire is directed into the page on the right side of the wire and out of the page on the left side. The field due to the current in the horizontal wire is out of the page above this wire and into the page below the wire. Thus, the two contributions to the total magnetic field have the same directions at points *B* (both out of the page) and *D* (both contributions into the page), while the two contributions have opposite directions at points *A* and *C*. The magnitude of the total magnetic field will be greatest at points *B* and *D* where the two contributions are in the same direction, and smallest at points *A* and *C* where the two contributions are in opposite directions and tend to cancel.

OQ30.6 **(i)** Answer (b). Magnetic field lines lie in horizontal planes and go around the wire clockwise as seen from above. East of the wire the field points horizontally south.

(ii) Answer (b). The direction of the magnetic field at a given point is determined by the direction of the conventional current that creates it.

***OQ30.7** **(i)** Answer (d). **(ii)** Answer (c). Current on each side of the frame produces magnetic field lines that wrap around the tubes. The field lines pass into the plane enclosed by the frame (away from you) and then return to pass back through the plane outside the frame (toward you).

OQ30.8 Answer (a). According to the right-hand rule, the magnetic field at point *P* due to the current in the wire is directed out of the page, and the magnitude of this field is given by Equation 30.14: $B = \mu_0 I/2\pi r$.

OQ30.9 Answers (c) and (d). Any point in region I is closer to the upper wire, which carries the larger current. At all points in this region, the outward directed field due the upper wire will have a greater magnitude than will the inward directed field due to the lower wire. Thus, the resultant field in region I will be nonzero and out of the page, meaning that choice (d) is a true statement and choice (a) is false. In region II, the field due to each wire is directed into the page, so their magnitudes add and the resultant field cannot be zero at any point in this region. This means that choice (b) is false. In region III, the field due to the upper wire is directed into the page while that due to the lower wire is out of the page. Since points in this region are closer to the wire carrying the smaller current, there are points in this region where the magnitudes of the oppositely directed fields due to the two wires will have equal magnitudes, canceling each other and producing a zero resultant field. Thus, choice (c) is true and choice (e) is false.

OQ30.10 Answer (b). Wires carrying currents in opposite directions repel. In regions II and III, the field due to the upper wire is directed into the page. The lower wire, with its current to the left, experiences a downward force in the field of the upper wire.

OQ30.11 Answers (b) and (c). In each case, electric charge is moving.

OQ30.12 Answer (a). The adjacent wires carry currents in the same direction.

OQ30.13 Answer (c). Conceptually, for there to be magnetic flux through a coil, magnetic field lines must pass through the area enclosed by the coil. The magnetic field lines do not pass through the areas of the coils in the *xy* and *xz* planes, but they do through the area of the coil in the *yz* plane. Mathematically, the magnetic flux is $\Phi_B = BA\cos\theta$, where θ is the angle between the normal to the area enclosed by the coil and the magnetic field. The flux is maximum when the field is perpendicular to the area of the coil. The flux is zero when there is no component of magnetic field perpendicular to the loop—that is, when the plane of the loop contains the *x* axis.

OQ30.14 The ranking is e > c > b > a > d. Express the fields in units of μ_0 (ampere/cm):

(a) for a long, straight wire,

$$\mu_0 I/2\pi r = \mu_0\left[3/2\pi(2)\right] = \mu_0\,[0.75/\pi]\,(\text{ampere/cm})$$

(b) for a circular coil,

$$N\mu_0 I/2r = \mu_0\left[(10)(0.3)/2(2)\right] = \mu_0\,[0.75]\ (\text{ampere/cm})$$

(c) for a solenoid,

$$N\mu_0 I/\ell = \mu_0\left[(1\ 000)(0.3)/200\right] = \mu_0\,[1.5]\ (\text{ampere/cm})$$

which is also

$$\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left[1.5\ \text{A}/(0.01\ \text{m})\right] = 0.19\times10^{-3}\ \text{T} = 0.19\ \text{mT}$$

(d) The field is zero at the center of a current-carrying wire.

(e) 1 mT is larger than 0.19 mT, so it is largest of all.

OQ30.15 The ranking is C > A > B. The magnetic field inside a solenoid, carrying current *I*, with *N* turns and length *L*, is $B = \mu_0 nI = \mu_0\left(\dfrac{N}{L}\right)I$.

Thus, $B_A = \dfrac{\mu_0 N_A I}{L_A}$, $B_B = \dfrac{\mu_0 N_A I}{2L_A} = \dfrac{1}{2}B_A$, and $B_C = \dfrac{\mu_0(2N_A)I}{L_A/2} = 4B_A$.

ANSWERS TO CONCEPTUAL QUESTIONS

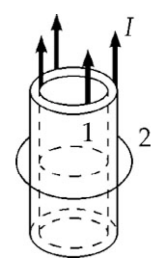
CQ30.1 No. The magnetic field created by a single current loop resembles that of a bar magnet – strongest inside the loop, and decreasing in strength as you move away from the loop. Neither is the field uniform in direction – the magnetic field lines loop through the loop.

CQ30.2 Yes. Either pole of the magnet creates a field that turns the atoms of the domains inside the iron to align their magnetic moments with the external field. Then the nonuniform field exerts a net force on each domain toward the direction in which the field is getting stronger.

A magnet on a refrigerator door goes through the same steps to exert a strong normal force on the door. Then the magnet is supported by a frictional force.

CQ30.3 The Biot-Savart law considers the contribution of each element of current in a conductor to determine the magnetic field, while for Ampère's law, one need only know the current passing through a given surface. Given situations of high degrees of symmetry, Ampère's law is more convenient to use, even though both laws are equally valid in all situations.

CQ30.4 Apply Ampère's law to the circular path labeled 1 in the picture. Because the current has a cylindrical symmetry about its central axis, the line integral reduces to the magnitude of the magnetic field times the circumference of the path, but this is equal to zero because there is no current inside this path; therefore, the magnetic field inside the tube must be zero. On the other hand, the current through path 2 is the current carried by the conductor; then the line integral is not equal to zero, so the magnetic field outside the tube is nonzero.



ANS. FIG. CQ30.4

CQ30.5 Magnetic field lines come out of north magnetic poles. The Earth's north magnetic pole is off the coast of Antarctica, near the south geographic pole. Straight up.

CQ30.6 Ampère's law is valid for all closed paths surrounding a conductor, but not always convenient. There are many paths along which the integral is cumbersome to calculate, although not impossible. Consider a circular path around but *not* coaxial with a long, straight current-carrying wire. Ampère's law is useful in calculating $\vec{\mathbf{B}}$ if the current in a conductor has sufficient symmetry that the line integral can be reduced to the magnitude of $\vec{\mathbf{B}}$ times an integral.

CQ30.7 Magnetic domain alignment within the magnet creates an external magnetic field, which in turn induces domain alignment within the first piece of iron, creating another external magnetic field. The field of the first piece of iron in turn can align domains in another iron sample. A nonuniform magnetic field exerts a net force of attraction on the magnetic dipoles of the domains aligned with the field.

CQ30.8 The shock misaligns the domains. Heating will also decrease magnetism (see Curie Temperature).

CQ30.9 Zero in each case. The fields have no component perpendicular to the area.

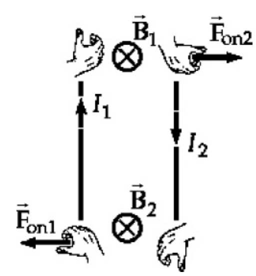
CQ30.10 (a) The third magnet from the top repels the second one with a force equal to the weight of the top two. The yellow magnet repels the blue one with a force equal to the weight of the blue one.

(b) The rods (or a pencil) prevent motion to the side and prevents the magnets from rotating under their mutual torques. Its constraint changes unstable equilibrium into stable.

(c) Most likely, the disks are magnetized perpendicular to their flat faces, making one face a north pole and the other a south pole. One disk has its north pole on the top side and the adjacent magnets have their north poles on their bottom sides.

(d) If the blue magnet were inverted, it and the yellow one would stick firmly together. The pair would still produce an external field and would float together above the red magnets.

CQ30.11 In the figure, the magnetic field created by wire 1 at the position of wire 2 is into the paper. Hence, the magnetic force on wire 2 is in direction (current down) × (field into the paper) = (force to the right), away from wire 1. Now wire 2 creates a magnetic field into the page at the location of wire 1, so wire 1 feels force (current up) × (field into the paper) = (force to the left), away from wire 2.



ANS. FIG. CQ30.11

CQ30.12 (a) The field can be uniform in magnitude.
Gauss's law for magnetism implies that magnetic field lines never start or stop. If the field is uniform in direction, the lines are parallel and their density stays constant along any one bundle of lines. Therefore, the magnitude of the field has the same value at all points along a line in the direction of the field. (b) The magnitude of the field could vary over a plane perpendicular to the lines, or it could be constant throughout the volume.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 30.1 The Biot–Savart Law

***P30.1** (a) Each coil separately produces field given by $B = \dfrac{N\mu_0 IR^2}{2\left(R^2 + x^2\right)^{3/2}}$ at the point halfway between them. Together they produce field

$$2B = \frac{N\mu_0 IR^2}{\left(R^2 + x^2\right)^{3/2}} = 4.50 \times 10^{-5}\ \text{T}$$

$$= \frac{50\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)I(0.012\ \text{m})^2}{\left[(0.012\ \text{m})^2 + (0.011\ \text{m})^2\right]^{3/2}}$$

$$= \frac{9.05 \times 10^{-9}\ \text{T}\cdot\text{m}^3/\text{A}}{4.31 \times 10^{-6}\ \text{m}^3} I$$

$$\rightarrow I = \frac{4.50 \times 10^{-5}\ \text{T A}}{2.10 \times 10^{-3}\ \text{T}} = \boxed{21.5\ \text{mA}}$$

(b) $\Delta V = IR = (0.021\,5\ \text{A})(210\ \Omega) = \boxed{4.51\ \text{V}}$

(c) $P = (\Delta V)I = (4.51\ \text{V})(0.021\,5\ \text{A}) = \boxed{96.7\ \text{mW}}$

P30.2 Imagine grasping the conductor with the right hand so the fingers curl around the conductor in the direction of the magnetic field. The thumb then points along the conductor in the direction of the current. The results are

(a) $\boxed{\text{toward the left}}$ (b) $\boxed{\text{out of the page}}$ (c) $\boxed{\text{lower left to upper right}}$

P30.3 The magnetic field is given by

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi(0.250\ \text{m})} = \boxed{1.60 \times 10^{-6}\ \text{T}}$$

P30.4 Model the tornado as a long, straight, vertical conductor and imagine grasping it with the right hand so the fingers point northward on the western side of the tornado (that is, at the observatory's location). The thumb is directed downward, meaning that the conventional current is downward. The magnitude of the current is found from $B = \mu_0 I/2\pi r\,a$ as

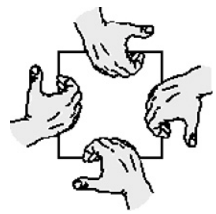
$$I = \frac{2\pi rB}{\mu_0} = \frac{2\pi\left(9.00 \times 10^3\ \text{m}\right)\left(1.50 \times 10^{-8}\ \text{T}\right)}{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}} = 675\ \text{A}$$

Thus, the current is $\boxed{675\ \text{A, downward}}$.

P30.5 (a) Use Equation 30.4 for the field produced by each side of the square.

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

where $\theta_1 = 45.0°,\ \theta_2 = -45.0°,$ and $a = \frac{\ell}{2}$



ANS. FIG. P30.5

Each side produces a field into the page. The four sides altogether produce

$$B_{\text{center}} = 4B = 4\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$= \frac{\mu_0 I}{\pi \ell/2}[\sin 45.0° - \sin(-45.0°)]$$

$$= \frac{2\mu_0 I}{\pi \ell}\left[\frac{2}{\sqrt{2}}\right] = \frac{2\sqrt{2}\mu_0 I}{\pi \ell}$$

$$B = \frac{2\sqrt{2}(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})}{\pi(0.400\ \text{m})}$$

$$= 2\sqrt{2} \times 10^{-5}\ \text{T} = \boxed{28.3\ \mu\text{T into the page}}$$

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})}{4(0.400\ \text{m})}$$

$$= \boxed{24.7\ \mu\text{T into the page}}$$

P30.6 Treat the magnetic field as that produced in the center of a ring of radius *R* carrying current *I*: from Equation 30.8, the field is $B = \frac{\mu_0 I}{2R}$.

The current due to the electron is

$$I = \frac{\Delta q}{\Delta t} = \frac{e}{2\pi R/v} = \frac{ev}{2\pi R}$$

so the magnetic field is

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0}{2R}\left(\frac{ev}{2\pi R}\right) = \frac{\mu_0}{4\pi}\frac{ev}{R^2}$$

$$= \left(\frac{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\right)\frac{\left(1.60\times 10^{-19}\ \text{C}\right)\left(2.19\times 10^{6}\ \text{m/s}\right)}{\left(5.29\times 10^{-11}\ \text{m}\right)^2}$$

$$= \boxed{12.5\ \text{T}}$$

P30.7 We can think of the total magnetic field as the superposition of the field due to the long straight wire, having magnitude $\frac{\mu_0 I}{2\pi R}$ and directed into the page, and the field due to the circular loop, having magnitude $\frac{\mu_0 I}{2R}$ and directed into the page. The resultant magnetic field is:

$$\vec{\mathbf{B}} = \left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R} = \left(1+\frac{1}{\pi}\right)\frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.00\ \text{A})}{2(0.150\ \text{m})}$$

$$= 5.52\times 10^{-6} = \boxed{5.52\ \mu\text{T into the page}}$$

P30.8 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\frac{\mu_0 I}{2\pi R}$ and directed into the page) and the field due to the circular loop (having magnitude $\frac{\mu_0 I}{2R}$ and directed into the page). The resultant magnetic field is:

$$\boxed{\vec{\mathbf{B}} = \left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R}\ \text{(directed into the page)}}$$

P30.9 Wire 1 creates at the origin magnetic field:

$$\vec{\mathbf{B}}_1 = \frac{\mu_0 I}{2\pi r}\ \text{right hand rule} = \frac{\mu_0 I_1}{2\pi a}$$  $$= \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}$$

(a) If the total field at the origin is $\frac{2\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} + \vec{\mathbf{B}}_2$ then the second wire must create field according to $\vec{\mathbf{B}}_2 = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} = \frac{\mu_0 I_2}{2\pi(2a)}$ .

Then $I_2 = \boxed{2I_1 \text{ out of the paper}}$.

(b) The other possibility is $\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{2\mu_0 I_1}{2\pi a}(-\hat{\mathbf{j}}) = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} + \vec{\mathbf{B}}_2$. Then,

$$\vec{\mathbf{B}}_2 = \frac{3\mu_0 I_1}{2\pi a}(-\hat{\mathbf{j}}) = \frac{\mu_0 I_2}{2\pi(2a)}$$  and $I_2 = \boxed{6I_1 \text{ into the paper}}$.

P30.10 The vertical section of wire constitutes one half of an infinitely long, straight wire at distance x from P, so it creates a field equal to

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right)$$

Hold your right hand with extended thumb in the direction of the current; the field is away from you, into the paper.

For each bit of the horizontal section of wire $d\vec{\mathbf{s}}$ is to the left and $\hat{\mathbf{r}}$ is to the right, so $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = 0$. The horizontal current produces zero field at P. Thus,

$$B = \boxed{\frac{\mu_0 I}{4\pi x} \text{ into the paper}}$$

P30.11 Every element of current creates magnetic field in the same direction, into the page, at the center of the arc. The upper straight portion creates one-half of the field that an infinitely long straight wire would create. The curved portion creates one-quarter of the field that a circular loop produces at its center. The lower straight segment also creates field $\frac{1}{2}\frac{\mu_0 I}{2\pi r}$.

The total field is

$$\vec{\mathbf{B}} = \left(\frac{1}{2}\frac{\mu_0 I}{2\pi r} + \frac{1}{4}\frac{\mu_0 I}{2r} + \frac{1}{2}\frac{\mu_0 I}{2\pi r}\right) \text{ into the page}$$

$$= \boxed{\frac{\mu_0 I}{2r}\left(\frac{1}{\pi} + \frac{1}{4}\right) \text{ into the plane of the paper}}$$

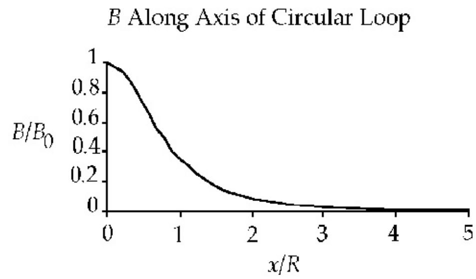
$$= \left(\frac{0.284\,15\mu_0 I}{r}\right) \text{ into the page}$$

P30.12 Along the axis of a circular loop of radius R,

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

or $$\frac{B}{B_0} = \left[\frac{1}{(x/R)^2 + 1}\right]^{3/2},$$

where $$B_0 \equiv \frac{\mu_0 I}{2R}.$$



ANS. FIG. P30.12

x/R	B/B_0
0.00	1.00
1.00	0.354
2.00	0.089 4
3.00	0.031 6
4.00	0.014 3
5.00	0.007 54

P30.13 We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\vec{\mathbf{s}}$ is at 0° or 180° to $\hat{\mathbf{r}}$, so $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = 0$. Thus, only the curved section of wire contributes to $\vec{\mathbf{B}}$ at *P*. Hence, $d\vec{\mathbf{s}}$ is tangent to the arc and $\hat{\mathbf{r}}$ is radially inward; so $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = |ds| 1 \sin 90° \otimes = |ds| \otimes$. All points along the curve are the same distance *r* = 0.600 m from the field point, so

$$B = \int_{\text{all current}} \left|d\vec{\mathbf{B}}\right| = \int \frac{\mu_0}{4\pi} \frac{I\left|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}\right|}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int |ds| = \frac{\mu_0}{4\pi} \frac{I}{r^2} s$$

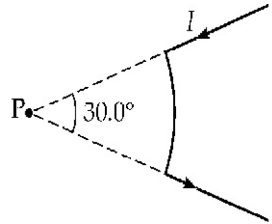
where *s* is the arc length of the curved wire,

$$s = r\theta = (0.600 \text{ m})(30.0°)\left(\frac{2\pi}{360°}\right) = 0.314 \text{ m}$$

Then,

$$B = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{(3.00\ \text{A})}{(0.600\ \text{m})^2}(0.314\ \text{m})$$

$$B = \boxed{262\ \text{nT into the page}}$$



ANS. FIG. P30.13

P30.14 (a) Above the pair of wires, the field out of the page of the 50.0-A current will be stronger than the $\left(-\hat{\mathbf{k}}\right)$ field of the 30.0-A current, so they cannot add to zero. Between the wires, both produce fields into the page. They can only add to zero below the wires, at coordinate $y = -|y|$. Here the total field is

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}$$  $$+\frac{\mu_0 I}{2\pi r}$$  :

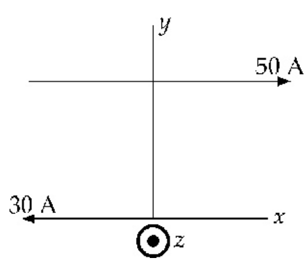
$$0 = \frac{\mu_0}{2\pi}\left[\frac{50.0\ \text{A}}{\left(|y| + 0.280\ \text{m}\right)}\left(-\hat{\mathbf{k}}\right) + \frac{30.0\ \text{A}}{|y|}\left(\hat{\mathbf{k}}\right)\right]$$

$$50.0|y| = 30.0\left(|y| + 0.280\ \text{m}\right)$$

$$50.0(-y) = 30.0(0.280\ \text{m} - y)$$

$$-20.0y = 30.0(0.280\ \text{m})$$

$$\boxed{y = -0.420\ \text{m}}$$



ANS. FIG. P30.14

(b) At $y = 0.100$ m the total field is

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}$$  $$+\frac{\mu_0 I}{2\pi r}$$  :

$$\vec{\mathbf{B}} = \left(\frac{4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}}{2\pi} \right)$$
$$\times \left(\frac{50.0\ \mathrm{A}}{(0.280 - 0.100)\ \mathrm{m}} \left(-\hat{\mathbf{k}}\right) + \frac{30.0\ \mathrm{A}}{0.100\ \mathrm{m}} \left(-\hat{\mathbf{k}}\right) \right)$$
$$= 1.16 \times 10^{-4}\ \mathrm{T}\left(-\hat{\mathbf{k}}\right)$$

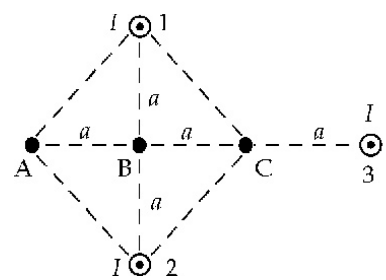
The force on the particle is

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$
$$= \left(-2 \times 10^{-6}\ \mathrm{C}\right)\left(150 \times 10^{6}\ \mathrm{m/s}\right)\left(\hat{\mathbf{i}}\right)$$
$$\times \left(1.16 \times 10^{-4}\ \mathrm{N \cdot s/C \cdot m}\right)\left(-\hat{\mathbf{k}}\right)$$
$$= \boxed{3.47 \times 10^{-2}\ \mathrm{N}\left(-\hat{\mathbf{j}}\right)}$$

(c) We require $\vec{\mathbf{F}}_e = 3.47 \times 10^{-2}\ \mathrm{N}\left(+\hat{\mathbf{j}}\right) = q\vec{\mathbf{E}} = \left(-2 \times 10^{-6}\ \mathrm{C}\right)\vec{\mathbf{E}}$,

so $\vec{\mathbf{E}} = \boxed{-1.73 \times 10^{4}\hat{\mathbf{j}}\ \mathrm{N/C}}$.

P30.15 Label the wires 1, 2, and 3 as shown in ANS. FIG. P30.15(a) and let the magnetic field created by the currents in these wires be $\vec{\mathbf{B}}_1$, $\vec{\mathbf{B}}_2$, and $\vec{\mathbf{B}}_3$, respectively.

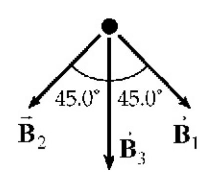


ANS. FIG. P30.15(a)

(a) At point A:

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$$

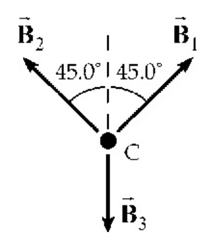
and $B_3 = \dfrac{\mu_0 I}{2\pi(3a)}$.



ANS. FIG. P30.15(b)

The directions of these fields are shown in ANS. FIG. P30.15(b). Observe that the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel while their vertical components both add onto $\vec{\mathbf{B}}_3$. Therefore, the net field at point A is:

$$B_A = B_1 \cos 45.0° + B_2 \cos 45.0° + B_3$$
$$= \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° + \frac{1}{3}\right]$$



ANS. FIG. P30.15(c)

$$B_A = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi\left(1.00\times10^{-2}\ \text{m}\right)}\left[\frac{2}{\sqrt{2}}\cos 45^\circ + \frac{1}{3}\right]$$

$$B_A = \boxed{53.3\ \mu\text{T toward the bottom of the page}}$$

(b) At point B: $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel, leaving

$$B_B = B_3 = \frac{\mu_0 I}{2\pi(2a)}$$

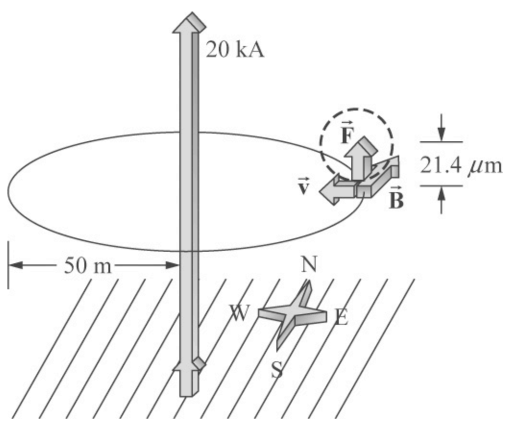
$$B_B = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi(2)\left(1.00\times10^{-2}\ \text{m}\right)}$$

$$= \boxed{20.0\ \mu\text{T toward the bottom of the page}}$$

(c) At point C: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi a}$ with the directions shown in ANS. FIG. P30.15(c). Again, the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel. The vertical components both oppose $\vec{\mathbf{B}}_3$ giving

$$B_C = 2\left[\frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}\cos 45.0^\circ\right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0^\circ - 1\right] = \boxed{0}$$

P30.16 (a) ANS. FIG. P30.16 shows the various vectors.



ANS. FIG. P30.16

(b) The upward lightning current creates field lines in counterclockwise horizontal circles.

$$B = \frac{\mu_0 I}{2\pi r} \text{ [righthand rule]}$$
$$= \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(20.0\times 10^{3}\ \text{A})}{2\pi(50.0\ \text{m})} \text{ north}$$
$$= 8.00\times 10^{-5}\ \text{T north}$$

The force on the electron is

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$$
$$= (-1.6\times 10^{-19}\ \text{C})(300\ \text{m/s west})\times(8.00\times 10^{-5}\ \text{T north})$$
$$= -(3.84\times 10^{-21}\ \text{N down}) = \boxed{3.84\times 10^{-21}\ \text{N up}}$$

(c) From Equation 29.3,

$$r = \frac{mv}{qB} = \frac{(9.11\times 10^{-31}\ \text{kg})\ (300\ \text{m/s})}{(1.60\times 10^{-19}\ \text{C})\ (8.00\times 10^{-5}\ \text{T})} = \boxed{2.14\times 10^{-5}\ \text{m}}.$$

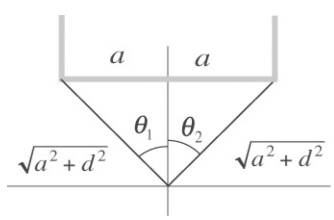
(d) $\boxed{\text{This distance is negligible compared to 50 m, so the electron does move in a uniform field.}}$

(e) Use Equation 29.4, $\omega = qB/m$, which is equal to $2\pi N/\Delta t$, where N is the number of revolutions:

$$N = \frac{qB\Delta t}{2\pi m} = \frac{(1.60\times 10^{-19}\ \text{C})(8.00\times 10^{-5}\ \text{T})(60.0\times 10^{-6}\ \text{s})}{2\pi\ (9.11\times 10^{-31}\ \text{kg})}$$
$$= \boxed{134 \text{ revolutions}}$$

P30.17 Apply the Equation 30.4, $B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 - \sin\theta_2)$, to each of the wires.

For the horizontal wire (H), $\sin\theta_1 = -\frac{a}{\sqrt{d^2+a^2}}$ and $\sin\theta_2 = \frac{a}{\sqrt{d^2+a^2}}$ because θ_1 measures to the wire's end point on the $-x$-axis and θ_2 measures to the wire's end point on the $+x$-axis. For the left vertical wire (VL) and the right vertical wire (VR), $\sin\theta_1 = \frac{d}{\sqrt{d^2+a^2}}$ and $\sin\theta_2 =$ 1 because both angles measure to the wire's end points on the $+y$-axis.



ANS. FIG. P30.17

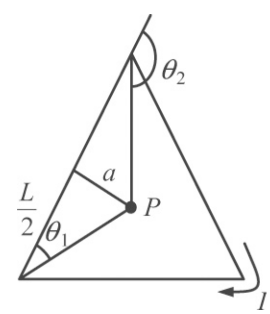
Take out of the page as the positive direction, and into the page as the negative direction. The field at the origin is

$$
\begin{aligned}
B_O &= |B_{VL}| - |B_H| + |B_{VR}| \\
&= \frac{\mu_0 I}{4\pi a}\left(1 - \frac{d}{\sqrt{d^2+a^2}}\right) - \frac{\mu_0 I}{4\pi d}\left[\frac{a}{\sqrt{d^2+a^2}} - \left(-\frac{a}{\sqrt{d^2+a^2}}\right)\right] \\
&\qquad + \frac{\mu_0 I}{4\pi a}\left(1 - \frac{d}{\sqrt{d^2+a^2}}\right) \\
&= \frac{\mu_0 I}{4\pi a}\left(2 - \frac{2d}{\sqrt{d^2+a^2}}\right) - \frac{\mu_0 I}{4\pi d}\left(\frac{2a}{\sqrt{d^2+a^2}}\right) \\
&= \frac{\mu_0 I}{2\pi ad}\left(d - \frac{d^2}{\sqrt{d^2+a^2}} - \frac{a^2}{\sqrt{d^2+a^2}}\right) \\
&= \frac{\mu_0 I}{2\pi ad}\left(d - \frac{d^2+a^2}{\sqrt{d^2+a^2}}\right) = \frac{\mu_0 I}{2\pi ad}\left(d - \sqrt{a^2+d^2}\right) \\
&= -\frac{\mu_0 I}{2\pi ad}\left(\sqrt{a^2+d^2} - d\right)
\end{aligned}
$$

The field is negative: magnetic field at the origin is $\boxed{\frac{\mu_0 I}{2\pi ad}\left(\sqrt{a^2+d^2} - d\right)}$ into the page.

P30.18 (a) We use Equation 30.4 in the chapter text for the field created by a straight wire of limited length. The sines of the angles appearing in that equation are equal to the cosines of the complementary angles shown in our diagram. For the distance a from the wire to the field point we have

$\tan 30° = \frac{a}{L/2}$, $a = 0.288\,7L$. One wire contributes to the field at P



ANS. FIG. P30.18(a)

$$
\begin{aligned}
B &= \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) = \frac{\mu_0 I}{4\pi(0.288\,7L)}(\cos 30° - \cos 150°) \\
&= \frac{\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \frac{1.50\mu_0 I}{\pi L}
\end{aligned}
$$

Each side contributes the same amount of field in the same direction, which is perpendicularly into the paper in the picture. So the total field is $3\left(\frac{1.50\mu_0 I}{\pi L}\right) = \boxed{\frac{4.50\mu_0 I}{\pi L}}$.

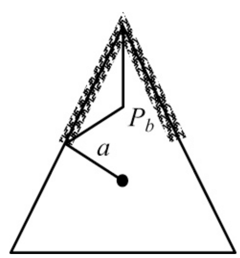
(b) As we showed in part (a), one whole side of the triangle creates field at the center $\frac{\mu_0 I(1.732)}{4\pi a}$. Now one-half of one nearby side of the triangle will be half as far away from point P_b and have a geometrically similar situation. Then it creates at P_b field

$$\frac{\mu_0 I(1.732)}{4\pi(a/2)} = \frac{2\mu_0 I(1.732)}{4\pi a}$$

The two half-sides shown crosshatched in the picture create at P_b field

$$2\left(\frac{2\mu_0 I(1.732)}{4\pi a}\right) = \frac{4\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \frac{6\mu_0 I}{\pi L}$$

The rest of the triangle will contribute somewhat more field in the same direction, so we already have a proof that the field at P_b is $\boxed{\text{stronger}}$.



ANS. FIG. P30.18(b)

P30.19 Assume that the wire on the right is wire 1 and that on the left is wire 2. Also, choose the positive direction for the magnetic field to be out of the page and negative into the page.

(a) At the point half way between the two wires,

$$B_{\text{net}} = -B_1 - B_2 = -\left[\frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2}\right] = -\frac{\mu_0}{2\pi r}(I_1 + I_2)$$

$$= -\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(5.00\times 10^{-2}\ \text{m})}(10.0\ \text{A}) = -4.00\times 10^{-5}\ \text{T}$$

or $B_{\text{net}} = \boxed{40.0\ \mu\text{T into the page}}$

(b) At point P_1,

$$B_{net} = +B_1 - B_2 = \frac{\mu_0}{2\pi}\left[+\frac{I_1}{r_1} - \frac{I_2}{r_2}\right]$$

$$B_{net} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi}\left[\frac{5.00\ \text{A}}{0.100\ \text{m}} - \frac{5.00\ \text{A}}{0.200\ \text{m}}\right]$$

$$= \boxed{5.00\ \mu\text{T out of page}}$$

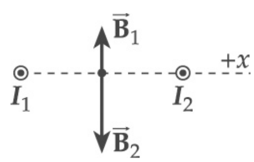
(c) At point P_2,

$$B_{net} = -B_1 + B_2 = \frac{\mu_0}{2\pi}\left[-\frac{I_1}{r_1} + \frac{I_2}{r_2}\right]$$

$$B_{net} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi}\left[-\frac{5.00\ \text{A}}{0.300\ \text{m}} + \frac{5.00\ \text{A}}{0.200\ \text{m}}\right]$$

$$= \boxed{1.67\ \mu\text{T out of page}}$$

P30.20 Call the wire carrying a current of 3.00 A wire 1 and the other wire 2. Also, choose the line running from wire 1 to wire 2 as the positive x direction.



ANS. FIG. P30.20(a)

(a) At the point midway between the wires, the field due to each wire is parallel to the y-axis and the net field is

$$B_{net} = +B_{1y} - B_{2y} = \mu_0(I_1 - I_2)/2\pi r$$

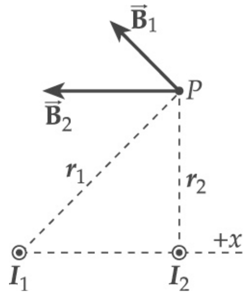
Thus,

$$B_{net} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(0.100\ \text{m})}(3.00\ \text{A} - 5.00\ \text{A}) = -4.00\times10^{-6}\ \text{T}$$

or $\quad B_{net} = \boxed{4.00\ \mu\text{T toward the bottom of the page}}$

(b) Refer to ANS. FIG. P30.20(b). At point P, $r_1 = (0.200\ \text{m})\sqrt{2}$ and B_1 is directed at θ_1 = +135°. The magnitude of B_1 is

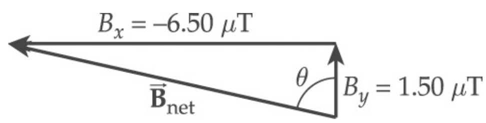
$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.00\ \text{A})}{2\pi(0.200\sqrt{2}\ \text{m})} = 2.12\ \mu\text{T}$$



ANS. FIG. P30.20(b)

The contribution from wire 2 is in the $-x$ direction and has magnitude

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})}{2\pi(0.200\ \text{m})} = 5.00\ \mu\text{T}$$



ANS. FIG. P30.20(c)

Therefore, the components of the net field at point P are:

$$B_x = B_1 \cos 135° + B_2 \cos 180°$$

$$= (2.12\ \mu\text{T})\cos 135° + (5.00\ \mu\text{T})\cos 180° = -6.50\ \mu\text{T}$$

and

$$B_y = B_1 \sin 135° + B_2 \sin 180° = (2.12\ \mu\text{T})\sin 135° + 0 = +1.50\ \mu\text{T}$$

Therefore,

$$B_{\text{net}} = \sqrt{B_x^2 + B_y^2} = 6.67\ \mu\text{T}$$

at $\theta = \tan^{-1}\left(\frac{|B_x|}{B_y}\right) = \tan^{-1}\left(\frac{6.50\ \mu\text{T}}{1.50\ \mu\text{T}}\right) = 77.0°$

in ANS. FIG. P30.20(c), which is 77.0° + 90.0° = 167.0° from the positive x axis. Therefore,

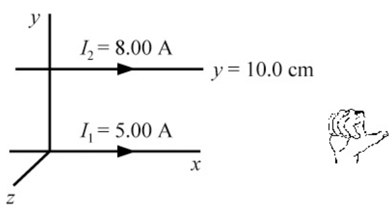
$$\vec{\mathbf{B}}_{\text{net}} = \boxed{6.67\ \mu\text{T at } 167.0° \text{ from the positive } x \text{ axis}}.$$

Section 30.2 The Magnetic Force Between Two Parallel Conductors

P30.21 Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

(a) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\mathbf{k}} = \frac{\left(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}\right)(5.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \boxed{1.00\times 10^{-5}\ \mathrm{T\ out\ of\ the\ page}}$$



ANS. FIG. P30.21(a)

(b) $$\vec{\mathbf{F}}_B = I_2\vec{\ell}\times\vec{\mathbf{B}} = (8.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\hat{\mathbf{i}}\times\left(1.00\times 10^{-5}\ \mathrm{T}\right)\hat{\mathbf{k}}\right]$$

$$= \left(8.00\times 10^{-5}\ \mathrm{N}\right)\left(-\hat{\mathbf{j}}\right)$$

ANS. FIG. P30.21(b)

$$\vec{\mathbf{F}}_B = \boxed{8.00\times 10^{-5}\ \mathrm{N\ toward\ the\ first\ wire}}$$

(c) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\left(-\hat{\mathbf{k}}\right) = \frac{\left(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}\right)(8.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}\left(-\hat{\mathbf{k}}\right)$$

$$= \left(1.60\times 10^{-5}\ \mathrm{T}\right)\left(-\hat{\mathbf{k}}\right)$$

ANS. FIG. P30.21(c)

$$\vec{\mathbf{B}} = \boxed{1.60\times 10^{-5}\ \mathrm{T\ into\ the\ page}}$$

(d) $$\vec{\mathbf{F}}_B = I_1\vec{\ell}\times\vec{\mathbf{B}} = (5.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\hat{\mathbf{i}}\times\left(1.60\times 10^{-5}\ \mathrm{T}\right)\left(-\hat{\mathbf{k}}\right)\right]$$

$$= \left(8.00\times 10^{-5}\ \mathrm{N}\right)\left(+\hat{\mathbf{j}}\right)$$

ANS. FIG. P30.21(d)

$$\vec{\mathbf{F}}_B = \boxed{8.00 \times 10^{-5}\text{ N towards the second wire}}$$

P30.22 (a) The force per unit length that parallel conductors exert on each other is, from Equation 30.12, $F/\ell = \mu_0 I_1 I_2 / 2\pi d$. Thus, if $F/\ell = 2.00 \times 10^{-4}\text{ N/m}$, I_1 = 5.00 A, and d = 4.00 cm, the current in the second wire must be

$$\begin{aligned} I_2 &= \frac{2\pi d}{\mu_0 I_1}\left(\frac{F}{\ell}\right) \\ &= \left[\frac{2\pi\left(4.00 \times 10^{-2}\text{ m}\right)}{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}\right)(5.00\text{ A})}\right]\left(2.00 \times 10^{-4}\text{ N/m}\right) \\ &= \boxed{8.00\text{ A}} \end{aligned}$$

(b) Since parallel conductors carrying currents in the same direction attract each other (see Section 30.2 in the textbook), the currents in these conductors which repel each other must be $\boxed{\text{opposite directions.}}$

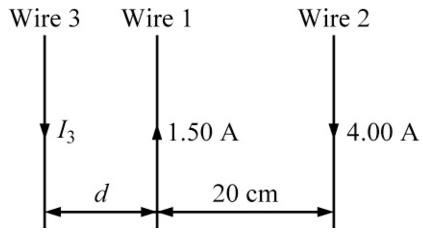
(c) From Equation 30.12, the force is directly proportional to the product of the currents. The result of reversing the direction of either of the currents and doubling the magnitude would be that the $\boxed{\text{force of interaction would be attractive and the magnitude of the force would double}}$.

P30.23 (a) From Equation 30.12, the force per unit length that one wire exerts on the other is $F/\ell = \mu_0 I_1 I_2 / 2\pi d$, where d is the distance separating the two wires. In this case, the value of this force is

$$\frac{F}{\ell} = \frac{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}\right)(3.00\text{ A})^2}{2\pi\left(6.00 \times 10^{-2}\text{ m}\right)} = \boxed{3.00 \times 10^{-5}\text{ N/m}}$$

(b) We can answer this question by consulting Section 30.2 in the textbook, or we can reason it out. Imagine these two wires lying side by side on a table with the two currents flowing toward you, wire 1 on the left and wire 2 on the right. The right-hand rule that relates current to field direction shows the magnetic field due to wire 1 at the location of wire 2 is directed vertically upward. Then, the right-hand rule that relates current and field to force gives the direction of the force experienced by wire 2, with its current flowing through this field, as being to the left, back toward wire 1. Thus, the force one wire exerts on the other is an $\boxed{\text{attractive}}$ force.

P30.24 Carrying oppositely directed currents, wires 1 and 2 repel each other. If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero. If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero. Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2. We answer part (b) first.



ANS. FIG. P30.24

(b) For the equilibrium of wire 3 we have

$$F_{1\text{ on }3} = F_{2\text{ on }3}: \qquad \frac{\mu_0(1.50\text{ A})I_3}{2\pi d} = \frac{\mu_0(4.00\text{ A})I_3}{2\pi(20.0\text{ cm}+d)}$$

$$1.50(20.0\text{ cm} + d) = 4.00d$$

$$d = \frac{30.0\text{ cm}}{2.50} = \boxed{12.0\text{ cm to the left of wire 1}}$$

(a) Thus the situation is possible in just $\boxed{\text{one way.}}$

(c) For the equilibrium of wire 1,

$$\frac{\mu_0 I_3(1.50\text{ A})}{2\pi(12.0\text{ cm})} = \frac{\mu_0(4.00\text{ A})(1.50\text{ A})}{2\pi(20.0\text{ cm})}$$

$$I_3 = \frac{12}{20}(4.00\text{ A}) = \boxed{2.40\text{ A down}}$$

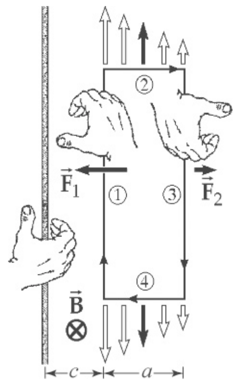
We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal-magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

P30.25 To the right of the long, straight wire, current I_1 creates a magnetic field into the page. By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (from Equation 30.12):

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{-a}{c(c+a)}\right]\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \frac{(4\pi\times10^{-7}\ \text{N/A}^2)(5.00\ \text{A})(10.0\ \text{A})(0.450\ \text{m})}{2\pi} \times \left(\frac{-0.150\ \text{m}}{(0.100\ \text{m})(0.250\ \text{m})}\right)\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = (-2.70\times10^{-5}\hat{\mathbf{i}})\ \text{N} = (-27.0\times10^{-6}\hat{\mathbf{i}})\ \text{N} = \boxed{-27.0\hat{\mathbf{i}}\ \mu\text{N}}$$



ANS. FIG. P30.25

P30.26 See ANS. FIG. P30.25. By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (from Equation 30.12)

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\hat{\mathbf{i}}$$

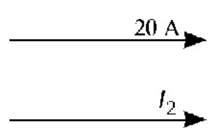
$$\vec{\mathbf{F}} = \boxed{\frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{a}{c(c+a)}\right] \text{ to the left}}$$

P30.27 To attract, both currents ($I_1 = 20.0$ A, and I_2) must be to the right. The attraction is described by (from Equation 30.12)

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

So
$$I_2 = \frac{F}{\ell}\frac{2\pi a}{\mu_0 I_1}$$

$$= (320\times10^{-6}\ \text{N/m})\left(\frac{2\pi(0.500\ \text{m})}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20.0\ \text{A})}\right) = 40.0\ \text{A}$$



ANS. FIG. P30.27

The zero-field point must lie between the two wires: this point cannot be above the upper wire or below the lower wire because the fields in these regions have the same direction, out of the page above the upper wire, and into the page below the lower wire. Let y represent the coordinate of the zero-field point above the lower wire; then, $r_1 = (0.500\text{ m}) - y$ and $r_2 = y$ represent the respective distances of currents I_1 and I_2 to the zero-field point. Taking the positive direction to be out of the page, at the zero-field point,

$$B = -B_1 + B_2$$

$$0 = -\frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2}$$

Eliminating and solving for r_1,

$$\frac{I_1}{r_1} = \frac{I_2}{r_2} \quad \to \quad r_1 = r_2\frac{I_1}{I_2} \quad \to \quad (0.500\text{ m}) - y = y\frac{I_1}{I_2}$$

Then,

$$(0.500\text{ m}) = y\left(\frac{I_1}{I_2} + 1\right)$$

$$y = \frac{(0.500\text{ m})}{\left(\frac{I_1}{I_2} + 1\right)} = \frac{(0.500\text{ m})}{\left(\frac{20.0\text{ A}}{40.0\text{ A}} + 1\right)} = \boxed{0.333\text{ m}}$$

P30.28 From Equation 30.12, we find the separation distance between the wires as

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \quad \to \quad a = \frac{\mu_0 I_1 I_2 \ell}{2\pi F_B}$$

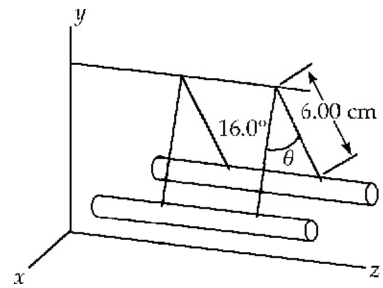
Substituting numerical values,

$$a = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(10.0\text{ A})(10.0\text{ A})(0.500\text{ m})}{2\pi(1.00\text{ N})}$$

$$= 1.00 \times 10^{-5}\text{ m} = 10.0\ \mu\text{m}$$

This is the required center-to-center separation distance of the wires, but the wires cannot be this close together. Their minimum possible center-to-center separation distance occurs if the wires are touching, but this value is $2r = 2(250\ \mu\text{m}) = 500\ \mu\text{m}$, which is much larger than the required value above. We could try to obtain this force between wires of smaller diameter, but these wires would have higher resistance and less surface area for radiating energy. It is likely that the wires would melt very shortly after the current begins.

P30.29 This is almost a standard equilibrium problem involving tension, weight, and a horizontal repulsive force; however, here we must consider the magnetic force per unit length and the weight per unit length. The tension makes an angle $\theta/2 = 8.00°$ with the vertical. The mass per unit length is $\lambda = mg/L$. The separation between the wires is $a = 2\ell\sin\theta/2$.



ANS. FIG. P30.29

(a) Because the wires repel, the currents are in opposite directions.

(b) For balance, the ratio of the horizontal tension component $T\sin\theta/2$ to the vertical tension component $T\cos\theta/2$ is equal to the ratio of the horizontal magnetic force per unit length F_B/L to the vertical weight per unit length F_g/L:

$$\frac{T\sin\theta/2}{T\cos\theta/2} = \frac{F_B/L}{F_g/L}$$

But,

$$F_B/L = IB\sin 90.0° = IB = I\frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I^2}{2\pi a}$$

$$F_g/L = \lambda g$$

Rearranging and substituting gives

$$\tan\theta/2 = \frac{\mu_0 I^2/2\pi a}{\lambda g} = \frac{\mu_0 I^2}{2\pi(2\ell\sin\theta/2)\lambda g}$$

Solving,

$$I^2 = \frac{4\pi\ell\lambda g}{\mu_0}(\tan\theta/2)(\sin\theta/2)$$

$$I^2 = \left[\frac{4\pi(0.060\ 0\ \text{m})(40.0\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}\right] \times(\tan 8.00°)(\sin 8.00°)$$

$$I = \boxed{67.8\ \text{A}}$$

(c) Smaller. A smaller gravitational force would be pulling down on the wires, requiring less magnetic force to raise the wires to the same angle and therefore less current.

Section 30.3 Ampère's Law

P30.30 From $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I$, $I = \dfrac{2\pi rB}{\mu_0} = \dfrac{2\pi\left(1.00\times10^{-3}\text{ m}\right)(0.100\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = \boxed{500\text{ A}}$.

P30.31 (a) From Ampère's law, the magnetic field at point a is given by $B_a = \dfrac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(1.00\text{ A})}{2\pi\left(1.00\times10^{-3}\text{ m}\right)} = \boxed{200\ \mu\text{T toward top of page}}$$

(b) Similarly at point b: $B_b = \dfrac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b. Taking out of the page as positive, $I_b = 1.00\text{ A} - 3.00\text{ A} = -2.00\text{ A}$, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(2.00\text{ A})}{2\pi\left(3.00\times10^{-3}\text{ m}\right)} = \boxed{133\ \mu\text{T toward bottom of page}}$$

P30.32 (a) $B_{\text{inner}} = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(900)\left(14.0\times10^{3}\text{ A}\right)}{2\pi(0.700\text{ m})} = \boxed{3.60\text{ T}}$

(b) $B_{\text{outer}} = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(900)\left(14.0\times10^{3}\text{ A}\right)}{2\pi(1.30\text{ m})} = \boxed{1.94\text{ T}}$

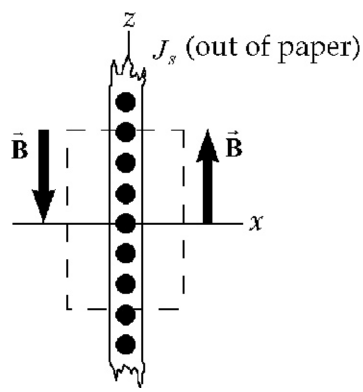
P30.33 Let the current I be to the right, in the positive x direction. The proton travels to the left, and is a distance d above the wire. Take up as the positive y direction. At the proton's location, the current creates a field $B = \dfrac{\mu_0 I}{2\pi d}$ in the positive z direction. The weight of the proton and the magnetic force are in balance:

$$mg\left(-\hat{\mathbf{j}}\right) + qv\left(-\hat{\mathbf{i}}\right)\times\frac{\mu_0 I}{2\pi d}\left(\hat{\mathbf{k}}\right) = 0$$

$$mg\left(-\hat{\mathbf{j}}\right) + \frac{qv\mu_0 I}{2\pi d}\left(\hat{\mathbf{j}}\right) = 0$$

$$d = \frac{qv\mu_0 I}{2\pi mg}$$
$$= \frac{(1.60\times10^{-19}\ \text{C})(2.30\times10^{4}\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.20\times10^{-6}\ \text{A})}{2\pi(1.67\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)}$$
$$= \boxed{5.40\ \text{cm}}$$

P30.34 We may regard the sheet as being composed of filaments of current $J_s d\vec{\mathbf{s}}$ directed out of the page. According to the Biot-Savart law, the field contribution at a point has the direction $d\vec{\mathbf{s}}\times\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ points from the current filament to the point. Consider the field contributions at an arbitrary point P to the right of the sheet. Draw a line normal to the sheet that passes through P. Consider the contributions to the field at P from two filaments that lie along the same vertical line and are equidistant from the normal (and P). The upper filament contributes $+z$ and $+x$ field components, but the lower filament $+z$ and $-x$ field components. The resulting field from both filaments points in the $+z$-direction. By similar reasoning, the magnetic field at any point on the left side of the sheet points in the $-z$ direction. These same arguments hold for any point within the sheet. Also, the same reasoning shows that for any pair of filaments that lie on the same vertical line, the magnetic field at a point midway between them is zero. Thus, the field has no horizontal component within the sheet.



ANS. FIG. P30.34

Therefore, each filament of current creates a contribution to the total field that is parallel to the sheet and perpendicular to the current direction. They create field lines straight up to the right of the sheet and straight down to the left of the sheet.

From Ampère's law applied to the suggested rectangle,

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I: \quad B\cdot 2\ell + 0 = \mu_0 J_s \ell$$

Therefore the field is uniform in space, with the magnitude

$$\boxed{B = \frac{\mu_0 J_s}{2}}$$

P30.35 (a) In $B = \dfrac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400\ \text{cm}}$.

(b) $\vec{\mathbf{B}} = \dfrac{\mu_0 I}{2\pi r_1}\hat{\mathbf{k}} + \dfrac{\mu_0 I}{2\pi r_2}\left(-\hat{\mathbf{k}}\right)$

so $B = \dfrac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi}\left(\dfrac{1}{0.398\,5\ \text{m}} - \dfrac{1}{0.401\,5\ \text{m}}\right)$

$= \boxed{7.50\ \text{nT}}$

(c) Call r the distance from cord center to field point and $2d = 3.00$ mm the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\frac{2d}{r^2 - d^2}$$

$$7.50 \times 10^{-10}\ \text{T}$$

$$= \left(2.00 \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})\frac{\left(3.00 \times 10^{-3}\ \text{m}\right)}{r^2 - \left(2.25 \times 10^{-6}\ \text{m}\right)^2}$$

so $r = \boxed{1.26\ \text{m}}$.

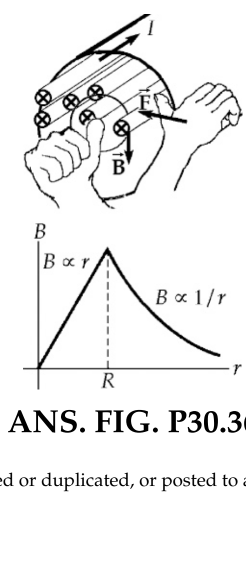
The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

P30.36 By Ampère's law, the field at the position of the wire at distance r from the center is due to the fraction of the other 99 wires that lie within the radius r.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I:$$

$$B \cdot 2\pi r = \mu_0\left[99I\left(\frac{\pi r^2}{\pi R^2}\right)\right] \quad \rightarrow \quad B = \frac{\mu_0 (99I)}{2\pi r}\left(\frac{r^2}{R^2}\right) = \frac{\mu_0 (99I)}{2\pi R}\left(\frac{r}{R}\right)$$



ANS. FIG. P30.36

The field is proportional to r, as shown in ANS. FIG. P30.36. This field points tangent to a circle of radius r and exerts a force $\vec{\mathbf{F}} = I\vec{\ell} \times \vec{\mathbf{B}}$ on the wire toward the center of the bundle. The magnitude of the force is

$$\frac{F}{\ell} = IB\sin\theta = I\left[\frac{\mu_0(99)I}{2\pi R}\left(\frac{r}{R}\right)\right]\sin 90° = \frac{\mu_0(99)I^2}{2\pi R}\left(\frac{r}{R}\right)$$

$$= \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(99)(2.00\ \text{A})^2}{2\pi\left(0.500\times10^{-2}\ \text{m}\right)}(0.400)$$

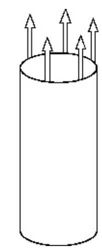
$$= 6.34\times10^{-3}\ \text{N/m}$$

(a) $\boxed{6.34\times10^{-3}\ \text{N/m}}$

(b) Referring to the figure, the field is clockwise, so at the position of the wire, the field is downward, and the force is [inward toward the center of the bundle].

(c) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is [greatest at the outer surface].

P30.37 We assume the current is vertically upward.



ANS. FIG. P30.37(a)

(a) Consider a circle of radius r slightly less than R. It encloses no current, so from

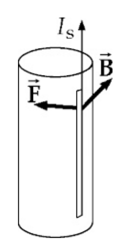
$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{inside}} \text{ gives } B(2\pi r) = 0,$$

we conclude that the magnetic field is [zero].

(b) Now let the r be barely larger than R. Ampère's law becomes

$$B(2\pi R) = \mu_0 I,$$

so $\boxed{B = \dfrac{\mu_0 I}{2\pi R} \text{ tangent to the wall}}$.



ANS. FIG. P30.37(b)

By the right-hand rule, the field direction is counterclockwise  (as seen from above).

(c) Consider a strip of the wall of horizontal width ds and length ℓ. Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.

The current it carries is $I_s = \dfrac{Ids}{2\pi R}$ up.

The force on it is

$$d\vec{\mathbf{F}} = I_s\vec{\ell} \times \vec{\mathbf{B}} = \frac{Ids}{2\pi R}\left(\ell\frac{\mu_0 I}{2\pi R}\right)\left[\widehat{\textbf{up}} \times \widehat{\textbf{into page}}\right]$$

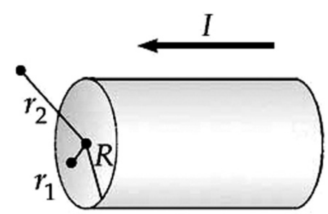
$$= \frac{\mu_0 I^2 \ell ds}{(2\pi R)^2}\widehat{\textbf{radially inward}}$$

The pressure on the strip, and therefore, everywhere on the cylinder, is

$$P = \frac{dF}{dA} = \frac{\mu_0 I^2 \ell ds}{(2\pi R)^2}\frac{1}{\ell ds} = \boxed{\frac{\mu_0 I^2}{(2\pi R)^2}\text{ inward}}$$

The pinch effect makes an effective demonstration when an aluminum can crushes itself as it carries a large current along its length.

P30.38 Take a circle of radius r_1 or r_2 to apply $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$, where for nonuniform current density $I = \int J dA$. In this case $\vec{\mathbf{B}}$ is parallel to $d\vec{\mathbf{s}}$ and the direction of J is straight through the area element dA, so Ampère's law gives



ANS. FIG. P30.38

$$\oint B ds = \mu_0 \int J dA$$

(a) For $r_1 < R$,

$$2\pi r_1 B = \mu_0 \int_0^{r_1} br(2\pi r dr) = \mu_0 2\pi b\left[\frac{r_1^3}{3} - 0\right]$$

and $B = \boxed{\frac{1}{3}\left(\mu_0 b r_1^2\right)\text{ (inside)}}$

(b) For $r_2 > R$,

$$2\pi r_2 B = \mu_0 \int_0^R br(2\pi r dr)$$

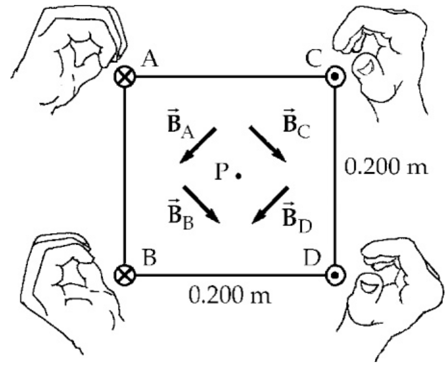
and $B = \boxed{\frac{\mu_0 b R^3}{3 r_2}\text{ (outside)}}$

P30.39 Each wire is distant from P by ($\ell = 0.200$ m)

$$r = \sqrt{\ell^2 + \ell^2}/2 = \ell/\sqrt{2}$$

and each wire produces a field at P of equal magnitude

$$B = \frac{\mu_0 I}{2\pi r}$$



ANS. FIG. P30.39

Carrying currents into the page, A produces at P a field to the left and downward at –135°, while B creates a field to the right and downward at –45°. Carrying currents out of the page, C produces a field downward and to the right at –45°, while D's contribution is downward and to the left. All horizontal components cancel; thus, all remaining components are vertically downward. The magnitude of the resulting field is

$$B_P = 4B\cos 45.0° = 4\frac{\mu_0 I}{2\pi r}\cos 45.0° = 4\frac{\mu_0 I}{2\pi\left(\ell/\sqrt{2}\right)}\frac{1}{\sqrt{2}} = \frac{2\mu_0 I}{\pi\ell}$$

$$= \frac{2\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(5.00\ \text{A})}{\pi(0.200\ \text{m})} = 2.00\times 10^{-5}\ \text{T}$$

The magnetic field is $\boxed{20.0\ \mu\text{T toward the bottom of the page}}$.

Section 30.4 The Magnetic Field of a Solenoid

P30.40 The magnetic field inside of a solenoid is $B = \mu_0 nI = \mu_0 (N/L)I$. Thus, the number of turns on this solenoid must be

$$N = \frac{BL}{\mu_0 I} = \frac{(9.00\ \text{T})(0.500\ \text{m})}{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(75.0\ \text{A})} = \boxed{4.77\times 10^4\ \text{turns}}$$

P30.41 The magnetic field at the center of a solenoid is $B = \mu_0 \frac{N}{\ell} I$, so

$$I = \frac{B}{\mu_0 n} = \frac{\left(1.00\times 10^{-4}\ \text{T}\right)(0.400\ \text{m})}{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1\,000)} = \boxed{31.8\ \text{mA}}$$

P30.42 In the expression $B = N\mu_0 I/\ell$ for the field within a solenoid with radius much less than 20 cm, all we want to do is increase N.

(a) Make the wire as long and thin as possible without melting when it carries the 5-A current. Then the solenoid can have many turns.

(b) As small in radius as possible with your experiment fitting inside. Then with a smaller circumference the wire can form a solenoid with more turns.

P30.43 (a) The field produced by the solenoid in its interior is given by

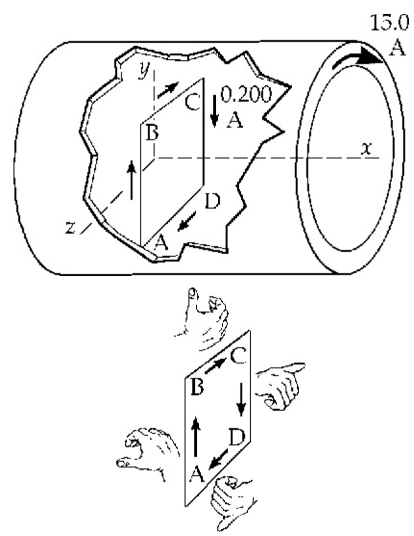
$$\vec{\mathbf{B}} = \mu_0 nI\left(-\hat{\mathbf{i}}\right) = \left(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}\right)\left(\frac{30.0}{10^{-2}\ \mathrm{m}}\right)(15.0\ \mathrm{A})\left(-\hat{\mathbf{i}}\right)$$

$$\vec{\mathbf{B}} = -\left(5.65\times 10^{-2}\ \mathrm{T}\right)\hat{\mathbf{i}}$$

The force exerted on side AB of the square current loop is

$$\left(\vec{\mathbf{F}}_B\right)_{\mathrm{AB}} = I\vec{\mathbf{L}}\times\vec{\mathbf{B}} = (0.200\ \mathrm{A}) \times\left[\left(2.00\times 10^{-2}\ \mathrm{m}\right)\hat{\mathbf{j}}\times\left(5.65\times 10^{-2}\ \mathrm{T}\right)\left(-\hat{\mathbf{i}}\right)\right]$$

$$\left(\vec{\mathbf{F}}_B\right)_{\mathrm{AB}} = \left(2.26\times 10^{-4}\ \mathrm{N}\right)\hat{\mathbf{k}}$$



ANS. FIG. P30.43

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of

| 226 μN directed away from the center of the loop |.

(b) From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid is | zero |. More formally, the magnetic dipole moment of the square loop is given by

$$\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}} = (0.200\ \mathrm{A})\left(2.00\times 10^{-2}\ \mathrm{m}\right)^2\left(-\hat{\mathbf{i}}\right) = -80.0\ \mu\mathrm{A\cdot m^2}\ \hat{\mathbf{i}}$$

The torque exerted on the loop is then

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}} = \left(-80.0\ \mu\mathrm{A\cdot m^2}\ \hat{\mathbf{i}}\right)\times\left(-5.65\times 10^{-2}\ \mathrm{T}\ \hat{\mathbf{i}}\right) = \boxed{0}$$

P30.44 The number of turns is $N = \dfrac{75.0 \text{ cm}}{0.100 \text{ cm}} = 750.$ We assume that the solenoid is long enough to qualify as a long solenoid. Then the field within it (not close to the ends) is $B = \dfrac{N\mu_0 I}{\ell}$, so

$$I = \frac{B\ell}{N\mu_0} = \frac{(8.00 \times 10^{-3} \text{ T})(0.750 \text{ m})}{750(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})} = 6.37 \text{ A}$$

The resistance of the wire is

$$R = \frac{\rho\ell_{\text{wire}}}{A} = \frac{(1.7 \times 10^{-8} \ \Omega\cdot\text{m})2\pi(0.050\ 0 \text{ m})750}{\pi(0.050\ 0 \times 10^{-2} \text{ m})^2} = 5.10 \ \Omega$$

The power delivered is

$$P = I\Delta V = I^2 R = (6.37 \text{ A})^2 (5.10 \ \Omega) = \boxed{207 \text{ W}}$$

The power required would be smaller if wire were wrapped in several layers.

P30.45 (a) From $R = \rho L/A$, the required length of wire to be used is $L = \dfrac{R \cdot A}{\rho}$. The total number of turns on the solenoid (that is, the number of times this length of wire will go around a 1.00 cm radius cylinder) is

$$N = \frac{L}{2\pi r} = \frac{R \cdot A}{2\pi r \rho} = \frac{(5.00 \ \Omega)\left[\pi(0.500 \times 10^{-3} \text{ m})^2/4\right]}{2\pi(1.00 \times 10^{-2} \text{ m})(1.7 \times 10^{-8} \ \Omega\cdot\text{m})}$$

$$= \boxed{9.2 \times 10^2 \text{ turns}}$$

(b) From $B = \mu_0 n I$, the number of turns per unit length on the solenoid is

$$n = \frac{B}{\mu_0 I} = \frac{4.00 \times 10^{-2} \text{ T}}{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(4.00 \text{ A})} = 7.96 \times 10^3 \text{ turns/m}$$

Thus, the required length of the solenoid is

$$L = \frac{N}{n} = \frac{9.2 \times 10^2 \text{ turns}}{7.96 \times 10^3 \text{ turns/m}} = 0.12 \text{ m} = \boxed{12 \text{ cm}}$$

Section 30.5 Gauss's Law in Magnetism

P30.46 (a) The magnetic flux through the flat surface S_1 is

$$(\Phi_B)_{\text{flat}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = B\pi R^2 \cos(180 - \theta) = \boxed{-B\pi R^2 \cos\theta}$$

(b) The net flux out of the closed surface is zero:

$$(\Phi_B)_{\text{flat}} + (\Phi_B)_{\text{curved}} = 0$$

Therefore,

$$(\Phi_B)_{\text{curved}} = \boxed{B\pi R^2 \cos\theta}$$

P30.47 The flux is defined as $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$

(a) The flux through the shaded face is $\Phi_B = B_x A_x + B_y A_y + B_z A_z$. The shaded square's area is in the *yz* plane, so it counts as an *x* component of area. Here $A_y = A_z = 0$. Then,

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = \left(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right) \text{ T} \cdot \left(2.50 \times 10^{-2} \text{ m}\right)^2 \hat{\mathbf{i}}$$

$$\Phi_B = 3.12 \times 10^{-3} \text{ T} \cdot \text{m}^2 = 3.12 \times 10^{-3} \text{ Wb} = \boxed{3.12 \text{ mWb}}$$

(b) For a closed surface, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$, so $(\Phi_B)_{\text{total}} = \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \boxed{0}$

P30.48 (a) $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA$ where A is the cross-sectional area of the solenoid. Then,

$$\Phi_B = \left(\frac{\mu_0 N I}{\ell}\right)\left(\pi r^2\right)$$

$$= \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(300)(12.0 \text{ A})}{0.300 \text{ m}}\left[\pi(0.012\ 5 \text{ m})^2\right]$$

$$= 7.40 \times 10^{-7} \text{ Wb} = \boxed{7.40\ \mu\text{Wb}}$$

(b) $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA = \left(\frac{\mu_0 N I}{\ell}\right)\left[\pi\left(r_2^2 - r_1^2\right)\right]$

$$\Phi_B = \left[\frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(300)(12.0 \text{ A})}{(0.300 \text{ m})}\right]$$

$$\times \pi\left[(8.00)^2 - (4.00)^2\right]\left(10^{-3} \text{ m}\right)^2$$

$$= \boxed{2.27\ \mu\text{Wb}}$$

Section 30.6 Magnetism in Matter

P30.49 (a) The Bohr magneton is

$$\mu_B = \left(9.27\times10^{-24}\,\frac{\text{J}}{\text{T}}\right)\left(\frac{\text{N}\cdot\text{m}}{1\text{ J}}\right)\left(\frac{1\text{ T}}{\text{N}\cdot\text{s/C}\cdot\text{m}}\right)\left(\frac{1\text{ A}}{\text{C/s}}\right)$$
$$= 9.27\times10^{-24}\,\text{A}\cdot\text{m}^2$$

The number of unpaired electrons is

$$N = \frac{8.00\times10^{22}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = \boxed{8.63\times10^{45}\ \text{e}^-}$$

(b) Each iron atom has two unpaired electrons, so the number of iron atoms required is

$$\frac{1}{2}N = \frac{1}{2}(8.63\times10^{45}) = 4.31\times10^{45}\ \text{iron atoms}$$

Thus,

$$M_{\text{Fe}} = \frac{(4.31\times10^{45}\ \text{atoms})(7\,900\ \text{kg/m}^3)}{8.50\times10^{28}\ \text{atoms/m}^3} = \boxed{4.01\times10^{20}\ \text{kg}}$$

P30.50 The magnetic moment of one electron is taken as one Bohr magneton μ_B. Let x represent the number of electrons per atom contributing and n the number of atoms per unit volume. Then $nx\mu_B$ is the magnetic moment per volume and the magnetic field (in the absence of any currents in wires) is $B = \mu_0 nx\mu_B = 2.00$ T.. Then

$$x = \frac{B}{\mu_0\mu_B n}$$
$$= \frac{2.00\ \text{T}}{\left(8.50\times10^{28}\ \text{m}^{-3}\right)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(9.27\times10^{-24}\ \text{N}\cdot\text{m/T}\right)}$$
$$= \boxed{2.02}$$

Additional Problems

P30.51 The magnetic field inside of a solenoid is $B = \mu_0 nI = \mu_0(N/L)I$. Thus, the current in this solenoid must be

$$I = \frac{BL}{\mu_0 N} = \frac{\left(2.00\times10^{-3}\ \text{T}\right)\left(6.00\times10^{-2}\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(30.0)} = \boxed{3.18\ \text{A}}$$

***P30.52** Call the wire along the x-axis wire 1 and the other wire 2. Also, choose the positive direction for the magnetic fields at point P to be out of the page. At point P,

$$B_{\text{net}} = +B_1 - B_2 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$$

Substituting numerical values,

$$B_{\text{net}} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2\pi}\left(\frac{7.00\ \text{A}}{3.00\ \text{m}} - \frac{6.00\ \text{A}}{4.00\ \text{m}}\right) = +1.67 \times 10^{-7}\ \text{T}$$

$$\vec{\mathbf{B}}_{\text{net}} = \boxed{0.167\ \mu\text{T out of the page}}$$

P30.53 (a) Suppose you have two 100-W headlights running from a 12-V battery, with the whole $I = \frac{P}{\Delta V} = \frac{200\ \text{W}}{12\ \text{V}} = 17\ \text{A}$ current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so $\mu \approx \mu_0$. Model the current as being from a long, straight wire. Then,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(17\ \text{A})}{2\pi(0.6\ \text{m})}\boxed{\sim 10^{-5}\ \text{T}}$$

(b) If the local geomagnetic field is 5×10^{-5} T, this is $\boxed{\sim 10^{-1}\ \text{times as large,}}$ enough to affect the compass noticeably.

P30.54 Use Equation 30.7 to find the field at a distance from a current loop equal to the radius of the loop:

$$B = \frac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{3/2}} = \frac{\mu_0 I a^2}{2\left(a^2 + a^2\right)^{3/2}} = \frac{\mu_0 I a^2}{2\left(2a^2\right)^{3/2}}$$

$$= \frac{\mu_0 I a^2}{2^{5/2} a^3} = \frac{\mu_0 I}{2^{5/2} a}$$

Solve for the current:

$$I = \frac{2^{5/2} a B}{\mu_0}$$

Let a be the radius of the Earth and substitute numerical values:

$$I = \frac{2^{5/2} R_E B}{\mu_0} = \frac{2^{5/2}\left(6.37 \times 10^6\ \text{m}\right)\left(7.00 \times 10^{-5}\ \text{T}\right)}{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}}$$

$$= 2.01 \times 10^9\ \text{A}$$

This current would instantly vaporize any wire of reasonable size. For example, if we imagine a 1.00-m segment of copper wire 10 cm in diameter, a *huge* wire, this current delivers over a terawatt of power to this short segment! Furthermore, the power delivered to such a wire wrapped around the Earth is on the order of 10^{20} W, which is larger than all of the solar power delivered to the Earth by the Sun.

P30.55 On the axis of a current loop, the magnetic field is given by

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. The magnetic field is directed away from the center, with a magnitude of

$$B = \frac{\mu_0 \omega R^2 q}{4\pi\left(x^2 + R^2\right)^{3/2}}$$

$$= \frac{\mu_0 (20.0 \text{ rad/s})(0.100 \text{ m})^2 \left(10.0 \times 10^{-6} \text{ C}\right)}{4\pi\left[(0.050\,0 \text{ m})^2 + (0.100 \text{ m})^2\right]^{3/2}}$$

$$= 1.43 \times 10^{-10} \text{ T} = \boxed{143 \text{ pT}}$$

P30.56 On the axis of a current loop, the magnetic field is given by

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. Therefore,

$$B = \frac{\mu_0 \omega R^2 q}{4\pi\left(x^2 + R^2\right)^{3/2}}$$

when $x = \dfrac{R}{2}$, then

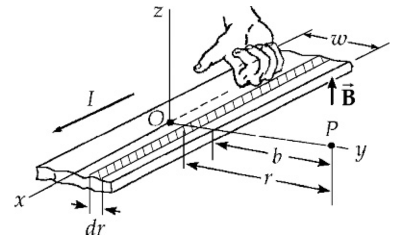
$$B = \frac{\mu_0 \omega R^2 q}{4\pi\left(\frac{5}{4} R^2\right)^{3/2}} = \boxed{\frac{\mu_0 q \omega}{2.5\sqrt{5}\pi R}}$$

P30.57 Consider a longitudinal filament of the strip of width *dr* as shown in the sketch. The contribution to the field at point *P* due to the current *dI* in the element *dr* is

$$dB = \frac{\mu_0 dI}{2\pi r}$$

where $dI = I\left(\frac{dr}{w}\right)$. Then,

$$\vec{\mathbf{B}} = \int d\vec{\mathbf{B}} = \int_b^{b+w} \frac{\mu_0 I}{2\pi w}\frac{dr}{r}\hat{\mathbf{k}} = \boxed{\frac{\mu_0 I}{2\pi w}\ln\left(1+\frac{w}{b}\right)\hat{\mathbf{k}}}$$



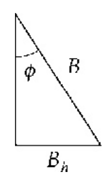
ANS. FIG. P30.57

P30.58 (a) The horizontal component of Earth's magnetic field is given by

$$B_h = B_{\text{coil}} = \frac{\mu_0 N I}{2R} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5)(0.600\ \text{A})}{0.300\ \text{m}}$$
$$= \boxed{12.6\ \mu\text{T}}$$

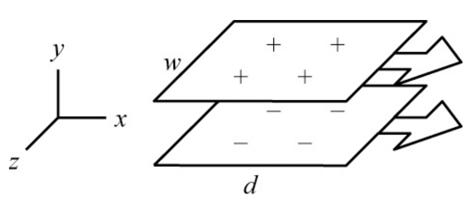
(b) Refer to ANS. FIG. P30.58. We obtain the total magnetic field from

$$B_h = B\sin\phi \rightarrow B = \frac{B_h}{\sin\phi} = \frac{12.6\ \mu\text{T}}{\sin 13.0°} = \boxed{56.0\ \mu\text{T}}$$



ANS. FIG. P30.58

P30.59 In ANS FIG. P30.59(a), the upper sheet acts as conventional current to the right. Consider a patch of the sheet of width w parallel to the z axis and length d parallel to the x axis. The charge on it, $\Delta q = \sigma w d$, passes a point in time interval $\Delta t = d/v$, so the current it constitutes is $\Delta q/\Delta t = \sigma w d/(d/v) = \sigma w v$ and the linear current density is $J_s = \sigma w v / w = \sigma v$.



ANS. FIG. P30.59(a)

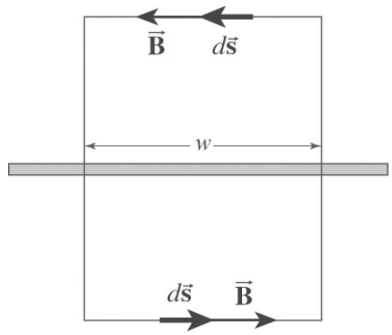
We may use Ampere's law to find the magnitude of the magnetic field produced by a sheet because of the translational symmetry along the z axis. In ANS. FIG. P30.59(b), we look at the upper sheet as it approaches us: the upper sheet (and z-axis) lies in a horizontal plane and the conventional current is out of the page. Choose a closed rectangular path of width w centered about the upper sheet. Because the current it out on the page, we expect the field to point to the right below the sheet and to the left above the sheet.

For the loop, the term $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ is non-zero along the sides parallel to the sheet and zero along the sides perpendicular to the sheet. From Ampere's law, we find the magnitude of the magnetic field on either side of the sheet:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$

$$B(2w) = \mu_0 (J_s w)$$

$$B = \frac{\mu_0 J_s}{2} = \frac{\mu_0 \sigma v}{2}$$



ANS. FIG. P30.59(b)

Therefore, the upper sheet creates field $\vec{\mathbf{B}} = \frac{\mu_0 J_s}{2}\hat{\mathbf{k}}$ above it and $\frac{\mu_0 J_s}{2}(-\hat{\mathbf{k}})$ below it. Similarly, the lower sheet in its motion toward the right constitutes conventional current toward the left. It creates magnetic field $\frac{1}{2}\mu_0 \sigma v(-\hat{\mathbf{k}})$ above it and $\frac{1}{2}\mu_0 \sigma v \hat{\mathbf{k}}$ below it.

(a) Between the plates, their fields add to

$$\mu_0 \sigma v(-\hat{\mathbf{k}}) = \boxed{\mu_0 \sigma v \text{ into the page}}.$$

(b) Above both sheets and below both, their equal-magnitude fields add to $\boxed{\text{zero}}$.

(c) The upper plate exerts no force on itself. The field of the lower plate, $\frac{1}{2}\mu_0\sigma v\left(-\hat{\mathbf{k}}\right)$ will exert a force on the current in the w by d section, given by

$$\vec{\mathbf{F}}_{\mathbf{B}} = I\vec{\ell}\times\vec{\mathbf{B}} = \sigma wvd\hat{\mathbf{i}}\times\frac{1}{2}\mu_0\sigma v\left(-\hat{\mathbf{k}}\right) = \frac{1}{2}\mu_0\sigma^2 v^2 wd\hat{\mathbf{j}}$$

The force per area is

$$\frac{\vec{\mathbf{F}}_{\mathbf{B}}}{wd} = \frac{1}{2}\frac{\mu_0\sigma^2 v^2 wd}{wd}\hat{\mathbf{j}}$$

$$= \boxed{\frac{1}{2}\mu_0\sigma^2 v^2 \text{ up toward the top of the page}}$$

(d) The electrical force on our section of the upper plate is

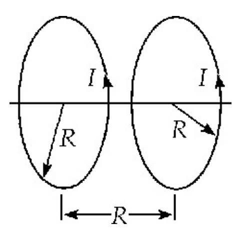
$$q\vec{\mathbf{E}}_{\text{lower}} = \left(\sigma wd\right)\frac{\sigma}{2\epsilon_0}\left(-\hat{\mathbf{j}}\right) = \frac{\sigma^2 wd}{2\epsilon_0}\left(-\hat{\mathbf{j}}\right)$$

The electrical force per area is $\frac{\sigma^2 wd}{2\epsilon_0 wd}$ down $= \frac{\sigma^2}{2\epsilon_0}$ down. To have $\frac{1}{2}\mu_0\sigma^2 v^2 = \frac{\sigma^2}{2\epsilon_0}$ we require

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$. We will find out in Chapter 34 that this speed is the speed of light. We will find out in Chapter 39 that this speed is not possible for the capacitor plates.

P30.60 (a) Use Equation 30.7 twice for the field created by a current loop

$$B_x = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$



ANS. FIG. P30.60

If each coil has N turns, the field is just N times larger.

$$B = B_{x1} + B_{x2} = \frac{N\mu_0 IR^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left[\left(R-x\right)^2+R^2\right]^{3/2}}\right]$$

$$\boxed{B = \frac{N\mu_0 IR^2}{2}\left[\frac{1}{\left(R^2+x^2\right)^{3/2}} + \frac{1}{\left(2R^2+x^2-2xR\right)^{3/2}}\right]}$$

(b) $$\frac{dB}{dx} = \frac{N\mu_0 IR^2}{2}\left[-\frac{3}{2}(2x)\left(x^2+R^2\right)^{-5/2} - \frac{3}{2}\left(2R^2+x^2-2xR\right)^{-5/2}(2x-2R)\right]$$

Substituting $x = \frac{R}{2}$ and canceling terms, $\boxed{\frac{dB}{dx} = 0}$.

$$\frac{d^2B}{dx^2} = \frac{-3N\mu_0 IR^2}{2}\left[\left(x^2+R^2\right)^{-5/2} - 5x^2\left(x^2+R^2\right)^{-7/2} + \left(2R^2+x^2-2xR\right)^{-5/2} - 5(x-R)^2\left(2R^2+x^2-2xR\right)^{-7/2}\right]$$

Again substituting $x = \frac{R}{2}$ and canceling terms, $\boxed{\frac{d^2B}{dx^2} = 0}$.

P30.61 We have a pair of Helmholtz coils whose separation distance is equal to their radius R. To find the magnetic field halfway between the coils on their common axis, we use Equation 30.7 to find the field produced on the axis of a loop the distance $x = R/2$ from its center:

$$B = 2\frac{\mu_0 IR^2}{2\left[(R/2)^2+R^2\right]^{3/2}} = \frac{\mu_0 IR^2}{\left[\frac{1}{4}+1\right]^{3/2}R^3} = \frac{\mu_0 I}{1.40R} \text{ for 1 turn}$$

For N turns in each coil,

$$B = \frac{\mu_0 NI}{1.40R} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)100(10.0\text{ A})}{1.40(0.500\text{ m})}$$

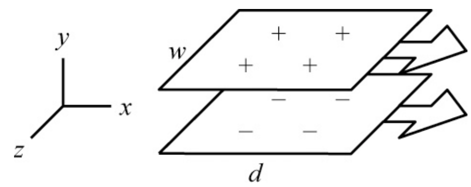
$$= 1.80\times10^{-3}\text{ T} = \boxed{1.80\text{ mT}}$$

P30.62 Model the two wires as straight parallel wires (!). From the treatment of this situation in the chapter text (refer to Equation 30.12), we have

(a) $$F_B = \frac{\mu_0 I^2 \ell}{2\pi a}$$

$$F_B = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(140\text{ A})^2\left[(2\pi)(0.100\text{ m})\right]}{2\pi\left(1.00\times10^{-3}\text{ m}\right)}$$

$$= \boxed{2.46\text{ N upward}}$$



ANS. FIG. P30.62

(b) Equation 30.7, $B_x = \dfrac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{3/2}}$ is the expression for the magnetic field produced a distance x above the center of a loop. The magnetic field at the center of the loop or on its axis is much weaker than the magnetic field just outside the wire. The wire has negligible curvature on the scale of 1 mm, so we model the lower loop as a long straight wire to find the field it creates at the location of the upper wire.

(c) The acceleration of the upper loop is found from Newton's second law:

$$\sum F = m_{\text{loop}} a_{\text{loop}} = F_B - m_{\text{loop}} g:$$

$$a_{\text{loop}} = \frac{F_B - m_{\text{loop}} g}{m_{\text{loop}}} = \frac{2.46\ \text{N} - (0.021\ 0\ \text{kg})(9.80\ \text{m/s}^2)}{(0.021\ 0\ \text{kg})}$$

$$= \boxed{107\ \text{m/s}^2\ \ \text{upward}}$$

P30.63 In the textbook Figure P30.63, wire 1 carries current along the x axis and wire 2 carries current along the y axis.

Choosing out of the page as the positive field direction, the field at point P is

$$B = B_1 - B_2 = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$$

$$= \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi}\left(\frac{5.00\ \text{A}}{0.400\ \text{m}} - \frac{3.00\ \text{A}}{0.300\ \text{m}}\right) = 5.00 \times 10^{-7}\ \text{T}$$

The result is positive; therefore, the field at P is

(a) $\boxed{0.500\ \mu\text{T}}$

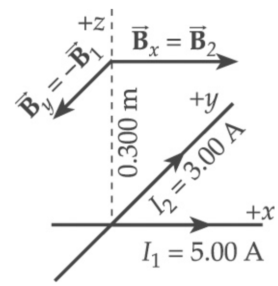
(b) $\boxed{\text{out of the page}}$

(c) At 30.0 cm above the intersection of the wires, the field components are as shown in ANS. FIG. P30.63, where

$$B_y = -B_1 = -\frac{\mu_0 I_1}{2\pi r}$$

$$= -\frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(5.00\ \text{A})}{2\pi(0.300\ \text{m})} = -3.33\times 10^{-6}\ \text{T}$$

and $B_x = B_2 = \dfrac{\mu_0 I_2}{2\pi r} = \dfrac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(3.00\ \text{A})}{2\pi(0.300\ \text{m})} = 2.00\times 10^{-6}\ \text{T}$



ANS. FIG. P30.63

The resultant field is

$$B = \sqrt{B_x^2 + B_y^2} = 3.89\times 10^{-6}\ \text{T} \quad \text{at} \quad \theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = -59.0°$$

or $\vec{\mathbf{B}} =$ $\boxed{3.89\ \mu\text{T in the } xy \text{ plane and at } 59.0° \text{ clockwise}}$ $\boxed{\text{from the } +x \text{ direction}}$

P30.64 (a) The magnetic field at the center of a circular current loop of radius R and carrying current I is $B = \mu_0 I / 2R$. The direction of the field at this center is given by the right-hand rule. Taking out of the page (toward the reader) as positive, the net magnetic field at the common center of these coplanar loops is

$$B_{\text{net}} = B_2 - B_1 = \frac{\mu_0 I_2}{2r_2} - \frac{\mu_0 I_1}{2r_1}$$

$$= \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2}\left(\frac{3.00\ \text{A}}{9.00\times 10^{-2}\ \text{m}} - \frac{5.00\ \text{A}}{12.0\times 10^{-2}\ \text{m}}\right)$$

$$= -5.24\times 10^{-6}\ \text{T} \rightarrow \left|B_{\text{net}}\right| = \boxed{5.24\ \mu\text{T}}$$

(b) By our convention above (out of the page is positive), the result of part (a) tells us that the net magnetic field is $\boxed{\text{into the page}}$.

(c) To have $B_{net} = 0$, it is necessary that $I_2/r_2 = I_1/r_1$, or

$$r_2 = \left(\frac{I_2}{I_1}\right) r_1 = \left(\frac{3.00\ \text{A}}{5.00\ \text{A}}\right)(12.0\ \text{cm}) = \boxed{7.20\ \text{cm}}$$

P30.65 (a) In $d\vec{\mathbf{B}} = \dfrac{\mu_0}{4\pi r^2} I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}$, the moving charge constitutes a bit of current as in $I = nqvA$. For a positive charge the direction of $d\vec{\mathbf{s}}$ is the direction of $\vec{\mathbf{v}}$, so $d\vec{\mathbf{B}} = \dfrac{\mu_0}{4\pi r^2} nqA(ds)\vec{\mathbf{v}} \times \hat{\mathbf{r}}$. Next, $A\ (ds)$ is the volume occupied by the moving charge, and $nA\ (ds) = 1$ for just one charge. Then,

$$\boxed{\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}}$$

(b) The magnitude of the field is

$$B = \frac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^7\ \text{m/s})}{4\pi(1.00 \times 10^{-3}\ \text{m})^2} \sin 90.0°$$

$$= \boxed{3.20 \times 10^{-13}\ \text{T}}$$

(c) The magnetic force on a second proton moving in the opposite direction is

$$F_B = q\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = (1.60 \times 10^{-19}\ \text{C})(2.00 \times 10^7\ \text{m/s}) \times (3.20 \times 10^{-13}\ \text{T})\sin 90.0°$$

$F_B = \boxed{1.02 \times 10^{-24}\ \text{N}}$ directed away from the first proton

(d) The electric force on a second proton moving in the opposite direction is

$$F_e = qE = \frac{k_e q_1 q_2}{r^2} = \frac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})^2}{(1.00 \times 10^{-3})^2}$$

$F_e = \boxed{2.30 \times 10^{-22}\ \text{N}}$ directed away from the first proton

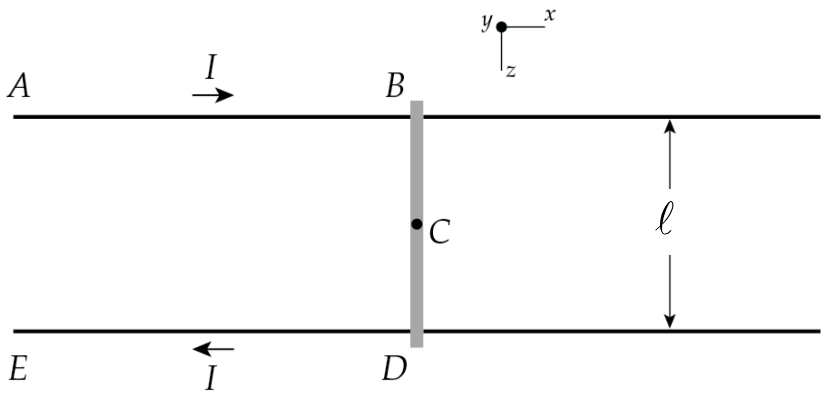
P30.66 (a) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(24.0\ \text{A})}{2\pi(0.017\ 5\ \text{m})} = \boxed{2.74 \times 10^{-4}\ \text{T}}$

(b) Because current is diverted through the bar, only half of each rail carries current, so the field produced by each rail is half what an infinitely long wire produces.

Therefore, at point C, conductor AB produces a field

$$\frac{1}{2}\left(2.74\times10^{-4}\ \text{T}\right)\left(-\hat{\mathbf{j}}\right),$$

conductor DE produces a field of $\frac{1}{2}\left(2.74\times10^{-4}\ \text{T}\right)\left(-\hat{\mathbf{j}}\right)$, BD produces no field, and AE produces negligible field. The total field at C is $\boxed{2.74\times10^{-4}\ \text{T}\left(-\hat{\mathbf{j}}\right)}$.



ANS. FIG. P30.66

(c) Under the assumption that the rails are infinitely long, the length of rail to the left of the bar does not depend on the location of the bar.

The force on the bar is

$$\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}} = (24.0\ \text{A})\left(0.035\,0\ \text{m}\hat{\mathbf{k}}\right)\times\left[5\left(2.74\times10^{-4}\ \text{T}\right)\left(-\hat{\mathbf{j}}\right)\right]$$

$$= 1.15\times10^{-3}\,\hat{\mathbf{i}}\ \text{N}$$

The field has magnitude

(d) $\boxed{1.15\times10^{-3}\ \text{N}}$ in the

(e) $\boxed{+x \text{ direction.}}$

(f) The bar is already so far from AE that it moves through nearly constant magnetic field.

Yes, length of the bar, current, and field are constant, so force is constant.

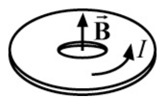
(g) The acceleration is $\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{(1.15\times 10^{-3}\ \text{N})\hat{\mathbf{i}}}{3.00\times 10^{-3}\ \text{kg}} = (0.384\ \text{m/s}^2)\hat{\mathbf{i}}$:

$$v_f^2 = v_i^2 + 2ax = 0 + 2(0.384\ \text{m/s}^2)(1.30\ \text{m})$$

so $\vec{\mathbf{v}}_f = \boxed{(0.999\ \text{m/s})\hat{\mathbf{i}}}$.

P30.67 Each turn creates a field of $\frac{\mu_0 I}{2R}$ at the center of the coil. In all, they create the field

$$B = \frac{\mu_0 I}{2}\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{50}}\right)$$



ANS. FIG. P30.67

Using a spreadsheet to calculate the sum, we have

$$B = \frac{\mu_0 I}{2}\left(\frac{1}{5.05} + \frac{1}{5.15} + \cdots + \frac{1}{9.95}\right)\left(\frac{1}{10^{-2}\ \text{m}}\right)$$
$$= \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})I}{2}(6.931347\ldots)(100\ \text{m}^{-1})$$

Therefore, $\boxed{B = 4.36\times 10^{-4} I, \text{ where } B \text{ is in teslas and } I \text{ is in amperes.}}$

P30.68 The central wire creates field $\vec{\mathbf{B}} = \frac{\mu_0 I_1}{2\pi R}$ counterclockwise. The curved portions of the loop feel no force since $\vec{\ell}\times\vec{\mathbf{B}} = 0$ there. The straight portions both feel $I\vec{\ell}\times\vec{\mathbf{B}}$ forces to the right, amounting to

$$\vec{\mathbf{F}}_B = I_2 2L\frac{\mu_0 I_1}{2\pi R} = \boxed{\frac{\mu_0 I_1 I_2 L}{\pi R} \text{ to the right}}$$

Challenge Problems

P30.69 (a) Let the axis of the solenoid lie along the y axis from $y=-\ell$ to y = 0. We will determine the field at position $y = x$: this point will be inside the solenoid if $-\ell < x < 0$ and outside if $x < -\ell$ or $x > 0$. We think of solenoid as formed of rings, each of thickness dy. Now I is the symbol for the current in each turn of wire and the number of turns per length is $\left(\frac{N}{\ell}\right)$. So the number of turns in the ring is $\left(\frac{N}{\ell}\right)dy$ and the current in the ring is $I_{\text{ring}} = I\left(\frac{N}{\ell}\right)dy$. Now, we use Equation 30.7 for the field created by one ring:

$$B_{\text{ring}} = \frac{\mu_0 I_{\text{ring}} a^2}{2\left[(x-y)^2 + a^2\right]^{3/2}}$$

where $x - y$ is the distance from the center of the ring, at location y, to the field point (note that y is negative, so $x - y = x + |y|$). Each ring creates a field in the same direction, along the y axis, so the whole field of the solenoid is

$$B = \sum_{\text{all rings}} B_{\text{ring}} = \sum \frac{\mu_0 I_{\text{ring}} a^2}{2\left[(x-y)^2 + a^2\right]^{3/2}} \to \int_{-\ell}^{0} \frac{\mu_0 I (N/\ell) a^2 dy}{2\left[(x-y)^2 + a^2\right]^{3/2}}$$

$$= \frac{\mu_0 I N a^2}{2\ell} \int_{-\ell}^{0} \frac{dy}{\left[(x-y)^2 + a^2\right]^{3/2}}$$

To perform the integral we change variables to $u = x - y$ and $dy = -du$. Then,

$$B = -\frac{\mu_0 I N a^2}{2\ell} \int_{x+\ell}^{x} \frac{du}{\left(u^2 + a^2\right)^{3/2}}$$

and then using the table of integrals in the appendix,

$$B = -\frac{\mu_0 I N a^2}{2\ell} \frac{u}{a^2\sqrt{u^2 + a^2}}\Bigg|_{x+\ell}^{x}$$

$$= -\frac{\mu_0 I N}{2\ell}\left[\frac{x}{\sqrt{x^2 + a^2}} - \frac{x+\ell}{\sqrt{(x+\ell)^2 + a^2}}\right]$$

$$= \boxed{\frac{\mu_0 I N}{2\ell}\left[\frac{x+\ell}{\sqrt{(x+\ell)^2 + a^2}} - \frac{x}{\sqrt{x^2 + a^2}}\right]}$$

(b) If ℓ is much larger than a and $x = 0$, we have

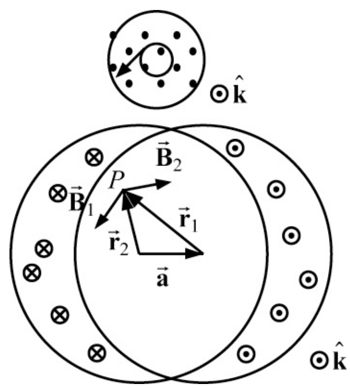
$$B \cong \frac{\mu_0 IN}{2\ell}\left[\frac{\ell}{\sqrt{\ell^2}} + 0\right] = \frac{\mu_0 IN}{2\ell}$$

This is just half the magnitude of the field deep within the solenoid. We would get the same result by substituting $x = -\ell$ to describe the other end.

P30.70 Consider first a solid cylindrical rod of radius R carrying current toward you, uniformly distributed over its cross-sectional area. To find the field at distance r from its center we consider a circular loop of radius r:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{inside}}$$

$$B 2\pi r = \mu_0 \pi r^2 J \qquad B = \frac{\mu_0 J r}{2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 J}{2}\hat{\mathbf{k}} \times \mathbf{r}$$



ANS. FIG. P30.70

Now the total field at P inside the saddle coils is the field due to a solid rod carrying current toward you, centered at the head of vector $\vec{\mathbf{d}}$, plus the field of a solid rod centered at the tail of vector $\vec{\mathbf{d}}$ carrying current away from you.

$$\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}} \times \vec{\mathbf{r}}_1 + \frac{\mu_0 J}{2}\left(-\hat{\mathbf{k}}\right) \times \vec{\mathbf{r}}_2$$

Now note $\vec{\mathbf{d}} + \vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_2$. Then,

$$\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}} \times \vec{\mathbf{r}}_1 - \frac{\mu_0 J}{2}\hat{\mathbf{k}} \times \left(\vec{\mathbf{d}} + \vec{\mathbf{r}}_1\right) = \frac{\mu_0 J}{2}\vec{\mathbf{d}} \times \hat{\mathbf{k}}$$

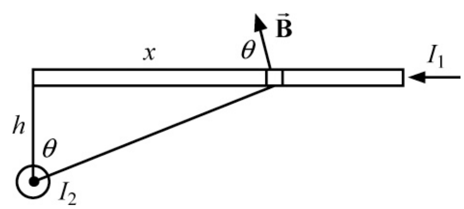
$$= \frac{\mu_0 J d}{2} \text{ down in the diagram}$$

P30.71 At a point at distance x from the left end of the bar, current I_2 creates magnetic field $\vec{\mathbf{B}} = \dfrac{\mu_0 I_2}{2\pi\sqrt{h^2 + x^2}}$ to the left and above the horizontal at angle θ where $\tan\theta = \dfrac{x}{h}$. This field exerts force on an element of the rod of length dx

$$d\vec{\mathbf{F}} = I_1 \vec{\ell} \times \vec{\mathbf{B}} = I_1 \frac{\mu_0 I_2 dx}{2\pi\sqrt{h^2 + x^2}} \underset{\text{right hand rule}}{\sin\theta}$$

$$= \frac{\mu_0 I_1 I_2 dx}{2\pi\sqrt{h^2 + x^2}} \frac{x}{\sqrt{h^2 + x^2}} \text{ into the page}$$

$$d\vec{\mathbf{F}} = \frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2 + x^2\right)}\left(-\hat{\mathbf{k}}\right)$$



ANS. FIG. P30.71

The whole force is the sum of the forces on all of the elements of the bar:

$$\vec{\mathbf{F}} = \int_{x=0}^{\ell} \frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2 + x^2\right)}\left(-\hat{\mathbf{k}}\right) = \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\int_0^{\ell} \frac{2x dx}{h^2 + x^2}$$

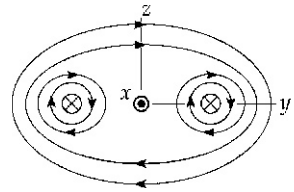
$$= \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi} \ln\left(h^2 + x^2\right)\Big|_0^{\ell}$$

$$= \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\left[\ln\left(h^2 + \ell^2\right) - \ln h^2\right]$$

$$= \frac{\left(10^{-7}\text{ N}\right)(100\text{ A})(200\text{ A})\left(-\hat{\mathbf{k}}\right)}{\text{A}^2} \ln\left[\frac{(0.500\text{ cm})^2 + (10.0\text{ cm})^2}{(0.500\text{ cm})^2}\right]$$

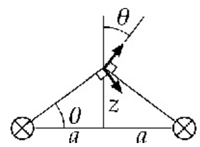
$$= 2 \times 10^{-3}\text{ N}\left(-\hat{\mathbf{k}}\right)\ln 401 = \boxed{1.20 \times 10^{-2}\text{ N}\left(-\hat{\mathbf{k}}\right)}$$

P30.72 (a) See ANS. FIG. P30.72(a).



(currents are into the paper)

ANS. FIG. P30.72(a)



at a distance *z* above the plane of the conductors

ANS. FIG. P30.72(b)

(b) By symmetry, the contribution of each wire to the magnetic field at the origin is the same, but the directions of the fields are opposite, so the total field is $\boxed{\text{zero}}$. We can see this from cancellation of the separate fields in ANS. FIG. P30.72(a).

(d) We choose to do part (d) first. At a point on the z axis, the contribution from each wire has magnitude $B = \dfrac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}$ and is perpendicular to the line from this point to the wire as shown in ANS. FIG. P30.72(b). Combining fields, the vertical components cancel while the horizontal components add, yielding

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}\sin\theta\right) = \frac{\mu_0 I}{\pi\sqrt{a^2+z^2}}\left(\frac{z}{\sqrt{a^2+z^2}}\right) = \frac{\mu_0 I z}{\pi\left(a^2+z^2\right)}$$

$$B_y = \frac{\left(4\pi\times10^{-7}\right)(8.00)z}{\pi\left[(0.0300)^2+z^2\right]} \quad \text{so}$$

$$\boxed{\vec{\mathbf{B}} = \frac{32\times10^{-7}z}{9\times10^{-4}+z^2}\,\hat{\mathbf{j}}, \text{ where } \vec{\mathbf{B}} \text{ is in teslas and } z \text{ is in meters.}}$$

(c) From part (d), taking the limit $z \to \infty$ gives $1/z \to 0$; so, the field is $\boxed{\text{zero}}$, as we should expect.

(e) The condition for a maximum is:

$$\frac{dB_y}{dz} = \frac{-\mu_0 I z(2z)}{\pi\left(a^2+z^2\right)^2} + \frac{\mu_0 I}{\pi\left(a^2+z^2\right)} = 0 \quad \text{or} \quad \frac{\mu_0 I}{\pi}\frac{\left(a^2-z^2\right)}{\left(a^2+z^2\right)^2} = 0$$

Thus, along the z axis, the field is a maximum at $\boxed{d = a = 3.00\text{ cm}}$.

(f) Using the equation derived in part (d), the value of the maximum field is

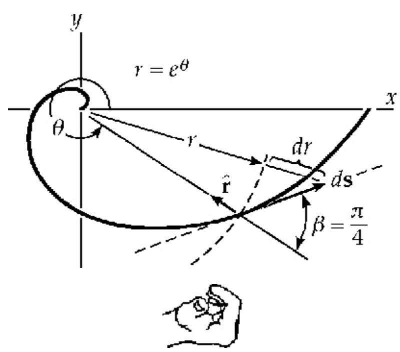
$$\vec{\mathbf{B}} = \frac{(32\times10^{-7})(0.030\ 0)}{9\times10^{-4}+(0.030\ 0)^2}\hat{\mathbf{j}}\text{ T} = 5.33\times10^{-5}\text{ T} = \boxed{53.3\hat{\mathbf{j}}\ \mu\text{T}}$$

P30.73 (a) From the shape of the wire,

$$r = f(\theta) = e^{\theta} \quad \rightarrow \quad \frac{dr}{d\theta} = e^{\theta} = r$$

and so we have

$$\tan\beta = \frac{r}{dr/d\theta} = \frac{r}{r} = 1 \rightarrow \beta = 45° = \pi/4$$



ANS. FIG. P30.73

(b) At the origin, there is no contribution from the straight portion of the wire since $d\vec{\mathbf{s}}\times\hat{\mathbf{r}} = 0$. For the field contribution from the spiral, refer to the figure. The direction of $d\vec{\mathbf{s}}\times\hat{\mathbf{r}}$ is out of the page. The magnitude $|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}| = \sin(3\pi/4)$ because the angle between $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$ is always 180° – 45° = 135° = $3\pi/4$.

Also, from the figure,

$$dr = ds\sin\pi/4 = ds/\sqrt{2} \rightarrow ds = \sqrt{2}dr$$

The contribution to the magnetic field is then

$$dB = \left|d\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{(4\pi)}\left|\frac{(d\vec{\mathbf{s}}\times\hat{\mathbf{r}})}{r^2}\right| = \frac{\mu_0 I}{(4\pi)}\frac{|d\vec{\mathbf{s}}|\sin\theta|\hat{\mathbf{r}}|}{r^2}$$

$$= \frac{\mu_0 I}{(4\pi)}\frac{\sqrt{2}dr}{r^2}\left[\sin\left(\frac{3\pi}{4}\right)\right]$$

The total magnetic field is

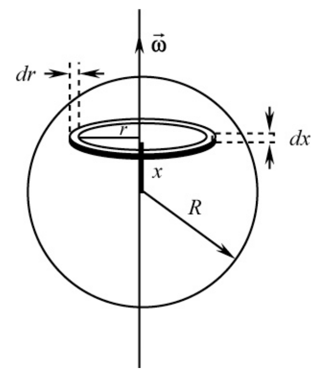
$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{\cancel{\sqrt{2}}\,dr}{r^2}\left[\frac{1}{\cancel{\sqrt{2}}}\right]\frac{1}{r^2} = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi} r^{-2}dr = -\frac{\mu_0 I}{4\pi}\left(r^{-1}\right)\Big|_{\theta=0}^{2\pi}$$

Substitute $r = e^{\theta}$: $B = -\frac{\mu_0 I}{4\pi}\left[e^{-\theta}\right]_0^{2\pi} = -\frac{\mu_0 I}{4\pi}\left[e^{-2\pi} - e^{0}\right] = \boxed{\frac{\mu_0 I}{4\pi}\left(1 - e^{-2\pi}\right)}$

$\boxed{\text{out of the page.}}$

P30.74 (a) Consider the sphere as being built up of little spinning ring elements of radius *r*, thickness *dr*, and height *dx*, centered on the rotation axis. Each ring holds charge *dQ*:

$$dQ = \rho dV = \rho(2\pi r dr)(dx)$$



ANS. FIG. P30.74

Each ring, with angular speed ω, takes a period $T = \omega/2\pi$ to complete one rotation. Thus, each ring carries current

$$dI = \frac{dQ}{T} = \frac{\omega}{2\pi}\left[\rho(2\pi r dr)(dx)\right] = \rho\omega r dr dx$$

The contribution of each ring element to the magnetic field at a point on the rotation axis a distance *x* from the center of the sphere is given by Equation 30.7:

$$dB = \frac{\mu_0 r^2 dI}{2\left(x^2 + r^2\right)^{3/2}}$$

Combining the above terms, the field contribution is of a ring element is

$$dB = \frac{\mu_0 \rho\omega r^3 dr dx}{2\left(x^2 + r^2\right)^{3/2}}$$

The contributions of all rings gives

$$B=\int_{x=-R}^{+R}\int_{r=0}^{\sqrt{R^2-x^2}}\frac{\mu_0\rho\omega}{2}\frac{r^3drdx}{\left(x^2+r^2\right)^{3/2}}$$

To evaluate the integral, let $v = r^2 + x^2$, $dv = 2rdr$, and $r^2 = v - x^2$.

$$B=\int_{x=-R}^{+R}\int_{v=x^2}^{R^2}\frac{\mu_0\rho\omega}{2}\frac{\left(v-x^2\right)dv}{2v^{3/2}}dx$$

$$=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[\int_{v=x^2}^{R^2}v^{-1/2}\,dv-x^2\int_{v=x^2}^{R^2}v^{-3/2}\,dv\right]dx$$

$$B=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[2v^{1/2}\Big|_{x^2}^{R^2}+\left(2x^2\right)v^{-1/2}\Big|_{x^2}^{R^2}\right]dx$$

$$=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[2\left(R-|x|\right)+2x^2\left(\frac{1}{R}-\frac{1}{|x|}\right)\right]dx$$

$$B=\frac{\mu_0\rho\omega}{4}\int_{-R}^{R}\left[2\frac{x^2}{R}-4|x|+2R\right]dx$$

$$=\frac{2\mu_0\rho\omega}{4}\int_{0}^{R}\left[2\frac{x^2}{R}-4x+2R\right]dx$$

$$B=\frac{2\mu_0\rho\omega}{4}\left(\frac{2R^3}{3R}-\frac{4R^2}{2}+2R^2\right)=\boxed{\frac{\mu_0\rho\omega R^2}{3}}$$

(b) From part (a), the current associated with each rotating ring of charge is

$$dI=\rho\omega rdrdx$$

The magnetic moment contributed by this ring is

$$d\mu=A\left(dI\right)=\left(\pi r^2\right)\left(\rho\omega rdrdx\right)=\pi\omega\rho r^3drdx$$

The total magnetic moment is

$$\mu=\pi\omega\rho\int_{x=-R}^{+R}\left[\int_{r=0}^{\sqrt{R^2-x^2}}r^3\,dr\right]dx=\pi\omega\rho\int_{x=-R}^{+R}\frac{\left(\sqrt{R^2-x^2}\right)^4}{4}dx$$

$$=\pi\omega\rho\int_{x=-R}^{+R}\frac{\left(R^2-x^2\right)^2}{4}dx$$

$$\mu = \frac{\pi\omega\rho}{4}\int_{x=-R}^{+R}\left(R^4 - 2R^2x^2 + x^4\right)dx$$

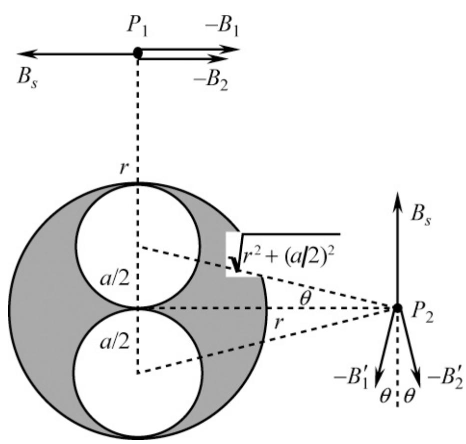
$$= \frac{\pi\omega\rho}{4}\left[R^4(2R) - 2R^2\left(\frac{2R^3}{3}\right) + \frac{2R^5}{5}\right]$$

$$\mu = \frac{\pi\omega\rho}{4}R^5\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \frac{\pi\omega\rho R^5}{4}\left(\frac{16}{15}\right) = \boxed{\frac{4\pi\omega\rho R^5}{15}}$$

P30.75 Note that the current I exists in the conductor with a current density $J = \frac{I}{A}$, where

$$A = \pi\left[a^2 - \frac{a^2}{4} - \frac{a^2}{4}\right] = \frac{\pi a^2}{2}$$

Therefore $J = \frac{2I}{\pi a^2}$.



ANS. FIG. P30.75

To find the field at either point P_1 or P_2, find B_s which would exist if the conductor were solid, using Ampère's law. Next, find B_1 and B_2 that *would* be due to the conductors of radius $\frac{a}{2}$ that *could* occupy the void where the holes exist. Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

(a) At point P_1,

$$B_s = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r},\ B_1 = \frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r - (a/2)\right)},\ \text{and}\ B_2 = \frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r + (a/2)\right)}$$

$$B = B_s - B_1 - B_2$$

$$= \frac{\mu J \pi a^2}{2\pi}\left[\frac{1}{r} - \frac{1}{4\left(r-(a/2)\right)} - \frac{1}{4\left(r+(a/2)\right)}\right]$$

$$B = \frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2 - a^2 - 2r^2}{4r\left(r^2-(a^2/4)\right)}\right]$$

$$= \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right] \text{ directed to the left}}$$

(b) At point P_2,

$$B_s = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r} \text{ and } B_1' = B_2' = \frac{\mu_0 J \pi (a/2)^2}{2\pi\sqrt{r^2 + (a/2)^2}}$$

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$B = B_s - B_1'\cos\theta - B_2'\cos\theta$$

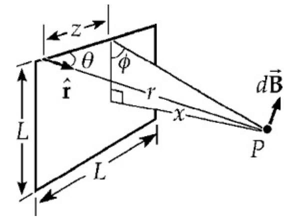
$$= \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r} - 2\left(\frac{\mu_0 J \pi a^2/4}{2\pi\sqrt{r^2+\left(a^2/4\right)}}\right)\frac{r}{\sqrt{r^2+\left(a^2/4\right)}}$$

$$B = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}\left[1 - \frac{r^2}{2\left(r^2+\left(a^2/4\right)\right)}\right] = \frac{\mu_0(2I)}{2\pi r}\left[1 - \frac{2r^2}{4r^2+a^2}\right]$$

$$= \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right] \text{ directed toward the top of the page}}$$

P30.76 By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B_P = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $\frac{L}{2}$. In order to calculate B_0, we use the Biot-Savart law and consider the plane of the square to be the yz plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by the integral form of the Biot-Savart law as

$$\vec{\mathbf{B}}_0 = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\ell}\times\hat{\mathbf{r}}}{r^2}$$



ANS. FIG. P30.76

From ANS. FIG. P30.76 we see that

$$r = \sqrt{x^2 + L^2/4 + z^2} \quad \text{and} \quad \left|d\vec{\ell} \times \hat{\mathbf{r}}\right| = dz \sin\theta = dz\sqrt{\frac{L^2/4 + x^2}{L^2/4 + x^2 + z^2}}$$

By symmetry all components of the field $\vec{\mathbf{B}}$ at P cancel except the components along x (perpendicular to the plane of the square); and

$$B_{0x} = B_0 \cos\phi \quad \text{where} \quad \cos\phi = \frac{L/2}{\sqrt{L^2/4 + x^2}}$$

Therefore,

$$\left|\vec{\mathbf{B}}_0\right| = B_{0x} = \frac{\mu_0 I}{4\pi}\int_0^{L/2} \frac{\sin\theta\cos\phi dz}{r^2}$$

and at P, $B_P = 8B_{0x}$.

Using the expressions given above for $\sin\theta$, $\cos\phi$, and r, we find

$$B_P = 8\left(\frac{\mu_0 I}{4\pi}\right)\int_0^{L/2} \frac{1}{L^2/4 + x^2 + z^2}\sqrt{\frac{L^2/4 + x^2}{L^2/4 + x^2 + z^2}}\,\frac{L/2}{\sqrt{L^2/4 + x^2}}dz$$

$$= \frac{\mu_0 IL}{\pi}\int_0^{L/2} \frac{dz}{\left(L^2/4 + x^2 + z^2\right)^{3/2}}$$

$$= \frac{\mu_0 IL}{8\pi}\frac{1}{\left(L^2/4 + x^2\right)}\frac{z}{\sqrt{L^2/4 + x^2 + z^2}}\Bigg|_0^{L/2}$$

$$= \frac{\mu_0 IL}{\pi}\frac{1}{\left(L^2/4 + x^2\right)}\left[\frac{L/2}{\sqrt{L^2/4 + x^2 + L^2/4}} - 0\right]$$

Therefore,

$$B_P = \frac{\mu_0 IL^2}{2\pi\left(x^2 + L^2/4\right)\sqrt{x^2 + L^2/2}}$$

P30.77 (a) From Equation 30.9, the magnetic field produced by one loop at the center of the second loop is given by

$$B = \frac{\mu_0 I R^2}{2x^3} = \frac{\mu_0 I\left(\pi R^2\right)}{2\pi x^3} = \frac{\mu_0 \mu}{2\pi x^3}$$

where the magnetic moment of either loop is $\mu = I\left(\pi R^2\right)$. Therefore,

$$\left|F_x\right| = \mu \frac{dB}{dx} = \mu \frac{d}{dx}\left(\frac{\mu_0 \mu}{2\pi x^3}\right) = \mu\left(\frac{\mu_0 \mu}{2\pi}\right)\left(\frac{3}{x^4}\right)$$

$$= \frac{3\mu_0\left(I\pi R^2\right)^2}{2\pi x^4} = \boxed{\frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4}}$$

(b) $$\left|F_x\right| = \frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4} = \frac{3\pi}{2}\frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(10.0\ \text{A}\right)^2\left(5.00\times 10^{-3}\ \text{m}\right)^4}{\left(5.00\times 10^{-2}\ \text{m}\right)^4}$$

$$= \boxed{5.92\times 10^{-8}\ \text{N}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P30.2 (a) toward the left; (b) out of the page; (c) lower left to upper right

P30.4 675 A, downward

P30.6 12.5 T

P30.8 $\vec{\mathbf{B}} = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R}$ (directed into the page)

P30.10 $\frac{\mu_0 I}{4\pi x}$ into the paper

P30.12 See ANS. FIG. P30.12

P30.14 (a) at $y = -0.420$ m; (b) $3.47 \times 10^{-2}\ \text{N}\left(-\hat{\mathbf{j}}\right)$; (c) $-1.73 \times 10^{4}\,\hat{\mathbf{j}}\ \text{N/C}$

P30.16 (a) See ANS. FIG. P30.16; (b) 3.84×10^{-21} N up; (c) 2.14×10^{-5} m; (d) This distance is negligible compared to 50 m, so the electron does move in a uniform field; (e) 134 revolutions

P30.18 (a) $\frac{4.50\mu_0 I}{\pi L}$; (b) stronger

P30.20 (a) 4.00 μT toward the bottom of the page; (b) 6.67 μT at 167.0° from the positive x axis

P30.22 (a) 8.00 A; (b) opposite directions; (c) force of interaction would be attractive and the magnitude of the force would double

P30.24 (a) The situation is possible in just one way; (b) 12.0 cm to the left of wire 1; (c) 2.40 A down

P30.26 $\frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{a}{c(c+a)}\right]$ to the left

P30.28 This is the required center-to-center separation distance of the wires, but the wires cannot be this close together. Their minimum possible center-to-center separation distance occurs if the wires are touching, but this value is $2r = 2(25.0\ \mu\text{m}) = 50.0\ \mu\text{m}$, which is much larger than the required value above. We could try to obtain this force between wires of smaller diameter, but these wires would have higher resistance and less surface area for radiating energy. It is likely that the wires would melt very shortly after the current begins.

P30.30 500 A

P30.32 (a) 3.60 T; (b) 1.94 T

P30.34 $\frac{\mu_0 J_s}{2}$

P30.36 (a) 6.34×10^{-3} N/m; (b) inward toward the center of the bundle; (c) greatest at the outer surface

P30.38 (a) $\frac{\mu_0 b r_1^2}{3}$ (for $r_1 < R$ or inside the cylinder);

(b) $\frac{\mu_0 b R^3}{3r_2}$ (for $r_2 > R$ or outside the cylinder)

P30.40 4.77×10^4 turns

P30.42 (a) Make the wire as long and thin as possible without melting when it carries the 5-A current; (b) As small in radius as possible with your experiment fitting inside. Then with a smaller circumference, the wire can form a solenoid with more turns.

P30.44 207 W

P30.46 (a) $-B\pi R^2 \cos\theta$; (b) $B\pi R^2 \cos\theta$

P30.48 (a) 7.40 μWb; (b) 2.27 μWb

P30.50 2.02

P30.52 0.167 μT out of the page

P30.54 This current would instantly vaporize any wire of reasonable size. For example, if we imagine a 1.00-m segment of copper wire 10 cm in diameter, a *huge* wire, this current delivers over a terawatt of power to this short segment! Furthermore, the power delivered to such a wire wrapped around the Earth is on the order of 10^{20} W, which is larger than all of the solar power delivered to the Earth by the Sun.

P30.56 $\frac{\mu_0 q\omega}{2.5\sqrt{5}\pi R}$

P30.58 (a) 12.6 μT; (b) 56.0 μT

P30.60 (a) $B = B_{x1} + B_{x2} = \frac{N\mu_0 I R^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left[(R-x)^2+R^2\right]^{3/2}}\right]$; (b) See P30.60(b) for full explanation.

P30.62 (a) 2.46 N upward; (b) Equation 30.7 is the expression for the magnetic field produced a distance x above the center of a loop. The magnetic field at the center of the loop or on its axis is much weaker than the magnetic field just outside the wire. The wire has negligible curvature on the scale of 1 mm, so we model the lower loop as a long straight wire to find the field it creates at the location of the upper wire; (c) 107 m/s^2 upward

P30.64 (a) 5.24 μT; (b) into the page; (c) 7.20 cm

P30.66 (a) 2.74×10^{-4} T; (b) $2.74\times10^{-4}\ \text{T}\left(-\hat{\mathbf{j}}\right)$; (c) Under the assumption that the rails are infinitely long, the length of rail to the left of the bar does not depend on the location of the bar; (d) 1.15×10^{-3} N; (e) $+x$ direction; (f) Yes, length of the bar, current, and field are constant, so force is constant; (g) $(0.999\ \text{m/s})\hat{\mathbf{i}}$

P30.68 $\dfrac{\mu_0 I_1 I_2 L}{\pi R}$ to the right

P30.70 See P30.70 for full explanation.

P30.72 (a) See ANS FIG P30.72(a); (b) zero; (c) zero; (d) $\vec{\mathbf{B}} = \dfrac{32\times10^{-7} z}{9\times10^{-4} + z^2}\hat{\mathbf{j}}$, where $\vec{\mathbf{B}}$ is in teslas and z is in meters; (e) $d = a = 3.00$ cm; (f) $53.3\hat{\mathbf{j}}\ \mu\text{T}$

P30.74 (a) $\dfrac{\mu_0 \rho\omega R^2}{3}$; (b) $\dfrac{4\pi\omega\rho R^5}{15}$

P30.76 See P30.76 for full explanation.